MEASURING THE LEVEL AND UNCERTAINTY OF TREND INFLATION

Elmar Mertens*

Abstract—Firmly anchored inflation expectations are widely viewed as playing a central role in the successful conduct of monetary policy. This paper presents estimates of trend inflation, based on a fairly broad information set, spanning survey expectations, realized inflation rates, and the term structure of interest rates. In order to assess whether inflation expectations are anchored, estimates not only the level of trend inflation but also the uncertainty surrounding future changes in trend inflation.

I. Introduction

Firmly anchored inflation expectations are widely viewed as playing a central role in the successful conduct of monetary policy. This paper presents estimates of trend inflation, based on a fairly broad information set, spanning survey expectations, realized inflation rates, and the term structure of interest rates. In order to assess whether inflation expectations are anchored, I estimate not only the level of trend inflation but also the uncertainty surrounding future changes in trend inflation.

While the task of monitoring inflation expectations is important, it is typically hampered by the lack of a comprehensive summary measure of inflation expectations. For example, a variety of survey measures exist that often differ from one another in several respects. Some indicators measure short-term expectations and some longer-term expectations of future inflation rates; some refer to changes in the Consumer Price Index (CPI), others to Gross Domestic Product (GDP) or Personal Consumption Expenditures (PCE). In some cases, the forecast horizon may only be vaguely specified; in others, the relevant price index may be left open. The series often report at different frequencies and typically have different starting points. As an alternative data source, inflation expectations may also be extracted from financial market data, like the levels of nominal interest rates. While all these measures are potentially valuable, they are also likely subject to noise and measurement errors and at times may convey conflicting signals.

To address this lack of a comprehensive measure, this paper estimates trend inflation from a time series model that combines a variety of data series: inflation rates, survey responses about future inflation, and nominal interest rates.

Received for publication October 11, 2011. Revision accepted for publication July 29, 2015. Editor: Mark W. Watson.

* Federal Reserve Board.

The views in this paper do not necessarily represent the views of the Federal Reserve Board, or any other person in the Federal Reserve System or the Federal Open Market Committee. I thank Todd Clark, Mike Dotsey, Keith Kuester, André Kurmann, David López-Salido, Jeremy Nalewaik, Edward Nelson, Seth Pruitt, John Roberts, Jeremy Rudd, Mark Watson, Min Wei, Jonathan Wright, and an anonymous referee, for comments and Kurt von Tish for editing assistance. Excellent research assistance has been provided by Benjamin Brookins, Will Gamber, and Christine Garnier. A supplemental appendix is available online at http://www.mitpressjournals.org/doi/suppl/10.1162/REST_a_00549.

The model allows one to merge subjective views about trend inflation, as they are embedded in survey responses, with statistical forecasts derived from historical data. The inclusion of forward-looking variables, like survey responses and financial market data, is of course central to the task of monitoring inflation expectations. By covering a variety of surveys and other indicator series, my application also needs to handle a fair amount of missing observations arising from the infrequent publication or limited availability of some series. Bayesian state-space methods allow me to easily combine data sampled over different periods and at different frequencies.

In the spirit of Beveridge and Nelson (1981), trend inflation is defined here as a long-run forecast for inflation, specifically headline PCE inflation, which is the preferred inflation measure of the Federal Reserve.1 In my multivariate setting, a central assumption is that the trend in headline PCE inflation moves in lockstep with the long-run forecasts of other inflation measures, derived, for example, from the CPI or the GDP deflator, as well as the trend component of survey expectations. My model remains, however, agnostic about structural features of the economy; among others, it does not require surveys to be efficient and unbiased predictors of future inflation (Kozicki & Tinsley, 2012; Chernov & Mueller, 2012), nor is a particular form of information frictions imposed (Colibin & Gorodnichenko, 2012, 2015), neither are no-arbitrage restrictions or the expectations theory of the term structure of interest rates assumed to hold (Haubrich, Pennacchi, & Ritchken, 2011; Cogley, 2005; Chernov & Mueller, 2012). Instead, the common-trend framework employed here requires only that survey errors and deviations between inflation rates computed from the different price baskets are stationary. Considering the permanent component of nominal yields, I find strong, but not perfect, comovement with trend inflation, likely reflecting permanent shocks to the real rate of interest or risk premiums.

Uncertainty in the trend of inflation expectations is measured by the volatility of trend shocks, which is allowed to vary over time as in the unobserved components model with stochastic volatility (UCSV) of Stock and Watson (2007).2

1 The Monetary Policy Report submitted to the Congress by the board of governors of the Federal Reserve System regularly describes the board’s outlook for inflation in terms of PCE prices, since its construction better reflects the changing composition of spending than other measures, like the CPI. McCully, Moyer, and Stewart (2007) also provide a detailed comparison of inflation measures derived from PCE and CPI prices.

2 Following Stock and Watson (2007), the UCSV model has been applied to univariate inflation data by numerous other studies, including Stock and Watson (2010), Cecchetti et al. (2007), Grassi and Proietti (2010), Shephard (2013), Cogley et al. (2015), and Cogley and Sargent (2015). See, for example, Mueller and Watson (2013) for an alternative, nonparametric approach to measuring the long-run level of inflation. Links between inflation expectations generated from the UCSV model and survey responses have been investigated, for example, by Henzel (2013), Nason and Smith (2014), and Mertens and Nason (2015).
When the volatility of trend shocks is low, the trend behaves like a constant and we can speak of well-anchored inflation expectations. When the volatility of trend shocks is high, inflation expectations will more likely become unmoored, and trend movements will be a major source of variations in actual inflation. By tracking time variation in the uncertainty measure, the model can document whether and to what extent inflation expectations have become unanchored at times in the past, as well as provide an estimate of the current risk of changes in trend inflation. The dynamic interaction between latent factors representing the level and uncertainty of trend inflation results in a nonlinear state space representation of the model that is handled by Markov chain Monte Carlo methods (MCMC) and a particle filter.

The specific contribution of this paper is to condition trend forecasts on a broad set of indicator variables covering more than a dozen variables, including surveys and financial market data, as well as realized inflation rates, and sampled at different intervals. In a multivariate generalization of Stock and Watson’s UCSV model, my empirical model combines monthly data series with less frequently sampled variables—notably surveys, but also the inflation measure derived from the quarterly GDP deflator—and handles missing observations arising from a lack of available data for many variables during earlier parts of the sample period. In particular, I show how different conditioning sets affect the contours of trend inflation in the United States since the 1960s. Furthermore, based on a particle filter that exploits the conditional linearity of the model’s state space, the relevance of individual indicator variables can be characterized by a particle-weighted version of the Kalman gain that is otherwise known only from linear models.

Model estimates of the time-varying importance of trend changes are, of course, central to the extraction of trend estimates from the data. In the unobserved components framework espoused by both Stock and Watson (2007) and this paper, this time variation is governed by a limited number of time-varying volatility parameters, which allows me to scale up the model to a multivariate application with almost two dozen variables and missing observations. In a related but somewhat different approach, Cogley and Sargent (2005) and Cogley, Primiceri, and Sargent (2010) measure trend inflation as the time-varying mean of inflation implied by VARs with time-varying parameters; owing to the large number of latent states in such a framework, the relevance of individual indicator variables can be characterized by a particle-weighted version of the Kalman gain that is otherwise known only from linear models.

A. The Beveridge-Nelson Trend with Stochastic Volatility

An important motivation for monitoring inflation expectations is to detect shifts in people’s belief about an economy’s nominal anchor (or lack thereof). The Beveridge-Nelson trend is particularly suited for this task, since it is an expectation of future inflation conditional on a current information set. As will be seen below, different information sets—for example, depending on whether they use forward-looking survey forecasts as opposed to realized inflation rates—can and will lead to different trend estimates.

Formally, the Beveridge-Nelson trend ($\tau_t$) of inflation ($\pi_t$) is identified as a forecast of inflation at the infinite horizon conditional on an information set ($\Omega_t$) (which will be discussed further below):

$$E(\pi_{t+\infty}|\Omega_t) = \tau_t.$$  \hfill (1)

Defining the trend measure as an expectation has profound consequences for the implied dynamics of inflation itself, since differencing equation (1) yields a unit root process for the trend,

$$\tau_t = \tau_{t-1} + E(\pi_{t+\infty}|\Omega_t) - E(\pi_{t+\infty}|\Omega_{t-1}) = \tau_{t-1} + \tilde{e}_t,$$ \hfill (2)

where the trend shocks, $\tilde{e}_t$, form a martingale-difference sequence.\(^3\) Thus, unless trend shocks were always 0, the

\(^3\) Notice that trend inflation is not a forecast of average inflation between now and some long-dated maturity, but rather the forecast of inflation at a long-dated point in time (namely, the infinite horizon), which allows equation (2) to abstract from roll-over issues as time evolves, since changes in trend inflation merely reflect changes in information, not changes in the forecast’s target date, which always remains equal to the infinite horizon.
trend follows a random walk. Actual inflation is assumed to be the sum of the trend and a stationary component, $\pi_t$, and inherits the random walk in the trend:

$$\pi_t = \tau_t + \tilde{\pi}_t, \quad \tilde{\pi}_t \sim I(0) \quad E(\tilde{\pi}_t) = 0. \quad (3)$$

Adopting the terminology of Cogley et al. (2010), $\tilde{\pi}_t$ will be called the inflation gap. The inflation gap is restricted to be stationary with otherwise arbitrary serial correlations.

Taken at face value, the notion of a random walk component in the inflation process could seem troubling. For instance, a nonstationary inflation process would imply that monetary policy has failed in its task of keeping inflation rates stable. An inflation model with a Beveridge-Nelson decomposition, as in equation (3), will always assign some weight to a nonstationary component in inflation; however, as long as the weight is small, the model might as well approximate a stationary process, and vice versa (Cochrane, 1991). Ultimately, it is an empirical question whether inflation contains a nonstationary component and how large it is, and, ideally, the analysis should neither preclude the possibility of well-anchored inflation expectations nor should such risks be ruled out. To address this issue, a key ingredient in the model is the assumption of a time-varying volatility of trend shocks:

$$\tilde{\varepsilon}_t \sim N(0, \tilde{\sigma}_t^2). \quad (4)$$

This specification allows for periods when trend shocks are essentially 0 and inflation is close to a stationary process, as well as for situations when inflation expectations may become unanchored and trend shocks are sizable. Similar to Stock and Watson (2007) and others, the log variance of trend shocks is assumed to follow a driftless random walk:

$$\log \tilde{\sigma}_t^2 = \tilde{\varphi}_t - \tilde{\varphi}_{t-1} + \tilde{\varphi}_t \tilde{\varepsilon}_t, \quad \tilde{\varepsilon}_t \sim N(0, 1). \quad (5)$$

As in Stock and Watson (2007) but different from the original Beveridge-Nelson procedure, the econometrician might not observe the $\tilde{\pi}_t$, and the trend $\tau_t$ is treated as an unobserved component. The econometrician is rather assumed to observe only a potentially limited set of indicators, like realized inflation rates or survey forecasts, stacked into a vector $Y_t$. The history $Y^t = \{Y_t, Y_{t-1}, Y_{t-2}, \ldots\}$ does not likely span $\tilde{\pi}_t$, which gives rise to a distinction between filtered and smoothed estimates, $\tau_{t|t'} = E(\tau_t|Y^t)$ and $\tau_{t|T} = E(\tau_t|Y^T)$, respectively. The central focus of this paper is the role of using different measurement vectors $Y_t$ in shaping these trend estimates. In particular, I compare the effect of conditioning trend estimates solely on measures of realized inflation against estimates derived from a $Y_t$ made up of explicitly forward-looking variables—in particular survey forecasts, or both.

To illustrate the potential effect of different information sets $Y^t$, consider the example of an economy with a newly established, credible inflation target that is different from average past inflation. In this case, the trend measure will crucially depend on the information set used to generate long-term inflation forecasts. When the information set contains knowledge about the new inflation target, as might be the case with survey forecasts, the Beveridge-Nelson trend would instantaneously adjust to the new target rate. If forecasts were generated by extrapolating solely from past inflation behavior, trend estimates should converge only over time to the new target, where the rate of convergence would depend on the length of the adjustment period to the new target regime and on the weight given by the forecast to more recent inflation data. Of course, the opposite might emerge if survey responses were not to fully reflect recent data, possible due to sticky information (Mankiw & Reis, 2002; Coibion & Gorodnichenko, 2015; Mertens & Nason, 2015). By using an estimated time-series model, this paper will invariably resort to generating forecasts by extrapolating from the past. But with the help of forward-looking information variables like surveys and financial market data, the procedure should also detect shifts in the inflation outlook not necessarily captured in realized inflation data.

B. Multivariate Data, Cointegration, and the Beveridge-Nelson Trend

This section describes the various indicators and data series used in the paper and the cointegration assumption that is made to link them to the inflation trend. The econometric model used to estimate the trend is described in the next section. All individual input series are listed in table 1; they can broadly be classified into four categories:

- INF: Realized inflation rates, like those derived from the CPI or the PCE deflator
- TRM: Trimmed-mean or median measures of realized inflation
- SRV: Survey forecasts of future inflation from the Survey of Professional Forecasters (SPF) or the Blue Chip and the Livingston surveys
- YLD: Yields on nominal, longer-term Treasury securities

To highlight some of the properties of my data set, figure 1 compares time-series data for select variables with realized inflation. The figure suggests some commonality in low-frequency movements of different measures of inflation and survey forecasts. Furthermore, deviations between nominal interest rates and inflation seem to be highly persistent and possibly nonstationary. Moreover, the figure illustrates that many indicators are not available at the monthly frequency, only at quarterly or semiannual intervals. Except for the Livingston survey, most survey measures are not available before 1980, and similarly so for the TRM data.

---

4 The original procedure by Beveridge and Nelson (1981) defines the trend explicitly with respect to the econometrician’s information set at time $t$ (corresponding to $\tau_t$). See also Harvey (1989), Oh, Zivot, and Creal (2008), and Morley (2011).
Table 1.—Data Description and Availability

<table>
<thead>
<tr>
<th>Since</th>
<th>Until</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>INF: Realized Inflation Rates</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PCE deflator</td>
<td>02/1959</td>
<td>02/2015</td>
</tr>
<tr>
<td>Core PCE deflator (excluding food and energy)</td>
<td>02/1959</td>
<td>02/2015</td>
</tr>
<tr>
<td>Consumer price index</td>
<td>02/1947</td>
<td>03/2015</td>
</tr>
<tr>
<td>GDP deflator</td>
<td>Q1/1947</td>
<td>Q4/2014</td>
</tr>
<tr>
<td><strong>TRM: Trimmed and Median Inflation Rates</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trimmed-mean PCE inflation</td>
<td>01/1978</td>
<td>02/2015</td>
</tr>
<tr>
<td>Trimmed-mean CPI inflation</td>
<td>01/1983</td>
<td>03/2015</td>
</tr>
<tr>
<td>Median CPI inflation</td>
<td>01/1983</td>
<td>03/2015</td>
</tr>
<tr>
<td><strong>SRV: Survey Expectations of Inflation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPF longer-term forecasts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PCE, next ten years</td>
<td>02/2007</td>
<td>02/2015</td>
</tr>
<tr>
<td>PCE, next five years</td>
<td>02/2007</td>
<td>02/2015</td>
</tr>
<tr>
<td>CPI, next ten yearsa</td>
<td>11/1979</td>
<td>02/2015</td>
</tr>
<tr>
<td>CPI, next five years</td>
<td>08/2005</td>
<td>02/2015</td>
</tr>
<tr>
<td>Blue Chip, five-to-ten years ahead</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPI</td>
<td>03/1986</td>
<td>03/2015</td>
</tr>
<tr>
<td>GDP/GNP deflator</td>
<td>03/1986</td>
<td>03/2015</td>
</tr>
<tr>
<td>SPF, average over the next four quarters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PCE</td>
<td>02/2007</td>
<td>02/2015</td>
</tr>
<tr>
<td>CPI</td>
<td>08/1981</td>
<td>02/2015</td>
</tr>
<tr>
<td>GDP/GNP deflatorb</td>
<td>11/1968</td>
<td>02/2015</td>
</tr>
<tr>
<td>Blue Chip, four quarters ahead</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPI</td>
<td>06/1980</td>
<td>03/2015</td>
</tr>
<tr>
<td>GDP/GNP deflator</td>
<td>06/1980</td>
<td>03/2015</td>
</tr>
<tr>
<td>Livingston, 6–12 months ahead</td>
<td>06/1960</td>
<td>12/2014</td>
</tr>
<tr>
<td><strong>YLD: Nominal Interest Rates</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10-year Treasury yield</td>
<td>04/1953</td>
<td>03/2015</td>
</tr>
<tr>
<td>20-year Treasury yield</td>
<td>04/1953</td>
<td>12/1986</td>
</tr>
<tr>
<td>30-year Treasury yield</td>
<td>10/1993</td>
<td>03/2015</td>
</tr>
<tr>
<td></td>
<td>02/1977</td>
<td>02/2002</td>
</tr>
<tr>
<td></td>
<td>02/2006</td>
<td>03/2015</td>
</tr>
</tbody>
</table>

For each category, trend levels are measured for the series indicated in bold; when categories are combined with INF data, the trend level is aligned with PCE headline inflation. The model uses monthly observations from January 1960 through March 2015, as far as available by mid-April 2015. SPF denotes the Survey of Professional Forecasts. Further details on data sources and individual variables are provided in the online appendix.

All variables are annualized and expressed as continuously compounded rates of change.
a Extended series, with semiannual observations based on Blue Chip and Livingston surveys prior to 1991. See the online appendix for details.
b Only fourth-quarter observations available before 1970:Q2.

And even while observations on nominal yields are generally available at high frequencies, some longer-term instruments, like thirty-year Treasury bonds, were not traded for several years during my sample period that starts in 1960.5 (Further details about the data set are provided in the online appendix.)

The TRM measures, constructed by the Federal Reserve Banks of Cleveland and Dallas, are measures of realized inflation that have been specifically designed to provide better gauges of underlying trend or core inflation; hence they have been categorized separately. Compared to their counterparts in the INF category, the TRM measures are less sensitive to large changes in the prices of individual categories of goods and services.

While trend inflation has been defined as a forecast of PCE headline inflation at the infinite horizon, none of the forward-looking indicators in the SRV and YLD categories provide a straight reading of such a forecast. In fact, most surveys refer rather to inflation derived from the CPI or the GDP deflator, and the price index relevant to bond investors is not even obvious. Furthermore, some surveys refer to short-term expectations and some to longer-term expectations. Specifically, I consider forecast horizons ranging from four-quarter-ahead inflation to average inflation over the next ten years; given the focus on trend inflation in this paper, forecast horizons shorter than a year have been omitted. One-year forecasts are still included, since they typically provide a longer history of observations than surveys of longer-term forecasts, which are at best available only for the later half of my sample.

5 A few candidate indicators have been omitted from the data set—for example, inflation forecasts from the Michigan survey, as well as measures of inflation compensation derived from Treasury inflation-protected securities (TIPS). Data from the Michigan survey were included in an earlier working paper version of this paper, but the common trend assumption proved strenuous to maintain (Mertens, 2011, figure 3). TIPS data have been available for only a little more than a decade, a period during which the underlying securities prices were subject to persistent shifts in liquidity premiums at least twice (during their early years of trading and then again during the financial crisis of 2008 and 2009), adding considerable noise to TIPS data (see, for example, Haubrich, Pennacchi, & Ritchken, 2008, or Bauer & Rudebusch, 2015).
To link these various indicators to the trend in PCE headline inflation, I assume that (at least with the exception of nominal yields) all variables listed in table 1 share a single common trend. Specifically, the trend components of different variables might differ only by a constant term—reflecting, for example, a potential bias in survey expectations or an average spread between inflation rates derived from different price baskets—but trend changes in different variables are all caused by the same scalar trend shock. The following assumptions are needed to ensure a common trend between the variables in INF, TRM, and SRV:

**Assumption 1.** The Beveridge-Nelson decomposition (3) applies to headline PCE inflation.

**Assumption 2.** Differences between the inflation measures in INF and TRM are stationary.

**Assumption 3.** Survey errors, defined as the difference between survey respondents’ forecasts for a given inflation measure and its subsequent realization, are stationary as well.

Taken together, these three assumptions imply not only cointegration between any of the inflation measures in INF and TRM, but also that the difference between any survey forecast in SRV and the current rate of PCE headline inflation is stationary—regardless of each survey’s underlying inflation measure and forecast horizon.6

Assumption 1 is a feature of the UCSV model of Stock and Watson (2007), which has proved highly useful in characterizing U.S. inflation. Under assumption 2, changes in (relative) prices can cause permanent differences to the levels of the different price indices underlying the INF and TRM measures, without leading to permanent differences in their rates of change, which seems empirically plausible.

---

6An impediment to the application of formal cointegration tests, which could otherwise be used to ascertain the validity of these assumptions, is the regular occurrence of missing data throughout my data set, and I have chosen to directly impose the common-trend assumption.
at least for the aggregate price baskets considered here.\textsuperscript{7} As in Grant and Thomas (1999), the stationarity of survey errors (assumption 3) imposes only a weak form of rationality in survey forecasts without precluding the possibility of bias or persistence in survey errors. My model thus nests cases where survey forecasts equal rational expectations, as in Koizicki and Tinsley (2012), or where they might reflect information frictions, as studied by Coibion and Gorodnichenko (2012, 2015), Nason and Smith (2014), and Mertens and Nason (2015).

Formally, the assumptions made above imply a multivariate Beveridge-Nelson decomposition for any vector of variables, \( Y_t \), from the categories INF, TRM, and SRV:

\[
Y_t = \tau_t + \tilde{Y}_t, \tag{6}
\]

with a stationary gap vector, \( \tilde{Y}_t \sim I(0) \), and a single shock responsible for changes in trend levels:

\[
\lim_{k \to \infty} E(Y_{t+k} | \Omega_t) = \tau_t = \tau_{t-1} + 1 \tilde{\epsilon}_t \tag{7}
\]

where \( \tilde{\epsilon}_t \sim N(0, \sigma_t^2) \), as in equation (5) above. A description of the assumed process for the gap vector \( \tilde{Y}_t \) will be deferred until the next section.

Notice that the gap levels are normalized to have zero mean, \( E(\tilde{Y}_t) = 0 \), while the initial trend levels of different variables, \( \tau_0 \), may differ up to a constant.\textsuperscript{8} Differences in \( \tau_0 \) allow the model to capture, for example, differences in average inflation rates arising from differences in the composition of price baskets (McCully et al., 2007) or from survey biases, which produce forecasts that are permanently too high or too low. For comparability, I will always report the element of \( \tau_t \) that corresponds to realized PCE headline inflation, as long as \( Y_t \) contains INF variables. If \( Y_t \) is made up of the TRM variables, the trend of trimmed PCE inflation will be reported, and in the case of \( Y_t \) consisting of the survey set SRV, the trend level will be aligned with the ten-year SPF forecast for PCE inflation.\textsuperscript{9}

Nominal yields on longer-term Treasury securities should also provide useful signals about trend inflation, since—without loss of generality—nominal interest rates can be written as the sum of expected inflation, a real interest rate, and a residual that will henceforth be labeled “risk premium.” Thus, via the expected-inflation term, trend inflation should also influence nominal yields, but other factors may affect the permanent component of nominal yields as well. For example, several studies have documented permanent shocks to the real rate of interest (see, e.g., Laubach & Williams, 2003; Clark & Koizicki, 2005, or, most recently, Hamilton et al., 2015). Others have also documented a persistent decline in term premiums over the last few decades, possibly linked to changes in the variability of trend inflation (Wright, 2011). As a consequence, two alternative assumptions about the permanent component of nominal yields will be considered:

Assumption 4a. Spreads between nominal longer-term yields are stationary.

Assumption 4b. Real rates and risk premiums are stationary.

While assumption 4a imposes merely a common trend on all variables in the YLD category, assumption 4b (together assumptions with 1, 2, and 3) ensures a common trend between nominal yields and other data. Based on assumption 4a, section IV characterizes a common trend in YLD data, whereas section V contrasts estimates of a common inflation trend in inflation, surveys, and yields that embeds assumption 4b with estimates from an extended model where trend inflation is only imperfectly correlated with the trend in nominal yields.

III. Empirical Model and Methods

Given the multivariate Beveridge-Nelson decomposition in equations (6) and (7), the dynamics of the gap vector \( \tilde{Y}_t \) remain to be specified. I assume that the gap dynamics are described by a VAR with fixed transition coefficients:

\[
A(L) \tilde{Y}_t = \tilde{\epsilon}_t, \quad \tilde{\epsilon}_t \sim N(0, \tilde{\Sigma}). \tag{8}
\]

Since the gap vector is assumed to be stationary, the roots of the lag-polynomial \( A(L) \) are required to lie outside the unit circle.

The constant-parameter case in equation (8) for the gap dynamics has been chosen as a baseline specification. A clear benefit of assuming time-invariant gap dynamics is the ability to handle a larger set of variables, with mixed-frequency observations and missing data over large parts of the sample. The online appendix also provides similar results from an alternative specification, used in Garnier et al. (2015), that lets the gap shocks also be inflicted by stochastic volatility. Garnier et al. (2015) also report estimates of a similar model that modifies equation (8) to incorporate both time variation in transition coefficients \( A(L) \) and shock variances \( \tilde{\Sigma}_t \); they find only small variation in the persistence of the gap VAR when applied to quarterly data consisting of three measures of realized inflation.

The state vector of the model consists of the log variance of trend shocks \( \hat{h}_t \) (see equation [5]), as well as a vector of “linear states” \( X_t = [\tau_t \tilde{Y}_t'] \). Conditional on the evolution of the trend-shock volatility, the data is described by a linear
system in $X_t$. For the sake of exposition, assume the VAR in equation (8) has a single lag, such that $A(L) = I - AL$:

$$X_t = \begin{bmatrix} I & 0 \\ 0 & A \end{bmatrix} X_{t-1} + \begin{bmatrix} 1 - \delta_t & 0 \\ 0 & 0 \end{bmatrix} \tilde{\Sigma} \frac{1}{2} w_t,$$

$$= AX_{t-1} + B_t w_t \tag{9}$$

where $w_t \sim N(0, I)$, and $\tilde{\Sigma}^{\frac{1}{2}}$ denotes an arbitrary factorization of the variance covariance matrix of the gap residuals. Let $Y^*_t = C X_t$, with $C = [I \ I]$, represent a hypothetical measurement vector, free of missing data, that is connected to the actual data vector $Y_t$ as follows:

$$Y_{t,i} = \begin{cases} Y^*_{t,i} & \text{if available} \\ 0 & \text{otherwise}, \end{cases} \tag{10}$$

where $Y^*_{t,i}$ and $Y_{t,i}$ denote the $i$th element of $Y^*_t$ and $Y_t$, respectively. The observer equation thus becomes

$$Y_t = C_t X_t \tag{11}$$

where the rows of $C_t$ are identical to the corresponding rows of $C$ if data on $Y_{t,i}$ are available, and 0 otherwise (Harvey, 1989). The model’s state space is described by the state equations (5) and (9) and the measurement equation (11). Owing to the interaction between $X_t$ and $\delta_t = \exp(0.5 \cdot \bar{h}_t)$ in equation (9), the state space is nonlinear. But conditional on $\bar{h}_t$, equations (9) and (11) represent a linear state space, which can be exploited for the computation of model estimates.

Two estimation techniques are used. First, MCMC methods are used to jointly estimate the various model parameters—like the transition coefficients $A$ of the gap VAR—and smoothed estimates of latent variables, notably the trend levels $\tau_t$ (details are described in the online appendix). Second, filtered estimates are derived from a particle filter that generalizes the Kalman filter applicable to equations (9) and (11) to the stochastic volatility case. For these filtered estimates, the constant parameters of the model are kept fixed at their posterior means generated by the MCMC method.

Given the conditionally linear structure of the model, a variant of the particle filter known as mixture Kalman filters (Chen & Liu, 2000) can be used (see also Creal, 2012, and Herbst & Schorfheide, 2014). This particle filter generates draws for the stochastic volatility component of the state vector (aka “particles”) and then uses a Rao-Blackwellization that analytically represents the distribution of $X_t$ conditional on data and the particle draw (further details are provided in the online appendix).

In addition to the filtered trend estimates, the particle filter also generates other objects of interest. In particular, the sensitivity of trend estimates to different input variables can be approximated by a weighted average of the Kalman gains for each particle. As shown in the online appendix, for each particle draw, indexed $(i)$, trend estimates are updated with a Kalman-filtering equation of the form

$$\tau_{t|t-1}^{(i)} = \frac{K_i^{(i)} (Y_t - Y_{t|t-1}^{(i)})}{1.0} \tag{12}$$

where $K_i^{(i)}$ is the $i$th particle’s Kalman gain. Denoting particle weights by $w_t^{(i)}$, filtered trend estimates are given by

$$\tau_{t|t} = \frac{\sum_i w_t^{(i)} \tau_{t|t-1}^{(i)}}{\sum_i w_t^{(i)}} \approx \tau_{t-1|t-1} + K_i (Y_t - Y_{t|t-1}) \tag{13}$$

with $K_i = \sum_i w_t^{(i)} K_i^{(i)}$ and $Y_{t|t-1} = \sum_i w_t^{(i)} Y_{t|t-1}^{(i)}$. The quality of the approximation in equation (13) depends on $K_i Y_{t|t-1} \approx \sum_i w_t^{(i)} Y_{t|t-1}^{(i)}$. For the application considered here, trend estimates constructed from the approximation in equation (13), not shown here, turn out to be very close to those obtained from the exact filter; the particle-weighted gains $K_i$ are thus highly useful for parsing the influence of individual input variables on trend estimates.

### IV. Trends in Inflation, Surveys, and Bond Yield Data

This section compares trend inflation measures extracted from realized inflation and surveys. Specifically, different estimates of level and uncertainty in trend inflation are constructed by conditioning on the different categories of input variables listed in table 1—INF, TRM, SRV, and YLD—and combinations thereof. In each case, the model described in section III is estimated over a monthly data set ranging from 1960 to early 2015, covering several complete cyclical episodes and different regimes for the conduct of monetary policy. (The data sources are described in the online appendix.)

#### A. Smoothed Estimates

Figure 2 compares smoothed MCMC estimates of trend level and uncertainty, $E(\tau_t | Y^T)$ and $E(\delta_t | Y^T)$, conditioned on different measurement vectors $Y^T$. The INF estimates shown in panel A tell a story that is familiar from the univariate UCSV estimates of Stock and Watson (2007). The INF trend peaks twice, in the mid-1970s and in 1980, at or near double-digit levels, before falling back toward about 4% by the mid-1980s. The 1970s and early 1980s are also known as the Great Inflation period, which ended with Paul Volcker’s disinflation. Consistent with the notion that inflation expectations became unanchored during this period, uncertainty in the INF trend also rises measurably before abating from the late 1980s onward. The rise in model estimates of trend uncertainty implies that changes in inflation largely

10 Denoting the posterior mean vector of all parameters $\theta^*$, the filtered trend estimates thus correspond to $\tau_{t|t} = E(\tau_t | Y^T, \theta^*)$. As shown in the online appendix, smoothed trend estimates that condition on $\theta^*$ turn out to be very similar to the smoothed MCMC estimates that integrate out the parameter vector $\theta$.

11 The persistent rise in U.S. inflation during the first half of the 1970s might also reflect the effects of price controls; see Garnier et al. (2015) for further discussion.
reflect shocks to trend inflation. Consequently, movements in the INF trend during the Great Inflation largely track the behavior of a simple twelve-month average of headline PCE inflation, as shown in panels b to d of figure 1. After the Volcker disinflation, the INF trend mostly hovers near 4% before falling further toward 2% during the 1990s. Notably, the INF measure of trend inflation dropped significantly, as measured by the pointwise 90% credible sets, below 2% during the last recession (2007–2009) and then again as of late 2012, though without generating a noticeable uptick in the uncertainty measure. Unfortunately, this most recent shortfall of the INF trend from 2% coincides with the release of the Federal Open Market committee’s (FOMC) statement from January 2012, when the committee noted that it judges a headline PCE inflation rate of 2% to be most consistent with its mandate.

Panels a and c of figure 2 also display corresponding estimates of trend level and uncertainty obtained from conditioning solely on the three measures of trimmed and median inflation contained in the TRM category. Not surprisingly, the TRM and INF measures of trend inflation display similar low-frequency patterns. However, a few differences stand out as well. First, the TRM measures are not available before 1978 and are thus silent about trend inflation during a good part of the Great Inflation, in particular, the oil crisis in 1973. Second, similar to the INF trend, the TRM measure peaks in 1980 but at a lower level—just 8% instead of nearly 10%. Third, overall changes in the TRM trend are less volatile, which is also mirrored by a lower trajectory of trend uncertainty in TRM shown in panel c. Nevertheless, the 90% credible sets around the level estimate are tighter than in the INF case, suggesting that the underlying TRM data provide a less noisy signal about trend inflation than the INF variables. Indeed, as shown in panel b of figure 2, the time series of observations for trimmed PCE inflation is particularly smooth and bears a strong resemblance to the TRM estimate of trend inflation. As will be confirmed further below, the estimates of trend inflation tend to take a particularly strong signal from this series. Panels b and d of figure 2 also display estimates of trend level and uncertainty that condition jointly on INF and TRM data; the joint information set will be denoted INFTRM. Prior to 1978, when TRM data are not available, the estimates resemble the INF measures; they are very similar to the TRM estimates thereafter. Henceforth these combined estimates will also be referred to as the “inflation-based” measures of trend inflation.
Trend estimates based solely on survey data (SRV) are shown in panel b of figure 2. Most of the survey data are not available prior to 1980; only the semiannual Livingston and the quarterly SPF survey for inflation in the GDP deflator inform the SRV estimates for the 1960s and 1970s (see also table 1). Compared to any of the inflation-based measures discussed before, the trend measure derived from surveys rises more belatedly during the first part of the Great Inflation and falls more slowly in the wake of the Volcker disinflation. This pattern is mirrored by the uncertainty estimates shown in panel d that rise only later and less vigorously than the inflation-based measure during the Great Inflation but then decline more slowly during the 1980s.

Strikingly, the trend levels generated by SRV and INFTRM are nearly identical only during the late 1970s and early 1980s; the SRV trend lies almost uniformly below the INFTRM measure prior to 1975 and above after 1982. In particular, the spread between the SRV trend and its inflation-based counterpart rises during the inflation scares of 1983, 1987, and 1994, identified by Goodfriend (1993, 2002). The SRV trend also runs significantly above the INFTRM measure during the 1990s. Over this period, the SRV trend declines only gradually from 4% to 2%, whereas the inflation-based trend fell quite swiftly right after the recession that ended in 1982. At the time, some policymakers also proposed a strategy of “opportunistic disinflations,” which would seek to attain lower levels of inflation only once inflation was pushed lower by cyclical fluctuations, instead of generally aiming for a lower long-run level of inflation (Meyer, 1996). Arguably, the belated decline in survey expectations during the 1990s could reflect some skepticism of survey respondents about policymakers’ intentions to push inflation permanently below 4%.

Generally the differences between the survey-based trend estimates and their inflation-based counterparts echo the empirical results of Coibion and Gorodnichenko (2012, 2015), Nason and Smith (2014), and Mertens and Nason (2015), which are consistent with models of informational frictions in survey responses. Overall, the 90% credible sets around the SRV trend are considerably tighter than for the inflation-based measure, reflecting the relative smoothness of the survey series compared to observations on realized inflation. (Trend estimates that condition jointly on inflation data and surveys are quite similar to these SRV estimates and will be discussed further below in the context of filtered estimates.)

The survey variables contained in the SRV set comprise both short-term and longer-term forecast horizons, where “longer term” refers to horizons beyond one year. An important motivation for including data on short-term survey forecasts is the lack of observations of longer-term survey forecasts prior to 1979. Figure 2e displays a trend estimate derived solely from longer-term surveys. Both the trend level and uncertainty are highly similar to the SRV estimates; not surprisingly, the level estimate derived from longer-term surveys also is a bit smoother. Until the mid-1980s, longer-term survey forecasts are available only from the SPF and only once per quarter; as a result, the uncertainty bands around the trend estimates are a bit wider than in the SRV case.

Turning to bond yield data, panels c and f of figure 2 present estimates of level and uncertainty in the common trend of nominal yields on longer-term Treasury securities (YLD). While the evolution of the YLD trend over the 1960s and 1970s bears some resemblance to the trend estimates derived from inflation and survey data, stark differences emerge after the late 1970s. By the end of the Great Inflation, the YLD trend has risen more persistently than the other measures, but over the following decades, the yields-base trend has also fallen much more. Whereas the SRV and INFTRM trends declined by about 6 percentage points from 1980 to the present, the YLD trend declined by about twice as much. In particular, while inflation- and survey-based measures of trend uncertainty seem to have largely stabilized at low levels over the last two decades, this has not been the case for the volatility of trend shocks to nominal bonds. As discussed before, changes in the permanent component of nominal yields need not only reflect changes in trend inflation but could also result from variations in the trend real rate or risk premiums (Laubach & Williams, 2003; Hamilton et al., 2015; Wright, 2011). Section V considers the implications of imposing a common trend restriction on yields as well as surveys and inflation data.

B. Filtered Estimates

The remainder of this section uses the particle filter discussed in section III to generate filtered estimates of trend level and uncertainty, as well as the particle-weighted Kalman gains that approximate the sensitivity of estimated trend levels to observations for the different input variables. The particle filter generates recursive updates for estimates of $\tau_t$ and $\sigma_t$ conditional on only the contemporaneous history of observations, denoted $Y_T$. In contrast, the estimates discussed so far represent smoothed MCMC estimates, $E(\tau_t|Y^T)$ and $E(\sigma_t|Y^T)$. In addition, these MCMC estimates integrate out constant model parameters, like the coefficients of the gap VAR. For the purpose of filtering, those model parameters are kept fixed at their posterior means (obtained from the MCMC estimation). Denoting the vector of the posterior means of the parameters by $\theta^*$, the filtered estimates thus correspond to $\bar{\tau}_{ij} = E(\tau_{ij}|Y^T, \theta^*)$ and $\bar{\sigma}_{ij} = E(\sigma_{ij}|Y^T, \theta^*)$. While the filtered estimates abstract from the effects of parameter reestimation, as well as from any issues related to data revisions, their comparison with the smoothed estimates allows me to ascertain at least in part the relevance of hindsight information embedded in the estimates discussed so far. Furthermore, in order to better elucidate the role of individual input variables, it is straightforward to compute particle-weighted Kalman gains for these filtered estimates.
Panels a and b of figure 3 compare filtered and smoothed estimates for trend level and uncertainty that condition on all inflation and survey measures in the categories INF, TRM, and SRV, dubbed INFTRMSRV. For consistency, the smoothed estimates are generated with a particle smoother that conditions on the same fixed parameter vector $\theta^*$ as the filtered estimates. As shown in the online appendix, smoothed estimates conditional on $\theta^*$ turn out to be quite similar to their MCMC counterparts for all of the information sets considered here.

Not surprisingly, the smoothed INFTRMSRV estimates display salient features of the smoothed SRV and TRM estimates discussed so far: trend level and uncertainty peak during the Great Inflation, but at lower values than when conditioned solely on INF data. After the mid-1980s, the trend estimate derived from the joint data declines at a pace similar to the survey-based measure, which is slower than the inflation-based estimates. Over the past decade, the INFTRMSRV measure has hovered closely around 2%, without displaying much of the fluctuation in the INF measure during that time. Foreshadowing the discussion of Kalman gains further below, this evidence suggests that the model takes more signal from (at least some of) the variables in the SRV and TRM categories than from the INF data.

Filtered and smoothed trend levels derived from the joint data set INFTRMSRV are quite similar except for a stark difference that emerges between 1975 and 1985. Smoothed estimates for this period run up to 2 percentage points below the filtered estimates—far outside the filter’s 90% credible set. Strikingly, a similar divergence does not occur for estimates based on the individual data categories, as evidenced by a comparison of the smoothed and filtered estimates derived from INFTRM and SRV in panel b of figure 2 and panel c of figure 3, respectively. The divergence between the INFTRMSRV estimates $\tau_{t|t}$ and $\tau_{t|T}$ for the years before and after 1980 reflects the absence of data for most surveys and the trimmed-mean (and median) measures of inflation prior to 1980. As will be seen next, the increase in the availability of cross-sectional signals substantially changes the weights placed by the filter on observations from different series starting around 1980. For the combined data set, INFTRMSRV, this also leads to a substantial switching between signals received from different data categories. The sizable deviation between $\tau_{t|t}$ and $\tau_{t|T}$ in the years before and after 1980 thus largely reflects the smoother’s backcasting of missing observations for TRM and SRV data prior to 1980 (instead of the usual reevaluation of signals received from earlier observations). The filtered trend estimates may have risen more in line with the smoothed measures during the 1970s if more data on survey forecasts and trimmed-mean rates had been available during that time and if those data had evolved roughly in line with the model’s backcasts.

C. The Role of Individual Input Variables

The trend estimates shown so far draw on different information sets, containing between three and nineteen input variables. Before turning to the question of which information set gives the “best” trend estimate in section VI, this section seeks to parse out which of the individual variables considered are most influential in shaping the trend estimates by considering their particle-weighted Kalman gains defined in section III. Like slope coefficients in a multivariate regression, these gains measure the marginal effect of an individual variable amid a cross-section of signals. The gain attached to a given variable thus depends on the composition of the measurement vector; for instance, the weight attached to core PCE inflation as part of the data vector INF is likely to be different (and smaller) than in the context of conditioning on all nineteen variables in INFTRMSRV.

For the trend estimates shown so far, the entire state space, including parameters like the coefficient of the gap $\varphi$ in equation (8), has been reestimated for the different information sets (e.g., INF, TRM, SRV). For comparison, the gain coefficients shown next are all generated by filtering the state space of the joint data vector INFTRMSRV but for different subvectors of data. As shown in the online appendix, results are similar to using the information-set specific state spaces.) Panels e and f of figure 3 display the filtered estimates of trend level and uncertainty derived for the different subcategories from this setup; the estimates paint a by-now-familiar picture.

Observations for many of the variables listed in table 1 are available only for certain months of the year. The results presented are thus broken down along typical months of the calendar year. In the case of INF data, the only distinction to be made between gains in different months of the year is whether it is the last month of a quarter, when a new observation for the GDP deflator has accrued. Panel a of figure 4 displays the INF gains for every January, when readings of headline and core PCE prices as well as the CPI are available, between 1960 and 2015. Not surprisingly, during this month, the filter takes the strongest signal from the core measure of PCE inflation that strips out the generally more volatile components food and energy. Still, monthly changes in core PCE prices are relatively noisy, and for every 1 percentage point surprise in core inflation, trend estimates reacted at most by 40 basis points during the Great Inflation and by just one-tenth of a percentage point over the previous

12 As shown in the online appendix, the same holds for estimates that condition on INF and TRM data separately. Furthermore, filtered SRV estimates display particularly wide credible sets prior to 2007—the 90% credible set shown in the figure is about 2 percentage points wide—when the first data for the SPF’s ten-year forecast of PCE inflation become available. As described in section II, initial trend levels of different variables are not restricted to be identical and, prior to 2007, these uncertainty bands largely reflect the model’s prior uncertainty about the difference between the trend level for the SPF forecast of PCE inflation over the next ten years and trend levels of the other survey variables that have shaped the filtered estimates. See the online appendix for an illustration.

13 The trend and gain measures discussed in this section all pertain to headline PCE inflation.
All levels are measured in annualized percentage points; uncertainty is measured by the standard deviation of a monthly trend shock. Shaded areas and thin lines depict 90% credible sets. Thick lines denote posterior means. All results were obtained from the particle filter described in section III. Estimates in panels e and f are generated from the INFTRMSRV state space. Thin vertical lines denote peaks and troughs of NBER recessions.
Figure 4.—Kalman Gains for Inflation-Based Trends

(a) January

(b) March

(c) TRM (March)

(d) INFTRM (March)

Particle-weighted Kalman gains for given calendar months computed from the particle filter described in section III. Estimates based on monthly data from 1960:M1 to 2015:M3. Kalman gains for missing data are set to 0 and not shown.

Gains are typically larger during the Great Inflation, since trend-shock volatility was higher during that time and incoming data were likely to provide a stronger trend signal; this will be a recurring feature across all information sets considered here.) When observations on the GDP deflator are available (as in the month of March; see panel b of the figure), they attract at least as much weight as core inflation, even driving out the latter quite a bit during the Great Inflation. However, this is not necessarily due to GDP prices being less noisy than core PCE prices but rather is a reflection of the fact that GDP data are collected as a three-month average and are thus inherently less noisy than monthly data.

14 Reflecting the vague prior on the initial trend level (as specified in the online appendix) the gains on the first data point, here in January 1960, are particularly large.

The trimmed-mean and median rates of inflation that are grouped together in the TRM category are all available at the monthly frequency, but not much before the early 1980s; results are thus shown for a single calendar month. The TRM trend seems largely influenced by data for the trimmed-mean rate of PCE inflation, not the data on trimmed-mean or median CPI inflation, which is much noisier (see panel b of figure 1). This conjecture is confirmed by figure 4c, where the gain of the trimmed-mean PCE inflation rate clearly dominates; the appropriate choice of trimming criterion is evidently nontrivial. Moreover, in comparison to the INF results, the gain on trimmed-mean PCE inflation is typically twice as large as the INF gain on core PCE. Panel d of the figure displays the gains obtained when all inflation measures are combined in the INFTRM category. The resulting pictures combine the previous results: core
PCE inflation and the GDP deflator (when available) are highly relevant, but only for the 1960s and 1970s when no TRM data are available; afterward the trend estimates are largely determined by trimmed-mean PCE inflation. To some extent, the outstanding role played by this particular trimmed-mean series is no wonder, since, as Dolmas (2005) described, the trimming procedure used by the Federal Reserve Bank of Dallas for the construction of the trimmed-mean PCE inflation rate has been specifically designed to approximate a (bandpass-filtered) trend measure akin to the Beveridge-Nelson trend used here.

The availability of survey data is most dispersed across different months of the year. Only the short-term Blue Chip surveys are published every month (since 1980), whereas SPF forecasts are compiled during the second month of every quarter. The Livingston survey and longer-term Blue Chip forecast are published only semiannually and in different months, but none of them is published in the same month as the SPF. All told, there are four typical months in a given calendar year. Neglecting cases like the month of January, when only short-term Blue Chip forecasts are available, the top panels of figure 5 display monthly survey gains for three typical calendar months. Overall, the filter takes strong signals about trend inflation from the combined survey data. Only the Livingston survey and the SPF for the GDP deflator are available in certain months prior to 1980; when they are, both attract sizable gains—comparable to the combined gains on core inflation and the GDP deflator for the INF trend. But after 1980, gains are spread out quite widely across different surveys. If anything, the Livingston survey receives only a marginal gain when combined with other surveys as of 1980 (see, for example, panel b). Remarkably, the gains on longer-term survey forecasts do not seem to dominate those on short-term forecasts. If anything, when longer-term Blue Chip forecasts are available alongside the monthly short-term forecasts—see panel d for the month of March—the filter even places a little less weight on the longer-term forecasts compared to the short-term surveys. This result stands in considerable contrast to the approach of Kozicki and Tinsley (2012) or Mertens and Nason (2015), who explicitly link survey forecasts to a model-implied inflation forecast, causing their filters to place higher-trend gains on longer-horizon forecasts. In contrast, while my model remains agnostic about the link between survey forecasts and realized inflation at any particular horizon, apart from imposing cointegration, the data do not lead the model to favor the trend signals received from survey forecasts with particular horizons.
Finally, the bottom panels of figure 5 display the corresponding gains for the combined data set (INFTRMSRV). The results from this horse race across different data categories largely reflect the earlier results. First, the filter places strong weights on the combined survey data relative to realized inflation measures while not consistently favoring any particular source when multiple survey indicators are available. Second, even next to multiple surveys, the trimmed-mean PCE inflation rate receives a strong weight from the filter.

V. Bond Yields and Trend Inflation

This section considers trend estimates conditioned on bond yields as well as surveys and inflation data, that is, on all variables listed in table 1. Panels a and b of figure 6 display estimates of trend level and uncertainty, which embed the assumption that all variables share the same common trend. As before, this requires assumptions 1, 2, and 3 to hold. But, reflecting the inclusion of the data on longer-term yields, assumption 4b is now required as well. The estimates, labeled “INFTRMSRV-YLD” in the figure, are very similar to those obtained from conditioning on surveys and inflation alone (“INFTRMSRV” in figure 3).

However, these common-trend estimates assume that real rates and risk premiums are stationary, contrary, for example, to evidence presented by Laubach and Williams (2003), Clark and Koizicki (2005), and Hamilton et al. (2015). Panel c depicts the same common trend as panel c, but now aligned to match the level of ten-year nominal yields, and with a credible set that also reflects uncertainty about the initial trend level for the ten-year rate. Deviations between data for the ten-year yield and this trend estimate are highly persistent, as yields run well above trend for about two decades from 1980 onward and then well below trend since the onset of the last recession in 2007. The high persistence in the yield gap is also reflected in substantially wider uncertainty about the location of the trend level for the ten-year rate. Permanent variations in real rates or risk premiums are likely small relative to their overall volatility, particularly at the monthly frequency. In this case, unit root tests are fraught with the so-called pileup problem (Stock & Watson, 1998), and their formal rejection of a permanent component in deviations between nominal yields and inflation data may need to be treated with caution.16

As a robustness check, an extended model is considered that maintains the assumption of a single common inflation trend representing the permanent component of inflation and survey data, as well as a further trend, common to nominal yields, that is correlated with the inflation trend. Specifically, denoting nominal yields by \( \tau_i \), the yield trend is specified as the sum of trend inflation (\( \tau_r \)) and a yield-specific martingale (\( \tau_r^* \)): \( \tau_i = \tau_r + \tau_r^* \). This trend component is assumed to be independent of shocks to trend inflation. By construction, \( \tau_r^* \) follows a random walk; analogous to the specification of trend inflation in equations (4) and (5), shocks to \( \tau_r^* \) are scaled by an independent stochastic volatility process.17

Like the other results shown in figure 6, the estimates are conditioned on all variables listed in table 1.

The extended model generates an inflation trend close to the common-trend version (panel a), while \( \tau_r^* \) soaks up much of the deviation between trend inflation and the yields data. This separate component in the yield trend captures the stylized facts that relative to inflation, nominal interest rates have been, loosely speaking, “too low” during the 1970s and “very high” during the 1980s (Taylor, 1999). Variations in \( \tau_r^* \) need not only reflect changes in long-term real rates, but also permanent changes in risk premiums affecting nominal longer-term bond yields (Wright, 2011). Estimates of \( \tau_r^* \) have persistently declined, by about 4 percentage points since 1985, and half of that since the onset of the last recession in 2007. As a by-product of the particle filtering algorithm described in the online appendix, LPSs can be derived to compare the common-trend model against the extended alternative; similar to univariate unit root tests, the LPS favor the common trend model.18 Nevertheless, the estimates of \( \tau_r^* \) document the existence of a highly persistent, common factor in bond yields that is separate from trend inflation.

VI. Trends as Forecasts

Clearly, there is a lot of commonality in the various trend estimates—except for the YLD trend in figure 1d—consistent with the notion that all are estimating the same common trend in the data sets INF, TRM, and SRV, as implied by assumptions 1 to 3. Differences in the various estimates, like the belated decline in the SRV trend after the Great Inflation, seem to reflect salient, but ultimately transitory, deviations inherent in the various data categories. While these differences may be useful to know about as stylized facts, they also beg the question: Which trend estimate is “best”? Unfortunately, the true trend is defined as an unobservable infinite-horizon forecast whose performance is hard to judge directly. Moreover, due to the nonoverlapping

15 All estimates shown in this section are based on the smoothed posterior distribution from the MCMC sampler; qualitatively similar results hold for filtered estimates as well.

16 Applied to deviations between the various nominal yields and realized inflation measures, for which continuous monthly data are available in my data set, standard unit root tests are not indicative of a permanent component in nominal yields different from trend inflation: a unit root in these data set, standard unit root tests are not indicative of a permanent component in deviations between nominal yields and inflation data may need to be treated with caution.16

17 The prior mean for the initial level of volatility in \( \tau_r^* \) has been set to 0.1, which is also at the order of what is suggested by the median-unbiased estimator of Stock and Watson (1998).

18 LPS is simply the sum of the particle-weighted contributions of every observation to the model’s log-likelihood. The LPS of the common trend model is \(-1.614.01\), which is a bit higher (better) than the LPS for the extended model at \(-1665.40\).
Figure 6.—Common Trend Estimates That Include Nominal Yields

All levels are measured in annualized percentage points; uncertainty is measured by the standard deviation of a monthly trend shock. Shaded areas and thin lines depict 90% credible sets. Thick lines denote posterior means. All results were obtained from the MCMC Gibbs sampler described in section III. Thin vertical lines denote peaks and troughs of NBER recessions.
data sets used in the estimation, likelihood measures are not meaningful to compare.

From the perspective of an applied econometrician, measures of trend inflation should be useful for forecasting future inflation. According to Faust and Wright (2013), centering forecasts around a measure of trend inflation might even be the most important ingredient in generating good predictions of future inflation rates. This section investigates the ability of the trend estimates to forecast average PCE headline inflation over different horizons, ranging from the next month to the next five years, $h = \{1, 12, 24, 36, 48, 60\}$. Forecast performance is measured by root-mean-squared errors

$$\text{RMSE}(h) = \sqrt{\frac{1}{T-h} \sum_{t=1}^{T-h} (\pi_{t+h} - \hat{\pi}_t)^2}$$

which are computed for the smoothed MCMC estimates, $\hat{\pi}_t = E(\tau_t | Y^T)$, as well as the particle-filtered trend estimates, $\hat{\pi}_t = E(\tau_t | Y^t, \theta^*)$; the significance of differential forecast performance is assessed with the test of Diebold and Mariano (1995).\(^{19}\) The results are shown in table 2.

![Table 2](http://direct.mit.edu/rest/article-pdf/98/5/950/1918361/rest_a_00549.pdf)

<table>
<thead>
<tr>
<th>Horizon (in months)</th>
<th>1</th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>48</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Smoothed Trend Estimates</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RMSE for INF</td>
<td>1.90</td>
<td>1.12</td>
<td>1.28</td>
<td>1.36</td>
<td>1.41</td>
<td>1.45</td>
</tr>
<tr>
<td>RMSE rel. to INF</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TRM</td>
<td>1.09***</td>
<td>1.17*</td>
<td>1.05</td>
<td>0.93</td>
<td>0.85</td>
<td>0.81</td>
</tr>
<tr>
<td>INFTRM</td>
<td>1.05***</td>
<td>1.00</td>
<td>0.96*</td>
<td>0.93**</td>
<td>0.92**</td>
<td>0.91**</td>
</tr>
<tr>
<td>SRV</td>
<td>1.17***</td>
<td>1.24**</td>
<td>1.10</td>
<td>1.06</td>
<td>1.07</td>
<td>1.09</td>
</tr>
<tr>
<td>SRV-LONG</td>
<td>1.13***</td>
<td>1.29**</td>
<td>1.10</td>
<td>0.98</td>
<td>0.92</td>
<td>0.90</td>
</tr>
<tr>
<td>INFTRMSRV</td>
<td>1.11***</td>
<td>1.24**</td>
<td>1.10</td>
<td>1.06</td>
<td>1.06</td>
<td>1.08</td>
</tr>
<tr>
<td>INFTRMSRV-YLD</td>
<td>1.16***</td>
<td>1.23**</td>
<td>1.09</td>
<td>1.05</td>
<td>1.05</td>
<td>1.08</td>
</tr>
</tbody>
</table>

**B. Filtered Trend Estimates**

| RMSE for INF        | 1.94 | 1.28 | 1.43 | 1.50 | 1.54 | 1.58 |
| RMSE rel. to INF    |      |      |      |      |      |      |
| TRM                 | 1.09*** | 1.08 | 0.99 | 0.90 | 0.84* | 0.81* |
| INFTRM              | 1.08*** | 1.07** | 1.03 | 1.01 | 1.00 | 0.99 |
| SRV                 | 1.18*** | 1.19** | 1.09 | 1.07 | 1.08 | 1.10 |
| SRV-LONG            | 1.15*** | 1.21** | 1.08 | 1.01 | 0.97 | 0.95 |
| INFTRMSRV           | 1.16*** | 1.20** | 1.11 | 1.09 | 1.10 | 1.12 |
| INFTRMSRV-YLD      | 1.17*** | 1.20** | 1.11 | 1.08 | 1.09 | 1.11 |

RMSE obtained from using different trend measures as forecasts of average PCE headline inflation over different horizons:

$$\text{RMSE}(h) = \sqrt{\frac{1}{T-h} \sum_{t=1}^{T-h} (\pi_{t+h} - \tilde{\pi}_t)^2}$$

with $\tilde{\pi}_{t+h} = \frac{1}{h} \sum_{j=0}^{h-1} \pi_{t+j}$.

Statistically significant differences in squared forecast errors—as computed from the test by Diebold and Mariano (1995)—at *10%, **5%, ***1%. Trend estimates derived from TRM, SRV-LONG, and SRV are available only starting from 1978:M1, 1979:M11, and 1960:M6, respectively. Relative RMSEs for these trend measures are computed based on the RMSE for the INF trend obtained from those subsamples. RMSEs shown in A, have been computed from smoothed MCMC estimates. B reflects filtered trend estimates computed from a particle filter that conditions on the posterior mean values of model parameters obtained from the MCMC sampler.

As seen in figure 1a, monthly changes in inflation are particularly volatile, and the one-month-ahead RMSE level for the INF trend amounts to almost 2 percentage points at an annualized rate. However, most practitioners are likely more interested in forecasting price changes over longer horizons that are less affected by high-frequency noise. The longer-horizon RMSE reported in table 2 is indeed lower—at levels comparable to levels known from other studies (Faust & Wright, 2013; Garnier et al., 2015)—but since they are measured over the entire period from 1960 to 2015, they are also affected by sustained misses that occurred in the run-up and aftermath of the Great Inflation.

When it comes to forecasting inflation beyond the next year, the results favor somewhat the TRM or INFTRM trends, but barely any of the other trend measures generate RMSE that are significantly larger than the INF trend. Reflecting the persistent deviations between SRV and INF trends discussed above, the SRV trend has been the worst forecaster of realized inflation over the last five decades. A survey-based trend constructed solely from longer-term surveys would have done a bit better than the INF trend at longer forecast horizons, in part since this trend did not rise quite as strongly toward the end of the Great Inflation; however, the evaluation period for this trend measure also excludes the run-up to the Great Inflation due to a lack of survey data. The results are broadly similar when considering either smoothed or filtered estimates.

In sum, inflation forecasts centered around inflation-based trends, at best including the trimmed-mean PCE measures driving the TRM trend, did best in the past. But a
survey-based trend did not perform significantly worse, even though the apparent sluggishness of the SRV data in catching up with the Great Inflation and the Volcker disinflation did not help. The somewhat worse forecasting performance of the SRV trend is also consistent with forecast errors arising from information frictions embedded in the formation of survey forecasts, as suggested by Coibion and Gorodnichenko (2012, 2015) and Mertens and Nason (2015).

VII. Conclusion

This paper presents estimates of the level and uncertainty of trend inflation, extracted from realized inflation rates, survey expectations, and the term structure of interest rates since 1960. By imposing a common trend assumption within or even across the different data categories considered, monthly estimates of trend inflation are extracted from a variety of indicator variables, many of which are either less frequently observed or use data that are available only over a limited time span. The common trend assumption is consistent with different notions of the expectations formation process embedded in surveys and nominal interest rates. Indeed, the survey-based trend estimates move more sluggishly before and after the Great Inflation, consistent with the informational frictions studied by Coibion and Gorodnichenko (2012, 2015). Notably, the common trend in nominal, longer-term interest rates deviates quite persistently from inflation- or survey-based estimates. While the notion of a single common trend in inflation, surveys, and nominal yields cannot be rejected, the evidence documents the existence of a highly persistent, yield-specific factor that is independent of trend inflation.

Considering the role of different data sources in shaping the trend estimates, the model takes a strong signal from the cross-section of survey inputs, but without necessarily placing particularly large weight on any particular survey forecasts. Among different measures of realized inflation, the trimmed-mean rate of PCE inflation stands out as a particularly strong signal of trend inflation. The high weight placed by the trend filter on trimmed-mean PCE inflation indicates the usefulness of exploiting information embedded in disaggregated price data for measuring trend inflation, as in Stock and Watson (2015).

REFERENCES


Laubach, Thomas, and John C. Williams, “Measuring the Natural Rate of Interest,” this review 85 (2003), 1063–1070.


——— “Modeling Inflation after the Crisis,” NBER working papers 16488 (2010).

