Mach Number Correction of Rectangular Duct Criticals

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ABSTRACT

The theory involved in accounting for the velocity dependence of acoustic cut-off frequencies in rectangular ducts is briefly summarized. This is followed by a comparison with reported duct critical frequencies in a flow test involving isolated cylinders in cross-flow. The agreement with the theory illustrates the importance of addressing the Mach number effects in moderate to high velocity services.

INTRODUCTION

The role of fluid velocity in formulating duct mode resonances, often referred to as the cut-off frequencies in isentropic uniform flow is well known, and readily developed from the convected wave equations [1-5]. While it is of little importance for

fluid velocities less than 40 m/s as in air ducting and many heat exchanger applications, published flow test in [6] covering a range of Mach numbers of 0.1 - 0.4, illustrate its potential significance. Operating experience in power plants likewise suggests the importance of understanding acoustic resonance conditions in steam lines [7], yet is not directly addressed in current industry guidance [8].

The flow tests presently under consideration [6], rely on carefully arranged, isolated cylinders in uniform, low turbulence cross-flow, to excite the acoustic duct modes sequentially as the fluid velocity is increased. This allows us to carefully examine the velocity dependence of the duct modes excited, with identifiable near constant Strouhal numbers of the cylinders of the providing the acoustic excitation.

DUCT CUT-OFF FREQUENCIES

The convected wave equations [1-5] involving the acoustic velocity potential or the linearized Euler equations, are used to quantify duct cut-off frequencies or resonances. In rectangular ducts, the velocity potential ϕ, as in [2] is described by

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=\frac{1}{c_0^2}\left(\frac{\partial}{\partial t}+V_x\frac{\partial}{\partial x}\right)^2\phi \qquad (1)$$

The right hand side of (1) is based on the total derivative of the velocity potential in the presence of idealized uniform flow, with duct coordinates {*x*,*y*,*z*}, mean fluid velocity V_x, and speed of sound, c_0.

Expanding the total derivative in (1), we now have

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}= \frac{1}{c_0^2}\left(\frac{\partial^2\phi}{\partial t^2}+2V_x\frac{\partial^2\phi}{\partial t\,\partial x}+V_x^2\frac{\partial^2\phi}{\partial x^2}\right) \tag{2}$$

This equation is combined with the trial solutions in (3) that are periodic in both time and duct coordinates, with frequency ω, and wave numbers $\{k_x, k_y, k_z\}$.

$$\phi = A\,e^{j(k_x x-\omega t)}\,\mathrm{Cos}(k_y y)\,\mathrm{Cos}(k_z z) \tag{3}$$

With the duct length considerably greater than the transverse dimensions, we shall focus on the transverse modes. Applying hard-wall boundary conditions given by

$$\left.\frac{\partial\phi(x,y,z)}{\partial y}\right|_{y=\pm H/2}=0 \;\text{ and }\; \left.\frac{\partial\phi(x,y,z)}{\partial y}\right|_{z=\pm W/2}=0\,, \tag{4}$$

to the trial solutions in (3) requires $k_y = m\pi/H$ and $k_z = n\pi/W$, with integer mode numbers {*m*,*n*}.

Substituting the trial solutions into the convected wave equation (2) and simplifying, results in the characteristic equation

$$(k_x^2 + k_y^2 + k_z^2) + [(\omega - V_x k_x)^2 / c_0^2 - k_x^2] = 0 \quad (5)$$

Upon defining $\beta_{m,n}^2 \equiv k_y^2 + k_z^2$, and rearranging allows us to write

$$(1 - M^2) k_x^2 + 2 k_x M (\omega / c_0) - [(\omega / c_0)^2 + \beta_{m,n}^2] = 0 , \quad (6)$$

which is readily solved, yielding the longitudinal wave number as

$$k_x(m,n) = -(M\omega / c_0) / \sqrt{1 - M^2} \pm \sqrt{(\omega / c_0)^2 - \beta_{m,n}^2 / (1 - M^2)} \quad (7)$$

When k_x is real, longitudinal propagation is allowed, and cut-off (or attenuated) otherwise.

In the absence of flow, the wave number $k_x(m,n)$ is zero for $\omega / c_0 = \beta_{m,n}$, with propagation cut-off at the traditional duct resonance condition

$$\omega(m,n) = c_0 \pi \sqrt{(m/H)^2 + (n/W)^2} \quad (8)$$

In the presence of uniform flow, the cut-off frequency is now

$$\omega(m,n) = c_0 \pi \sqrt{(n/H)^2 + (m/W)^2} / (1 - M^2)^{0.5} , \quad (9)$$

with wave propagation allowed for $\omega > \omega(m,n)$.

Before comparing these predictions with those observed in [6], one can consider a finite element solution of the acoustic wave equation in terms of the acoustic pressure in the absence of flow as in [6]. The Helmholtz pressure wave equation is

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}=\frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} \tag{10}$$

With solutions of the form $p=\varphi e^{-j\omega t}$, now (10) reduces to

$$\nabla^2\varphi+k^2\varphi=0 \tag{11}$$

where $k=\omega/c_0$, with duct cut-off frequencies ω, and speed of sound c_0. Using (11) together with the hardwall boundary conditions $\varphi(x,y)\big|_{y=\pm H/2}=0$ and $\varphi(x,y)\big|_{z=\pm W/2}=0$, one can compute the cut-off frequencies ω, and mode shapes φ, using finite element methods described in [9,10].

FLOW TEST COMPARISON

The duct dimensions used in the flow test [6] are shown in Fig. 1, with transverse dimensions of the duct width *W*=127 mm and height *H*=254 mm, with cylinders having diameter *D*=12.7 mm. The reported Strouhal number as $St(m,0)=D\,f(m,0)/V$, is taken as the mean of those reported in Table 1 of [6] as *St*=0.189 for Reynolds number of 31,000 to 137,000. This Strouhal estimate agrees with the empirical correlations in [11] commonly used with smooth cylinders in cross flow. The fluid temperature is not stated, so we have assumed it to equal 20 degrees C, which corresponds to c_0=343 m/*s*.

The reported duct resonances presented in Table 1, represent single and multiple cylinders, which are carefully positioned to coincide with the acoustic particle

velocity antinodes, so as to engage the first three duct modes in sequential fashion with increasing fluid velocity. The frequencies reported for a given mode number are averaged to provide mean estimates of each frequency of each mode. The root mean square (RMS) of the fluctuations about the mean are less than 0.5% for each mode. In the case of the single cylinder located midway at y/H=0 (a pressure node), the m=2 mode is not excited, and is omitted from the averaging process. The changes in the absolute temperature associated with the reported pressure drop are less than 0.22% at 100 m/s and 0.58% at 150 m/s, and is omitted from consideration.

The mean duct criticals for the half-dozen cylinder arrangements tested, are plotted in Fig. 2, versus the transverse mode number m, with and without the Mach number correction. The finite element calculations, shown as a dashed line, coincide with the analytical solutions in (8). The fluid velocity associated with the modes excited by vortex shedding acoustics is shown adjacent to the mode number to facilitate association of the fluid velocity and this cylinder shedding rate associated with the modes excited.

In Table 1, the predicted duct criticals (9) versus mode number together with the mean fluid velocity at each excited duct resonance are summarized together with the frequency deviation from the mean for each mode. Clearly the transverse duct resonances, based on the convected wave equation agree with the flow test data. The criticals without the Mach number correction, over-estimate those observed by 0.4% for

velocities of 45 m/s (M=0.13), increasing to 7.5% at 127 m/s (M=0.37). This suggests that the Mach number correction is minimal in most industrial applications.

CONCLUSIONS

This rather brief comparison of the duct criticals intends to increase awareness of the convected wave equation where high velocity fluids are encountered. We began with the convective wave equation in (1) and the transverse resonance conditions in (9), with and without the Mach number correction to exemplify the importance of the Mach number in moderate to high velocity, duct acoustics.

While the flow test considered pertain specifically to an atmospheric wind tunnel test, it provides definitive evidence of Mach number effects with increasing velocity, thus serving to validate the analysis. The convected wave equation may also be expressed in terms of acoustic pressure and particle velocity, but requires careful attention to the boundary conditions when less than ideal flow conditions are present. Applications involving high velocity fluids in cylindrical ducts are covered in [1-5], while those involving cross-flow heat exchangers as in [12,13] are beyond the scope of the present article and subject to ongoing study.

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160 **Figure Captions List**

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Fig. 1 Duct Coordinates showing side and end view and representative cylinder placements in [6]

Fig. 2 Comparison of duct criticals with and without uniform flow Mach No. correction

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163 **Table Caption List**

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Table 1 Frequency of transverse duct modes for single and multiple cylinders [6]

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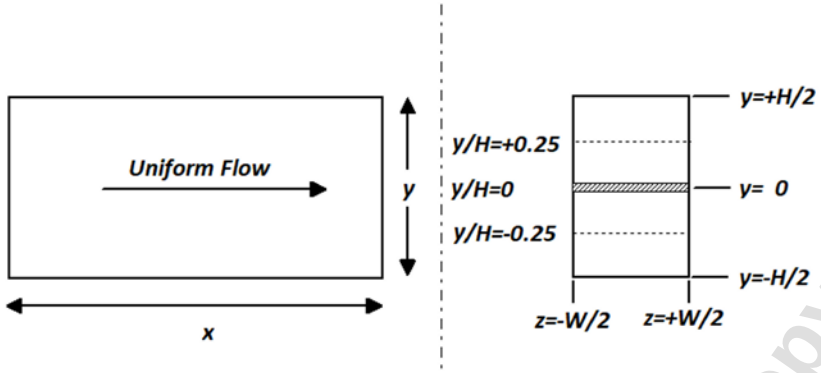
167 Fig. 1



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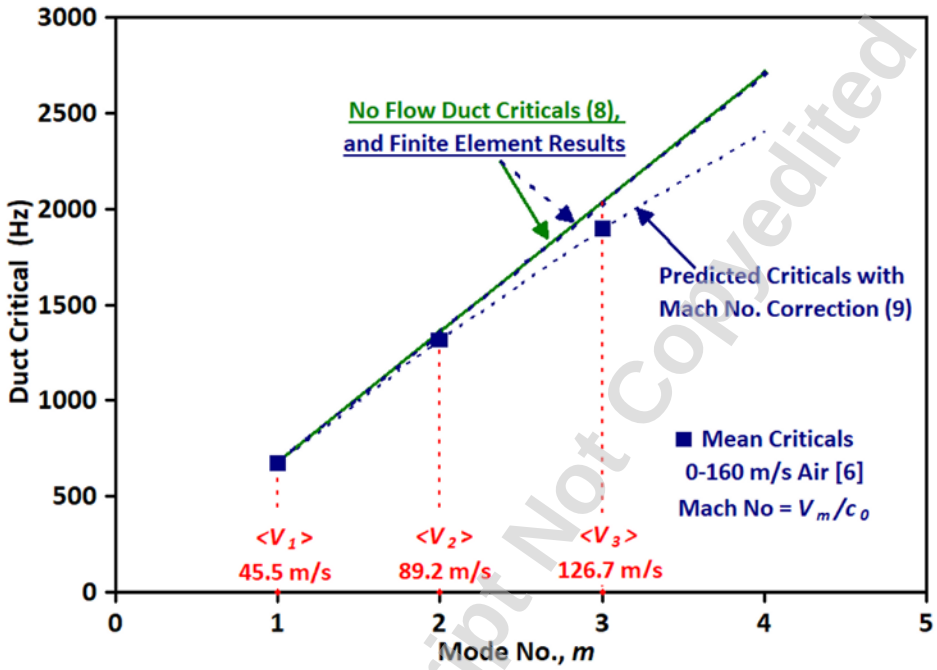
170 Fig. 2



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172 Table 1

Y/H Cylinder Location	**First Transverse Mode; Mach=0.131 *m*=1**	**Second Transverse Mode; Mach=0.259 *m*=2**	**Third Transverse Mode; Mach=0.370 *m*=3**	**St Nos. Reported *m*=1; n=0**
-	**(Hz)**	**(Hz)**	**(Hz)**	**-**
0	678	*Not excited*	1884	0.190
0.25	681	1326	1886	0.189
0 , 0.25	679	1332	1889	0.193
0.25 , -0.25	678	1324	1880	0.194
0, 0.25 & -0.25	679	1332	1879	0.187
0, 0.167 & -0.167	677	1326	1926	0.184
RMS Scatter	**0. 1%**	**0.1%**	**0.4%**	**0.7%**
$\langle f(m,n)=0 \rangle$	**679**	**1330**	**1891**	**0.189**

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