CORRIGENDA.

THE STRONG AND WEAK CONVERGENCE OF FUNCTIONS OF GENERAL TYPE


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I am indebted to Drs. Birnbaum and Orlicz for pointing out that the hypothesis of Theorem 1 is insufficient. The function $f$ is to be assumed to belong to class $Q^1$ instead of to the wider class $Q$. The remaining theorems are unaffected, since Theorem 1 is used only for functions of the class $Q^1$. For an interesting extension of the theorem see Birnbaum and Orlicz, "Über Approximation im Mittel", Studia Math., 2 (1930), 197-206.

ON THE SUMMABILITY OF FOURIER SERIES


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In Theorem 4, that

$$\phi(t) = \frac{1}{2} \{ f(x+t) + f(x-t) - 2x \} \to 0 \ (C, \beta)$$

as $t \to 0$ whenever the Denjoy-Fourier series of $f(x)$ is summable $(C, a)$ to $s$, where $\beta > a + 1$ and $a \geq -1$, my proof for the case $-1 \leq a < 0$, $\beta < 1$, was incomplete*. The proof may be completed by the following lemma, valid also for $0 \leq a < 1$.

**LEMMA.** If $\sum a_n \cos nt$ is the Denjoy-Fourier series of $\phi(t)$ and $a_n = o(n^\gamma)$, then

$$\phi_\beta(t) = \beta \sum a_n \gamma_\beta(nt),$$

for $t \neq 0$, where $\beta > a + 1$ and $-1 \leq a < 0$.

**Proof.** Suppose that $0 < \beta < 2$. Then, writing

$$\gamma_{\beta, \varepsilon}(x) = \int_0^{1-\varepsilon} (1-u)^{\beta-1} \cos xu \, du,$$

we have, since $(1-u)^{\beta-1}$ is of bounded variation in $(0, 1-\varepsilon)$,

$$\int_0^{1-\varepsilon} (1-u)^{\beta-1} \phi(tu) \, du = \sum a_n \gamma_{\beta, \varepsilon}(nt).$$

Moreover, for a given $t \neq 0$, it is easily verified that $\gamma_{\beta, \varepsilon}(nt) = O(\varepsilon^\alpha) + O(n^{-\beta})$, and hence

$$\sum a_n \{ \gamma_{\beta}(nt) - \gamma_{\beta, \varepsilon}(nt) \} = \sum_{n \leq t^{1/\varepsilon}} o(n^\alpha) O(\varepsilon^\alpha) + \sum_{n > t^{1/\varepsilon}} o(n^{-\beta}) = o(\varepsilon^{\beta-1}).$$

The result follows by letting $\varepsilon \to 0$.

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* The existence of $\phi_\beta(t) = \beta \int_0^1 (1-u)^{\beta-1} \phi(tu) \, du$ was in fact established only for almost every $t$.

† See Pollard, 15, and Hobson, 13, (2), 385.