13.3 Visualization and Rationality

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Summary

The epistemic function of visualization was always controversial. This chapter is organized around the important, but often blurred, distinction between external and internal visualization. *External* visualizations are visual aids such as pictures, graphics, or diagrams, which are designed to help people to reason accurately. The question is whether such external visualizations are useful or harmful when people solve epistemic problems. *Internal* visualizations are picture-like mental images, a special form of mental representation, characterized as analogous to representations that arise in the mind as a result of actual visual perception. The question is whether such visual mental images are useful or harmful in rational reasoning. The chapter presents the empirical findings from cognitive psychology, cognitive brain research, and computational modeling.

1. Two Types of Visualization

Visualization is an important concept in the cognitive sciences. Many experiments show that mental visualization can help remembering objects or events, or when objects must be mentally inspected or manipulated (Kosslyn, 1980). Such visualizations are correlated with activity in early visual cortices, which might explain the experienced similarity between mental visualization and visual perception (Kosslyn, 1994). However, there is also evidence that visualization can disrupt cognitive processes and impede thinking. For instance, individuals on the autism spectrum often posit that they use visualization for tasks that typically developing individuals perform verbally (Kunda & Goel, 2011). In her famous autobiography *Thinking in Pictures*, Temple Grandin (1995/2006) describes how her visual thinking style supported her practice but also resulted in problems in dealing with more abstract thoughts. Certain psychotic drugs that suppress visual mental images (e.g., benzodiazepine) lead to better reasoning performance if the problems would otherwise elicit visual images (Pompéia, Manzano, Pradella-Hallinan, & Bueno, 2007). And the role of visualization in mathematics is still controversial (Arana, 2016; Mancosu, 2005). Some scholars assert that it helps. For instance, the influential mathematician Felix Klein (1979) claimed that “mathematics is not merely a matter of understanding but quite essentially a matter of imagination” (p. 207). Other eminent mathematicians argued that visualization is a nuisance in getting mathematical insight or proofs. The most prominent advocate of this view was Hilbert, who stated that deduction should be independent of figures in order to be rigorous (Hilbert, 1899).

This chapter focuses on the relationship between epistemic rationality and visualization. The first concept, *epistemic rationality*, is concerned with belief acquisition, formation, and revision. A central component of these processes is reasoning, a cognitive process that leads from given premises to a rationally justified conclusion, that is, a new belief or, at least, a belief that is made explicit in the process of inference. To be epistemically rational, the conclusion should conform to certain normative ideals, which often come from logic or other formal systems for rational reasoning. In chapter 5.4 by Gazzo Castañeda and Knauff (this handbook), it is shown that classical logic might not always be the best normative standard for everyday human reasoning. Yet, understanding how humans reason logically is still important for a comprehensive cognitive theory of human rationality.

The second concept, *visualization*, is often used in a very broad sense. Here I use it with two specific meanings. *External* visualizations are visual aids such as pictures, graphics, or diagrams, which are designed to help people to reason accurately. The question is whether such external visualizations are useful or harmful when people have problems to solve or make inferences. *Internal* visualizations are picture-like mental images, a special form of mental representation, characterized as analogous to representations that arise in the mind as a result of visual perception. The question is whether such visual mental images are epistemically useful or harmful in human logical reasoning. Does visualization help people in drawing rationally justified inferences, in reasoning accurately and without logical errors? In other words, what are the epistemic benefits and limitations of mental imagery in human reasoning? To answers these questions, the chapter reports empirical evidence from cognitive psychology, cognitive brain research, and computational modeling. It closes with some general corollaries on the epistemic role of visual thinking in human rationality.

2. Epistemology of Pictures, Graphics, and Diagrams

Philosophers, logicians, and mathematicians have always had an ambiguous relationship with external visualization, that is, graphics or diagrams (Krämer, 2016). Take, for example, the following problem: imagine a square with another square inside it, where the corners of the inner square are exactly at the midpoints of the sides of the outer square. What is the size of the inner square in comparison to the outer square? It will be hard for you to solve this problem in your head alone. Now the picture in figure 13.3.1 is presented to you. You will notice that you can now solve the problem more easily—the size of the outer square is exactly twice as large as that of the inner one, because the four triangles in the corners of the outer square together cover exactly the inner square. You can easily see this if you imagine folding over the corner triangles along the sides of the inner square.

Pedagogues and teachers often use such visualizations in their classes, because they believe that they help students to understand complex mathematical issues (Kadunz & Yerushalmy, 2015). Yet, in geometry, diagrams often have been distrusted and considered inadequate and inappropriate for constructing proofs (Giaquinto, 2016). Imagine, for instance, that I have been just a bit sloppy with the drawing—then it could be that the four triangles do not exactly fit into the inner square, and you may not understand the general mathematical principle. Apparently, the formal laws of geometry allow you to say something about all squares and rectangular triangles, independently from the concrete visualization—your insights are a priori and hence independent from perceptual experience, as Kant would say. The long and interesting history of the controversies about the epistemology of visualization in mathematics is described in Arana (2016), Giaquinto (2016), and Mancosu (2005).

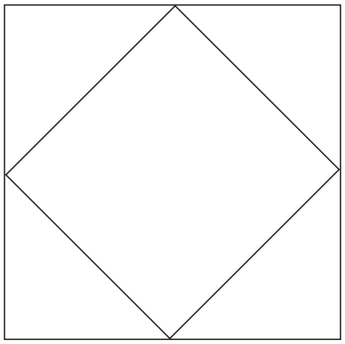


Figure 13.3.1
What is the size of the inner square in relation to the outer square?

Since logical reasoning is a particularly important foundation of epistemic rationality, it is not surprising that philosophers and logicians have paid particular attention to the relation between logic and visualization. On the one hand, up to the turn of the 20th century, mathematicians and logicians were concerned with the formal and normative dimension of visualization in logical reasoning. Euler and Venn developed logical systems for reasoning with diagrams. Most notably, Peirce introduced the notion of *iconicity* in his nonverbal logic. An *icon*, in Peirce's semiotics, is a sign that is perceptibly similar to the object it represents. More recently, Shin (1994) has devised a formal syntax and semantics of particular forms of Venn diagrams that has also inspired much research in artificial intelligence. A good overview of this work is given in chapter 13.1 by Jamnik (this handbook).

On the other hand, the majority of scientists and philosophers questioned the importance of visualizations for logical reasoning. Driven by Ryle (1949) and notably the later Wittgenstein (1953/2001), this formal movement resulted in picture-free logical systems that are inextricably linked to language. Since then, the dogma for most logicians and philosophers has been that rational reasoning relies on the sentences of a language from which further valid sentences of this language are derived. The language need not be a natural language but rather can be any language that has a formal grammar, such as propositional or predicate logic. Thus, external visualizations (e.g., diagrams) are often accepted as aids to foster logical intuitions but not as an independent tool for conducting logical proofs.

The empirical evidence from cognitive research is similarly ambiguous. On the one hand, some experiments show that diagrams can help in logical thinking. Bauer and Johnson-Laird (1993) presented their participants conditional reasoning problems as sentences and additionally clarified the logical connections using different types of diagrams. The diagrams were relatively abstract line-drawings that represented the logical relations between different sets of entities in a spatial way. In another experimental condition, there were no diagrams; the tasks were presented only as sentences. The participants solved the tasks better with diagrams than without. On the other hand, Stenning (2002) has explored in detail the logical role of visual presentation formats in Hyperproof, a tutorial system developed by the logicians Barwise and Etchemendy (1994) to teach students formal logic through language and visualization.

Stenning (2002) found that students with poorer logical abilities use visualizations, whereas students with better logic skills use more nonvisual reasoning strategies. In fact, strong "visualizers" could benefit the most from learning to use abstract strategies instead of visualization. Similar results were reported by Sato, Sugimoto, and Ueda (2017), who explored the usefulness of different grades of concreteness in conditional reasoning. They either presented photos of real objects to their participants, or the objects were presented more abstractly in an annotated virtual environment. Reasoning performance was better with the more abstract objects in the annotated virtual environment. The visual details of the photos seem to distract people from what is actually relevant for performing the inference. Similar results were found in spatial cognition research, where people have to make inferences to understand verbal route descriptions or navigate through photorealistic virtual environments (Meilinger, Knauff, & Bülthoff, 2008).

Overall, the cognitive advantages and disadvantages of external visual aids seem to depend on the iconicity, concreteness, and abstractness of the form of representation. Too many visual details seem to hinder the process of reasoning, whereas more abstract diagrams that aid in understanding the underlying logical structure of the problem can help people to reason more accurately and to grasp the intuition of the proof, that is, to understand why the theorem and even the proof are correct (see chapter 13.1 by Jamnik, this handbook). In any case, the common intuition that it is always helpful for understanding and reasoning to make abstract issues more concrete in pictures or diagrams is certainly wrong. Images, photos, and other highly visual presentation formats can actually lead to the deterioration of epistemic rationality. A possible explanation is that people learn better when pictures are excluded from a learning system rather than included. The reason is that visual overload may result in redundancy, for instance, if texts and images are presented simultaneously, which can hinder learning and reasoning effectiveness (Mayer, 2009; Schnotz & Bannert, 2003). This also agrees with the *seductive details effect*: textbooks are often enriched with nice-looking pictures or other eye-catching details that are hardly relevant for meaningful learning. But empirical findings show that students display poorer learning outcomes when they learned with this kind of seductive information (Eitel, Bender, & Renkl, 2019; Harp & Mayer, 1998).

3. Epistemology of Visual Mental Imagery

We have seen that the epistemic role of external visualization for human reasoning is complex. But how helpful are *internal* visualizations for logical reasoning? This is even more difficult, as we will now see. The idea of thinking in the mind's eye already existed long before it received attention in psychology. In philosophy, the concept already appears in the oldest writings about cognition—the works of Plato and Aristotle (Nigel, 2018). Later, philosophers like Descartes, Locke, and Hume also developed different thoughts about the structure and function of mental imagery. Although their conceptions were very different, all had the idea that human thought relies on pictorial mental images (Thomas, 1997/2019; Tye, 1991). This idea was also supported by the introspective reports of scientists, for example, the distinguished Austrian physicist Ludwig Boltzmann, who wrote, "I am of the opinion that the task of theory consists in constructing an image of the external world that exists purely internally and must be our guiding star in thought and experiment" (Boltzmann, 1890, p. 33). Similarly, the chemist August Kekulé reported that visualization had suggested to him that the benzene molecule might have a cyclic structure (Kekulé, 1865). Today, some contemporary scholars even allege that vivid imagination was crucial in solving the key problems of chemistry in the 19th century (Rocke, 2010).

In psychology, several academic debates revolved around the role of visual images in human cognition. On the one hand, in 1910, studies by Cheves Perky revealed that people often merge mental images and what is actually seen. Obviously, visualizations can be so similar to real perceptions that they can be mistaken for the latter (Perky, 1910). On the other hand, in particular the "Würzburg School" argued that thinking is possible without imagination. These researchers conducted several experiments in which they asked the participants what had happened in their mind when solving different kinds of reasoning problems. Not one of them reported experiencing visual images. Karl Bühler, one of the main figures of this school, concluded that thinking is possible without seeing in the mind's eye. Human thinking, he argued, often is "pure thinking," which is independent from concrete imagination or visualization (Bühler, 1909). Other authors have questioned this approach and had no doubt that thinking calls for visual imagination (e.g., Titchener, 1909).

Later, psychologists avoided the concept of visual mental imagery, given the hostile criticism it had received from behaviorists (Watson, 1913). In his famous "manifesto," Watson (1913) argued that the controversy about visual mental images was a prime example of the maladies of psychology for which behaviorism would be the therapy. For Watson, the speculations about mental images in the mind (as all other mentalistic concepts) were only "old wives' tales" and nothing more than the sentimental "dramatizing" of verbally mediated memories (Watson, 1928).

This radically changed with the rise of cognitive psychology, which put mental representations and processes center stage. One of the most striking arguments for the role of visual mental images in human cognition was the seminal experiment by Kosslyn, Ball, and Reiser (1978), who asked people to memorize and visualize the map of a fictitious island. In a later memory test, the time to mentally move from one object to another was proportional to the distance between the objects on the map. Such results were taken as evidence that visual mental images are a distinct kind of mental representation, which is similar to representations resulting from visual perception (e.g., Pearson, Naselaris, Holmes, & Kosslyn, 2015).

Other researchers are, on the whole, dismissive of the role of visualization in human thought. Largely in parallel to the arguments of logicians against external diagrams for logical reasoning, these scholars argue that mental reasoning is solely based on propositional representations as a language of thought. Visual imagery is a mere epiphenomenon that does not play a causal role in mental processes (e.g., Adler & Rips, 2008; Pylyshyn, 2002; Rips, 1994). Under the label "mental imagery debate," these controversies addressed some of the most fundamental questions at the frontiers of the cognitive sciences. They are still not resolved. In the following, the empirical results from the area of logical reasoning are reported.

3.1 Results from Cognitive Psychology

A pioneering study on the role of visual images in logical reasoning was carried out by De Soto, London, and Handel (1965), who argued that reasoners visualize the content of reasoning problems in a mental image and then "read off" the conclusion by inspecting this visual image. Following this idea, several authors assumed that if reasoning relies on visual mental images, then reasoning with problems that are easy to visualize should be easier and accompanied by better performance than reasoning with problems that are hard to visualize. Yet, until recently, the evidence was equivocal. Some researchers found that the ease of visualization helps people to reason accurately, whereas others did not find such an effect. A detailed overview of these ambiguous findings is given in Knauff (2013).

Research from our own laboratory helps to resolve the inconsistency in the previous results and draws an opposite picture to what the mental imagery account suggests: visual mental images can actually impede reasoning and hinder the process of inference. If the content of a reasoning process is easy to imagine visually, people need more time and make more errors than with less visual problems (Knauff, 2013). In Knauff and Johnson-Laird (2002), we empirically identified four sorts of reasoning problems: (1) *visuospatial problems*, which are easy to envisage visually and spatially; (2) *visual problems*, which are easy to envisage visually but hard to envisage spatially; (3) *spatial problems*, which are hard to envisage visually but easy to envisage spatially; and (4) *abstract problems*, which are hard to envisage either visually or spatially. We then asked our participants to solve reasoning problems of these four sorts and measured the error rates and how much time they needed to solve the problem. A theory based on visual mental images would predict that visual (and probably visuospatial) problems are easier to solve. Our prediction, however, was that problems that elicit visual images containing details that are irrelevant to an inference should impede the process of reasoning. Our findings supported this prediction: in several experiments, we found that the ease of visualization impaired reasoning: participants were significantly slower with these problems than with the other sorts of problems. In other experiments, we also found that visualization results in lower reasoning accuracy: people make more errors with visual problems than with the other sorts of problems (Knauff, 2009; Knauff & May, 2006). This is called the *visual impedance effect* (Knauff, 2013; Knauff & Johnson-Laird, 2002).

The visual impedance effect was corroborated in research from several other groups. For instance, studies have shown that one of the main causes for dyslexia (a reading deficit) is the strong tendency of dyslexics to represent the information from a text in a visual, rather than a verbal, way. In a series of experiments, Bacon and coworkers could show that dyslexics are also handicapped in logical reasoning with highly visual materials. The authors explain this with the visual impedance effect (Bacon & Handley, 2010; Bacon, Handley, & McDonald, 2007). Panagiotidou, Serrano, and Moreno-Rios (2018) asked adult participants with reading deficits to solve reasoning problems, which were represented either visually or verbally. They also found a strong visual impedance effect and argue that the visual materials help to study the reasoning abilities of people with and without reading deficits. Tse, Ragni, and Lösch (2017) found that the forming of excessive visual images induced by the premises can impede reasoning and that this interacts with the number of relations used in the problem. Gazzo Castañeda and Knauff (2013) used the well-known Verbalizer–Visualizer Questionnaire (VVQ; Richardson, 1977) to identify two groups of individuals with different cognitive styles. One group consisted of "visualizers," with a strong tendency toward reporting visualization during thinking, and the second group consisted of "verbalizers," with a strong bias toward reporting verbalization during thinking. We found that visualizers showed a strong visual impedance effect with the visual problems but not with nonvisual problems. This effect was stronger than in the nonvisualizers. In Mathias, Vogel, and Knauff (2019), we found similar effects of visual thinking styles in the learning of minimally invasive surgery.

3.2 Results from Cognitive Neuroscience

Several studies showed that areas of the primary and secondary visual cortex are active during visual mental imagery. This supports the assumption that visual perception and visual imagination also overlap on the cortical level (Kosslyn, Ganis, & Thompson; 2001; Kosslyn & Thompson, 2003). Other studies explored the neural correlates of human reasoning. A core result is that reasoning problems activate a complex and widespread network of cortical areas, covering regions in the frontal, temporal, and parieto-occipital lobes (Goel, 2007; Knauff, 2009; Prado, Chadha, & Booth, 2011).

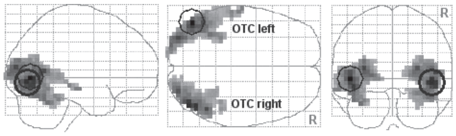
Our group was the first that studied the connection between reasoning, visual mental imagery, and the visual cortices. In one functional magnetic resonance imaging (fMRI) study, we used the same materials as Knauff and Johnson-Laird (2002). While lying in the scanner, participants saw the visual, the visuospatial, the spatial, and the abstract problems and had to evaluate conclusions by mouse clicks (Knauff, Fangmeier, Ruff, & Johnson-Laird, 2003). Two results were important: first, during the solving of all problems, we found activity in areas of the parietal cortex. This indicates that people solved all kinds of problems by constructing and inspecting spatially organized mental representations, which are more abstract than visual images (Knauff, 2013). However, the second result was that only the visual problems resulted in additional activity in the early visual cortex. These areas are related to visual mental imagery in the literal sense, because they are the only regions of the brain that are retinotopically organized, that is, a stimulation of the retina is (almost) analogically represented in these brain areas. Obviously, these areas can be activated not only by visual perception but also by the process of thought. Our interpretation is that, first, the visual problems activate visual mental images in these visual areas but, second, as we found before, these mental images can impede the process of reasoning.

In another study, we used transcranial magnetic stimulation (TMS; Hamburger et al., 2018). TMS uses strong magnetic fields to hinder the neural processing in a well-defined cerebral area and to study how this suppression influences participants' performance in cognitive tasks. In the experiment, we applied the magnetic field to the primary visual cortex. Our assumption was that this should hinder participants' ability to create visual mental images in the visual cortex, and this in turn should help them to reason better than with the visual images they would spontaneously construct, if the activity in these areas were not suppressed. That is exactly what we found. When people could not visualize the content of the reason problem, they reasoned better—which is exactly the opposite of what the visualization theory would assume. In another TMS study, we applied the TMS signal to the parietal cortex and found that this hinders people's reasoning performance. They reasoned less accurately when they could not construct spatial mental models in the parietal cortex (Ragni, Franzmeier, Maier, & Knauff, 2016).

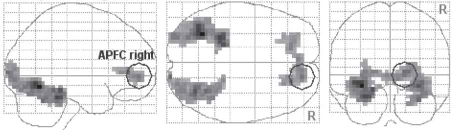
We also used patient studies to explore the visual impedance effect. In one of the studies, we asked congenitally totally blind people to solve our reasoning problems. People who are blind from birth do not experience visual mental images. However, they are as good as sighted people in constructing and utilizing spatial representations. Again, the participants solved visual, visuospatial, spatial, and abstract problems. Interestingly, the visual impedance effect disappeared in those participants. Obviously, congenitally totally blind people are immune to the visual impedance effect, as they do not construct impeding visual mental images (Knauff & May, 2006).

To more precisely study the relation between visualization and reasoning processes, we conducted a further fMRI study (Fangmeier & Knauff, 2009; Fangmeier, Knauff, Ruff, & Sloutsky, 2006). This was the first experiment that was conducted in a way that allowed us to distinguish between different phases of a reasoning process. We distinguished between three different phases: first, people have to understand the premises. Second, they have to integrate the information from the premises into one unified mental representation. Third, they are confronted with a conclusion and have to say whether or not this conclusion logically follows from the given premises. As shown in figure 13.3.2, we obtained different patterns of neural activity for all three phases. In the first phase, we found activity in the primary visual cortex, although all activities related to the purely visual presentation of premises were excluded. In the second phase, this activity was still visible, together with some other activities that were related to executive control, and so forth. In the third phase, which is the actual core of reasoning, something interesting happened: now the activity in the visual cortex disappeared, but another activity in a large cluster in the parietal cortex arose. We argued that this pattern of neural activation explains why people experience visual mental images during reasoning. During the reading and processing of the premises, such visual mental images are present in the visual cortex. This might then help people to maintain the information active in working memory. Hence, people have the feeling that they can see the reasoning problem in their inner eye. However, during the actual reasoning, this activity was not present anymore. Instead, cortical areas were active that are related to spatial processing from different modalities. Typically, people do not become aware of such spatial representations, because they do not have conscious access to representations and processes in the parietal cortex (for details, see Knauff, 2013).

Premise processing phase



Integration phase



Conclusion evaluation phase

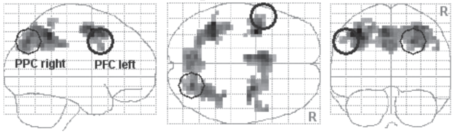


Figure 13.3.2

Neural activation in the three phases of reasoning (from Fangmeier et al., 2006).

3.3 Results from Computational Cognitive Modeling

The idea that reasoning is linked to visualization also aroused researchers' interest in developing computer programs of imagery-based inference. A classic example is the WHISPER system, which could observe and manipulate diagrams to solve problems in the blocks-world environment (Funt, 1980). Kosslyn and Shwartz (1977) programmed a system in which the locations of objects were represented metrically in a matrix of filled and unfilled cells, simulating the use of visual images. Today, many different imagery-based approaches in artificial intelligence exist, notably in the area of diagrammatic reasoning, where researchers seek to use visual images to solve complex problems. A good overview is given in chapter 13.1 by Jamnik (this handbook).

Some computer scientists have developed systems for logical reasoning with visual mental images and with more abstract spatial representations. These computer simulations are helpful in exploring whether visual mental images or spatial representations are more useful on the algorithmic level of human reasoning. One such system was developed by Berendt (1996), who tried to simulate results from our research group. In these experiments, we showed that in the case of ambiguous sets of premises, people prefer one model but ignore other interpretations. The *preferred model* is the one that is easier to construct and manipulate in working memory (Knauff, 1999; Knauff & Ragni, 2011; Knauff, Rauh, & Schlieder, 1995; Rauh et al., 2005). Berendt could show that a system that uses visual mental images is able to reconstruct some of these preferences (Berendt, 1996). However, Schlieder (1999) developed another computational model that just used the order of starting points and endpoints of the object's edges. This means that this representation is spatial in nature—it just needs ordering information—and it is more parsimonious than Berendt's visual system, which uses excessive visual details to perform the inference. Nevertheless, the abstract spatial model was better able to reconstruct the empirical data than the visual model (Schlieder & Berendt, 1998).

In Ragni and Knauff (2013), we developed a computational theory and implemented computer program for reasoning with spatial mental models. The PRISM system can reconstruct many empirical findings from the literature on human reasoning. The basic idea is that working memory is conceptualized as a spatial array of filled and unfilled cells. A spatial scanning mechanism inspects this array to find new information, which is not explicitly given in the premises. In this way, the system can draw logically valid conclusions without representing visual details. It can also reconstruct why people sometimes err and draw inferences that are invalid from a logical point of view. Based on this work, two approaches tried to simulate the visual impedance effect in computational systems (Albrecht, Schultheis, & Fu, 2015; Boeddinghaus, Ragni, Knauff, & Nebel, 2006). The approaches are an interesting starting point, although still much work has to be done to understand the algorithmic details of the visual impedance effect.

4. Corollaries and Future Work

The goal of this chapter was to develop a better empirical understanding of the epistemic role of visualization in human rationality. This is important, as the field is full of speculations from folk psychology and misleading subjective experiences. Sometimes these may even bias scientists toward certain theories. So, what can we really learn from this chapter?

The first corollary is that introspection can be fatally misguiding. This is by no means new. It has long been known that people do not have conscious access to most of their cognitive processes.

The second corollary is that the visual impedance effect provides a new view on the ongoing debates on the epistemic role of visualization in logical reasoning and rationality. Most of these controversies have taken place in philosophy, mathematics, and other areas addressing the *normative function* of visualization. Instead, the present chapter is concerned with *actual* human reasoning. As such, it is part of the goal to develop a comprehensive *descriptive* theory of human thinking. Such a theory also accounts for errors, that is, deviations from what we normatively would call rational.

The third corollary is that visual mental imagery in reasoning is not a mere epiphenomenon: it has a causal power. But this effect is not in the direction that the orthodox idea of visual imagery suggests. Visualization does not help people to reason; actually, it can hinder human reasoning. The reason is that, in the "foreground" of conscious experience, people seem indeed to envisage the content of a reasoning problem in a visual image, at least if the situation is easily visualizable. Since the visual cortex is linked to visual experience, these mental images might be pushed into consciousness. However, in the "background" of conscious experience, the actual logical work is based on more abstract mental representations and processes. Hence, premises that almost automatically evoke vivid visual images initiate a "detour" via the visual cortex, which requires additional time but is useless for the actual reasoning process that works on these abstract representations. This detour may cause the visual impedance effect and may also cause the low utility of external visualizations in the form of pictures or other highly visual illustrations (Knauff, 2013). This largely resembles the normative view of most logicians and philosophers that reasoning should be independent of visualization in order to be rational.

The fourth corollary concerns the impact of the current results for education in logical reasoning. This chapter does not underestimate the power of mental visualization in training and education. We know from many different areas that mental imagery can accelerate learning and leads to better performance in different tasks (Nigel, 2018). The point in this chapter is, however, that the power of visualization depends on many different factors. In particular, very concrete, vivid mental imagery must be distinguished from more abstract spatial representations. Although many people are confident that the former is important for human cognition, it might actually be the case that the facilitating effects of imagination are mostly caused by more abstract—albeit not consciously experienced—spatial representations and processes.

The fifth corollary is that the relation between external and internal visualizations is complex. Sometimes the former can trigger the latter. And this means that it is important how much visual detail is given in external visualizations such as diagrams, figures, pictures, and so forth. Too many seductive visual details in external visualizations may trigger the construction of visual mental images, which can impede cognition. But if external visualizations really focus on the information relevant for the task, they can support thinking and learning (see chapter 13.1 by Jamnik, this handbook). Teachers and pedagogues should carefully consider this important trade-off between eye-catching, nice-looking pictures and meaningful learning.

The sixth corollary from this chapter is that still much work has to be done in the field. On the one hand, logical reasoning is a very particular domain of human cognition. Hence, we probably cannot simply transfer the corollaries from this chapter to all kinds of problems. On the other hand, however, the reported results challenge the positive image that visualization has in science and folk psychology. I hope that this result will trigger further research that goes beyond the area of logical reasoning and human rationality.

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The Handbook of Rationality

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Citation:

The Handbook of Rationality
Edited by: Markus Knauff, Wolfgang Spohn
DOI: 10.7551/mitpress/11252.001.0001
ISBN (electronic): 9780262366175
Publisher: The MIT Press
Published: 2021

Funding for the open access edition was provided by the MIT Libraries Open Monograph Fund.

The MIT Press

The MIT Press would like to thank the anonymous peer reviewers who provided comments on drafts of this book. The generous work of academic experts is essential for establishing the authority and quality of our publications. We acknowledge with gratitude the contributions of these otherwise uncredited readers.

This book was set in Stone Serif and Stone Sans by Westchester Publishing Services.

Library of Congress Cataloging-in-Publication Data

Names: Knauff, Markus, editor. | Spohn, Wolfgang, editor.
Title: The handbook of rationality / edited by Markus Knauff and Wolfgang Spohn.
Description: Cambridge : The MIT Press, 2021. | Includes bibliographical references and index.
Identifiers: LCCN 2020048455 | ISBN 9780262045070 (hardcover)
Subjects: LCSH: Reasoning (Psychology) | Reason. | Cognitive psychology. | Logic. | Philosophy of mind.
Classification: LCC BF442 .H36 2021 | DDC 153.4/3—dc23
LC record available at https://lccn.loc.gov/2020048455