13.2 Rational Reasoning about Spatial and Temporal Relations

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Summary

Reasoning about spatial and temporal relations is ubiquitous in everyday life. It is important that such reasoning processes proceed rationally; otherwise, we will arrive at the wrong place or appear there at the wrong time. To demonstrate the specifics of the underlying cognitive processes, human spatial and temporal reasoning is compared to formal approaches from artificial intelligence (AI). The underlying cognitive processes have particular characteristics: First, they are *qualitative*, that is, metrical information is taken into account only when necessary. Second, they realize *resource-bounded rationality*, that is, due to limited resources, only parsimonious mental representations are built: often a preferred mental model is constructed and, based on it, putative conclusions are drawn. Third, they are *sensitive to the mental effort* problems require, that is, often mental models are constructed that require a small number of operations only. This often leads to rational spatial and temporal inferences but can also lead to systematic errors.

1. Space and Time Are Fundamental in Everyday Life

Humans are always within the realm of space and time. But we are not always consciously aware of both dimensions of our existence. Kant (1781/1787/1995, A23/B38) claimed that space and time are not empirical (in the sense that "abstract" space or time is not perceivable by our senses) but rather "a priori representations." We also cannot perceive abstract space and time directly but become aware of the reality of space and time whenever we navigate, communicate, or reason about our environment or about events in time. In the following, we start by considering a simple spatial reasoning problem concerning four entities and consisting of two pieces of relational information. Please read the following information and try to answer the question afterward.

> You are now at the train station.
>
> Your bicycle is in front of the north entrance of the church.
>
> The church is north of the train station.
>
> What, if anything, can be inferred from the given information?

An immediate conclusion, that is, an inference that can easily be drawn, is that you have to head north to find your bicycle. Analogously, let us assume that you know that

> The complete decoding of the human genome took place after the first moon landing.
>
> The invention of the computer took place before the first moon landing.

It is easy for you to infer from these two statements that the computer was invented before the complete decoding of the human genome took place. With some background knowledge, such as that computers are nowadays often involved in scientific progress, you can even form a putative conclusion that the invention of the computer was very likely important in the complete decoding of the human genome.

These two examples are similar to a multitude of other problems we encounter in everyday life. Let us consider some characteristics of such problems: first, the relations are qualitative, that is, no numerical information is given, and you do not need any numerical information or may have even thought about it. Second, all information is relevant to solving the problem. Third, in our example, the four terms in the spatial problem and the three events in the temporal problem were relatively simple, but it is not hard to imagine that additional information may make such problems more difficult to solve.

In solving such problems, you may have had the impression that you formed a vivid mental picture or that you used an abstract relational representation of the described spatial situation. Or, instead, you may have applied logical inference rules (e.g., a transitivity rule) to the two relations “in front of (the north entrance of)” and “north of” (despite being nonidentical, they do not differ in their direction), without forming an analogous model-like representation, to infer that you have to head north. Or you may have applied a heuristic to generate a conclusion. An example of such a heuristic might be to use the only cardinal direction relation (“north”) mentioned in the given information, giving you the general direction in your conclusion. While for this sample problem, each type of inference process can be applied, all leading to the same answer, it is very unlikely that a human reasoner employs all of these processes. It is much more plausible that there are specific processes that most reasoners tend to use. Probably an important observation is that humans often use qualitative spatial and temporal relations, which do not represent exact, metrical information in a continuous way. Rather, they only represent those relations that are necessary for the inference. This allows humans to represent spatial and temporal information even if exact information is not available or would require too much effort to acquire. An advantage of qualitative relations is that they do not require any artificial external metrical scale (Freksa, 1992). Therefore, qualitative spatial and temporal relations are more practical in an environment where exact information is missing. A qualitative relation can be considered an “abstraction summarizing similar quantitative states into one qualitative characterization” (Moratz & Ragni, 2008, p. 76; see also Rauh et al., 2005). In this sense, qualitative descriptions are compressed representations and are much more useful in a complex environment for an agent that has a bounded rationality only.

An underlying assumption of this chapter is that the way in which human reasoning proceeds depends on the need to be applicable and useful in a complex world, given that cognitive resources, such as working memory, are limited. We will consider the underlying processes of reasoning about spatial and temporal beliefs both from a *normative* perspective (i.e., looking at logical formalisms capturing idealized knowledge representation and reasoning) and from a *descriptive* perspective (i.e., how humans actually represent knowledge and reason about it). By contrasting what *ought to be* and what *is*, it is possible to identify specific differences between formal and human reasoning. The chapter addresses this question on a *formal* level, by introducing artificial intelligence (AI) systems based on qualitative relations; on an *empirical* level, to gain access to human reasoning; and on an *algorithmic* level, by explaining the cognitive processes via cognitive modeling.

2. Formal Models for Reasoning about Spatial and Temporal Relations

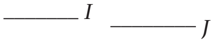
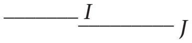
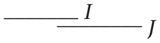
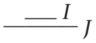
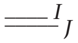
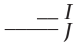
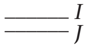
Since it is natural for humans to draw inferences such as the ones above, we often consider these problems as simple. From a formal perspective, however, they are certainly not. In the past years, cognitively inspired calculi (e.g., the double-cross calculus; Zimmermann & Freksa, 1996) have been introduced that help us to reconstruct how the example problems can be solved. Two famous calculi are the interval calculus introduced by Allen (1983) and the region connection calculus introduced by Randell, Cui, and Cohn (1992).

The interval calculus was initially developed for reasoning with temporal relations but was soon transferred to spatial reasoning (Guesgen, 1989). In the spatial domain, the calculus works with solid objects. The calculus can also be extended into more complex formalisms (Ligozat, Mitra, & Condotta, 2004). An event can be represented by its start time and its end time (see table 13.2.1). Accordingly, two events can be qualitatively related by the relations between the start and end times of the two events, for example, that the first event ends before the second starts, or that the first event overlaps with the second, or that both events start at the same time, or that both finish at the same time, or that the first event takes place during the second event. If the qualitative relations between start and end points are known, then 13 different relations can be identified (see table 13.2.1). Imprecise knowledge about the start and end times of the events is represented by disjunctions of these 13 relations (e.g., an event finishes before the other starts *or* they overlap).

The region connection calculus (RCC) has been developed in the spatial domain and is similar to the interval calculus, but instead of orientational information, it just uses topological information, that is, the relations between the boundaries of objects (e.g., two regions can be disjoint, or they can overlap, or the first region can be a proper part of the second, or vice versa, or they can be identical). The calculus using these five basic topological relations is called RCC-5. If the boundaries of the regions are relevant too (e.g., when two countries share a common border), then three more relations can be formed: a region touches another from the outside or they are disjoint (and not even their borders overlap), or the first region is a proper part of the second and touches the latter’s border or not, or this relation holds for the second region. With the resulting eight relations, we have the calculus RCC-8 (see figure 13.2.1). Based on these base relations, one can again express indeterminate

Table 13.2.1

The 13 qualitative interval relations of the interval calculus, their names for temporal reasoning, natural language expressions for the spatial domain, and an interval representation

Temporal	Spatial	Relation	Inverse	Pictorial Representation
before	*I* lies to the left of *J*	*I* b *J*	*J* bi *I*	
meets	*I* touches *J* at the left	*I* m *J*	*J* mi *I*	
overlaps	*I* overlaps *J* from the left	*I* o *J*	*J* oi *I*	
during	*I* is completely in *J*	*I* d *J*	*J* di *I*	
starts	*I* lies left-justified in *J*	*I* s *J*	*J* si *I*	
finishes	*I* lies right-justified in *J*	*I* f *J*	*J* fi *I*	
equals	*I* equals *J*	*I* e *J*	*J* e *I*	

Note: Adapted from Knauff (1999).

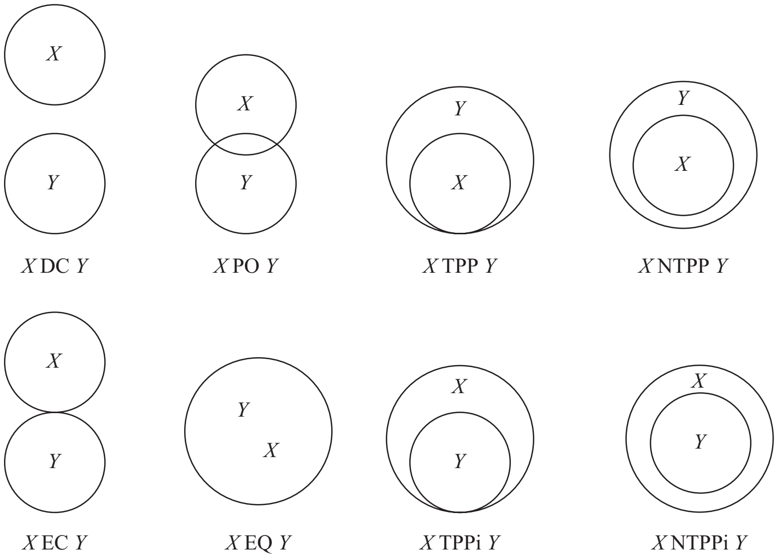


Figure 13.2.1

The region connection calculus with eight base relations (RCC-8). *X* and *Y* are regions. The eight base relations are DC = disconnected, PO = partially overlaps, (N)TPP = (non)tangential proper part, EC = externally connected, EQ = equal, and the two inverse relations NTTPi and TPPi.

descriptions reflecting imprecise knowledge (e.g., that a region contains the other or that they are disjoint).

These are just two of many approaches that have been suggested in AI for reasoning in a rational way with spatial and temporal relations. These approaches can serve as normative accounts of how correct reasoning with qualitative spatial and temporal relations should proceed. They have many interesting formal characteristics and have been used in many domains of application, for example, in robotics (Moratz & Ragni, 2008), plan generation (e.g., Westphal, Dornhege, Wölfl, Gissler, & Nebel, 2011), sea navigation (e.g., Wolter et al., 2008), and language processing (e.g., Mast, Wolter, Klippel, Wallgrün, & Tenbrink, 2014). These calculi are of interest in this chapter also because many cognitive psychologists have used them to compare human reasoning with reasoning in technical systems (Knauff, Rauh, Schlieder, & Strube, 1998; Rauh et al., 2005; Schlieder & Berendt, 1998) and also to determine the correct solutions to inference problems. The results of these empirical studies show that the formal approaches capture some aspects of human reasoning but that people also quite often reason with spatial and temporal relations differently, and there are cross-cultural similarities as well as differences (e.g., Knauff & Ragni, 2011).

The problem with many formal calculi is that they can be computationally expensive (for an overview, see Renz & Nebel, 2007), that is, the number of simple computations and/or the amount of memory space needed to solve the problem rise exponentially with the size of the problem description. This makes “large” problems practically intractable. A central empirical finding, however, is that, in contrast to formal approaches, humans do not consider all possible models but rather try to find a useful specific model, especially when the verbal description is indeterminate (i.e., allows for multiple models). In the following, we describe this and related findings from empirical research and computational modeling.

3. Cognitive Findings on Spatial and Temporal Reasoning

Let us first look at the behavioral level by studying which conclusions people draw from a set of given premises. Such questions have been in the center of reasoning research for several decades, and many effects are so reliable that they have been replicated across many studies. Here are a few of these robust findings:

(1) The *indeterminacy effect*: if a problem description is indeterminate, that is, if it allows for multiple scenarios, the problems are on average more difficult to reason through. Indeterminate descriptions can appear quite often in everyday life (e.g., in cases where information is missing). This effect has been observed repeatedly and has been explained via both mental models and mental logic approaches (e.g., Johnson-Laird & Byrne, 1991; Kießner & Ragni, 2017; Van der Henst, 2002).

(2) The *relational complexity effect*: if a problem description contains relations of a higher arity (e.g., "between," which is a ternary relation), or the relational information needs to be decomposed into facts involving simpler relations, then the reasoning difficulty increases. Hence, relational complexity influences the mental representation that is constructed (e.g., Goodwin & Johnson-Laird, 2005; Halford, Wilson, & Phillips, 1998; Jahn, Knauff, & Johnson-Laird, 2007).

(3) The *visual impedance effect*: if a problem description uses relations that are easier to visualize than more abstract relations, then this visualizability can impede reasoning (e.g., Knauff, 2013). This indicates that the mental representation most human reasoners use is rather abstract and relational and less a visual image (e.g., Pylyshyn, 2002).

(4) The *continuity effect*. Consider the following two descriptions:

(*continuous*): *A* is left of *B*; *B* is left of *C*; *C* is left of *D*.

(*discontinuous*): *A* is left of *B*; *C* is left of *D*; *B* is left of *C*.

In the first description (*continuous*), each new premise connects directly to the previous one, such that the new information can be easily mentally integrated into a new representation. In the second description (*discontinuous*), the information cannot be directly related. The first type of description is easier for reasoners to process than the second (e.g., Knauff et al., 1998; Nejasmic, Bucher, & Knauff, 2015). Since the information given is identical, an explanation for the difference is that for the continuous problem, there is an immediate successive integration of the premise information into a mental model, while in the discontinuous case, the processing requires to first store two independent pieces of information.

(5) The *relevance of information effect*: if reasoners receive information that is partially irrelevant for the conclusion (e.g., because the conclusion is already given), then such problems are easier than problems with the same amount of information where all of it is relevant (e.g., Schaeken, Girotto, & Johnson-Laird, 1998; Van der Henst, 1999). This indicates that spatial and temporal reasoning difficulty depends on the amount of relevant information, not on the total amount of information, and that the model contains only the relevant information.

These five effects (for an overview, see, e.g., Ragni, 2008) have been repeatedly reported and support the hypothesis that spatial beliefs are represented by mental models based on the principle of minimal operations (see below). We will later see that an explanation for how humans draw inferences is that often, instead of constructing all possible models, they construct just one specific mental model, the "preferred" model. This does not only reduce the memory load incurred by *storing* models but also reduces the mental effort that comes with *constructing* models. This resource awareness is a realization of *bounded rationality*. Bounded rationality is an adaptation of our mind to these limitations, for example, of the available information and of our cognitive resources (see Gigerenzer & Selten, 2002; Simon, 1955). Hence, another robust result is the following:

(6) The *preference effect*. Consider the following example from Ragni and Knauff (2013):

The Ferrari is parked to the left of the Porsche.
The Beetle is parked to the right of the Porsche.
The Porsche is parked to the left of the Hummer.
The Hummer is parked to the left of the Dodge.

The description allows for the following three models:

Model 1: Ferrari Porsche Beetle Hummer Dodge
Model 2: Ferrari Porsche Hummer Beetle Dodge
Model 3: Ferrari Porsche Hummer Dodge Beetle

> While logically, all three arrangements are models of the situation described in the premises, experimental studies showed that human reasoners accept model 1 (which has been called the preferred mental model) more often and faster than model 2, which they in turn accept more often and faster than model 3 (which are called alternative mental models). To make logically correct inferences, the construction of *all* alternative models is relevant, because an alternative to the preferred model can be a counterexample, which would make the inference based on the preferred model wrong. For example, the previous premises might be presented to reasoners, and then they must answer the question, "Is the Beetle (necessarily) parked to the left of the Dodge?" In this case, most people do construct the first model but not the other two models. Hence, they give a wrong answer.

The preference effect has been reported for the first time in Knauff, Rauh, and Schlieder (1995). These authors also suggested the term "preferred mental model" to explain the cognitive foundations of the preference effect. Since then, the preference effect was repeatedly found in many experiments and has been replicated several times (e.g., Knauff, 2013; Knauff et al., 1998; Ragni & Knauff, 2013; Rauh et al., 2005; Schultheis, Bertel, & Barkowsky, 2014). We return to this in the next section.

4. Neural Findings and Their Impact for a Theory of Spatial and Temporal Reasoning

The way our brain processes spatial and temporal information can explain how and why we deviate from formal approaches to reasoning in AI. For instance, we know that, in contrast to formal models of computation, like Turing machines, the human mind is not monolithic, that is, there is no single storage space; rather, for different tasks, different regions are relevant. Core regions (cf. chapter 1.4 by Goel, this handbook) identified for spatial and temporal reasoning are the prefrontal cortex (especially the dorsolateral and the left prefrontal cortex), mainly implementing control functions (e.g., the integration of different premises; Fangmeier, Knauff, Ruff, & Sloutsky, 2006), and the parietal cortex, mainly storing the mental model (Crone et al., 2009; Knauff, 2006; Ragni, Franzmeier, Wenczel, & Maier, 2014). A recent meta-analysis (Wertheim & Ragni, 2018) of functional magnetic resonance imaging studies was able to identify the involvement of two frontal areas (Brodmann areas 6 and 45, with a role in complex planning and semantic understanding, respectively) and one parietal area (Brodmann area 7). For spatial reasoning, especially the superior parietal lobe (SPL) is relevant.

The theory of preferred mental models (Ragni & Knauff, 2013) claims that different phases in spatial reasoning exist: first constructing the preferred mental model and then searching for counterexamples by constructing alternative mental models. Now, a study shows that disturbing the information processing in SPL by transcranial magnetic stimulation (TMS) leads to a decrease in people's ability to construct the preferred mental model, but the stimulation of the SPL has no impact on the generation of alternative models, that is, models 2 and 3 above (Ragni, Franzmeier, Maier, & Knauff, 2016). This supports that these processes are different, because they are realized differently on the neural level. Additionally, the study supports the claim that bounded rationality is a relevant factor. Reasoners with a smaller working memory capacity made more errors in accepting the preferred and alternative models, which are correct models. Reasoners with a smaller working memory capacity are more susceptible to disturbance effects by TMS and subsequently accept the preferred model less often. In fact, the disturbance effect by application of TMS was inversely correlated with working memory capacity: it was lower for participants with greater working memory capacity and higher for those with smaller working memory capacity.

The results show that reasoning about spatial and temporal relations depends on the preferred mental models that are formed in the parietal cortex and on control processes that are relevant for integrating new information and are located in the frontal regions. Finally, limited working memory capacity can explain why not all mental models are constructed.

5. Computational Models and the Relevance of Complexity for Rationality

We now turn to cognitive computational systems that aim to model, reconstruct, and explain the processes underlying human reasoning. Some of these cognitive computational systems for spatial reasoning use Cartesian coordinate systems (Johnson-Laird & Byrne, 1991; Schultheis & Barkowsky, 2011), whereas others use discrete representational structures such as lists or grids (e.g., Krumnack, Bucher, Nejasmic, Nebel, & Knauff, 2011; PRISM, Ragni & Knauff, 2013). Some systems assume the construction of mental models while others rely on visual images (e.g., Casimir; Schultheis & Barkowsky, 2011). Yet others use abstract relational information (e.g., PRISM), or are inspired by the linguistic task of encoding and reencoding spatial relations (Krumnack et al., 2011).

Those computational models that successfully solve problems like humans do shed some light on the underlying cognitive processes. For instance, most cognitive models assume that human reasoners construct mental models and make transitive inferences based on these models. For the visual impedance effect, current cognitive models assume that people initially create a visual mental image but then have to extract the relevant spatial information from this image, which can be time-consuming and error-prone (Knauff, 2013). Others try to explain the effect by assuming that the visual relations activate additional features in declarative memory (Albrecht, Schultheis, & Fu, 2015).

There exist only a few systems for *temporal* relational reasoning, and these are based on mental models. The cognitive processes are similar to those in processing spatial relations: humans construct preferred mental models and then vary them to test putative conclusions (Johnson-Laird & Byrne, 1991; Juhos, Quelhas, & Johnson-Laird, 2012). Kelly and Khemlani (2019) have demonstrated that many features that have been identified in spatial reasoning also transfer to durative temporal relations, for instance, one event happening during another event. Examples of features that transfer from the spatial to the temporal domain are that the underlying mental models are iconic, that they rely both on intuitive and on deliberative processes, and that problems eliciting more models have a higher reasoning difficulty. Moreover, the consideration of cognitive costs can explain why some reasoners are able to mentally simulate alternative models that falsify the conclusion suggested by the preferred model.

We already mentioned that the findings from research on spatial and temporal reasoning can be explained within the conception of *bounded rationality*. In particular, in cases where several models are possible, humans systematically prefer one model while ignoring others (Knauff et al., 1998; Ragni & Knauff, 2013; Rauh et al., 2005). The explanation is that such preferred mental models are constructed according to principles of *operational parsimony*: the human mind builds the mental model that needs the smallest number of cognitive operations.

An important point that these findings demonstrate is that human reasoners do not generate all possible mental models of given information with equal ease and equal likelihood. And this in turn shows that logical formalisms such as formal qualitative calculi (see section 2 above), which do not make any difference between mental models, are descriptively inadequate.

The theory of preferred mental models, in contrast, assumes that reasoners successively insert each new object from the premises into their mental model, filling a slot that is not already occupied by another object, such that the resulting model still matches the premises. This process has been algorithmically realized in a computational model called PRISM (Ragni & Knauff, 2013), which indeed implements bounded rationality and preference effects, based on the idea of simple spatial operations such as inserting an object into a mental model or changing the position of a mental focus. The number of such operations needed for solving a problem forms a measure of its cognitive complexity. By considering the number of operations in a spatial working memory representation, this measure can explain the preference effect and which alternative model is constructed when (Ragni & Knauff, 2013). The number of such operations explains which mental model is constructed and inspected, as well as which putative conclusion is considered in the reasoning process. If the number of operations is high, often only a single model is constructed (Ragni & Knauff, 2013). Which model is constructed depends on the specific way the information is mentally processed (e.g., whether the information is presented in a continuous way, as in the continuity effect), on background knowledge (e.g., whether we have a preferred arrangement of objects, say, of the silverware on a table), and on the underlying spatial domain. For example, it is easier to mentally insert objects in large-scale scenarios (e.g., cardinal directions; Ragni & Becker, 2010) than in small-scale ones (e.g., left–right relations among fruits on a table; Ragni & Knauff, 2013).

The preference effect is ubiquitous and has been replicated for small-scale spaces (Ragni, Fangmeier, Webber, & Knauff, 2006; Rauh et al., 2005), for large-scale spaces (with respect to the cardinal directions; Ragni & Becker, 2010; Schultheis & Barkowsky, 2011), for topological relations (even cross-culturally; Knauff & Ragni, 2011), and also for other reasoning domains (e.g., Johnson-Laird, 2006). That the preference effect is so prevalent indicates that in the domain of spatial and temporal relational reasoning, rationality is bounded not only by working memory limitations but also by limitations on the number of mental operations one can perform. Of course, the two are related, but we will not discuss this here (see Knauff, 2013). In the following, some further heuristics of human spatial and temporal reasoning are described.

6. Further Characteristics of Human Spatial and Temporal Reasoning

Although many kinds of spatial and temporal relations exist, a particularly important kind of such relations are *transitive inferences*. In such inferences, a conclusion "*A* is north of *C*" is drawn from the premises "*A* is north of *B*" and "*B* is north of *C*." When humans try to solve such inferences, they often fall prey to illusions even for nonspatial relations such as blood-relatedness: many reasoners infer from "*A* is blood-related to *B*" and "*B* is blood-related to *C*" that *A* is blood-related to *C*. In fact, there is a type of counterexample known to almost all reasoners: *A* is the father, *B* is the daughter, and *C* is the mother. Still, the internal mental representation and the cognitive inference system prompt many human reasoners to make this inference (Goodwin & Johnson-Laird, 2008). Such illusions demonstrate that our internal inference mechanism is overeager to apply transitivity. And they demonstrate that we internally represent even nonspatial information in a way that allows drawing transitive inferences based on spatially organized models. Many of the following results are from experiments with transitive relations.

6.1 Human Reasoning about Space and Time Employs Mental Models and Manifests Preference Effects

There is broad agreement among current theories and computational models that spatial and temporal beliefs are represented by mental models (Krumnack et al., 2011; Ragni & Knauff, 2013; Schultheis & Barkowsky, 2011; Johnson-Laird & Byrne, 1991), which are analogical representations of states of affairs. But there is also some disagreement about how much visual information is present in mental models and in what form it is represented (e.g., Pylyshyn, 2002). Experimental findings indicate that our internal representation is rather abstract-relational and less visual. In fact, if additional visual features must be encoded, an impedance effect on reasoning can occur (Knauff, 2013). The underlying representation is again an abstraction comprising only the information relevant for the specific task. This principle of minimizing cognitive effort holds also for cases where multiple models are possible: usually only one particular model is constructed, which comprises relevant beliefs. Given limited cognitive resources and time pressure, this parsimony is rational. In the following, results are reported that support this claim.

6.2 Human Reasoning Is Influenced by Problem Content and Embodiment

Experimental studies have shown that problem content and the reasoner's background information about the given domain can have strong effects on the construction of the preferred mental model (e.g., Rauh, 2000). Accordingly, the mechanisms for the construction of spatial representations even extend to other domains (e.g., mathematical cognition). In the spatial–numerical association of response codes (SNARC) effect, small numbers are responded to faster with the left hand, and large numbers show a similar proclivity with regard to right-sided responses and presentations (Hubbard, Piazza, Pinel, & Dehaene, 2005). This indicates that people internally represent numbers in a spatial ordering from left to right by ascending size. For relational reasoning, the same effect has been identified for the relational information "*A* is left of *B*" and "*B* is left of *C*" in responding with the left hand for items further left and with the right hand for items further right (Prado, Van der Henst, & Noveck, 2008). Thus, even for arbitrary relational information, our internal mental representation is not independent from our body; hence, our "embodiment" matters.

6.3 Human Rationality Is Bounded

Empirical findings demonstrate that human reasoners often do not generate all possible models but only a specific one—the preferred mental model. Additionally, not all alternative models are constructed, only small modifications of the preferred model. Findings from neuroscience demonstrate that mental processes are not monolithic: alternative mental models are generated in other brain regions than the preferred mental model. Working memory capacity in turn is a limiting factor for the construction of mental models. An analysis of core empirical effects demonstrates that problems that are more difficult (e.g., the discontinuous case or multiple-model problems) require more mental operations than easier problem descriptions—supporting an operational parsimony. Together, working memory limitations and operational parsimony are core aspects constituting *bounded rationality* in humans.

7. Conclusion

We can draw several conclusions from this chapter. Probably the most important are that human reasoning about spatial and temporal relations deviates from classical conceptions of rationality and that it is multifaceted: we do not simply apply a few general principles. Beyond the limited information capacity of our senses, human rationality is bounded in a twofold way: first, our working memory capacity is limited, forcing our brains to construct and manipulate specific mental models; second, our computing capacity is limited, whence our brains cannot exhaustively survey all possibilities—this operational parsimony explains many, if not all, of the cited effects. For estimating operational effort, there is a cognitive complexity measure that is based on the number of simple operations and explains especially why some models are constructed and others are neglected.

While humans have sometimes been considered irrational, they are actually not: they are quite sensible about how much mental effort—given their limited resources—they do invest. Often a first mental model is sufficient to understand the information, and very often finding an optimal path or time plan takes too much effort. Optimality is not necessary in our everyday lives; avoiding mental depletion is—it can be dangerous. From a strictly normative perspective, humans may not always find the correct inference—but they often find one that suffices.

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