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INVESTMENT SELECTION UNDER UNCERTAINTY FOR PRODUCING OPERATIONS

By

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One of the notable features of the modern oil industry is the constant pressure for increased efficiency to meet the challenge of growing costs and increased competition. With increasingly large capital outlays for producing operations, it has become critical that investment be made as effective as possible. For this reason, considerable attention has been given to techniques for insuring maximum benefits from dollar expenditures. It is to our industry's credit that many quantitative methods for dealing with the myriad of assumptions and variable factors of producing operations have received wide acceptance.

One of the most serious and challenging decisions facing a producing company executive is that of selecting from the investment opportunities available the combination which will provide the highest returns. Since budget funds are usually limited, this selection must be made on the basis of economic and operational merit. The degree of merit can be determined only from the information available to the executive and from his experience.

The treatment of risk in capital budgeting is critical since its measurement is largely subjective in evaluation assumptions and managers' preference for it is variable. Procedures for considering risk range from the simple discounting of expected values of investment returns, to the more complex determination of the utility of the investment. The main disadvantages in existing procedures are the suppression of information and the computational complexity. In brief, uncertainty is a fact of life in producing. We must live with it, and make financial plans accordingly. In dealing with risk, it is the results that should be considered. We must rule out unnecessary risks, but still aim for the high returns.

THE INVESTMENT OPPORTUNITY

Let us restrict the investment opportunities facing a company to producing operations. For convenience, I have grouped them into the following seven categories:

1. Unproven acreage acquisition
2. Proven acreage acquisition
3. Exploratory drilling
4. Development drilling
5. Lease automation and centralization
6. Production development
7. Field processing

For experimental purposes, a hypothetical "average" producing company, or producing division, was assumed to have a number of investment proposals in each of these categories in a given year. Assuming that the operator has limited budget funds, it is desired to allocate the funds available among the projects to select the combination of projects which will provide the optimum, best assured return under all projected conditions. Since it is essential to develop all aspects of the company's operations, corporate objectives are assumed to demand a preassigned level of investment in the projects of each category.

Profit indicators are assumed to have been calculated for all projects. The most effective indicators are considered to be the discounted cash flow rate of return, (or internal rate of return), and the expected value ratio (the revenue stream discounted at the cost of capital, or some target rate of interest, divided by investment cost). We can bypass the controversy as to which of these indicators is better. They are both widely accepted; and their benefits and drawbacks are known. I have used them together to get the good judgement qualities of both. The D.C.F. rate of return is the main decision criterion for the budget allocation scheme, and a limiting value of expected value ratio is applied. However, the indicators can be used interchangably depending on the individual.

The computed profit indicators present the most likely returns from the investment projects. However, any review of the investment proposals of completed projects is bound to reveal that these preliminary values are not often realized. Most executives familiar with a producing area can predict with much greater accuracy the range of likely outcomes for the various projects. The return that they would expect to get from the project might fall between two extreme values. In this range the computed return should refer to the most likely value.

This type of information can only be derived by the executive's experience with similar projects. In this way, the range of likely values can be considered as a probability distribution, or density function, of the returns from an investment project. For purposes of predicting, such a density function should be empirical, based on the available data. However, if the information is largely subjective, too scarce, or not amenable to such an empirical distribution, it may be assumed to belong to an appropriate known functional form, such as the Lognormal or Normal distribution.

The range and frequency of return values of the investment projects will be readily evident from such distributions. Having these return values for all the investment projects, we are now in a position to allocate the budget funds among the projects.

ALLOCATION AND EXPECTED OUTCOME

Formulation

In this problem, the returns are not known with certainty, and it is desired to select the investment projects to be included in the budget on the basis of two or more parameters. For these reasons, the simple convention of ranking the investments in order of rates of return cannot be followed. Instead, we must allocate funds to projects according to the objective of maximizing the total return on investment, subject to the constraints that (a) the total budget funds available will not be exceeded, and (b) that the expected value ratio will be at least as high as some preassigned level. Selection of projects for a capital budget, unlike other allocation problems, requires a direct accept-reject decision as to whether or not funds are to be invested in a project.

The structure of the investment allocation problem is not really new. Problems of this type have been discussed by Dantzig (2) (the "knapsack" problem) and by Cord (1). The problem of maximizing return, subject to budget and expected value constraints, requires two transformations of the initial data. First, since it is more meaningful to deal with dollar values than with dimensionless figures, the variable to be maximized should be the product of the rate of return and the project cost. Secondly, since expected value is to be a minimizing constraint, it is best transformed to a fractional value. This is done simply by dividing the investment project cost by the expected value, to produce an expected value fraction. (This fraction is treated as a weighted average, weighted by the ratio of project cost to total budget.)

The problem may now be stated as follows:

Maximize: $$R = \sum_{i=1}^{N} r_i X_i$$

Subject to:

(1) $X_i = 0$ or 1

(2) $$\sum_{i=1}^{N} c_i X_i \leq B$$

(3) $$\sum_{i=1}^{N} e_i X_i \leq E$$

Where,

c_i = investment project cost of i^{th} project

r_i = product of rate of return and investment cost, c_i, of the i^{th} project

e_i = expected value fraction of the i^{th} project (weighted average)

X_i = a variable constrained to the values 0 or 1, depending on whether the ith project is included in the budget

R = total return

B = total budgetfunds available

E = upper bound on expected value fraction

By introducing a Lagrange multiplier, the problem may be reformulated as a recurrence relationship which can be solved by dynamic programming.

$$f_N(B') = \max_{\substack{0 \leq X_N \leq 1 \\ 0 \leq B' \leq B}} \left[r_N X_N - \lambda e_N X_N + f_{N-1}(B' - c_N X_N) \right]$$

In this relationship, the capital budgeting problem is considered as a multistage allocation problem in which each project is, in effect, a separate stage. B' is incrementally increased from 0 to B for each project and a maximum return is calculated for each of its values. Successive projects are included in the budget if the value added, in combination with the previously included projects, is greater than it would be without the project in question. The Lagrange multiplier is changed after the final stage, and the process is repeated until the constriants are satisfied.

The dynamic programming approach to the problem of allocation of the capital budget provides the advantages of flexibility and versatility in application. Varying numbers of different constraints may be handled and various levels of optimum policies and returns may be obtained. Dynamic programming also provides a full sensitivity analysis, and the computational procedure is rapid and effective.

<u>Projection of results under uncertainty</u>

The optimum combination of investment projects was selected on the basis of the most likely rate of return values for the projects, as determined by economic evaluation. Due to the great uncertainty in the return, it would be of great benefit in investment planning to be able to project what outcome could be anticipated from the various projects. Returns have been considered to be known as probability distributions, either empirical or known forms, which are determined from the nature and environment of the projects. With this knowledge, execution of the projects may be simulated by the Monte Carlo method of sampling from the appropriate density function. Monte Carlo simulation involves unrestricted random generation of events, placed in the established frequency patterns for returns and for successes. A sample trial may be considered as a simulated "execution" of the project in question. However, since the procedure is rather simple, many trials for each project may be quickly carried out. The results of a number of trials can be used as a basis for prediction of the future returns from any one project or from the entire budget group. Also, the effect and dimension of the risk involved is readily evident. In this way, we may make the most accurate prediction allowed by the available information, of the results of investment planning, and take appropriate action.

The Return Functions

In the return functions we know, as a minimum, the two extreme return values, the most likely value, or the mode, and also, the shape of the functional form, either from available information or from general experience. The extreme values of the returns, as determined by the manager, can be considered as one standard deviation on either side of the mean. This assumption seems reasonable at this stage since some 68 per cent of the values of the distribution will fall within this range. If the range in returns, given initial success, were greater than this, it is doubtful if the project would be considered except in some exceptional cases. Monte Carlo simulation may be carried out in this range of the distribution for each project as a basis for prediction of returns.

If an empirical distribution is not available for an investment project, one of several known distributions may be used. Many excellent papers have been presented on the applicability of such distributions to various problems. The Lognormal distribution has received wide use in the analysis of economic data and in mineral deposit evaluation, due to its premise of multiplicative growth. A most notable contribution was made by Kaufman (3), who showed that the size and frequency of oil and gas fields could be predicted by the lognormal distribution. Following his reasoning, the returns from field or production development projects may be considered to be lognormally distributed.

Returns from other projects where data is incomplete may, for various reasons, be best approximated by the Normal distribution. Projects involving construction and installation of new equipment or facilities may best fall into this category.

The "Success" Function

Many investment projects facing a producing company are uncertain not only in the measure of return, but also in whether any success is realized at all. This is the case in exploratory operations, as exemplified by the long odds against a wildcat well making a discovery. Monte Carlo simulation can again be of assistance to the investment planner in the prediction of exploration success.

In a broad oil and gas region, such as the Southwestern States, many prospective areas have been outlined where potentially hydrocarbon-bearing horizons are known. Let us consider the area where each of these horizons is prospective as a hydrocarbon-bearing reservoir as a cell, such that everywhere in this cell, equal probabilities exist for commercial discovery from this particular target horizon. Cells may overlap in areas where several target horizons occur, thereby enhancing the probability of a hit. These probabilities of success can again be assigned by the executive familiar with operations in the area in question.

Under these conditions, wildcat drilling in the individual cells may be assigned a binomial probability of success, and each wildcat test would correspond to a single trial. Monte Carlo sampling may now be carried out on the geometric distribution formed in this way. From the results, the success and risks of the exploration program may be predicted. When the simulated return values are applied to the successful projects, the returns from the exploration program, and hence from the entire budget can be derived.

Assimilation

We have now simulated the operating system, producing a series of trials of our budget allocation. Frequency distributions may be constructed for the returns from the entire budget or for any one project. Following convention, the expected value of the budget can be indicated by the mean of these distributions of returns, and the variances of the returns can be used as measures of uncertainty

Simulation used in this fashion can be of significant assistance as a device to check a company's investment planning. If we can test a plan on the basis of our best information concerning the frequency of all possible outcomes, we are in a much better position to evaluate the returns subject to the risks and inherent dangers in it. Also, any changes in the plan, or any manipulation of those factors subject to management control may be tried out experimentally on paper before committing company funds.

The sequence of techniques presented here represents an initial application of a rather new tool and a slightly different approach to an old problem. Investment planning is becoming increasingly important in our industry. The dynamic programming approach to the allocation of funds fills an important need in this area; it provides the plan of expenditure leading to the highest profits. Subsequent testing will indicate any inherent dangers in the plan. These techniques use only the manager's experience and the available information. However, the presentation of the best spending plan can assure the decision maker that the information has been used with maximum efficiency.

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CAPITAL INVESTMENT ALLOCATION MODEL

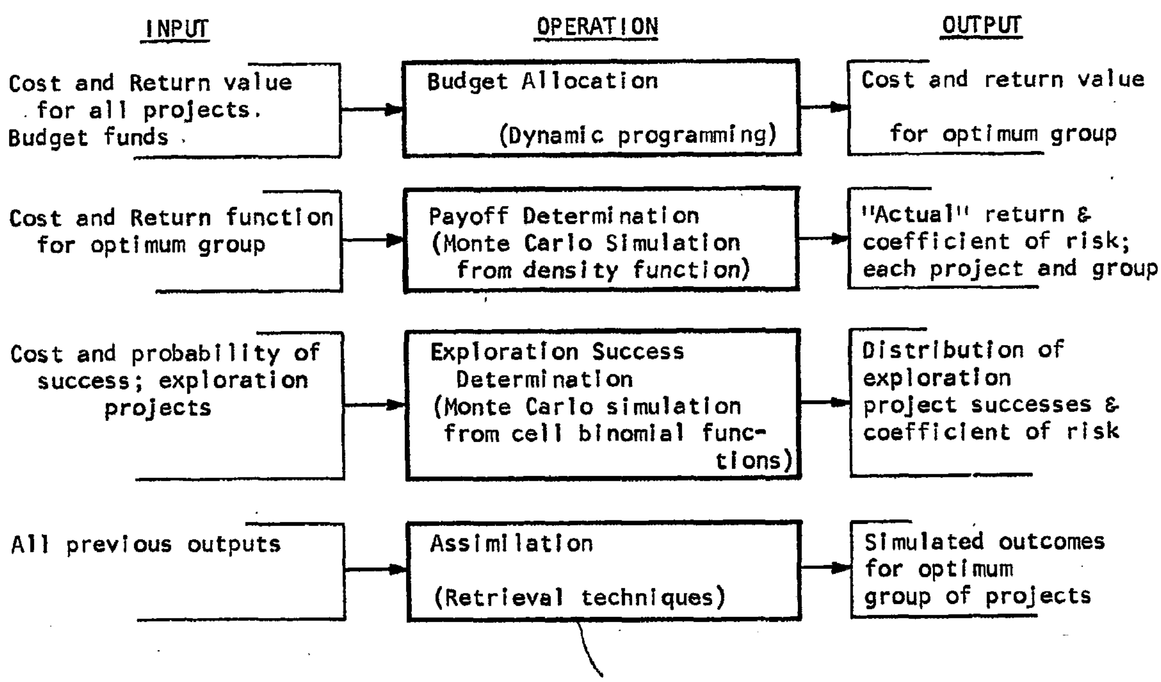

Figure 1

Given the results of economic evaluation of all investment projects as inputs, the operations may be regarded as "black boxes" through which the respective outputs are determined.