SPE 1099

THE INFLUENCE OF PROJECT PROFITABILITY ON COMPANY GROWTH

BY

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The financial structure of a large company is quite complex. We are able, however, to follow the trend of such elements as the annual net cash income I (which is the income after paying all costs, including overhead and taxes), the annual net profit P (which is the annual net cash income minus writeoffs) and the dividend paid out. Other characteristics of a company, such as the average annual earning power i and the market value V, are not readily detectable but are related in some manner to the aforementioned elements.

In an attempt to establish mathematical relationships between the various elements a simple model of a company (Figure 1) was studied. This model starts with an investment of \$1000 in a project which yields \$400 annually for three years. (It is assumed in the model that the incomes are received at the end of each year.) From the definition of earning power, present value income discounted at earning power i equals investment, or $D_i(1200) = 1000$, the deferment factor D_i is found to be 0.83. Using deferment factor tables, the value of the earning power i of this project is found to be 10 percent. All incomes derived from the first investment are reinvested in projects with the same characteristics as the first (i = 10 percent, three year constant income). The annual cash income first shows an irregular trend but, as shown in the top curve of Figure 2, after some time this trend stabilizes and the growth rate of I becomes constant. For complete reinvestment of cash incomes the following important relation is true:

$$\text{growth rate } g = \text{earning power } i. \qquad (1)$$

If, however, as shown in the lower curve of Figure 2, seven percent of I is paid out as dividend, then the reinvestment ratio b is 0.93 and, although the cash income after stabilization obtains a constant growth rate g, this rate is less than the earning power i. The relation between b, i and g is expressed by

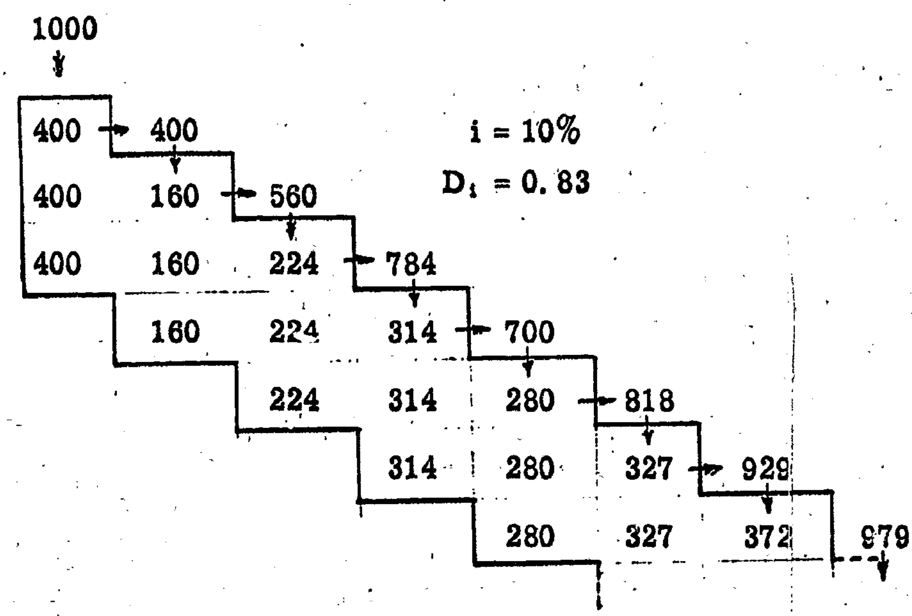

1000
i = 10%
D₁ = 0.83
400 400
400 160 560
400 160 224 784
160 224 314 700
224 314 280 818
314 280 327 929
280 327 372 979

figure 1

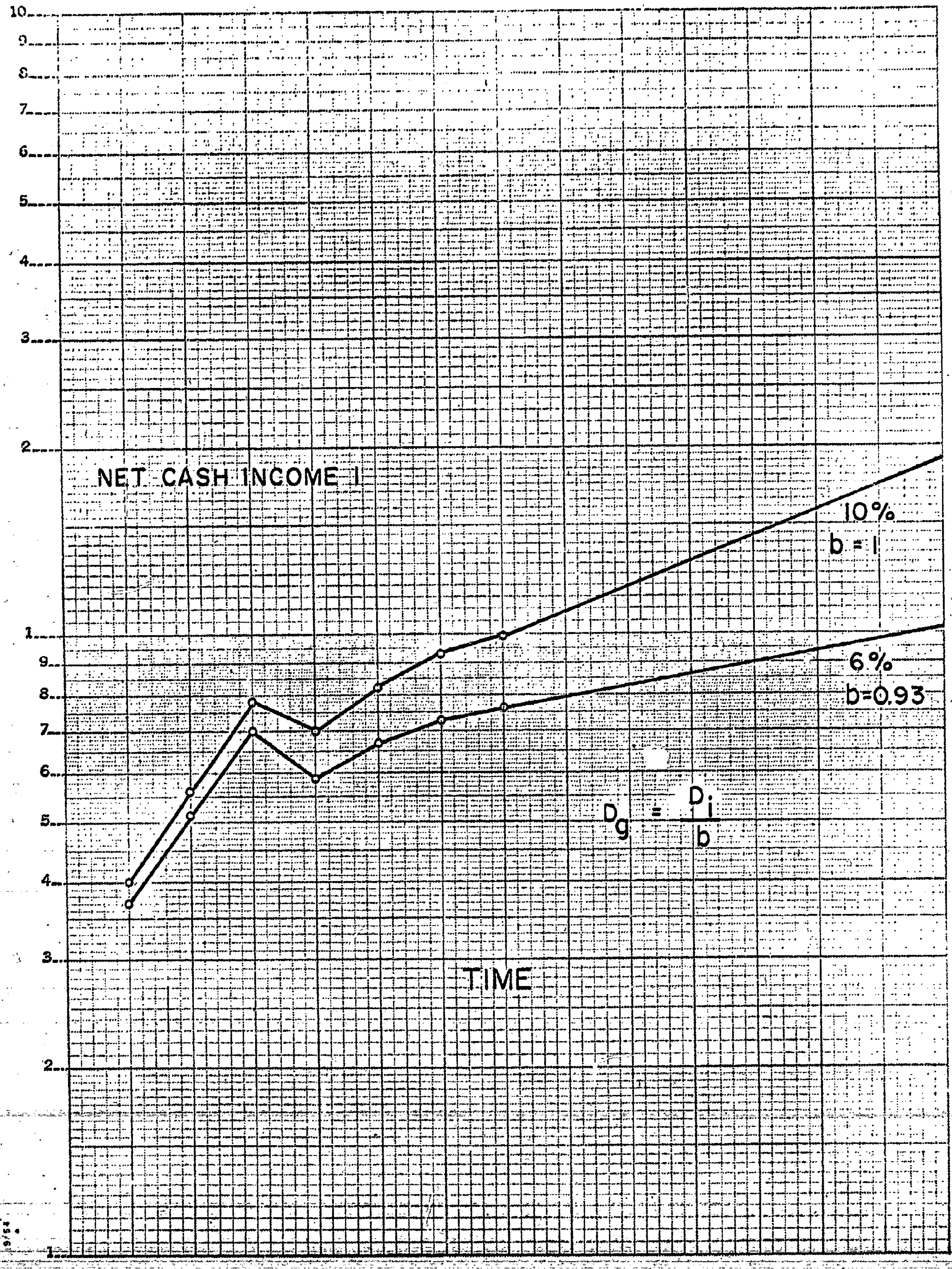


figure 2

$$D_g = \frac{D_i}{b} . \quad (2)$$

The proof of (2) is omitted here but is not hard to find.

To show the meaning of (2), we relate it to our model with $i = 10$ percent and a life for every project of three years with constant income rate. From tables, D_i (the deferment factor based on a discount rate of $i = 10$ percent) is found to be 0.83. With $b = 0.93$, from (2) it is found that D_g (the deferment factor based on a discount rate of g) is 0.89. Again from the same tables, it is read that the growth rate g is six percent.

Equation (2) has certain shortcomings in its applicability. First, it determines the growth rate g of the cash income I, which is of limited interest. Also, the relation between i, g and b is not uniquely determined, but because the deferment factors D_i and D_g enter into it, this relation depends on the time sequence of the project (income type). Therefore, two other, more revealing elements were studied in the model. One, the annual profit P is, as stated before, equal to I minus an amount written off for amortization of previous investments. As is well known, the methods by which these amortizations are made is immaterial because it is only a bookkeeping procedure which does not affect the amount of money available for reinvestment. The other element is the present value of all future incomes derived from past investments, discounted at a rate equal to the earning power i. This is symbolized by M. Note that M is not the same as the market value V of a company, but a relation exists between the two which is subject to further study. In Figure 3 the growth rates of M, I and P are shown for constant earning power i and reinvestment rate b. They are all the same for long term income. If, however, the income type suddenly changes from long term to short term, the stability is disturbed even if i and b are kept the same. A study of the model revealed that if e (which is the ratio of dividend to net profit) is kept a constant, I, M and P will behave as shown in Figure 3. This means that under this condition the trend of M is unaffected by a change in income type, P is hardly affected, whereas I shows a large jump before returning to the same growth rate as before.

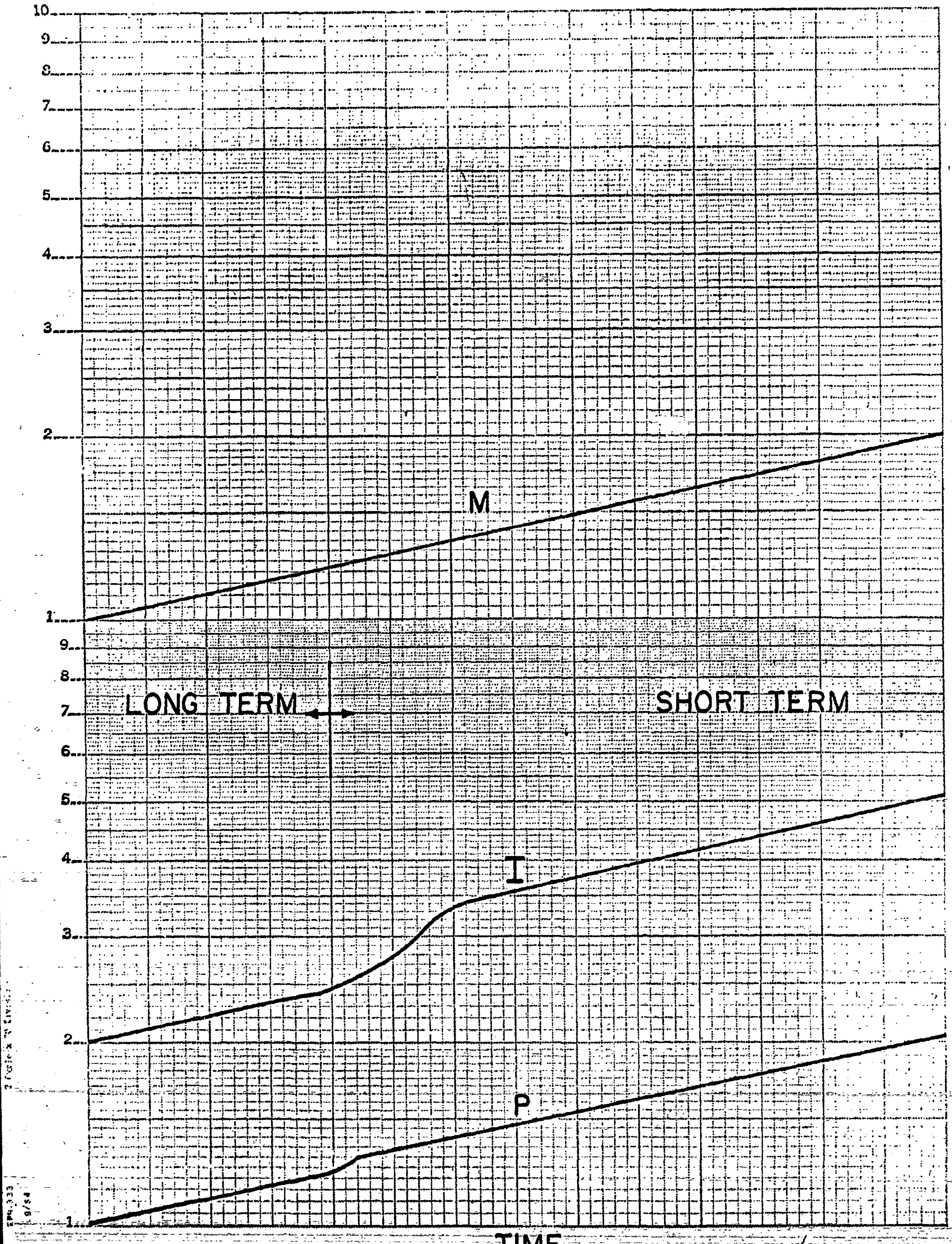


figure 3

Therefore, if we now define g as the growth rate of the net annual profit P; then, provided that the ratio e = dividend/net profit is kept constant, this growth rate is unaffected by the income type of the projects (long or short term, declining or constant rate) and can be proven to be a function of only i and e by the equation

$$g = i(1 - e), \tag{3}$$

which is an approximation. Equation (3) shows how the growth rate of P, which is equal to the growth rate of M, is related to the average earning power i of the company under equilibrium conditions. The growth rate g of the annual net profit P can be determined from a plot of data contained in the 10 year survey of a company stockholder's report. The average value of the dividend ratio e = dividend/net profit can also be determined from these data. Then equation (3) will give an approximate figure for the company earning power. For instance, if g is found to be six percent per year and e = 0.5, then $i = g/(1 - e) = 12$ percent. A further relation

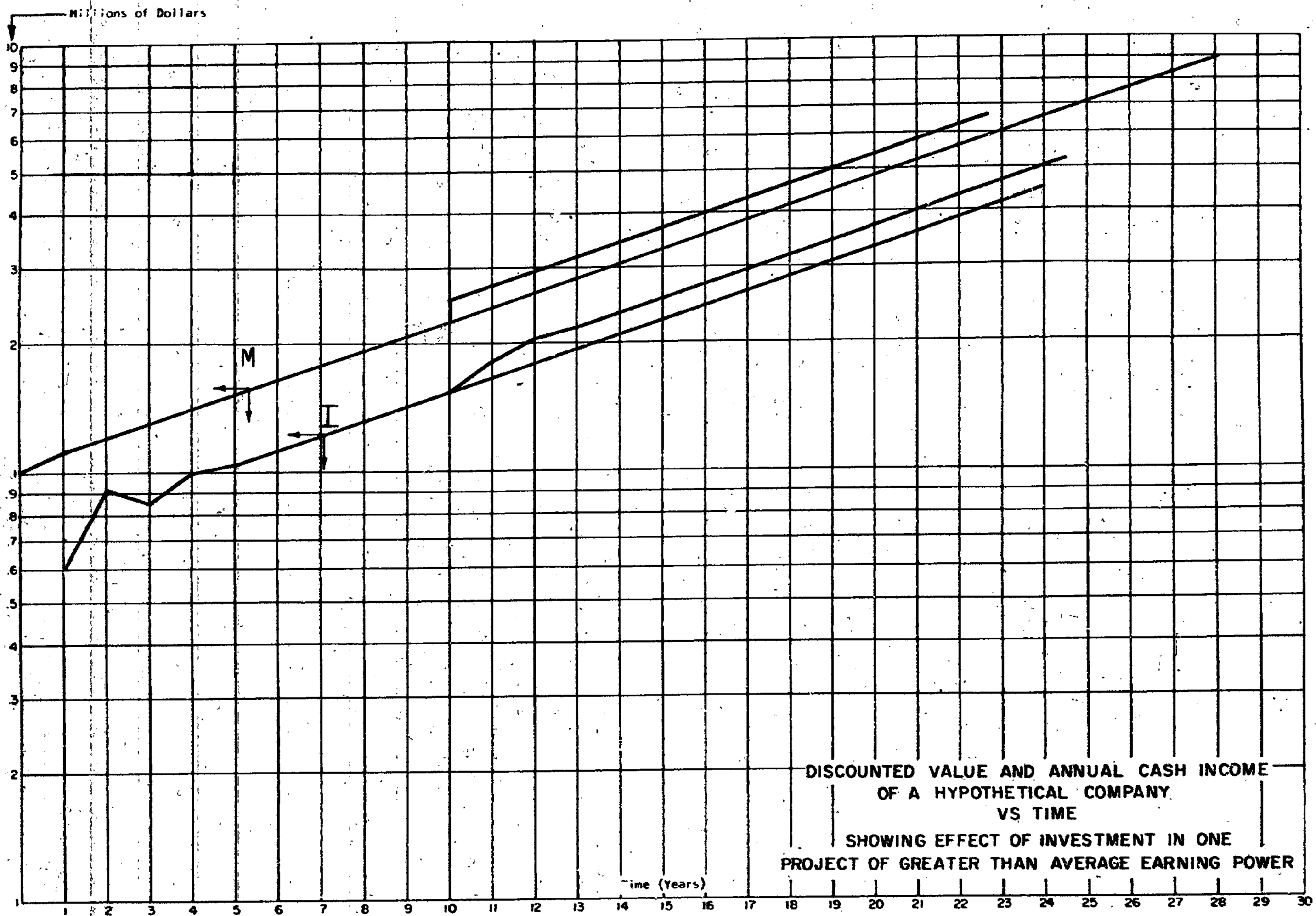
$$M = P\frac{1+i}{i}, \tag{4}$$

which is also an approximation, may be useful in further studies.

In Figure 4 the influence of a single project on the financial position of a company is shown. Let the present value of the profit from this project, discounted at average company earning power be, say, \$1000. Then, at the moment of investment the value of M increases by \$1000, but the future growth is unaffected as long as the average earning power of future investments remains the same.

In the table below the symbols and relations of the various elements are summarized.

i = earning power	$e = \frac{\text{div.}}{P}$
g = growth rate	$g = i(1 - e)$
I = annual cash income	$D_i = \frac{I - P}{I}$
P = annual profit	M = p.v. future incomes
$b = \frac{I - \text{div.}}{I} = \frac{D_i}{D_g}$	$M = P\frac{1+i}{i}$



Millions of Dollars
10
9
8
7
6
5
4
3
2
1
.9
.8
.7
.6
.5
.4
.3
.2
.1
M
I
DISCOUNTED VALUE AND ANNUAL CASH INCOME OF A HYPOTHETICAL COMPANY VS TIME
SHOWING EFFECT OF INVESTMENT IN ONE PROJECT OF GREATER THAN AVERAGE EARNING POWER
Time (Years)
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

FIGURE 4

Concluding, a simple model of a company was shown to be suitable for detecting mathematical relations between certain characteristic properties as given by equations (1) to (4). Several questions have been left unanswered even in the model, and difficulties will occur when proceeding from the simple to the more complex model. However, we believe that the approach given here is a promising one. To quote the political economist Irving Fisher (Trans. of Conn. Academy, 1892):

"The economic world is a misty region. The first explorers used unaided vision. Mathematics is the lantern by which what before was dimly visible now looms up in firm, bold outlines. The old phantasmagoria disappear. We see better. We also see further."