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The Handbook of Rationality

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4.4 Probability Logic

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Summary

This chapter presents probability logic as a rationality framework for human reasoning under uncertainty. Selected formal-normative aspects of probability logic are discussed in the light of experimental evidence. Specifically, probability logic is characterized as a generalization of bivalent truth-functional propositional logic ("classical logic," for short), as being connexive, and as being nonmonotonic. The chapter discusses selected argument forms and associated uncertainty propagation rules. Throughout the chapter, the descriptive validity of probability logic is compared to classical logic, which was used as the gold standard of reference for assessing the rationality of human deductive reasoning in the 20th century.

1. Probability Logic Is a Generalization of Classical Logic

Probability logic as a rationality framework combines probabilistic reasoning with logical rule-based reasoning and studies formal properties of uncertain argument forms.

Among various approaches to probability logic (for overviews, see, e.g., Adams, 1975, 1998; Coletti & Scozzafava, 2002; Demey, Kooi, & Sack, 2019; Haenni, Romeijn, Wheeler, & Williamson, 2011; Hailperin, 1996), this chapter reviews selected formal-normative aspects of probability logic in the light of experimental evidence. The focus is on probability logic as a generalization of the classical propositional calculus ("classical logic," for short; for probabilistic generalizations of quantified statements, see, e.g., Hailperin, 2011; Pfeifer & Sanfilippo, 2017, 2019). The generalization is obtained (i) by the use of probability functions and (ii) by the introduction of the conditional event as a logical object, which is not expressible within classical logic. This generalization is currently most frequently investigated from a psychological point of view (see, e.g., Elqayam, Bonnefon, & Over, 2016; Oaksford & Chater, 2010; Pfeifer & Kleiter, 2009), and it is thus most suitable for discussing both empirical and normative aspects of probability logic as a rationality framework. The empirical focus is on investigating general patterns of human reasoning under uncertainty and not on data modeling.

Classical logic is bivalent as it deals with *true* and *false* as the two truth-values, which are assigned to propositional variables (denoted by uppercase letters *in italics*; for classical logic, see also chapter 3.1. by Steinberger, this handbook). Truth tables can be used to define logical connectives like *conjunction* ("A and B," denoted by "$A \,\&\, B$"), *disjunction* ("A or B," denoted by "$A \vee B$"), *negation* ("not-A," denoted by "$\sim A$"), or the *material conditional*, which is the disjunction $\sim A \vee B$. Probability logic generalizes classical logic by using the whole real-valued unit interval from 0 to 1 instead of just two truth-values: truth-value functions are generalized by probability functions. While classical logic is truth-functional, probability logic is only partially truth-functional as, usually, in the conclusion, even from point-valued premise probabilities, a probability interval is obtained. Probability functions can be used to formalize a real or an ideal agent's degrees of belief in propositions formed by logical connectives (e.g., in $A \,\&\, B$, $A \vee B$, or $\sim A$). Moreover, conditional probability functions can measure the degree of belief in a *conditional event*, that is, $p(B\,|\,A)$. The conditional event $B\,|\,A$ is a three-valued logical entity that is *true* if $A \,\&\, B$ is true, *false* if $A \,\&\, \sim B$ is true, and *void* (or undetermined) if $\sim A$ is true.[1] Since the conditional event cannot be expressed by a two-valued proposition, it is by definition not propositional and constitutes a further generalization of classical logic. Using the betting interpretation of probability, "true" means that the bet is won, "false" means that the bet is lost, and "void" means that the bet is called off (i.e., you get your money back).

Note that $p(A) = 0$ does not imply that A is logically impossible (i.e., a logical contradiction $\bot$). However, $p(\bot)$ is necessarily equal to zero. As it does not make sense to

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add $\perp$ to your stock of beliefs (or to bet on a conditional whose antecedent is $\perp$), it is obvious why "$A \mid \perp$" is undefined in the coherence approach.[2] The semantics of the conditional event matches the participants' responses in the truth table task (see, e.g., Wason & Johnson-Laird, 1972; see also chapter 4.6 by Oberauer & Pessach, this handbook): most people judge that (i) $A \& B$ confirms the conditional "If A, then B," (ii) $A \& {\sim}B$ disconfirms it, and (iii) ${\sim}A$ is irrelevant for "If A, then B." Under the material-conditional interpretation, one would expect that rational people judge that ${\sim}A$ confirms the conditional. As this expectation was violated, the response pattern (i)–(iii) was pejoratively called the "defective truth table." Within the rationality framework of probability logic, however, this response is perfectly rational, as it matches the semantics of the conditional event (see, e.g., Kleiter, Fugard, & Pfeifer, 2018; Over & Baratgin, 2017; Pfeifer & Tulkki, 2017b; see also chapter 6.2 by Over & Cruz, in this handbook).

2. Conditional Probability, Zero-Probability Antecedents, and Paradoxes

Traditionally, the conditional probability of B given A is defined by

(1) $p(B \mid A) =_{\text{def.}} p(A \& B)/p(A)$, if $p(A) > 0$.

Condition $p(A) > 0$ here serves to avoid fractions over zero. But what if $p(A) = 0$? Then the conditional probability is undefined or default assumptions about $p(B \mid A)$ are made. For example, some approaches suggest, by default, to equate $p(B \mid A)$ with 1 in this case (e.g., Adams, 1998, p. 57, footnote 5). However, this leads to wrong results, since then also $p({\sim}B \mid A) = 1$, which violates the basic probabilistic principle $p(B \mid A) + p({\sim}B \mid A) = 1$ (for a discussion, see Gilio, 2002). Moreover, from a practical point of view, if $p(B \mid A)$ is left undefined, counterintuitive consequences may follow. Consider, for example, the following "paradox of the material conditional":

(2) B; therefore, *if A, then B*.

Argument (2) consists of a premise B and a conditional as the conclusion. Under the material-conditional interpretation, (2) is *logically valid* (i.e., there is no model in which the premise set is true while the conclusion is false). However, natural-language instantiations can appear counterintuitive (substitute, for example, "The moon is made of green cheese" for A and "The sun will shine in Vienna" for B). This mismatch between the logical validity of (2) and its counterintuitive instantiations constitutes the paradox. From a logical point of view, a logically valid argument remains logically valid whatever the instantiations are. If (2) is formalized in probability logic, however, the paradox is blocked when the conditional is represented by a conditional probability; then the corresponding argument is probabilistically noninformative, that is, for all probability values x:

(3) $p(B) = x$; therefore, $0 \leq p(B \mid A) \leq 1$ is coherent for all probability values x.

An argument is *probabilistically noninformative* if the premise probabilities do not constrain the probability of the conclusion. More technically, probabilistic noninformativeness means that for all coherent probability assessments of the premises, the tightest coherent probability bounds on the conclusion coincide with the unit interval $[0, 1]$.

What does "coherent" mean here? An assessment p on an arbitrary family $\mathbf{K}$ of conditional events is *coherent* if and only if, for any combination of bets on a finite subset of conditional events in $\mathbf{K}$, it cannot happen that the values of the random gain,[3] when at least one bet is not called off, are all positive or all negative ("no Dutch book"; see also chapter 4.1 by Hájek & Staffel, this handbook). In the coherence-based approach, the avoidance of Dutch books is in the case of unconditional events equivalent to the solvability of a specific system of linear equations. This solvability reflects the existence of at least one probability distribution on a suitable partition of the constituents (i.e., the possible cases), which is compatible with the initial probability assessment. In geometrical terms, a probability assessment on n events can be represented by a prevision point $\mathbf{p}$ (i.e., a vector in $[0, 1]^n$) and the set of constituents by a set Q of binary points (i.e., of vectors in $\{0, 1\}^n$). Then, $\mathbf{p}$ is coherent if and only if $\mathbf{p}$ belongs to the convex hull of Q (de Finetti, 1974). In the case of conditional events, coherence amounts to the solvability of a suitable finite sequence of systems of linear equations (see, e.g., Biazzo & Gilio, 2000; Coletti & Scozzafava, 2002).

If the conditional in (2) is represented by the probability of the material conditional, the paradox is inherited. For all probability values x:

(4) $p(B) = x$; therefore, $x \leq p({\sim}A \vee B) \leq 1$ is coherent.

Probability values strictly less than x in the conclusion of (4) are of course not coherent. Note that in general, (3) is probabilistically noninformative for all positive premise probabilities (i.e., $p(B) > 0$). For the extreme case $p(B) = 1$, when conditional probabilities are defined by (1), then $p(B \mid A) = 1$ or is undefined. This is obvious, since if $p(B) = 1$, then $p(A \& B) = p(A)$; therefore, by (1),

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$p(B|A) = p(A \& B)/p(A) = p(A)/p(A) = 1$, provided $p(A) > 0$. If $p(A) = 0$, then $p(B|A)$ is undefined. This result is counterintuitive and does not match the experimental data: people interpret (3) as probabilistically noninformative, even in the case of $p(B) = 1$ (Pfeifer & Kleiter, 2011). However, in *coherence-based probability logic* (see, e.g., Coletti & Scozzafava, 2002; Gilio, Pfeifer, & Sanfilippo, 2016), where $p(B|A)$ is primitive and problems with zero-probability antecedents are avoided, $0 \leq p(B|A) \leq 1$ is coherent even in the extreme case $p(B) = 1$ (for a detailed proof, see Pfeifer, 2014). This example shows that the evaluation of the rationality of a probabilistic inference depends on whether the underlying probability concept allows for dealing with zero-probability antecedents or not. In the framework of coherence, the probabilistic noninformativeness of argument (3) holds for all probability values of the premises; however, for approaches that are based on (1), it holds only for positive probabilities.

3. From the Truth Table Task to the Probabilistic Truth Table Task

In the 20th century, the dominating rationality framework in the psychology of deductive reasoning was logic. Prominent examples are Braine and O'Brien's "mental logic" (1998; see also chapter 3.2 by O'Brien, this handbook), the "mental rule theory" by Rips (1994), and Johnson-Laird's (1983) theory of mental models (see chapter 2.3 by Johnson-Laird, this handbook). The rationality framework of the former two theories is derived from classical logical proof theory (which is syntactic and "rule-based"), whereas the latter one is based on logical model theory (which is semantic and "model-based"). According to these logic-based rationality frameworks, people are rational if they use logically valid rules of inference (like modus ponens) or if they build mental models that are inspired by truth tables. With the advent of the "new paradigm psychology of reasoning," which is characterized by using probabilistic rationality frameworks instead of logic, not only did the evaluation of the rationality of human inference change but also the task paradigms were adapted. The abovementioned truth table task, for example, became a *probabilistic* truth table task (PTTT, for short) to investigate how people interpret conditionals (Evans, Handley, & Over, 2003; Oberauer & Wilhelm, 2003; see also chapter 4.6 by Oberauer & Pessach, this handbook). From a probability-logical point of view, the PTTT presented the following premises to the participants:

(5) $p(A \& B) = x_1$, $p(A \& {\sim}B) = x_2$, $p({\sim}A \& B) = x_3$, and $p({\sim}A \& {\sim}B) = x_4$.

Then the participants were asked to infer their degree of belief in the conditional "If *A*, then *B*" based on the probabilistic information given in (5). The main experimental result obtained from this task was that most participants gave as their response values consistent with $x_1/(x_1 + x_2)$, which corresponds to the conditional-probability interpretation of the conditional ($p(B|A)$; this is consistent with the "Ramsey test" as described in chapter 6.2 by Over & Cruz, this handbook). A significant minority responded with x_1, which corresponds to the conjunction interpretation of the conditional ($p(A \& B)$). Under the material-conditional interpretation, one would expect $x_1 + x_3 + x_4$ as the most frequent response type in this task. However, experimental evidence for this hypothesis was negligible. When the task was given several times to the same participants, among those who did not use conditional-probability responses in the first PTTT tasks, a "shift of interpretation" was observed: over the course of the experiment, these participants "shifted" to the conditional-probability interpretation. In the last tasks of the experiment, about 80% of the responses were consistent with the conditional-probability interpretation. This indicates that conditional probability is a key building block for a rationality framework for human reasoning about conditionals under uncertainty (see Fugard, Pfeifer, Mayerhofer, & Kleiter, 2011; Pfeifer, 2013).

In the PTTT, mostly indicative conditionals ("if–then" formulations) with "abstract" materials were used (like "If the figure shows a *square*, then the figure is *red*"). Interestingly, the finding that conditional probability is the best predictor for the data was also replicated for a larger variety of conditionals: causal conditionals ("If *cause*, then *effect*"), counterfactual conditionals ("If *A were the case*, then *B would be the case*"; Over, Hadjichristidis, Evans, Handley, & Sloman, 2007; Pfeifer & Stöckle-Schobel, 2015), and abductive conditionals ("If *effect*, then *cause*"; Pfeifer & Tulkki, 2017a).

The next sections explain why probability logic validates basic connexive principles and why it is nonmonotonic under the conditional-probability interpretation of the conditional.

4. Probability Logic Is Connexive

Connexive logics are motivated by the idea that there should be some connection between antecedents and consequents of conditionals in the sense that they should not contradict each other. Connexive logics are alternatives to (classical) logic as they are neither contained in, nor proper extensions of, it (for an overview,

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see Wansing, 2020). They include, for example, Aristotle's theses:

(AT1) $\sim$ (if $\sim A$, then A),

and

(AT2) $\sim$ (if A, then $\sim A$).

Under the material-conditional interpretation, (AT1) and (AT2) are contingent in logic (i.e., (AT1) is logically equivalent to $\sim A$ and (AT2) is logically equivalent to A). Thus, within logic, it is rational to say that it depends on the truth-value of A whether (AT1) and (AT2) are true. Indeed, (AT1) and (AT2) are not tautologies in logic. Experimental data suggest, however, that people believe that (AT1) and (AT2) must be true (Pfeifer, 2012). Probability logic allows for validating the rationality of (AT1) and (AT2). First, look at the conditionals (in terms of conditional probabilities): by coherence, $p(A \mid \sim A)$ and $p(\sim A \mid A)$ must be equal to zero. Second, the corresponding conditionals are negated by negating their consequents, that is, $p(\sim A \mid \sim A)$ and $p(\sim\sim A \mid A)$ (= $p(A \mid A)$), respectively. Since probability 1 is the only coherent assessment for $p(\sim A \mid \sim A)$ and $p(A \mid A)$, (AT1) and (AT2) are validated. This matches the experimental data (Pfeifer, 2012; Pfeifer & Tulkki, 2017b).

Boethius's theses are another instance of connexive principles. Like Aristotle's theses, Boethius's theses can be justified within probability logic (but not within classical logic). Boethius's theses are (the arrow denotes a conditional):

(BT1) $(A \rightarrow B) \rightarrow \sim(A \rightarrow \sim B)$

and

(BT2) $(A \rightarrow \sim B) \rightarrow \sim(A \rightarrow B)$.

Under the narrow-scope negation interpretation of negating conditionals,[4] the antecedent of (BT1) is interpreted in probability logic by $p(B \mid A) > x$ (for some threshold $x > .5$) and its consequent by $p(\sim\sim B \mid A) > x$. Since B is logically equivalent to $\sim\sim B$, (BT1) holds in probability logic. Analogously, (BT2) is validated in probability logic. Under the material-conditional interpretation, neither (BT1) nor (BT2) hold in general: (BT1) and (BT2) are both logically equivalent to A. Moreover, Abelard's first principle, which is another connexive principle, can be rationally justified in probability logic:

(AFP) $\sim((A \rightarrow B) \,\&\, (A \rightarrow \sim B))$.

Since, in general, $p(B \mid A) + p(\sim B \mid A) = 1$, it cannot be the case that both $p(B \mid A)$ and $p(\sim B \mid A)$ are "high" (i.e., at least strictly greater than .5). Therefore, (AFP) is validated in probability logic. However, under the material-conditional interpretation, (AFP) is logically equivalent to A. Aristotle's and Boethius's theses and Abelard's first principle are intuitively plausible principles that hold in connexive and in probability logic but not in classical logic. Aristotle's theses have received strong experimental support (Pfeifer, 2012; Pfeifer & Tulkki, 2017b). For the other connexive principles, future empirical research is needed.

5. Probability Logic Is Nonmonotonic

Nonmonotonic reasoning is about retracting conclusions in the light of new evidence. For example, from "If this animal is a bird (B), then this animal can fly (F)," one would not want to conclude "If this animal is a bird and a penguin ($B \,\&\, P$), then this animal can fly (F)." Classical logic, however, is monotonic: adding premises to a logically valid argument can only lead to an increase but never to a decrease of the conclusion set (for an overview, see, e.g., Antoniou, 1997; Strasser & Antonelli, 2019). Therefore, conclusions cannot be retracted in classical logic. Under the material-conditional interpretation, the abovementioned argument is logically valid. The argument form is called "monotonicity" (or "premise strengthening"):

(MON) $\sim B \vee F$ logically implies $\sim(B \,\&\, P) \vee F$.

In probability logic, however, the corresponding argument form is probabilistically noninformative, and monotonicity is therefore blocked, that is, for all probability values x:

(6) $p(F \mid B) = x$; therefore, $0 \leq p(F \mid (B \,\&\, P)) \leq 1$ is coherent.

Basic rationality principles for retracting conclusions in the light of new evidence are concentrated around System P (Kraus, Lehmann, & Magidor, 1990). The principles of System P are broadly considered to be minimal rationality requirements for any system of nonmonotonic reasoning. It is therefore a key system for reasoning in general. Various different semantics were developed for nonmonotonic reasoning systems, some of which are probability-logical (see, e.g., Adams, 1975; Gilio, 2002; Goldszmidt & Pearl, 1996; Hawthorne & Makinson, 2007; Schurz, 1997). Psychologically, Pfeifer and Kleiter (2005, 2006b, 2010) and Pfeifer and Tulkki (2017b) present experimental data supporting the descriptive validity of the coherence-based probability semantics of System P (Gilio, 2002; for experimental studies on the possibilistic semantics of System P, see, e.g., Benferhat, Bonnefon,

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& Da Silva Neves, 2005; Da Silva Neves, Bonnefon, & Raufaste, 2002). In the coherence semantics, default conditionals like "If A, then *normally* B"[5] are interpreted as "high" coherent conditional probabilities that may also be imprecise (e.g., interval-valued probabilities). For each rule of System P, Gilio (2002) proved the probability propagation rules, which describe how the probabilities of the premises are propagated to the conclusion. As an example, consider the *and*-rule. For all probability intervals $[x_L, x_U]$ and $[y_L, y_U]$:

(AND) $x_L \leq p(B|A) \leq x_U$ and $y_L \leq p(C|A) \leq y_U$; therefore, $\max\{0, x_L + y_L - 1\} \leq p((B \& C)|A) \leq \min\{x_U, y_U\}$ is coherent.

It can easily be seen that even in cases where the premise probabilities are point-valued (i.e., $x_L = x_U$ and $y_L = y_U$), the probability of the conclusion is usually interval-valued. In the extreme case where the premise probabilities are equal to 1, the only coherent conclusion probability is also equal to 1.

Experimental data suggest that most people infer coherent interval responses in inference tasks corresponding to (AND). The majority of those people who violate coherence violate the lower bound (Pfeifer & Kleiter, 2005). These data speak also against the common misunderstanding that people are unable to perform probabilistic reasoning because of the high frequency of "conjunction fallacies" allegedly committed in Tversky and Kahneman's (1983) well-known Linda task. The conjunction fallacy consists in ranking the probability of a conjunction $(B \& C)$ as higher than that of one of its conjuncts (C). In the context of (AND), this would mean that the *upper* probability bound on the conclusion is violated. In the experimental data on (AND), however, the people who violated the coherent interval violated the *lower* probability bound (Pfeifer & Kleiter, 2005). Even if the terms "and" and "probability" are mentioned, that does not mean that actual conjunction probabilities are investigated in the Linda task: the participants might not interpret the task as a task about conjunction probabilities.

Concerning the other rules of System P, a strong agreement between the participants' interval responses and the coherent intervals was observed (Pfeifer & Kleiter, 2005, 2006b, 2010; Pfeifer & Tulkki, 2017b). Moreover, the majority of the participants correctly understood that (6) and that contraposition (e.g., for all probability values x: if $p(B|A) = x$, then $0 \leq p(\sim A|\sim B) \leq 1$ is coherent) are probabilistically noninformative. This is interesting as these argument forms are logically valid under the material-conditional interpretation, but they cannot be validated without further assumptions in a nonmonotonic reasoning system. Adding monotonicity or contraposition to System P, for example, would make System P monotonic (which is undesirable of course). Transitivity is also probabilistically noninformative (i.e., for all probability values x and y: if $p(B|A) = x$ and $p(C|B) = y$, then $0 \leq p(C|A) \leq 1$ is coherent), and its addition to System P would make it monotonic. Experimental data suggest that people interpret the task material of Transitivity (presumably for pragmatic reasons) as Cumulative Transitivity (CUT) of System P (Pfeifer & Kleiter, 2006b, 2010): CUT changes the premises of Transitivity by conjunctively adding the antecedent of the first premise to the antecedent of the second premise. For all probability values x and y:

(CUT) $p(B|A) = x$, $p(C|(A \& B)) = y$; therefore, $xy \leq p(C|A) \leq xy + 1 - x$ is coherent (Gilio, 2002).

Note that the probability propagation rules of (CUT) coincide with those of the probabilistic modus ponens (Pfeifer & Kleiter, 2006a); for all probability values x and y:

(MP) $p(B) = x$, $p(C|B) = y$; therefore, $xy \leq p(C) \leq xy + 1 - x$ is coherent.

This close relationship between (CUT) and (MP) is explained by the fact that unconditional probabilities are defined in probability logic by the following principle:

(7) $p(A) =_{\text{def.}} p(A|\top)$, where "$\top$" denotes a logical tautology.

By replacing A with $\top$ in (CUT) and by (7), we obtain (MP). Modus ponens is one of the most frequently investigated argument forms in the psychology of reasoning. Its nonprobabilistic version is usually endorsed by most participants (Evans, Newstead, & Byrne, 1993). The clear majority of responses in tasks on probabilistic modus ponens support the predictions by probability logic (Pfeifer & Kleiter, 2009; Pfeifer & Tulkki, 2017b).

6. Concluding Remarks

This chapter characterized probability logic as a generalization of logic and explained the importance of zero-probability antecedents. Probability logic is connexive and nonmonotonic. It is a powerful tool for investigating the rationality of reasoning in a unified and systematic way. Experimental studies support its descriptive validity. Future normative and descriptive research should include nested and compound conditionals. Probabilistic modus ponens and other argument forms, for example, were recently generalized to deal with nested conditionals and

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compounds of conditionals (Gilio, Pfeifer, & Sanfilippo, 2020; Sanfilippo, Pfeifer, & Gilio, 2017; Sanfilippo, Pfeifer, Over, & Gilio, 2018). Interestingly, the uncertainty propagation rules for modus ponens involving *nested* conditionals coincide with those of nonnested modus ponens (MP). For instance, consider the following nested instance of modus ponens: from "The cup breaks if dropped" ($D \to B$) and "If the cup breaks if dropped, then the cup is fragile" ($(D \to B) \to F$) infer "The cup is fragile" (F). Here, the lower bound on the degree of belief in the conclusion F equals the product of the degrees of belief in the premises, and the upper bound on F equals the sum of the lower bound on F plus 1 minus the degree of belief in $D \to B$, which coincides with the uncertainty propagation rules of the nonnested (MP) (for details, see Sanfilippo et al., 2017). In this approach, which exploits the notions of conditional random quantities and conditional previsions, the law of import–export does not hold (Gilio & Sanfilippo, 2014; Sanfilippo, Gilio, Over, & Pfeifer, 2020), which is key to blocking Lewis's (1976) notorious triviality results. Lewis's triviality results show that sentences like $(D \to B) \to F$ must not be simply interpreted by $p(F|(B|D))$. Rather, for properly investigating such structures, a richer formal structure is required. Future work is needed to assess the psychological plausibility of this approach.

Finally, one might wonder why various nonclassical logics—which validate intuitively plausible rationality principles—were broadly neglected in the psychological literature, even if they were available already for decades. The reasons might be due to research traditions. This chapter proposed probability logic as a normatively and descriptively appealing rationality framework for human reasoning, which combines (i) the requirement of plausible *qualitative* logical principles (like nonmonotonicity and connexivity) with (ii) the expressibility of *quantitative* degrees of beliefs for investigating reasoning and argumentation under uncertainty.

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Notes

1. Note that the conditional event must not be nested: neither A nor B in $B|A$ may contain occurrences of "|", because of Lewis's (1976) triviality results. For nested conditionals and logical operations among conditionals, more complex structures are needed to avoid triviality, which go beyond the scope of this chapter (see, e.g., the theory of "conditional random quantities"; Gilio, Pfeifer, & Sanfilippo, 2020; Gilio & Sanfilippo, 2014; Sanfilippo, Gilio, Over, & Pfeifer, 2020; Sanfilippo, Pfeifer, & Gilio, 2017; Sanfilippo, Pfeifer, Over, & Gilio, 2018).

2. Popper functions, however, allow for conditioning on contradictions (for a discussion, see Coletti, Scozzafava, & Vantaggi, 2001).

3. In betting terms, you evaluate the probability p of an event E on the understanding that, for each real number s, you are willing to pay ps with the proviso that you will receive either s or 0 according to whether E happens or does not happen, respectively. The random gain (or "net gain") G is the difference between what you receive and what you pay. Thus, $G = s - ps$, when E is true, and $G = -ps$, when E is false. For a probability assessment on a family of events **F**, the associated random gain is the sum of the random gains of each bet on all events in **F**.

4. For a wide-scope negation interpretation of negating conditionals in probability logic, see Gilio, Pfeifer, and Sanfilippo (2016) and Pfeifer and Sanfilippo (2017).

5. Here, the scope of "normally" is the whole conditional and not just the consequent B. In other words, this means "The conditional 'If A, then B' holds by default" or "Normally, B follows from A."

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