SPECULATION AND RISK SHARING WITH NEW FINANCIAL ASSETS*

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I investigate the effect of financial innovation on portfolio risks when traders have belief disagreements. I decompose traders’ average portfolio risks into two components: the uninsurable variance, defined as portfolio risks that would obtain without belief disagreements, and the speculative variance, defined as portfolio risks that result from speculation. My main result shows that financial innovation always increases the speculative variance through two distinct channels: by generating new bets and by amplifying traders’ existing bets. When disagreements are large, these effects are sufficiently strong that financial innovation increases average portfolio risks, decreases average portfolio comovements, and generates greater speculative trading volume relative to risk-sharing volume. Moreover, a profit-seeking market maker endogenously introduces speculative assets that increase average portfolio risks. JEL Codes: G11, G12, D53.

I. INTRODUCTION

In recent years financial markets have seen a proliferation of new financial assets, such as different types of futures, swaps, options, and more exotic derivatives. According to the traditional view of financial innovation, these assets facilitate diversification and risk sharing.1 However, this view does not take into account that market participants might naturally disagree about how to value financial assets. The thesis of this article is that belief disagreements change the implications of financial innovation for portfolio risks. In particular, belief disagreements naturally lead

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1. Cochrane (2005, 56) summarizes this view as follows: “Better risk sharing is much of the force behind financial innovation. Many successful new securities can be understood as devices to share risks more widely.”

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to speculation, which represents a powerful economic force by which financial innovation increases portfolio risks.

An example is offered by the recent crisis. Assets backed by pools of subprime mortgages (e.g., subprime collateralized debt obligations) became highly popular in the run-up to the crisis. One role of these assets is to allocate the risks to market participants who are best able to bear them. The safer tranches are held by investors that are looking for safety (or liquidity), and the riskier tranches are held by financial institutions that are willing to hold these risks at some price. Although these assets (and their credit default swaps) should have served a stabilizing role in theory, they became a major trigger of the crisis in practice when a fraction of financial institutions realized losses from their positions. Importantly, the same set of assets also generated considerable profits for some market participants, which suggests that at least some of the trades on these assets were speculative.2

What becomes of the risk-sharing role of new assets when market participants use them to speculate on their different views?

To address this question, I analyze the effect of financial innovation on portfolio risks in a model that features both the risk-sharing and the speculation motives for trade. Traders with income risks take positions in a set of financial assets, which enables them to share and diversify some of their background risks. However, traders have belief disagreements about asset payoffs, which induces them to take speculative positions as well. I assume traders have mean-variance preferences over net worth. In this setting, a natural measure of portfolio risk for a trader is the variance of her net worth. I define the average variance as an average of this risk measure across all traders. I further decompose the average variance into two components: the uninsurable variance, defined as the variance that would obtain if there were no belief disagreements, and the speculative variance, defined as the residual amount of variance that results from speculation. I model financial innovation as an expansion of the set of assets available for trade. My main result characterizes the effect of financial innovation on each component of the average variance. In line with the traditional view, financial innovation always decreases the uninsurable variance because new assets increase the possibilities for risk sharing. Theorem 1 shows that

2. Lewis (2010) provides a detailed description of investors that took a short position on housing-related assets in the run-up to the recent crisis.
financial innovation also always increases the speculative variance. Moreover, when belief disagreements are sufficiently large, this effect is sufficiently strong that financial innovation increases the average variance.

My analysis identifies two distinct channels by which financial innovation increases portfolio risks. First, new assets generate new risky bets because traders might disagree about their payoffs. Second, and more subtly, new assets also amplify traders' risky bets on existing assets. The intuition behind this channel is the hedge-more/bet-more effect. To illustrate this effect, consider the following example of two currency traders taking positions in the Swiss franc and the euro. Traders have different views about the franc but not the euro, perhaps because they disagree about the prospects of the Swiss economy but not the euro zone. First, suppose traders can only take positions on the franc. In this case, traders do not take too large speculative positions because the franc is affected by several sources of risks, some of which they don’t disagree about. Traders must bear all of these risks, which makes them reluctant to speculate. Next suppose the euro is also introduced for trade. In this case, traders complement their positions in the franc by taking opposite positions in the euro. By doing so, traders hedge the risks that also affect the euro, which enables them to take purer bets on the franc. When traders are able to take purer bets, they also take larger and riskier bets. Consequently, the introduction of the euro in this example increases portfolio risks even though traders do not disagree about its payoff.

My analysis also has implications for portfolio comovements and trading volume. Risk sharing typically increases portfolio comovements (see, for instance, Townsend 1994). In contrast, speculation tends to reduce comovements because traders with different beliefs tend to take opposite positions. Consequently, as formalized in Proposition 1, financial innovation that increases average portfolio risks also decreases average portfolio comovements. Another prediction of the traditional risk-sharing view is that trading volume is mainly driven by traders’ portfolio rebalancing or liquidity needs. In contrast, trading volume in my model is partly driven by speculation. Moreover, Proposition 2 illustrates that (under appropriate assumptions) financial innovation generates greater speculative trading volume relative to risk-sharing volume whenever it increases average portfolio risks. A growing literature has invoked belief disagreements to
explain the large trading volume observed in financial markets (see Hong and Stein 2007). My analysis suggests that this literature also makes implicit predictions about portfolio risks. If speculation is the main source of trading volume in some new assets, then those assets are likely to increase portfolio risks.

The results so far take the new assets as exogenous. In practice, new assets are endogenously introduced by agents with profit incentives. A sizable body of literature emphasizes risk sharing as a major driving force in endogenous financial innovation (see Allen and Gale 1994; Duffie and Rahi 1995). A natural question is: to what extent is the risk-sharing motive for financial innovation robust to the presence of belief disagreements? I address this question by introducing a profit-seeking market maker that innovates new assets for which it subsequently serves as the intermediary. The market maker’s expected profits are proportional to traders’ willingness to pay to trade the new assets. Thus, traders’ speculative trading motive and their risk-sharing motive create innovation incentives. Theorem 2 shows that when belief disagreements are sufficiently large, the endogenous assets maximize the average variance among all possible choices. Intuitively, the market maker innovates speculative assets that enable traders to bet most precisely on their disagreements, completely disregarding the risk-sharing motive.

A natural question concerns the normative implications of these results. In the baseline setting, financial innovation generates a Pareto improvement even when it increases portfolio risks. However, this conclusion can be overturned in two natural variants: one in which belief disagreements emerge from psychological distortions, and another one in which the environment is associated with externalities. In both settings, a measure of traders’ average portfolio risks emerge as the central object in welfare analysis, providing some normative content to Theorems 1 and 2. That said, I do not make any policy recommendations because financial innovation might also affect welfare through various other channels not captured in this model.3

3. Among other things, even pure speculation driven by financial innovation can provide some social benefits by making asset prices more informative. On the other hand, there is also a large literature that emphasizes various additional channels by which financial innovation can reduce welfare. Hart (1975) and Elul (1995) show that new assets that only partially complete the market may make all agents worse off in view of general equilibrium price effects. Stein (1987) shows that speculation driven by financial innovation can reduce welfare through
The rest of the article is organized as follows. The next subsection discusses the related literature. Section II introduces the basic environment and uses a simple example to illustrate the main channels. Section III characterizes the equilibrium, presents the main result, and discusses its implications for portfolio comovements and trading volume. Section IV presents the results about endogenous financial innovation. Section V discusses the welfare implications, and Section VI concludes. The Appendix contains the omitted derivations and proofs.

I.A. Related Literature

My article belongs to a sizable literature on financial innovation and security design (see, in addition to the above-cited papers, Van Horne 1985; Miller 1986; Duffie and Jackson 1989; Merton 1989; Ross 1989; Cuny 1993; Demange and Laroque 1995; DeMarzo and Duffie 1999; Athanasoulis and Shiller 2000, 2001; Tufano 2003). This literature, with the exception of a few recent publications (some of which are discussed shortly), has not explored the implications of belief disagreements for financial innovation. For example, in their survey of the literature, Duffie and Rahi (1995) note that “one theme of the literature, going back at least to Working (1953) and evident in the Milgrom and Stokey (1982) no-trade theorem, is that an exchange would rarely find it attractive to introduce a security whose sole justification is the opportunity for speculation.” The results of this article show that this observation does not apply if traders’ belief differences reflect their disagreements as opposed to private information.

My article is most closely related to the work of Brock, Hommes, and Wagener (2009), who also emphasize the hedge-more/bet-more effect and identify destabilizing aspects of financial innovation. The papers are complementary in the sense that they use different ingredients and focus on different aspects of instability. First, their main ingredient is reinforcement learning informational externalities. I abstract away from these channels by focusing on an economy with single good (hence, no relative price effects) and belief disagreements (hence, no information). More recently, Rajan (2006), Calomiris (2009), and Korinek (2012) argue that financial innovation might exacerbate agency problems; Gennaioli, Shleifer, and Vishny (2012) emphasize neglected risks associated with new assets; and Thakor (2012) emphasizes the unfamiliarity of new assets. The potentially destabilizing role of speculation is also discussed in Stiglitz (1989), Summers and Summers (1989), Stout (1995), and Posner and Weyl (2013).
(that is, traders choose their beliefs according to a fitness measure). In contrast, my analysis applies regardless of how beliefs and disagreements are formed. Second, they focus on the dynamic instability of prices. I focus on portfolios instead of prices; more specifically, my notion of instability is an increase in portfolio risks.

The hedge-more/bet-more effect also appears in Dow (1998), who analyzes financial innovation in the context of market liquidity with asymmetric information. He considers the introduction of a new asset that makes arbitrage less risky. In view of the hedge-more/bet-more effect, this induces arbitrageurs to trade more aggressively. The main result is that more aggressive arbitrage could then reduce welfare by exacerbating adverse selection. In contrast, I analyze the effect of the hedge-more/bet-more effect on portfolio risks, and I show that financial innovation can increase these risks even in the absence of informational channels.

Other closely related papers include Weyl (2007) and Dieckmann (2011), which emphasize that increased trading opportunities might increase portfolio risks when traders have distorted or different beliefs. Weyl (2007) notes that cross-market arbitrage might create risks when investors have mistaken beliefs. Dieckmann (2011) shows that rare-event insurance can increase portfolio risks when traders disagree about the frequency of these events. The contribution of my article is to systematically characterize the effect of financial innovation on portfolio risks for a general environment with belief disagreements and mean-variance preferences. I also analyze endogenous financial innovation and show that it is partly driven by the speculation motive for trade. In recent work, Shen, Yan, and Zhang (2013) emphasize that endogenous financial innovation is also directed toward mitigating traders’ collateral constraints.

Finally, there is a large finance literature that analyzes the implications of belief disagreements for asset prices or trading volume. A very incomplete list includes Lintner (1969), Rubinstein (1974), Miller (1977), Harrison and Kreps (1978), Varian (1989), Harris and Raviv (1993), Kandel and Pearson (1995), Zapatero (1998), Chen, Hong, and Stein (2003), Scheinkman and Xiong (2003), Geanakoplos (2010), Cao (2011), Kubler and Schmedders (2012), and Simsek (2013). The main difference from this literature is the focus on the effect of belief disagreements on the riskiness of traders’ portfolios, rather than the volatility or the level of prices.
II. BASIC ENVIRONMENT AND MAIN CHANNELS

Consider an economy with a single period and a single consumption good, which will be referred to as a dollar. The uncertainty is captured by a $k \times 1$ vector of random variables, $\mathbf{v} = (v_1, \ldots, v_k)$. There are a finite number of traders denoted by $i \in I$. Each trader is endowed with $e_i$ dollars. She also receives an additional $w_i(\mathbf{v}) = (\mathbf{W}_i)\mathbf{v}$ dollars, where $\mathbf{W}_i$ is a $k \times 1$ vector, which captures her background risks. The presence of background risks generate the risk-sharing motive for trade in this economy. The following assumption about traders’ beliefs generates an additional speculation motive for trade.

**ASSUMPTION 1.** Trader $i$’s prior belief for $\mathbf{v}$ has a normal distribution, $N(\mu^v_i, \Lambda^v)$, where $\mu^v_i \in \mathbb{R}^k$ is the mean vector and $\Lambda^v$ is the $k \times k$ positive definite covariance matrix. Traders agree to disagree in the sense that each trader knows all other traders’ beliefs.

The first part of the assumption says that traders can potentially have different beliefs about the mean of the risks, $\mathbf{v}$. Traders are also assumed to agree on the variance. This feature ensures closed-form solutions but otherwise does not play an important role. The important ingredient is that traders have different beliefs about asset valuations; whether these differences come from variances or means is not central. The second part of Assumption 1 ensures that belief differences correspond to disagreements as opposed to traders’ private information. This part circumvents the well-known no-trade theorems (e.g., Milgrom and Stokey 1982). A growing literature suggests that disagreements of this type can explain various aspects of financial markets (see Hong and Stein 2007).

Traders in this economy cannot directly take positions on the underlying risks, $\mathbf{v}$. But they can do so indirectly by trading a finite number of risky financial assets, $j \in J$. Asset $j$ pays $a^j(\mathbf{v}) = (\mathbf{A}^j)\mathbf{v}$ dollars, where $\mathbf{A}^j$ is a $k \times 1$ vector. Let $\mathbf{A} = [\mathbf{A}^j]_{j \in J}$ denote the $k \times |J|$ matrix of asset payoffs, which is assumed to have full rank (so that assets are not redundant). These assets

4. In a continuous time setting with Brownian motion, Bayesian learning would immediately reveal the objective volatility of the underlying process. In contrast, the mean is much more difficult to learn, lending some additional credibility to Assumption 1. In view of this observation, the common belief for the variance, $\Lambda^v$, can also be reasonably assumed to be the same as the objective variance of $\mathbf{v}$. 
can be thought of as futures whose payoffs are linear functions of their underlying assets or indices. However, the economic insights generalize to nonlinear derivatives (such as options) and more exotic new assets.

Each asset \( j \) is in fixed supply normalized to 0, and it is traded in a competitive market at price \( p^j \). Each trader can take unrestricted positive or negative positions on this asset denoted by \( x^j_i \in \mathbb{R} \). The trader invests the rest of her endowment in cash, which is supplied elastically and delivers a gross return normalized to 1. The trader’s net worth can then be written as:

\[
n_i = e_i + W_i'\mathbf{v} + x_i'(A'\mathbf{v} - \mathbf{p}).
\]

Here, the vectors, \( \mathbf{p} = (p^1, \ldots, p^J)' \) and \( \mathbf{x}_i = (x^1_i, \ldots, x^J_i)' \), respectively denote the prices and the trader’s positions for all risky assets. The trader maximizes subjective expected utility over net worth. Her utility function takes the constant absolute risk aversion (CARA) form. Because the underlying risks, \( \mathbf{v} \), are normally distributed, the trader’s optimization reduces to the usual mean-variance problem:

\[
\max_{\mathbf{x}_i} E[n_i] - \frac{\theta_i}{2} \text{var}[n_i].
\]

Here, \( \theta_i \) denotes the trader’s absolute risk aversion coefficient, and \( E[\cdot] \) and \( \text{var}[\cdot] \) respectively denote the mean and the variance of the trader’s portfolio according to her beliefs.

The equilibrium in this economy is a collection of risky asset prices, \( \mathbf{p} \), and traders’ portfolios, \( \{\mathbf{x}_i\}_i \), such that each trader’s portfolio solves problem (2) and prices clear asset markets, that is, \( \sum x^j_i = 0 \) for each \( j \in J \). I capture financial innovation as an expansion of the set of traded assets. Before I turn to the general characterization, I use a simple example to illustrate the main effects of financial innovation on portfolio risks.

5. The results of this article apply as long as traders’ portfolio choice can be reduced to the form in equation (2) over net worth. An important special case is the continuous-time model in which traders have time-separable expected utility preferences (which are not necessarily CARA), and asset returns and background risks follow diffusion processes. In this case, the optimization problem of a trader at any date can be reduced to the form in equation (2) (see Ingersoll 1987). The only caveat is that traders’ reduced-form coefficients of absolute risk aversion, \( \{\theta_i\}_i \), are endogenous. Hence, in the continuous-time setting, the results apply at a trading date conditional on \( \{\theta_i\}_i \).
II.A. An Illustrative Example

Suppose there are two traders with the same coefficients of risk aversion, that is, $I = \{1, 2\}$ and $\theta_1 = \theta_2 = \theta$. The uncertainty is captured by two uncorrelated random variables, $v_1, v_2$. Traders' background risks depend on a combination of the two random variables. Moreover, they are perfectly negatively correlated with one another, that is:

$$w_1 = v \quad \text{and} \quad w_2 = -v,$$

where $v = v_1 + \alpha v_2$ for some $\alpha \neq 0$.

As a benchmark, suppose traders have common beliefs about both $v_1$ and $v_2$ given by the distribution $N(0,1)$. In this benchmark, first consider the case in which there are no assets, that is, $J = \emptyset$. In this case, there is no trade and traders' net worths are given by:

$$n_1 = e_1 + v \quad \text{and} \quad n_2 = e_2 - v.$$

Traders' net worths are risky because they are unable to hedge their background risks. Next suppose a new asset is introduced whose payoff is perfectly correlated with traders' background risks, $a^1 = v$. In this case, traders' equilibrium portfolios are given by $x^1_1 = -1$ and $x^1_2 = 1$ (and the equilibrium price is $p^1 = 0$). Traders' net worths are constant,

$$n_1 = e_1 \quad \text{and} \quad n_2 = e_2.$$

Thus, with common beliefs, financial innovation enables traders to hedge and diversify their idiosyncratic risks.

Now suppose traders have belief disagreements about some of the uncertainty in this economy. In particular, traders have common beliefs for $v_2$ given by the distribution $N(1,0)$. They also know that $v_1$ and $v_2$ are uncorrelated. However, they disagree about the distribution of $v_1$. Trader 1's prior belief for $v_1$ is given by $N(\varepsilon, 1)$, and trader 2's belief is given by $N(-\varepsilon, 1)$. The parameter $\varepsilon$ captures the level of the disagreement. I use this specification to illustrate the two channels by which financial innovation increases portfolio risks.

1. New Assets Generate New Bets. Suppose asset 1 is available for trade. Because traders disagree about the mean of $v_1$, they also disagree about the mean of the asset payoff, $a^1 = v_1 + \alpha v_2$. In this case, it is easy to check that traders' portfolios are
In particular, traders’ positions deviate from the common belief benchmark in view of their disagreement. Their net worths can also be calculated as:

\[ n_1 = e_1 + \frac{\varepsilon v_1 + \alpha v_2}{\theta \left(1 + \alpha^2\right)} \quad \text{and} \quad n_2 = e_2 - \frac{\varepsilon v_1 + \alpha v_2}{\theta \left(1 + \alpha^2\right)}. \]

If \( \varepsilon > \theta(1 + \alpha^2) \), then traders’ net worths are riskier than the case in which no new asset is introduced. Intuitively, trader 1 is so optimistic about the asset’s payoff that she takes a positive net position, even though her background risk covaries positively with the asset payoff. Consequently, the new asset increases portfolio risks by generating a new bet.

2. New Assets Amplify Existing Bets. Next suppose a second asset with payoff, \( \alpha^2 = v_2 \), is also available for trade. Note that traders do not disagree about the payoff of this asset. Nonetheless, this asset also increases portfolio risks through the hedge-more/bet-more effect. To see this, consider traders’ equilibrium portfolios given by:

\[
\begin{bmatrix}
    x_1^1 \\
    x_1^2
\end{bmatrix} = \left[ -1 + \frac{\varepsilon}{\theta}, -\alpha \frac{\varepsilon}{\theta} \right] \quad \text{and} \quad \begin{bmatrix}
    x_2^1 \\
    x_2^2
\end{bmatrix} = \left[ 1 - \frac{\varepsilon}{\theta}, \alpha \frac{\varepsilon}{\theta} \right].
\]

Note that traders’ positions on asset 1 deviate more from the common belief benchmark relative to the earlier single-asset setting. Traders’ net worths are also riskier and given by:

\[ n_1 = e_1 + \frac{\varepsilon}{\theta} v_1 \quad \text{and} \quad n_2 = e_2 - \frac{\varepsilon}{\theta} v. \]

Intuitively, asset 1 by itself provides the traders with only an impure bet on the risk, \( v_1 \), because its payoff also depends on the risk, \( v_2 \), on which traders do not disagree. To take speculative positions, traders must also hold these additional risks, which makes them reluctant to bet (captured by the \( 1/1 + \alpha^2 \) term in equation (3)). When asset 2 is also available, traders take positions on both assets to take a purer bet on the risk, \( v_1 \). When traders are able to purify their bets, they also amplify them, which in turn leads to greater portfolio risks.
III. FINANCIAL INNOVATION AND PORTFOLIO RISKS

I next characterize the equilibrium and decompose traders’ portfolio risks into the uninsurable and the speculative variance components. The main result, presented later in this section, describes the effect of financial innovation on these two components.

Combining equations (1) and (2) and Assumption 1, the trader’s portfolio choice problem can be written as:

\[
\max_{\mathbf{x}_i} \mathbf{x}_i' (\mu_i - \mathbf{p}) - \frac{1}{2} \theta_i (2 \mathbf{x}_i' \lambda_i + \mathbf{x}_i' \Lambda \mathbf{x}_i).
\]

Here, \( \mu_i \equiv \mathbf{A}' \mu_i^y \) (which is a \(|J| \times 1\) vector) denotes the trader’s belief for means of asset payoffs; \( \Lambda \equiv \mathbf{A}' \Lambda^y \mathbf{A} \) (which is a \(|J| \times |J|\) matrix) denotes the variance of asset payoffs; and \( \lambda_i = \mathbf{A}' \Lambda^y \mathbf{W}_i \) (which is a \(|J| \times 1\) vector) denotes the covariance of asset payoffs with the trader’s background risk. By solving this problem, the trader’s demand for risky assets is given by \( \mathbf{x}_i = \Lambda^{-1} \left( \frac{1}{\bar{\theta}} (\mathbf{u}_i - \mathbf{p}) - \lambda_i \right) \). Aggregating over all traders and using market clearing, asset prices are given by:

\[
\mathbf{p} = \frac{1}{|I|} \sum_{i \in I} \left( \frac{\bar{\theta}}{\theta_i} \mathbf{u}_i - \bar{\theta} \lambda_i \right),
\]

where \( \bar{\theta} \equiv \left( \sum_{i \in I} \theta_i^{-1} / |I| \right)^{-1} \) is the harmonic mean of traders’ absolute risk aversion coefficients. Intuitively, an asset commands a higher price if traders are on average optimistic about its payoff, or if on average it covaries negatively with traders’ background risks.

Using the prices in equation (5), the trader’s equilibrium portfolio is also solved in closed form as:

\[
\mathbf{x}_i = \mathbf{x}_i^R + \mathbf{x}_i^S, \quad \text{where} \quad \mathbf{x}_i^R = -\Lambda^{-1} \tilde{\lambda}_i \quad \text{and} \quad \mathbf{x}_i^S = \Lambda^{-1} \tilde{\mu}_i / \bar{\theta}_i,
\]

and

\[
\tilde{\lambda}_i = \lambda_i - \frac{\bar{\theta}}{\bar{\theta}_i} \frac{1}{|I|} \sum_{i \in I} \lambda_i,
\]

(8) \[ \tilde{\mu}_i = \mu_i - \frac{1}{|I|} \sum_{i \in I} \frac{\bar{\theta}}{\theta_i} \mu_i. \]
Here, (loosely speaking) the expression $\tilde{j}_i$ captures the covariance of the trader’s background risk relative to the average trader, and the expression $\tilde{\mu}_i$ captures her relative optimism. Note that the trader’s portfolio has two components. The first component, $x_{i}^R$, is the portfolio that would obtain if there were no belief disagreements (i.e., if $\tilde{\mu}_i = 0$ for each $i$). Hence, I refer to $x_{i}^R$ as the trader’s risk-sharing portfolio. The optimal risk-sharing portfolio is determined by traders’ background risks and their risk tolerances. The second component, $x_{i}^S$, captures traders’ deviations from this benchmark in view of their belief disagreements. Hence, I refer to $x_{i}^S$ as the speculative portfolio.

Equations (5)–(8) complete the characterization of equilibrium in this economy. The main goal of this article is to analyze the effect of financial innovation on portfolio risks. Given the mean-variance framework, a natural measure of portfolio risk for a trader $i$ is the variance of her net worth. I consider an average of this measure across all traders, the average variance, defined as follows:

$$\Omega = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\theta} \text{var}(n_i) = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\theta} (W_i^\Lambda^\top W_i + 2x_i^\top \lambda_i + x_i^\top \Lambda x_i).$$

(9)

Note that the portfolio risk of a trader is calculated according to traders’ (common) belief for the variance, $\Lambda^\top$. Note also that traders that are relatively more risk-averse are given a greater weight in the average. Intuitively, this weighting captures the risk-sharing benefits from transferring risks from traders with high $\theta_i$ to those with low $\theta_i$.

I use $\Omega$ as my main measure of average portfolio risks for two reasons. First, Section V shows that $\Omega$ is a measure of welfare in this economy when traders’ welfare is calculated according to a common belief (as opposed to their own heterogeneous beliefs). The second justification is provided by the following result.

**Lemma 1.** The risk-sharing portfolios, $\{x_{i}^R\}_{i}$, minimize the average variance, $\Omega$, among all feasible portfolios:

$$\min_{\{x_{i} \in \mathbb{R}^d\}} \Omega \quad \text{s.t.} \sum_{i} x_{i} = 0.$$

Note that absent belief disagreements, the risk-sharing portfolios are the same as equilibrium portfolios (see equation (6)). Thus, Lemma 1 implies that the equilibrium portfolios minimize the average variance when traders have the same beliefs.
Thus, it is natural to take $\Omega$ as the measure of risks, and to characterize the extent to which it deviates from the minimum benchmark when traders have belief disagreements.

To this end, I let $\Omega^R$ denote the minimum value of problem in Lemma 1 and refer to it as the \textit{uninsurable variance}. I also define $\Omega^S = \Omega - \Omega^R$ and refer to it as the \textit{speculative variance}. This provides a decomposition of the average variance, $\Omega = \Omega^R + \Omega^S$. The Appendix characterizes the two components in terms of the exogenous parameters as:

$$
\Omega^R = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\theta} \left( W_i^\prime \Lambda^\prime W_i - \tilde{\lambda}_i \Lambda^{-1} \tilde{\lambda}_i \right) \quad \text{and} \quad \Omega^S = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\theta} \left( \frac{\tilde{\mu}_i}{\theta_i} \Lambda^{-1} \frac{\tilde{\mu}_i}{\theta_i} \right).
$$

(10)

Loosely speaking, the uninsurable variance is lower when the assets provide better risk-sharing opportunities, captured by larger $\tilde{\lambda}_i$ (in vector norm), whereas the speculative variance is greater when the assets feature greater belief disagreements, captured by larger $\tilde{\mu}_i$.

\textbf{III.A. Comparative Statics of Portfolio Risks}

I next present the main result. Let $z(\hat{J})$ denote the equilibrium variable $z$ when the set of assets is given by $\hat{J}$.

\textbf{THEOREM 1.} (Financial Innovation and Portfolio Risks). Let $J_O$ and $J_N$ denote, respectively, the sets of old and new assets.

(i) Financial innovation always reduces the uninsurable variance, that is:

$$
\Omega^R(J_O \cup J_N) - \Omega^R(J_O) \leq 0.
$$

(ii) Financial innovation always increases the speculative variance, that is:

$$
\Omega^S(J_O \cup J_N) - \Omega^S(J_O) \geq 0.
$$

(iii) Suppose traders’ beliefs are given by $\mu_{i,D} = m_0^y + Dm_i^y$ for all $i$, where $m_0^y$ is a vector that captures the average belief, $\{m_i^y\}_i$ are vectors that satisfy $\sum m_i^y = 0$, and $D \geq 0$ is a parameter that scales belief disagreements. Suppose also that the inequality in part (ii) is strict for some $D > 0$. Then, there exists $\hat{D} \geq 0$ such that financial
innovation strictly increases the average variance, \( \Omega(J_O \cup J_N) > \Omega(J_O) \), if and only if \( D > D' \).

The first part, which is a corollary of Lemma 1, formalizes the traditional view of financial innovation by establishing that new assets always provide some risk-sharing benefits. The second part establishes that speculation creates a second force that always pushes in the opposite direction. The third part shows that when belief disagreements are large, the speculation force is sufficiently strong that financial innovation increases average portfolio risks.

Next I provide a sketch proof for the second part, which is useful to develop the intuition for the result. Consider an economy that is identical to the original economy except that there are no background risks (i.e., \( W_i = 0 \) for all \( i \in I \)), so that the only motive for trade is speculation. The proof in the Appendix shows that the average variance in this economy is identical to the speculative variance in the original economy. Thus, it suffices to show that financial innovation increases average portfolio risks in the hypothetical economy.

Recall that the Sharpe ratio of a portfolio is defined as the expected portfolio return in excess of the risk-free rate (which is normalized to 0) divided by the standard deviation of the portfolio return. Traders in the hypothetical economy perceive positive Sharpe ratios because they think various assets are mispriced. Define a trader’s speculative Sharpe ratio as the Sharpe ratio of her equilibrium portfolio in this economy. Using equations (5)–(8), this is given by:

\[
Sharpe^S_i = \frac{(x_i^S)'(\mu_i - p)}{\sqrt{(x_i^S)' \Lambda x_i^S}} = \sqrt{\frac{\mu_i^S \Lambda^{-1} \mu_i}{\lambda}}.
\]

Next consider the trader’s portfolio return given by \( n_i/e_i \) (where recall that \( e_i \) is the trader’s initial endowment). The standard deviation of this return can also be calculated as:

\[
\sigma_i^S = \frac{1}{e_i} \sqrt{(x_i^S)' \Lambda x_i^S} = \frac{1}{\theta i/e_i} \sqrt{\frac{\mu_i^S \Lambda^{-1} \mu_i}{\lambda}}.
\]

Note that the ratio, \( \theta_i/e_i \), provides a measure of trader \( i \)'s coefficient of relative risk aversion. Thus, combining the two expressions above gives the familiar result that the standard deviation of the portfolio return is equal to the Sharpe ratio of the optimal portfolio
divided by the coefficient of relative risk aversion (see Campbell and Viceira 2002). Intuitively, if a trader finds a risky portfolio with a higher Sharpe ratio, then she exploits this opportunity to such an extent that she ends up with greater portfolio risks.

The main result can then be understood through the lens of this textbook result. Financial innovation increases each trader’s speculative Sharpe ratio because it expands the traders’ betting possibilities frontier. That is, when the asset set is \( J_O \cup J_N \), traders are able to make all the speculative trades they could make when the asset set is \( J_O \), and some more. Importantly, new assets expand the betting frontier through the two channels emphasized before. First, new assets generate new bets, which creates higher expected returns (thereby increasing the numerator of the speculative Sharpe ratio). Second, new assets also enable traders to purify their existing bets (thereby reducing the denominator of the speculative Sharpe ratio). Once a trader obtains a higher speculative Sharpe ratio, she also undertakes greater speculative risks, providing a sketch proof for the main result.

III.B. Comparative Statics of Portfolio Comovements

Although Theorem 1 focuses on portfolio risks, it also naturally has implications for portfolio comovements. One signature of effective risk sharing is that traders’ net worths or consumption tend to comove (see Townsend 1994). In contrast, speculation naturally reduces comovements because traders with different beliefs tend to take opposite positions. To the extent that financial innovation facilitates speculation, it could also decrease comovements. The next result makes this intuition precise for the case in which traders share the same risk aversion coefficients. To state the result, let 

\[
\Psi = 1/(|I|^2 - |I|) \sum_{i_1 \in I, i_2 \in I \setminus \{i_2\}} \text{cov}(n_{i_1}, n_{i_2})
\]

denote the average covariance between two randomly selected traders’ net worths.\(^6\)

6. When traders differ in their risk aversion, even pure risk sharing might reduce comovements. To see this, consider an example with two traders with \( \theta_1 = 0 < \theta_2 \) who have the same beliefs (so there is no speculation). Suppose traders’ background risks are the same: \( w_1 = w_2 = v_1 + v_2 \), where \( v_1, v_2 \) are uncorrelated random variables. Consider the introduction of an asset with payoff \( a_1 = v_1 \). Then, the equilibrium features the risk-neutral trader holding all of the risk, \( v_1 \), even though this reduces the covariance of traders’ net worths.
PROPOSITION 1. (Financial Innovation and Portfolio Comove-
ments). Suppose $\forall i \in I$ for each $i$. Then, financial innovation
strictly increases the average variance, $\Omega(J_O \cup J_N) > \Omega(J_O)$,
if and only if it strictly decreases the average covariance,
$\Psi(J_O \cup J_N) < \Psi(J_O)$.

This result follows from the observation that the variance of
aggregate net worth, $\text{var}(\sum_{i \in I} n_i)$, is a linear combination of the
average variance, $\Omega$, and the average covariance, $\Psi$. Because this
is an endowment economy, the variance of aggregate net worth is constant. Therefore, an increase in $\Omega$ is associated with a de-
crease in $\Psi$.

Theorem 1 and Proposition 1 together predict that when
belief disagreements are large, financial innovation increases
portfolio risks and decreases portfolio comovements. These re-
results can be tested directly in some contexts in which traders’
net worths can be measured. For example, some Scandinavian
countries collect data on individuals’ wealth. The changes in
the riskiness and comovement of traders’ wealth can then be
linked to measures of financial innovation. In other contexts, it
might be possible to obtain close proxies of net worth. For in-
stance, if traders’ consumption can be measured, then changes
in consumption risks and comovements can be used to assess the
net effect of financial innovation.

III.C. Comparative Statics of Trading Volume

A notable feature of new financial assets is the large trading
volume they generate. Theorem 1 also has implications for trad-
ing volume. This is because traders’ portfolios are closely related
to their portfolio risks as follows:

$$\Omega^R = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\theta} \left( w_i \Lambda^v w_i - (x_i^R)' \Lambda x_i^R \right) \quad \text{and} \quad \Omega^S = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\theta} (x_i^S)' \Lambda x_i^S,$$
(11)

where the derivation follows by equations (6) and (10). Intuitively, the two components of portfolios, $[x_i^R, x_i^S]$, capture the extent to which the assets are used for the corresponding motive for trade. Loosely speaking, uninsurable variance is lower when traders’ risk-sharing portfolios are larger (in vector norm), whereas the speculative variance is greater when traders’ speculative portfolios are larger. As financial innovation reduces
and increases $\Omega^S$, it tends to increase both the risk-sharing and speculative positions, contributing to trading volume.

More subtly, equation (11) also suggests that the net effect of financial innovation on $\Omega$ depends on the relative size of the risk-sharing and the speculative trading volume in new assets. The next result formalizes this intuition for the case in which new assets are uncorrelated with existing assets. To state the result, define, respectively, the risk-sharing and the speculative trading volumes in a new asset, $j_N$, as:

$$T_{j_N,S} = \left( \frac{1}{|I|} \sum_i \frac{\theta_i}{\bar{\theta}} \left( x_{i,j_N}^S \right)^2 \right)^{1/2}$$

and

$$T_{j_N,R} = \left( \frac{1}{|I|} \sum_i \frac{\theta_i}{\bar{\theta}} \left( x_{i,j_N}^R \right)^2 \right)^{1/2}.$$ 

Note that trading volume is defined to be a weighted (and quadratic) average of traders’ positions, with a greater weight given to traders that are more risk averse.

**Proposition 2.** (Financial Innovation and Trading Volume). Suppose that a single new asset, $j_N$, is introduced whose payoff is uncorrelated with old assets (that is, $(A')' A A^j_N = 0$ for all $j \in J_O$). Then, financial innovation increases the average variance, $\Omega(J_O \cup J_N) > \Omega(J_O)$, if and only if $T_{j_N,S} > T_{j_N,R}$, that is, the new asset leads to a greater speculative trading volume than risk-sharing trading volume.

Proposition 2 and equation (11) have two empirical implications. First, these results provide an alternative method to test Theorem 1 when traders’ net worths are not observable but their risk-sharing and speculative positions can be separately identified. Though this is not easy, it might be possible in some contexts to estimate (or at the very least to bound) traders’ risk-sharing positions. This requires some knowledge of traders’ background risks but no knowledge of their beliefs (see equation (6)). Traders’ speculative positions can then be obtained as deviations of their

7. When new and old assets are correlated, trading volume in new assets does not fully reflect the changes in portfolio risks. Nonetheless, trading volume often provides a useful diagnostic tool, even in these more general cases. To see this, consider the case in Section II.A in which a second asset, $a^2 = v_2$, is introduced and is correlated with the first asset. This asset generates some speculative trading volume, that is, $x_{i,2}^R \neq 0$ for each $i$, and also leads to an increase in speculative variance. In contrast, the asset generates no risk-sharing trading volume, that is, $x_{i,2}^S = 0$ for each $i$, and has no impact on the risk-sharing variance.
actual positions from the estimated risk-sharing positions. To
give one example, risk sharing typically requires individuals to
take short positions in stocks of companies in the industry in
which they work. Thus, given an individual $i$ and stock $j$ in the
same industry, a reasonable upper bound on the risk-sharing
position might be $x_{ij}^{R} \leq 0$. If we observe the individual taking a
long position, $x_{ij} > 0$, this also implies a lower bound on the specu-
lative position, $x_{ij}^{S} \geq x_{ij}$. Empirical studies have in fact found that
individuals invest considerably in own company and profession-
ally close stocks (see Doskeland and Hvide 2011).

The second implication of Proposition 2 and equation (11) is
that belief disagreements generate joint predictions regarding
portfolio risks and trading volume. A growing literature has
emphasized belief disagreements as a potential explanation for
the large trading volume observed in financial markets (see Hong
and Stein 2007). The foregoing analysis suggests that this litera-
ture is also making implicit predictions about portfolio risks. If
speculation is the main source of trading volume in some new
assets, then those assets are likely to increase portfolio risks.

IV. Endogenous Financial Innovation

The analysis so far has taken the set of new assets as exogen-
ous. In practice, many financial products are introduced endogen-
ously by economic agents with profit incentives. A large literature
has emphasized risk sharing as a major driving force for endogen-
ous financial innovation (see Allen and Gale 1994; Duffie and
Rahi 1995).8 A natural question is to what extent the risk-sharing
motive for financial innovation is robust to the presence of belief
disagreements. To address this question, this section endogenizes
the asset design by introducing a profit seeking market maker.

Suppose the economy initially starts with no assets. A
market maker introduces the assets for which it then serves as
the intermediary. The market maker is constrained to introduce
$|J| \leq m$ assets, but is otherwise free to choose the assets’ loading
on the underlying risks (recall that $a_j'(v) = (A_j')^T v$ for each $j$). The

8. Risk sharing is one of several drivers of financial innovation emphasized by
the previous literature. Other factors include mitigating agency problems, redu-
cing asymmetric information, minimizing transaction costs, and sidestepping taxes
and regulation (see Tufano 2003). These other factors, though clearly important,
are left out to focus on the tension between risk sharing and speculation.
asset design, $A = [A^1, A^2, ..., A^J]$, affects the trader’s relative covariance and relative optimism (see equations (7) and (8)) according to \( \tilde{\lambda}_i(A) = A^\top \Lambda^\top \tilde{w}_i \) and \( \tilde{\mu}_i(A) = A^\top \tilde{\mu}_i \), where the deviation terms \( \tilde{w}_i \) and \( \tilde{\mu}_i \) are defined in equation (18) in the Appendix.

The market maker’s innovation incentives are determined by the surplus from trading, that is, traders’ willingness to pay to trade relative to keeping their endowments. The Appendix derives the total surplus as:

$$
\sum_i \frac{\theta_i}{2} \left( \frac{\tilde{\mu}_i(A)}{\theta_i} - \tilde{\lambda}_i(A) \right) \Lambda^{-1} \left( \frac{\tilde{\mu}_i(A)}{\theta_i} - \tilde{\lambda}_i(A) \right).
$$

Intuitively, traders are willing to pay to trade assets that facilitate better risk sharing (i.e., larger \( \tilde{\lambda}_i(A) \)) or those that generate greater belief disagreements (i.e., larger \( \tilde{\mu}_i(A) \)).

In practice, the market maker extracts some surplus by charging commissions or bid–ask spreads. I abstract away from institutional detail and assume that the market maker extracts a constant fraction of the total surplus. With this assumption, the market maker chooses the asset design, $A$, to maximize the objective function in equation (12). The next result characterizes the optimal asset design. Note that many choices of $A$ represent the same trading opportunities (and thus also generate the same profits). Thus, suppose without loss of generality that the market maker’s choice is subject to the following normalizations:

$$
\Lambda = A^\top \Lambda^\top A = I \quad \text{and} \quad \left( (\Lambda^\top)^{1/2} A \right)_j \geq 0 \quad \text{for each } j \in J.
$$

**Proposition 3. (Optimal Asset Design).** Suppose the $k \times k$ matrix

$$
\sum_i \frac{\theta_i}{2} \left( (\Lambda^\top)^{-1/2} \tilde{\mu}_i^\top \frac{\theta_i}{\theta_i} - (\Lambda^\top)^{1/2} \tilde{w}_i \right) \left( (\Lambda^\top)^{-1/2} \tilde{\mu}_i^\top \frac{\theta_i}{\theta_i} - (\Lambda^\top)^{1/2} \tilde{w}_i \right)'
$$

is nonsingular. Then, the design, $A$, is optimal if and only if the columns of the $|J| \times |J|$ matrix, $(\Lambda^\top)^{1/2} A$, are the eigenvectors corresponding to the $|J|$ largest eigenvalues of the matrix in equation (14). If the eigenvalues are distinct, then the optimal

9. Here, $(\Lambda^\top)^{1/2}$ denotes the unique positive definite square root of the matrix, $\Lambda^\top$. The first condition in equation (13) normalizes the variance of assets to be the identity matrix, $I$. This condition determines the column vectors of the matrix, $(\Lambda^\top)^{1/2} A$, up to a sign. The second condition resolves the remaining indeterminacy by adopting a sign convention for these column vectors.
design is uniquely determined by this condition along with the normalizations in equation (13). Otherwise, the design is determined up to a choice of the \(|J|\) largest eigenvalues.

This result generalizes the results in Demange and Laroque (1995) and Athanasoulis and Shiller (2000) to the case with belief disagreements. Importantly, equations (12) and (14) show that financial innovation is partly driven by the size and the nature of traders’ belief disagreements. This is because innovation incentives are generated by both speculation and risk-sharing motives for trade. I next present the main result of this section, which characterizes the optimal design in two extreme cases.

**Theorem 2. (Endogenous Innovation and Portfolio Risks).** Suppose traders’ beliefs are given by 
\[ \mu_{i,D}^\tau = \mu_0^\tau + Dm_i^\tau \]
for all \(i\), where \(m_0^\tau\) is a vector that captures the average belief, \(\{m_i^\tau\}_i\) are vectors that satisfy \(\sum m_i^\tau = 0\), and \(D \geq 0\) is a parameter that scales belief disagreements. For each \(D\), suppose the matrix in equation (14) is nonsingular with distinct eigenvalues. Let 
\[ \Omega_D(A) \]
be the average variance as a function of the asset design, and 
\[ A_D \]
the optimal design (characterized in Proposition 3).

(i) For \(D = 0\), the market maker innovates assets that minimize the average variance:
\[ A_0 \in \arg \min_A \Omega_0(A) \]
subject to the normalizations in (13).

(ii) Suppose there are at least two traders with different beliefs, that is, \(\mu_{i_1}^\tau \neq \mu_{i_2}^\tau\) for some \(i_1, i_2 \in I\). As \(D \to \infty\), the market maker innovates assets that maximize the average variance. That is, the limit of the optimal asset design, 
\[ A_\infty \equiv \lim_{D \to \infty} A_D, \]
and the limit of the scaled average variance, 
\[ \Omega_\infty(A) \equiv \lim_{D \to \infty} 1/D^2 \Omega_D(A), \]
exist and they satisfy
\[ A_\infty \in \arg \max_A \Omega_\infty(A) \]
subject to the normalizations in (13).

Without belief disagreements, the market maker innovates assets that minimize average portfolio risks in this economy, as illustrated by the first part of the theorem. The second part

10. The assumption of distinct eigenvalues can be relaxed at the expense of additional notation.
provides a sharp contrast to this traditional view. When belief disagreements are sufficiently large, the market maker innovates assets that maximize average portfolio risks, completely disregarding risk sharing. Among other things, this result might explain why most of the macro futures markets proposed by Shiller (1993) have not been adopted in practice, despite the fact that in principle they are very useful for risk-sharing purposes.

The intuition for Theorem 2 is that with large disagreements, speculation becomes the main motive for trade and the main source of profits for the market maker (see equation (12)). As this happens, the market maker introduces assets that enable the traders to bet most precisely on their different beliefs (see equation (14)). As a by-product, the market maker also maximizes average portfolio risks.

V. Welfare Implications

Although Theorems 1 and 2 establish that financial innovation may increase portfolio risks, they do not reach any welfare conclusions. In fact, financial innovation in the baseline setting results in a Pareto improvement if traders’ welfare is calculated according to their own beliefs. This is because each trader perceives a large expected return from her speculative positions in new assets, which justifies the additional risks that she is taking.11 Next I consider two variants of the baseline setting in which this welfare conclusion can be overturned.

V.A. Inefficiencies Driven by Belief Distortions

The first setting concerns an interpretation of disagreements as distortions stemming from various psychological biases emphasized in behavioral finance (see Barberis and Thaler 2003). If individuals’ beliefs are heterogeneously distorted, then

11. Note that the reason for efficiency in this setting is almost the opposite of what has been emphasized in much of the previous literature on financial innovation. In particular, traders’ welfare gains do not come from a decrease in their risks but from an increase in their perceived expected returns. Relatedly, it is not clear whether these perceived returns should be viewed as welfare gains because they are driven by belief disagreements. Although all traders perceive large expected returns, at most one of these expectations can be correct. In fact, a growing theory literature has argued that Pareto efficiency is not the appropriate welfare criterion for environments with belief disagreements (see Brunnermeier, Simsek, and Xiong 2012 and the references therein).
they would naturally come to have belief disagreements. In this case, traders’ welfare should ideally be evaluated according to a common objective belief, as opposed to their own distorted beliefs. However, there is a practical difficulty because the planner might not know the objective belief. In Brunnermeier, Simsek, and Xiong (2012), we propose a belief-neutral welfare criterion that circumvents this difficulty. In particular, we take any convex combination of agents’ beliefs as a reasonable objective belief, and we say that an allocation is belief-neutral inefficient if it is inefficient according to all reasonable beliefs.

To apply the belief-neutral criterion in this model, consider an arbitrary convex combination of traders’ beliefs: $N(\mu^*_h, \Lambda^*)$, where $\mu^*_h = \sum_i h_i \mu^*_i$, $h_i \geq 0$ and $\sum_i h_i = 1$. The social welfare under this belief (denoted by subscript $h$) is measured by the sum of traders’ certainty equivalent net worths: $\sum_{i \in L} (E_h[n_i] - \frac{\theta_i}{2} \text{var}_h(n_i))$. Using expressions (1) and (9) along with market clearing, the social welfare can be calculated as:

$$E_h \left[ \sum_{i \in L} e_i + w_i \right] - \frac{\theta}{2} \Omega.$$ 

The first component of welfare is agents’ expected endowment, which is exogenous in the sense that it does not depend on portfolios. The second and the endogenous component is proportional to average variance, $\Omega$. Consequently, financial innovation is belief-neutral inefficient if and only if it increases $\Omega$.

Intuitively, trading in this economy does not generate expected aggregate net worth since it simply redistributes wealth. Hence, trading affects social welfare only through portfolio risks.

12. More specifically, any allocation $\tilde{x}$ that yields a higher value of this expression than $x$ can also be made to Pareto dominate $x$ (under belief $h$) with appropriate ex ante wealth transfers.

13. More precisely, the equilibrium is belief-neutral Pareto inefficient as long as the average variance deviates from its minimum, $\Omega > \Omega^R$. However, it might be difficult for the planner to implement the minimum, $\Omega^R$, as this would require monitoring that each trader holds exactly the risk-sharing portfolio, $x^R$. Realistically, the planner might have to decide whether to allow unrestricted trade in new assets. A planner subject to this restriction would conclude that financial innovation is belief-neutral inefficient if and only if it increases $\Omega$. Note that Assumption A1 facilitates the belief-neutral welfare analysis by ensuring that traders agree on the variance. The welfare conclusions extend to the case in which there are relatively small disagreements on the variance.
When portfolio risks increase, social welfare decreases according to each trader’s belief (or their convex combinations). Put differently, each trader believes her welfare is increasing at the expense of other traders. Consequently, a planner can conclude that financial innovation is inefficient without taking a stand on whose belief is correct.

V.B. Inefficiencies Driven by Externalities

The welfare implications of the baseline setting can also be overturned if traders’ choices are associated with externalities. Such externalities naturally emerge when traders correspond to financial intermediaries. These intermediaries might take socially excessive risks either because of fire sale externalities (see, for example, Lorenzoni 2008), or because they enjoy explicit or implicit government protection (see, for example, Rajan 2010). Perhaps for these reasons, much existing regulation in the financial system is concerned with restricting intermediaries’ portfolio risks. To the extent that financial innovation increases these risks, it could lead to inefficiencies.

I next illustrate these types of inefficiencies in a version of the model in which traders are under government protection. Suppose each trader, $i$, corresponds to a financial intermediary with limited liability. An intermediary whose net worth falls below zero, $n_i < 0$, is forced into bankruptcy, and its creditors potentially face losses. However, there is a new agent, the government, which bails out the creditors of an intermediary that enters bankruptcy (for reasons that are outside this model). For simplicity, suppose the government makes the creditors whole by paying $-n_i > 0$. In addition, the government inflicts a nonpecuniary punishment to the bankrupt intermediary that is equivalent to a loss of $-n_i$ dollars. This assumption considerably simplifies the analysis by ensuring that the equilibrium remains unchanged despite the limited liability feature. It is also a conservative assumption because it eliminates the additional portfolio risks that would stem from the usual risk-shifting motive and enables me to focus on speculative risks.

The welfare analysis is potentially different than the baseline setting because the government is also affected by the intermediaries’ portfolio choices. Suppose the government is risk-neutral and its belief about the underlying risks is given by $N(\mu_g, \Lambda^v)$. The government’s expected welfare loss, which
also corresponds to the social cost of bailouts in this setting, is given by:

\[ \sum_{i=1}^{N} \Pr_g(n_i < 0)E_g[-n_i|n_i < 0]. \]

In particular, the government’s loss depends on a measure of intermediaries’ average portfolio risks that is similar to (but not the same as) the average variance, \( \Omega \). Intuitively, portfolio risks matter because they determine the extent to which financial intermediaries will need government assistance. Because financial innovation can increase these risks, it can also increase the government’s loss, illustrating the negative externalities. Moreover, financial innovation is inefficient, even in the usual Pareto sense, if it increases the government’s loss more than it increases intermediaries’ perceived private benefits.

In both settings analyzed in Sections V.A and V.B, a measure of average portfolio risks emerge as the central object in welfare analysis, providing some normative content to Theorems 1 and 2. These analyses should be viewed as partial exercises, characterizing the welfare effects of financial innovation that operate through portfolio risks. In particular, I do not take a strong normative stance because the model is missing some important ingredients that could change the welfare arithmetic. Among other things, speculation driven by financial innovation could provide additional social benefits by making asset prices more informative. Assessing the net welfare effect of financial innovation is an important question which I leave for future research.

VI. CONCLUSION

This article analyzed the effect of financial innovation on portfolio risks in a standard mean-variance setting with belief disagreements. When disagreements are large, financial innovation increases average portfolio risks, decreases average portfolio comovements, and generates relatively more speculative trading volume than risk-sharing volume. Moreover, financial

14. Note that bailouts represent negative-sum transfers in this economy (as opposed to zero-sum) because of the simplifying assumption that the government inflicts a nonpecuniary punishment on bankrupt intermediaries. Bailouts are also likely to be negative-sum transfers in practice although possibly for different reasons, for example, tax distortions.
innovation is endogenously directed toward speculative assets that increase average portfolio risks.

These results show that belief disagreements can overturn the traditional views regarding the relationship between financial innovation and portfolio risks. A natural question is how large belief disagreements should be for this analysis to be practically relevant. I address this question in Simsek (2011) by considering a calibration of the model in the context of the national income markets proposed by Shiller (1993). These assets could in principle facilitate the sharing of income risks among different countries. Athanasoulis and Shiller (2001) analyze these assets in the context of G7 countries and argue that they would reduce individuals’ consumption risks. Using exactly their data and calibration, I find that small amounts of belief disagreements about the gross domestic product growth rates of G7 countries (much smaller than implied by the Philadelphia Fed’s Survey of Professional Forecasters) imply that these assets would actually increase average consumption risks. Although this calibration exercise is promising, it is far from conclusive. I leave empirical analysis of the model for future research.

**APPENDIX: OMITTED DERIVATIONS AND PROOFS**

I first note a couple of identities that will be useful in some of the proofs. Consider vectors, \( z_i \), that satisfy \( \sum_i z_i = 0 \). Then:

\[
\sum_i z_i^\prime \Lambda^{-1} \mu_i = \sum_i z_i^\prime \Lambda^{-1} \tilde{\mu}_i \quad \text{and} \quad \sum_i z_i^\prime \Lambda^{-1} \theta_i \lambda_i = \sum_i z_i^\prime \Lambda^{-1} \theta_i \tilde{\lambda}_i,
\]

(15)

which follow from equations (7) and (8). In words, the belief and the covariance terms, \( \mu_i \) and \( \theta_i \lambda_i \), in the sums can be replaced by the deviation terms, \( \tilde{\mu}_i \) and \( \tilde{\lambda}_i \).

**Proof of Lemma 1**

Using equation (9), the first-order conditions are \( \Lambda x_i + \lambda_i = \gamma \tilde{\lambda}_i \) for each \( i \in I \), where \( \gamma \) is a vector of Lagrange multipliers. Note that \( x_i^R = -\Lambda^{-1} \tilde{\lambda}_i \) satisfies these first-order conditions for the Lagrange multiplier \( \gamma = (\sum_{i\in I} \lambda_i) / |I| \). It follows that \( \{x_i^R\}_i \) is the unique solution to the problem.
Derivation of Equation (10)

Plugging \( x_i^R = -\Lambda^{-1}\tilde{\lambda}_i \) into equation (9) implies:

\[
|I|\Omega^R = \sum_i \frac{\theta_i}{\theta} \left( W_i \Lambda^v W_i + \tilde{\lambda}_i \Lambda^{-1} \tilde{\lambda}_i \right) - 2 \sum_i \tilde{\lambda}_i \Lambda^{-1} \frac{\theta_i \tilde{\lambda}_i}{\theta},
\]

\[
= \sum_i \frac{\theta_i}{\theta} \left( W_i \Lambda^v W_i + \tilde{\lambda}_i \Lambda^{-1} \tilde{\lambda}_i \right) - 2 \sum_i \tilde{\lambda}_i \Lambda^{-1} \frac{\theta_i \tilde{\lambda}_i}{\theta},
\]

which in turn yields the expression for \( \Omega^R \). Here, the second line uses the identity in (15) with \( z_i = \tilde{\lambda}_i \).

Next consider the residual, \( |I|\Omega^S = |I|(\Omega - \Omega^R) \). Using equations (6) and (9), this is given by:

\[
\sum_i \frac{\theta_i}{\theta} x_i^S \Lambda x_i + 2 \sum_{i \in I} \frac{\theta_i}{\theta} x_i^S \tilde{\lambda}_i + \sum_{i \in I} \frac{\theta_i}{\theta} \Lambda^{-1} \tilde{\lambda}_i
\]

\[
= \sum_{i \in I} \frac{\theta_i}{\theta} \left( \tilde{\mu}_i - \tilde{\lambda}_i \right) \Lambda^{-1} \left( \tilde{\mu}_i - \tilde{\lambda}_i \right) + 2 \sum_{i \in I} \frac{\theta_i}{\theta} \Lambda^{-1} \frac{\theta_i \tilde{\lambda}_i}{\theta} + \sum_{i \in I} \frac{\theta_i}{\theta} \Lambda^{-1} \tilde{\lambda}_i
\]

\[
= \sum_{i \in I} \frac{\theta_i}{\theta} \left[ \left( \tilde{\mu}_i - \tilde{\lambda}_i \right) \Lambda^{-1} \left( \tilde{\mu}_i + \tilde{\lambda}_i \right) \right] + \sum_{i \in I} \frac{\theta_i}{\theta} \Lambda^{-1} \tilde{\lambda}_i,
\]

which in turn yields the expression for \( \Omega^S \). Here, the second line uses the identity in (15) with \( z_i = \frac{\tilde{\mu}_i}{\theta_i} - \tilde{\lambda}_i \).

Proof of Theorem 1

**PART (I).** By definition, \( \Omega^R \) is the optimal value of the minimization problem in Lemma 1. Because financial innovation expands the constraint set of this problem, it also decreases the optimal value, proving \( \Omega^R(J_0 \cup J_N) \leq \Omega^R(J_0) \).

**PART (II).** The proof proceeds in three steps. First, equation (6) implies that the speculative portfolio, \( x_i^S \), solves the following problem:

\[
\max_{x_i \in \mathbb{R}^n} \left( \tilde{\mu}_i \right) \Lambda x_i - \frac{\theta_i}{2} x_i^S \Lambda x_i,
\]

which corresponds to the mean-variance problem of the trader in the hypothetical economy described in the main text. Moreover, the speculative variance, \( \Omega^S \), is found by averaging the variance costs for each trader at the solution to this problem, that is, \( \Omega^S = 1/|I| \sum_i \frac{\theta_i}{\theta} \left( x_i^S \right)^\Lambda x_i^S \). Second, financial innovation relaxes the constraint set of problem (16).
Consequently, each trader $i$ obtains a higher objective value. Third, because the problem is a quadratic optimization, the linear and the quadratic terms at the optimum have a constant proportion, in particular, $(\bar{\mu}_i)'x_i^S = 2\frac{\theta_i}{\theta_i} (x_i^S)' \Lambda x_i^S$. Consequently, financial innovation increases the quadratic term for each trader, proving $\Omega^S(J_O \cup J_N) \geq \Omega^R(J_O)$.

**PART (III).** Note that $\bar{\mu}_i = DA'\tilde{m}_i$, where $\tilde{m}_i$ is defined as in equation (8). Thus, the speculative variance can be written as (see equation (10)):

$$\Omega^S = D^2 \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\theta_i} \left( (A'\tilde{m}_i)' \Lambda^{-1} A'\tilde{m}_i \right).$$

Hence, the difference, $\Omega^S(J_O \cup J_N) - \Omega^S(J_O)$, is proportional to $D^2$. Because it is positive for some $D > 0$, the difference is also strictly increasing in $D$. In contrast, the difference, $\Omega^R(J_O) - \Omega^R(J_O \cup J_N)$, does not depend on $D$. Consequently, there exists $\bar{D} \geq 0$ such that $\Omega^S(J_O \cup J_N) - \Omega^R(J_O \cup J_N) > \Omega^R(J_O)$ iff $D > \bar{D}$. This in turn implies $\Omega(J_O \cup J_N) > \Omega(J_O)$ iff $D > \bar{D}$, completing the proof.

**Proof of Proposition 2**

Because asset $j_N$ is uncorrelated with the remaining assets, the matrices $\Lambda$ and $\Lambda^{-1}$ are both block diagonal. By equation (6), the introduction of asset $j_N$ does not affect the positions on the remaining assets. This observation along with equation (11) implies:

$$\Omega^R(J_O) - \Omega^R(J_O \cup J_N) = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\theta_i} (x_{i,j_N}^R)' \Lambda_{i,j_N,j_N} x_{i,j_N}^R = \Lambda_{i,j_N,j_N} (T_{i,j_N}^R)^2,$$

and similarly $\Omega^S(J_O \cup J_N) - \Omega^S(J_O) = \Lambda_{i,j_N,j_N} (T_{i,j_N}^S)^2$. The proof follows from combining these expressions.

**Derivation of the Surplus in Equation (12)**

Using equation (6), traders’ net worth, $n_i$, can be written as:

$$n_i = e_i - x_i'p + W_i'v + \left( \frac{\tilde{\mu}_i(A)}{\theta_i} - \tilde{\nu}_i(A) \right)' \Lambda^{-1} A'v.$$
The certainty equivalent of this expression is given by:

\[ e_i - x_i'p + W'_i\mu_i^y + \left( \frac{\tilde{\mu}_i(A)}{\theta_i} - \tilde{\lambda}_i(A) \right)' \Lambda_i^{-1} \mu_i - \frac{\theta_i}{2} W'_i \Lambda^v W_i \]

\[ - \frac{\theta_i}{2} \left( \frac{\tilde{\mu}_i(A)}{\theta_i} - \tilde{\lambda}_i(A) \right)' \Lambda_i^{-1} \left( \frac{\tilde{\mu}_i(A)}{\theta_i} - \tilde{\lambda}_i(A) \right) - \frac{\theta_i}{2} \left( \frac{\tilde{\mu}_i(A)}{\theta_i} - \tilde{\lambda}_i(A) \right)' \Lambda_i^{-1} \lambda_i(A). \]

(17)

In contrast, the trader’s certainty equivalent payoff in autarky is given by

\[ e_i + W'_i\mu_i^y - \frac{\theta_i}{2} W'_i \Lambda^v W_i. \]

Subtracting this expression from equation (17), the surplus for each trader can be written as:

\[ - x_i'p + \left( \frac{\tilde{\mu}_i(A)}{\theta_i} - \tilde{\lambda}_i(A) \right)' \Lambda_i^{-1} \left( \mu_i(A) - \theta_i \lambda_i(A) \right) \]

\[ - \frac{\theta_i}{2} \left( \frac{\tilde{\mu}_i(A)}{\theta_i} - \tilde{\lambda}_i(A) \right)' \Lambda_i^{-1} \left( \frac{\tilde{\mu}_i(A)}{\theta_i} - \tilde{\lambda}_i(A) \right). \]

Summing this expression across all traders, using market clearing, \( \sum_i x_i'p = 0 \), and also using the identities in (15) for \( z_i = \tilde{\mu}_i(A)/\theta_i - \tilde{\lambda}_i(A) \), the total surplus is given by equation (12).

**Proof of Proposition 3**

Note that the market maker affects the traders’ relative covariance and the relative optimism according to \( \tilde{\lambda}_i(A) = A' \Lambda^v \tilde{W}_i \) and \( \tilde{\mu}_i(A) = A' \tilde{\mu}_i^y \), where:

(18) \[ \tilde{W}_i = W_i - \frac{\theta_i}{\theta_i + 1} \sum_{j\neq i} W_i \] and \( \tilde{\mu}_i^y = \mu_i^y - \frac{1}{\theta_i + 1} \sum_{j\neq i} \frac{\theta_i}{\theta_i + 1} \mu_j^y \).

It is useful to consider the market maker’s optimization problem in terms of a linear transformation of assets, \( B = (\Lambda^v)^{1/2} A \), where \( (\Lambda^v)^{1/2} \) is the unique positive definite square root matrix of \( \Lambda^v \). Note that choosing \( B \) is equivalent to choosing \( A \). The normalizations in equation (13) can be written in terms of \( B \) as \( B'B = I \) and \( B'_j \geq 0 \) for each \( j \). Using these normalizations, the surplus in equation (12) can also be written as:

(19) \[ \sum_i \frac{\theta_i}{2} \left( (\Lambda^v)^{-1/2} \tilde{\mu}_i^y - (\Lambda^v)^{1/2} \tilde{W}_i \right)' B \left( (\Lambda^v)^{-1/2} \tilde{\mu}_i^y - (\Lambda^v)^{1/2} \tilde{W}_i \right) \]

\[ = tr(B'\Gamma B) = \sum_j (B')' \Gamma B', \]
where \( \Gamma \) denotes the matrix defined in equation (14). Here, the second line uses the matrix identity \( \text{tr}(XY) = \text{tr}(YX) \) and the linearity of the trace operator.

Next consider the alternative problem of choosing \( B \) to maximize (19) subject to the constraint \( (B^j)'B^j = 1 \) for each \( j \), which is implied by the stronger constraint \( B'B = I \). The first-order conditions for this problem are given by \( \Gamma B^j = \gamma' B^j \) for each \( j \) where \( \gamma' \in \mathbb{R}_+ \) are Lagrange multipliers. It follows that the solution, \( \{B^j\}_j \), corresponds to eigenvectors of the matrix, \( \Gamma \), with corresponding eigenvalues \( \{\gamma'\}_j \). Plugging this into equation (19), the surplus is given by \( P_j \gamma' \). Thus, the objective value is maximized iff \( \{\gamma'\}_j \) correspond to the \( J \) largest eigenvalues of the matrix, \( \Gamma \). If the \( J \) largest eigenvalues are unique, then the solution, \( \{B^j\}_j \), is also uniquely characterized as the corresponding eigenvectors (along with the normalizations \( (B^j)'B^j = 1 \) and \( B^j_1 \geq 0 \)). If the \( J \) largest eigenvalues are not unique, then the solution, \( \{B^j\}_j \), is uniquely determined up to a choice of these eigenvalues.

Finally, consider the original problem of maximizing the expression in (19) subject to the stronger constraint, \( B'B = I \). Because \( \Gamma \) is a symmetric matrix, its eigenvectors are orthogonal. This implies that any solution to the alternative problem, \( \{B^j\}_j \), also satisfies the stronger constraint \( B'B = I \). It follows that the solutions to the two problems are the same, completing the proof.

**Proof of Theorem 2**

**PART (i).** Let \( \Pi(A) \) denote the surplus defined in equation (12). Note that \( \tilde{\mu}_i(A) = DA\tilde{m}_i^\gamma \), where \( \tilde{m}_i^\gamma \) is defined as in equation (8). Thus, \( D = 0 \) implies \( \tilde{\mu}_i(A) = 0 \), which in turn implies \( \Pi(A) = \sum_i c_2 \lambda_i(A) \Lambda^{-1}(A) \) for some \( c_1 \geq 0 \) and \( c_2 > 0 \) (see equation (10)). Thus, maximizing \( \Pi(A) \) is equivalent to minimizing \( \Omega^R(A) \). Moreover, \( \tilde{\mu}_i(A) = 0 \) also implies that \( \Omega^R(A) = \Omega(A) \). It follows that the optimal design minimizes \( \Omega(A) \).

**PART (ii).** Consider the alternative problem of choosing \( A \) to maximize the scaled surplus, \( 1/D^2 \Pi(A) \), subject to equation (13). The limit of the objective value is given by:

\[
\lim_{D \to \infty} \frac{1}{D^2} \Pi(A) = \lim_{D \to \infty} \sum_i \frac{\theta_i}{2} \left( \tilde{\mu}_i(A) \right)' \Lambda^{-1} \left( \frac{\tilde{\mu}_i(A) - \tilde{\lambda}_i(A)}{D} \right)
\]

(20)

\[
\quad = \sum_i \frac{\theta_i}{2} \frac{(A\tilde{m}_i^\gamma)'}{\theta_i} \Lambda^{-1} A\tilde{m}_i^\gamma.
\]

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where the first line uses equation (12) and the second line uses $\tilde{q}_i = DA^0 \tilde{m}_i$. Because the limit is finite, the alternative problem has a maximum for each $D$ over the extended space $D \in \mathbb{R}_+ \cup \{\infty\}$. Moreover, by the maximum theorem, the solution is upper hemicontinuous. By Proposition 3 (and the assumption in the problem statement), the solution, $A_D$, is unique and bounded for any finite $D$. It follows that the limit, $A_\infty = \lim_{D \to \infty} A_D$, exists and maximizes the expression in (20). Next consider the limit of the scaled average variance:

$$\Omega_\infty(A) = \lim_{D \to \infty} \left( \frac{\Omega^S_D(A)}{D^2} + \frac{\Omega^H_D(A)}{D^2} \right) = \frac{1}{|I|} \sum_i \frac{\theta_i}{\tilde{\theta}_i} \frac{(A^0 \tilde{m}_i')'}{\tilde{\theta}_i} \Lambda^{-1} \frac{(A^0 \tilde{m}_i')'}{\tilde{\theta}_i},$$

where the second equality uses equation (10) and $\tilde{q}_i = DA^0 \tilde{m}_i$. Combining this expression with equation (20) completes the proof.

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