The Difference Between Nominal and Effective Interest Tables And Nominal and Effective Rates of Return

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Abstract

Several different types of interest tables are commonly used by petroleum engineers. Conventional (or year-end) and midyear tables are based on an effective interest rate i, while continuous interest tables are expressed as a function of a nominal rate j. To avoid confusion, a rate of return i calculated using midyear or year-end tables should be called an effective rate of return, while a rate of return j calculated using continuous interest tables should be called a nominal rate of return. When either type of rate is known, the other can be determined readily from a simple equation.

Conversion factors to adjust receiving income other than at the end of a period are derived and their use discussed.

Introduction

The use of interest tables seems to be quite simple. However, there are a number of different types of commonly used interest tables that give different results. For instance, for a single problem one engineer using conventional interest tables might calculate a rate of return of 41 per cent, another using midyear tables might calculate a rate of return of 55 per cent, while still another using continuous interest tables might calculate a rate of return of 44 per cent. This occasionally can result in considerable confusion when two negotiating companies use different types of interest tables, or when different departments of a company use different types of tables.

The purpose of this paper is to point out the relationship between the various types of interest tables, to show how results compare, and to show how answers obtained using one type of table can be converted to equivalent answers from other types of tables.

Nomenclature and Symbols

In addition to the different types of tables in common use, there are a number of ways to express compound interest to account for different patterns of income or to simplify the solution of different types of interest problems. A set of interest tables, of any type, may include a compound interest table, a present worth table, several annuity tables, a sinking fund table, a partial payment table, an equivalent nominal rate table, auxiliary tables, or other tables.

Such interest tables are related through simple mathematical expressions. Unfortunately, there is no uniformity in what we call these factors, or tables. Identical tables, obtained from different sources, may have completely different, apparently unrelated, names (i.e., present value vs principal, which will amount to a given sum).[1,2] On the other hand, similar nomenclature is often used with the different types of tables. Thus, a set of conventional interest tables and set of continuously compounded interest tables might both include a table labeled "present worth", which will contain different factors within the tables.

A similar lack of standardization exists with regard to the mathematical symbols used in interest equations.[1-5] Such lack of standardization might cause a casual user of interest tables to become confused. The nomenclature and symbols used in this paper are similar to those used by the Financial Publishing Co.[5]

Conventional Compound-Interest Equations and Tables

Conventional interest tables are based on the premise that interest is compounded periodically and that income, or payments, is received at the end of each period. The following are the common ways that compound interest can be expressed.

Amount of $1 at Compound Interest

The amount of compound interest is often called the "amount of 1", the "single payment compound amount", or the "amount". It shows how $1 at compound interest will grow. Expressed as an equation, it is

$$(\text{Amount of } 1)_I = s = (1+i)^n \;, \quad . \quad . \quad (1)$$

where i = the uniform rate of interest, fraction

n = the number of periods of interest conversions,

and I indicates conventional interest equations, where income is received and compounded at the end of each period.

The amount at the end of each period is obtained by multiplying the amount at the beginning of the period by the ratio of increase $(1+i)$. A compound interest table can be constructed for any rate by successive multiplication by the ratio of increase.

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[1]References given at end of paper.

Amount of $1 Per Period (Amount of Annuity)

An annuity is a series of periodic investments or payments. The "amount of annuity" shows how $1 deposited periodically at compound interest will grow. All deposits are made at the end of a period. At the end of each period, $1 has just been deposited and has not yet grown. The amount of $1 per period may be obtained by adding the "amount of $1" for ($n-1$) periods plus $1. Expressed as an equation, it is

$$(\text{Amount of 1 per period})_I = S_n$$
$$= 1 + (1+i) \ldots (1+i)^{n-1}$$
$$= \sum_{n=0}^{n-1} (1+i)^n \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2\text{-A})$$
$$S_n = \frac{(1+i)^n - 1}{i} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2\text{-B})$$

Sinking Fund (Annuity Worth $1 in the Future)

A sinking-fund table shows the amount that must be deposited periodically to grow to $1 in the future. It is the reciprocal of the "amount of $1 per period". If $1 deposited periodically for n periods will grow to $25 at the end of n periods, then obviously $.04 deposited periodically for n periods at the same rate of interest will grow to $1. Expressed as an equation,

$$(\text{Sinking fund})_I = \frac{1}{S_n} = \frac{i}{(1+i)^n - 1} \quad . \; . \; . \quad (3)$$

Present Worth of $1 (Present Value)

Present value is the worth today of $1 due some time in the future. It is the amount that must be deposited at compound interest today to grow to $1 in the future. If $1 deposited today will grow to $\$s$ after n periods, then $\$(1/s)$ deposited at compound interest today will grow to $1 in the future. If $1 deposited today will grow to $\$s$ after n periods, then $\$(1/s)$ deposited today at the same rate of interest will grow to $1 after n periods. Thus, present value is the reciprocal of the "amount of $1". Expressed as an equation,

$$(\text{Present worth of } \$1)_I = s^{-1} = (1 + i)^{-n} \quad . \; . \quad (4)$$

A table of present-worth factors can be constructed easily for any interest rate i by successive multiplication by the ratio of decrease $(1+i)^{-1}$. Present-worth factors are commonly used by engineers in many economic problems.

Present Worth of $1 per Period (Present Worth of an Annuity)

This table shows what $1 received periodically for n future periods is worth today. It shows the sum of the present value of a series of future $1 payments. Expressed as an equation,

$$(\text{Present worth of } \$1 \text{ per period})_I = a_n$$
$$= \frac{1-(1+i)^{-n}}{i} \quad , \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

or

$$a_n = (1+i)^{-1} + (1+i)^{-2} + \ldots + (1+i)^{-n}$$
$$= \sum_{n=1}^{n} (1+i)^{-n} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

This table is useful in evaluating investments that produce a constant yearly income. The factor in the table is numerically equal to the time required to return an investment (the payout time). Thus, for a constant yearly income received for n years,

$$\text{Payout time} = a_n = \frac{1-(1+i)^{-n}}{i} \quad . \; . \; . \; . \; . \quad (7)$$

Partial Payment (Annuity Whose Present Worth is $1)

This table shows the amount that must be received periodically for n periods to be worth $1 today. It shows the periodic payment necessary to pay off a loan of $1, or the amount of periodic payments which could be obtained for n periods from a $1 investment. If a series of $1 future payments has a present value of $\$S$ today, then a series of $\$(1/S)$ future payments has a value of $1 today. The partial payment is therefore the reciprocal of the present worth of $1/period. Expressed as an equation,

$$(\text{Partial payment})_I = \frac{1}{a_n} = \frac{i}{1-(1+i)^{-n}} \quad . \; . \; . \quad (8)$$

The amount of partial payment is also equal to the amount of the sinking fund plus the rate of interest, or

$$\frac{1}{a_n} = \frac{1}{S_n} + i \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

The difference between the sinking fund and the partial payment should be noted. The amount of a sinking fund represents the amount that must be set aside periodically to grow to $1 in the future. The amount of partial payment represents the amount that must be set aside to periodically grow to a future amount whose present value is $1. The partial payment table is commonly used by bankers in calculating loans. It is the table you can use to determine that your friendly car dealer is probably charging you only 2 per cent per month on the unpaid balance of your loan.

Auxiliary Tables

Auxiliary tables are generally included in a set of interest tables to aid in calculating interest for fractions, or multiples of a unit period. The auxiliary tables are for measuring the amount of growth for time intervals other than a unit period. The tables use the same formulas as other tables, except n is expressed in fractions or multiples of a unit period.

$$(\text{Amount of 1})_I = s = (1+i)^m \quad , \; . \; . \; . \; . \quad (10)$$
$$(\text{Present value})_I = s^{-1} = (1+i)^{-m} \quad , \; . \; . \; . \quad (11)$$

where m = fractions, or multiples, of a unit period.

Compounding Other Than Annually (Nominal Rate and Effective Rate)

It is often desirable to compound interest at more frequent intervals than yearly periods. For instance, a bank may pay interest on deposits quarterly, and will receive payments on loans monthly. In these instances, the period does not represent one year, but a fraction of a year. When the period is for other than one year, the uniform rate of interest will not represent an annual rate. An interest rate can be expressed as a *nominal annual rate or nominal rate.* The nominal rate is the rate per period times the number of periods per year. For example, an interest rate of ½ per cent per period is equal to a nominal annual interest rate of ½ per cent if interest is compounded yearly, a nominal rate of 1½ per cent if interest is compounded quarterly, or a nominal rate of 6 per cent if interest is compounded monthly.

When a *nominal interest rate* is used, the nominal rate j is divided by the number of periods per year p, and the total number of periods is equal to the number of years times the periods per year. For instance, for a nominal rate j,

$$\text{Amount of } 1 = \left(1+\frac{j}{p}\right)^{pn} \quad , \; . \; . \; . \; . \; . \quad (12)$$

where j = nominal yearly rate
n = number of years (instead of the number of conversion periods)
p = number of interest conversion periods per year
j/p = uniform rate of interest per period
np = number of interest conversion periods.

If interest is compounded continuously, p is equal to infinity. From series expansions, the following identity can be obtained.

$$\lim_{p \to \infty} \left(1 + \frac{j}{p}\right)^{pn} = e^{jn} \quad , \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and, by substitution in Eqs. 1, 4 and 5,

$$(\text{Amount of } 1)_{\text{II}} = e^{jn} \quad , \quad . \quad . \quad . \quad . \quad . \quad (14)$$

$$(\text{Present worth of } 1)_{\text{I}} = e^{-jn} \quad , \quad . \quad . \quad . \quad . \quad . \quad (15)$$

$$(\text{Present worth of } 1/\text{period})_{\text{II}} = \sum_{1}^{n} e^{-jn} = \frac{1 - e^{-jn}}{e^{j} - 1} \quad , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where subscript II indicates continuous compounding, income received at the end of a period.

Eqs. 14 through 16 are used in calculating continuously compounded interest tables, and give the compound amount at the end of a period, or the present worth of income received at the end of each period.

Equal amounts left to grow at compound interest at the same nominal rate, but compounded for a different number of periods per year, will vary considerably over a long period, as shown in Table 1. Continuous compounding closely approximates monthly compounding.

Table 1 shows that \$1 compounded continuously for one year at a nominal rate of 6 per cent will grow to \$1.06184. However, \$1 compounded yearly at a rate of 6.184 per cent will also grow to \$1.06184 at the end of one year, and to \$403.43 at the end of 100 years. For example,

$$e^{(0.6)\,(100)} = (e^{0.6})^{100} = (1.06184)^{100} = 403.43.$$

Thus, a nominal annual rate of 6 per cent compounded continuously is equal to an effective annual rate of 6.184 per cent compounded yearly. The effective annual rate, or effective rate, is the rate i compounded yearly that is equivalent to some nominal rate j compounded for any period. Note from Table 1 that for any given nominal rate j the effective rate i will vary, depending upon the number of compounding periods per year p. The nominal rate j and the effective rate i are related by the following equations:

$$(1+i) = \left(1 + \frac{j}{p}\right)^{p} = (e^{j})_{p=\infty} \quad , \quad . \quad . \quad . \quad . \quad (17)$$

$$\text{Effective rate} = i = \left[\left(1 + \frac{j}{p}\right)^{p} - 1\right] = (e^{j} - 1)_{p=\infty} \, , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

TABLE 1—AMOUNT OF \$1 LEFT AT COMPOUND INTEREST AT 6% NOMINAL RATE j

Year (n)	Period of Compounding: Yearly	Semiannually	Monthly	Continuously
1	1.0600	1.0609	1.0617	1.06184
5	1.3382	1.3439	1.3488	1.3499
10	1.7908	1.8061	1.8194	1.8221
20	3.2072	3.2620	3.3101	3.3201
50	18.420	19.219	19.934	20.086
100	339.30	369.34	397.37	403.43
Effective rate (i)	6%	6.09%	6.17%	6.184%

$$\text{Nominal rate} = j = p\left[(1+i)^{1/p} - 1\right]$$

$$= \left[\ln(1+i)\right]_{p=\infty} \quad , \quad . \quad . \quad . \quad . \quad . \quad (19)$$

where p = number of compounding periods per year.

Tables showing the relationship of j to i, or of i to j, are often included in a set of interest tables. A plot of the relationship between i and j, over a wide range of values, is shown in Fig. 1.

The distinction between nominal rate and effective rate is important. Continuous interest tables are constructed for nominal rates j. Midyear and conventional (year-end) tables are based on an effective rate i. Previously it was stated that a calculated rate of return of 55 per cent using midyear tables was equivalent to a calculated rate of return of 44 per cent using continuous interest tables. The rate of return calculated from midyear tables (55 per cent) is an effective rate. The rate of return calculated using continuous interest tables (44 per cent) is a nominal rate. In this case, the nominal rate (44 per cent) can be converted to an effective rate by Eq. 18.

$$i = e^{j} - 1 = e^{0.44} - 1 = 0.5527.$$

Thus, a nominal rate of return of 44 per cent is equivalent to an effective rate of return of 55 per cent. Unfortunately, we rarely specify whether a rate of return is a nominal rate or an effective rate. This can result in a misunderstanding when we are dealing with another company which uses a different type of interest table.

Receipt of Income Other Than at the End of a Period

Eqs. 1 through 16 are all based on the assumption that income, or payments, is received at the end of a period. Income from producing oil and gas properties is usually received monthly (some individuals claim that income is received continuously). Using monthly periods in the interest calculation is tedious, requires extensive tables, and is seldom used. Several methods which simulate a monthly receipt of income but utilize yearly tables are commonly used in the petroleum industry.

Value of Money Received Uniformly During a Year

Money received throughout a period has a greater value to the recipient than the same amount of money received at the end of the period, since it can be reinvested sooner. For instance, if an amount of $\$(1/p)$ is received and reinvested p times per year at some nominal rate j, then from Eq. 2,

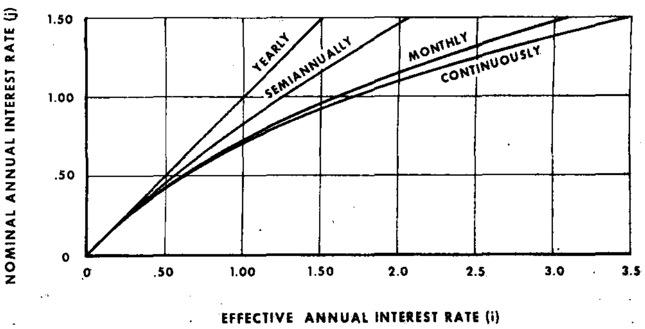

Fig. 1—Relationship between effective rate i and nominal rate j for various compounding periods.

$$\text{Relative increase in value} = a = \left(\frac{1}{p}\right)\left[\frac{\left(1+\frac{j}{p}\right)^{p}-1}{j/p}\right]$$

$$= \frac{\left(1+\frac{j}{p}\right)^{p}-1}{j} = \frac{i}{j} \quad , \quad . \ . \ . \ . \ . \ . \quad (20)$$

or

$$(a)_{p=\infty} = \frac{i}{\ln(1+i)} = \frac{e^{j}-1}{j} \quad . \ . \ . \ . \ . \ . \ . \ . \quad (21)$$

These equations give the relative increase in value when money is received uniformly throughout an entire period instead of at the end of the period. Multiplying the factors shown in the above equations by the present value that the money would have if it were received at the end of a period, gives the actual present value of the money (assuming that the money is reinvested immediately upon receipt at a rate i or j).

The above factors apply only when single, whole periods are involved. They are not valid if fractional, or multiple, periods are included in an interest calculation. If an amount of $\$(1/p)$ is received and reinvested mp times, then the actual value of the money reinvested after m years is equal to

$$am = \left(\frac{1}{p}\right)\left[\frac{\left(1+\frac{j}{p}\right)^{mp}-1}{j/p}\right] = \frac{\left(1+\frac{j}{p}\right)^{mp}-1}{j} \ ,$$

and the relative increase in value of the money reinvested is equal to

$$a = \frac{\left(1+\frac{j}{p}\right)^{mp}-1}{jm} = \left[\frac{(1+i)^{m}-1}{m\ \ln(1+i)}\right]_{p=\infty}\left[\frac{e^{jm}-1}{jm}\right]_{p=\infty} , \quad . \ . \ . \ . \ . \ . \ . \quad (22)$$

where

m = fractional part, or multiple of a year.

The conversion factors contained in Eq. 22 are numerically equal to those in Eqs. 20 and 21 when $m = 1$. When income taxes are included in an economic calculation, it is necessary for the selected calculation periods to correspond to fiscal years instead of successive 12-month periods. This usually results in a fractional initial period. In this situation, the conversion factors shown in Eq. 22 must be used in calculating interest factors rather than the commonly used factors shown in Eqs. 20 and 21.

Conversion of Interest Equations and Tables

The above factors can be used to convert Eqs. 1 through 9 or Eqs. 14 through 16 to account for uniform receipt of income throughout a period instead of at the end of the period. For instance,

$$(\text{PW of } 1)_{\text{I}} = (1+i)^{-n} \quad , \quad . \ . \ . \ . \ . \ . \ . \quad (4)$$

$$(\text{PW of } 1)_{\text{IV}} = (1+i)^{-n}\left[\frac{(1+i)^{m}-1}{m\ \ln(1+i)}\right] \quad . \ . \ . \quad (23)$$

where I is periodic compounding, income received at the end of nth period, and IV is periodic compounding, income received throughout the nth period; and

$$(\text{PW of } 1)_{\text{II}} = e^{-jn} \quad , \quad . \ . \ . \ . \ . \ . \ . \ . \quad (14)$$

$$(\text{PW of } 1)_{\text{III}} = e^{-jn}\left[\frac{e^{jm}-1}{jm}\right] \quad , \quad . \ . \ . \ . \ . \quad (24)$$

$$(\text{PW of } 1)_{\text{III}} = e^{-jn}\left[\frac{e^{j}-1}{j}\right]_{(\text{for } m=1)} \quad , \quad . \ . \ . \quad (25)$$

where II is continuous compounding, income received at the end of each period, and III is continuous compounding income received uniformly throughout each period.

Eq. 25 is the equation usually presented for continuous compounding where income is received uniformly throughout each period. However, it applies only for whole periods, and is not a valid equation for a fractional period. For example, when $j = 0.2$, $n = ½$,

$$(\text{PW of } 1)_{\text{III}} \neq e^{-0.1}\left[\frac{e^{0.2}-1}{0.2}\right] = \left[0.904837\right]\left[1.107\right]$$
$$= 1.00165.$$

Note that the equations grouped above are identical except for the conversion factor enclosed in the brackets. Conversion factors can be used to construct special interest tables, or they can be used to convert answers found using conventional interest tables to account for a uniform receipt of income, or other special conditions. The following equations can be utilized to convert values determined using conventional interest tables.

$$\text{Amount}_{(\text{special conditions})} = \text{Conversion Factor} \times \text{Amount}_{(\text{year end})} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (26)$$

$$\text{Present Worth}_{(\text{special conditions})} = \text{Conversion Factor} \times \text{Present Worth}_{(\text{year end})} \quad . \ . \ . \ . \ . \quad (27)$$

It is not usually necessary to correct each factor in the table. Unless fractional or multiple periods are involved, only the answer needs to be corrected.

Midyear Tables

A commonly used method to account for monthly receipt of income is to assume that each year's income is received at midpoint of the year. The following equations are used to construct midyear interest tables.

$$(\text{Amount of } 1)_{\text{VI}} = (1+i)^{n-\frac{1}{2}} = (1+i)^{n} \cdot (1+i)^{-\frac{1}{2}} \ , \quad . \ . \ . \ . \ . \ . \quad (28)$$

$$(\text{Present Worth})_{\text{VI}} = (1+i)^{-(n-\frac{1}{2})} = (1+i)^{-n} \cdot (1+i)^{\frac{1}{2}} \ , \quad . \ . \ . \ . \ . \ . \ . \quad (29)$$

$$(\text{PW of } 1/\text{period})_{\text{VI}} = \sum_{1}^{n} (1+i)^{-(n-\frac{1}{2})} = (1+i)^{\frac{1}{2}}\left[\frac{1-(1+i)^{-n}}{i}\right] , \quad (30)$$

where VI is yearly compounding, income received at midpoint of each year.

Eqs. 28, 29 and 30 differ from Eqs. 1, 4 and 5 by the factors $(1+i)^{\pm\frac{1}{2}}$. Thus, midyear interest tables and conventional interest tables differ only by a simple factor, and are readily convertible. Both sets of tables are based on an effective rate i.

Conversion using Simple Interest

A third method to convert monthly income to an equivalent yearly income is to assume that the income received each month draws simple interest at a rate of $100i/12$ per cent per month. The first month's income draws 11 month's simple interest; the second month's income draws 10 months' simple interest, etc. For the total period there will be 66 months' simple interest on the monthly income

(1/12 yearly income). At the end of the year, the relative value of the income will be increased by a factor equal to

$$a_i = 1 + \left[\frac{1}{12} \times 66 \times \frac{i}{12}\right] = 1 + \frac{66}{144} i$$
$$= (1 + 0.4583i),$$

and

$$PW_{(\text{Income received monthly})} = (1 + 0.4583i) \times PW_{(\text{year end})} \quad . . . . (31)$$

Comparative Value of Various Conversion Factors

In the oil industry, where income is usually received monthly, the most accurate conversion factor is given by Eq. 22 for $p = 12$. However, all other factors are approximately equal to this value and can be substituted without causing appreciable error. Typical values of yearly conversion factors in Eqs. 23 through 31 are shown in Table 2. The assumption that income is received at midyear is essentially the same as the assumption that income is received and reinvested continuously. Either assumption will closely simulate monthly receipt of income. At low interest rates, the approximate conversion factors $\pm(1+0.5i)$ could be used without appreciable error.

Value of Money Received Nonuniformly During a Year

Income from producing oil and gas wells usually declines with time. If the pattern of income variation throughout a period can be described mathematically, it is possible to derive equations that will correctly describe the relative increase in value of the money received nonuniformly throughout the year rather than at year end. The derived conversion factor can then be used to correct the conventional-interest equations for non-uniform receipt of income. While such conversion factors will increase the theoretical accuracy of a solution, their use is rarely justified. The error introduced by assuming uniform receipt of income throughout a period is generally quite small except at high interest rates or for cases of extreme variation of income throughout the year. In fact, the error introduced by such an assumption will often offset errors introduced by other assumptions. For example, if income is received monthly and declines at a constant instantaneous monthly decline rate d, the following conversion factor equation can be derived:

$$(a_j)_{\text{Declining Income}} = \frac{(e^d - 1)\,(e^{12d+j} - 1)}{(e^{d+j/12} - 1)\,(e^{12d} - 1)} \quad . \quad . \quad (32)$$

Fig. 2 shows a plot of this equation at various values of d for three values of j. Values along the y axis are the correct conversion factors for monthly receipt of income with no decline during the year. The crossed points on the y axis are the calculated conversion factors, assuming that income is received continuously and uniformly throughout the year. The difference between the solid and dashed curves represents the error introduced by assuming that income is received continuously and uniformly. Fig. 2 shows that the assumption that income is received continuously and uniformly is equivalent to the assumption that income is received monthly and that it declines at an instantaneous rate of 4.2 per cent per month (approximately 40 per cent per year).

TABLE 2—FACTORS TO CONVERT CONVENTIONAL INTEREST TABLES TO ACCOUNT FOR THE RECEIPT OF INCOME THROUGHOUT A PERIOD

Effective Rate	Conversion Factors [based on an effective rate i]			
	Midyear	Monthly	Continuous	Monthly Simple Int.
(i)	$(1+i)^{1/2}$	$(i/j)_{p=12}$	$(i/j)_{p=\infty}$	$(1+0.4583i)$
1	1.00499	1.00458	1.00499	1.00458
3	1.0148	1.0137	1.0149	1.0137
6	1.0296	1.0272	1.0297	1.0275
10	1.0488	1.0456	1.0492	1.0458
20	1.0954	1.0886	1.0970	1.0917
100	1.4142	1.4015	1.4427	1.4583

Use of Conversion Factors

Table 3 shows the basic compound interest equations and modifications that can be obtained by applying appropriate conversion factors. The equations in this table are not shown in their usual form. The purpose of Table 3 is to emphasize that the different equations for either a nominal rate j or an effective rate i differ only by a simple conversion factor. When applying the equations to determine interest factors, we should look for cancellation of terms between the basic interest equations and the conversion factor equations.

The conversion factor equations shown in Table 3 are all based on the assumption that income is received uniformly throughout a period. If income is received nonuniformly, we can substitute properly derived conversion equations for those shown in Table 3 and obtain correct interest equations for any pattern of income.

Deferment Factors

Deferment factors are the lazy man's way to calculate the present value of future income. If the income to be received at future periodic intervals can be expressed in some mathematical relationship with time, then it is possible to relate such an expression to the interest calculation to obtain a single discount factor, or deferment factor, that applies to the total future income.

$$\text{Deferment factor} = \frac{\text{Present value of future income}}{\text{Future income}},$$

or

$$\text{Present worth} = \text{Deferment factor} \times \text{Future income} \quad . . . . (33)$$

If income is received in equal periodic amounts throughout the life of a project (constant yearly income), the

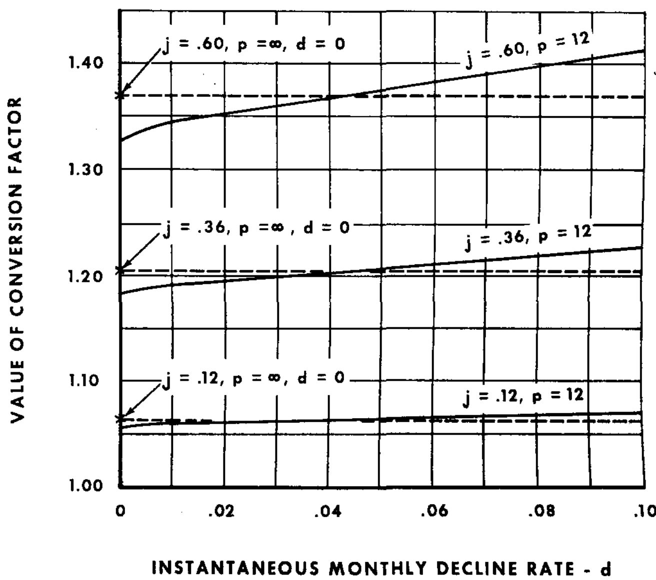

Fig. 2—Conversion factors if income is received monthly and declines at various rates.

TABLE 3—CHART FOR CONVERSION OF INTEREST TABLES

					AMOUNT OF 1	PW OF 1	AMT OF 1/PERIOD	PW OF 1/PERIOD	SINKING FUND	PARTIAL PAYMENT
			SYMBOL		C	D	G	H	J	K
			FUNCTION OF D		$\frac{1}{D}$	D	$\frac{1-D}{iD}$	$\frac{1-D}{i}$	$\frac{iD}{1-D}$	$\frac{i}{1-D}$
			FUNCTION OF H		$\frac{1}{1-iH}$	$1-iH$	$\frac{H}{1-iH}$	H	$\frac{1}{H}-i$	$\frac{1}{H}$
	METHOD OF COMPOUNDING	TIME INCOME RECEIVED	COMPOUNDING DURING PERIOD METHOD	RATE						
I	PERIODICALLY	END OF PERIOD	–	–	$(1+i)^n$	$(1+i)^{-n}$	$\frac{(1+i)^n-1}{i}$	$\frac{1-(1+i)^{-n}}{i}$	$\frac{i}{(1+i)^n-1}$	$\frac{i}{1-(1+i)^{-n}}$
II	CONTINUOUSLY	END OF PERIOD	–	–	e^{jn}	e^{-jn}	$\frac{e^{jn}-1}{e^j-1}$	$\frac{1-e^{-jn}}{e^j-1}$	$\frac{e^j-1}{e^{jn}-1}$	$\frac{e^j-1}{1-e^{-jn}}$
III	CONTINUOUSLY	DURING PERIOD	CONTINUOUSLY	j	$\frac{1}{a_j}\left[e^{jn}\right]$	$a_j\left[e^{-jn}\right]$	$\frac{1}{a_j}\left[\frac{e^{jn}-1}{e^j-1}\right]$	$a_j\left[\frac{1-e^{-jn}}{e^j-1}\right]$	$a_j\left[\frac{e^j-1}{e^{jn}-1}\right]$	$\frac{1}{a_j}\left[\frac{e^j-1}{1-e^{-jn}}\right]$
IV	PERIODICALLY	DURING PERIOD	CONTINUOUSLY	i	$\frac{1}{a_i}\left[(1+i)^n\right]$	$a_i\left[(1+i)^{-n}\right]$	$\frac{1}{a_i}\left[\frac{(1+i)^n-1}{i}\right]$	$a_i\left[\frac{1-(1+i)^{-n}}{i}\right]$	$a_i\left[\frac{i}{(1+i)^n-1}\right]$	$\frac{1}{a_i}\left[\frac{i}{1-(1+i)^{-n}}\right]$
V	PERIODICALLY	DURING PERIOD	CONTINUOUSLY	k	$\frac{1}{a_k}\left[(1+i)^n\right]$	$a_k\left[(1+i)^{-n}\right]$	$\frac{1}{a_k}\left[\frac{(1+i)^n-1}{i}\right]$	$a_k\left[\frac{1-(1+i)^{-n}}{i}\right]$	$a_k\left[\frac{i}{(1+i)^n-1}\right]$	$\frac{1}{a_k}\left[\frac{i}{1-(1+i)^{-n}}\right]$
VI	PERIODICALLY	MIDDLE OF PERIOD	PERIODICALLY	i	$\left[(1+i)^{-1/2}\right]\left[(1+i)^n\right]$	$\left[(1+i)^{1/2}\right]\left[(1+i)^{-n}\right]$	$\left[(1+i)^{-1/2}\right]\left[\frac{(1+i)^n-1}{i}\right]$	$\left[(1+i)^{1/2}\right]\left[\frac{1-(1+i)^{-n}}{i}\right]$	$\left[(1+i)^{1/2}\right]\left[\frac{i}{(1+i)^n-1}\right]$	$\left[(1+i)^{-1/2}\right]\left[\frac{i}{1-(1+i)^{-n}}\right]$

ADDITIONAL EQUATIONS

i = effective interest rate $= (1+\frac{j}{p})^p - 1 = (e^j - 1)_{p=\infty}$

j = nominal interest rate $= p\left[(1+i)^{1/p} - 1\right] = \left[\ln(1+i)\right]_{p=\infty}$

k = safe rate of interest.

NOMENCLATURE-

p = number of interest conversions per year

m = fractional portion of a year

n = number of periods [years]

CONVERSION FACTORS- Note- Where conversion factors occur, look for cancellation of terms.

1. To convert PW when income is received uniformly throughout a period instead of at the end of the period, or to convert amount when an investment is made uniformly throughout a period instead of at the end of the period.
2. To convert amount when an investment is made uniformly throughout a period instead of at the beginning of the period.
3. To convert amount when income is received uniformly throughout a period instead of at the end of the period.

$$a_i = \frac{(1+i)^m - 1}{m \ln(1+i)} = \left[\frac{i}{\ln(1+i)}\right]_{m=1} = \left[\frac{i}{j}\right]_{m=1}$$

$$a_j = \frac{e^{mj}-1}{mj} = \left[\frac{e^j-1}{j}\right]_{m=1} = \left[\frac{i}{j}\right]_{m=1}$$

$$a_k = \frac{(1+k)^m-1}{m\ln(1+k)} = \left[\frac{k}{\ln(1+k)}\right]_{m=1}$$

$$b_i = \frac{a_i}{(1+i)} \qquad b_j = \frac{a_j}{e^j} \qquad b_k = \frac{a_k}{(1+k)}$$

$$c_i = \frac{1}{a_i} \qquad c_j = \frac{1}{a_j} \qquad c_k = \frac{1}{a_k}$$

COMMENTS-

$$\left[\lim_{n\to\infty}\left(\frac{1-(1+i)^{-n}}{i}\right) = \frac{1}{i}\right]_{I} \quad \left[\lim_{n\to\infty}\frac{1-e^{-jn}}{j} = \frac{1}{j}\right]_{III} \quad \left[\lim_{n\to\infty}(1+i)^{1/2}\left(\frac{1-(1+i)^{-n}}{i}\right) = \frac{(1+i)^{1/2}}{i}\right]_{VI} \quad \left[\lim_{n\to\infty}\left(\frac{i}{\ln(1+i)}\right)\left(\frac{1-(1+i)^{-n}}{i}\right) = \frac{1}{\ln(1+i)}\right]_{IV}$$

deferment factor is equal to the present worth of 1 per period divided by the total number of periods:

$$(\text{Deferment factor})_{\text{I}} = \frac{a_n}{n} = \frac{1-(1+i)^{-n}}{in} \quad . \quad . \quad . \quad (34)$$

If income is received uniformly and continuously throughout the life of a project, the deferment-factor equation must contain an appropriate conversion factor to account for the continuous receipt of income.

$$(\text{Deferment factor})_{\text{IV}} = \frac{a_n}{n} = \frac{1-(1+i)^{-n}}{in}\left(\frac{i}{\ln(1+i)}\right)$$

$$= \frac{1-(1+i)^{-n}}{n\ \ln(1+i)} \quad , \quad . \quad (35)$$

or

$$(\text{Deferment factor})_{\text{III}} = \frac{1-e^{-jn}}{jn} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

If income is not received uniformly, but declines at a constant rate, or in some other manner which can be expressed mathematically, equations can be determined for the deferment factor, and interest tables can be constructed. Unfortunately, to cover all cases requires an infinite number of special deferment-factor tables. To avoid this difficulty, deferment factors are often expressed as a family of curves. This reduces accuracy but greatly increases the utility of the factors.

Deferment-factor curves and other short-cut methods of discounting are easily misused unless one is familiar with the set of curves and how they were constructed. *In addition, they will not provide an answer for unusual patterns of income.* If income taxes are included in the evaluation, irregular income patterns almost always will result. Consequently, *deferment factor curves are rarely applicable,* although they are commonly used. Short-cut methods of discounting should be used with caution.

Conclusions

1. Different types of interest tables result in different answers.
2. "Conventional" and "midyear" interest tables are expressed as factors of an *effective* annual rate i.
3. "Continuous" interest tables are expressed as factors of a nominal annual rate j.
4. There is a simple mathematical relationship between i and j.
5. The assumption that income is received at the midpoint of a period is approximately the same as assuming that income is received continuously throughout a period, or that it is received monthly.
6. To avoid confusion, a rate of return should be expressed as an *effective* rate of return or a *nominal* rate of return.

Nomenclature

a = conversion factor to correct present worth when income is received throughout a period instead of at the end of a period

i = uniform rate of interest per period = effective yearly interest rate when yearly periods are used

j = nominal yearly interest rate

k = safe interest rate

m = fraction, or multiple, of a yearly period

n = number of periods of interest conversions = number of years for an "effective" yearly interest rate

p = number of compounding periods per year

(I-VI) = subscripts referring to differences in time of interest compounding and time income is received (see Table 3).

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APPENDIX

Finding Interest Factors Beyond the Range of Tables

Occasionally the engineer will need to use interest factors not included within the range of available interest tables. Such factors can be obtained readily by using a hand calculator and the tables available.

1. Find the amount to which \$1,000 will grow in 20 years at 6 per cent nominal interest rate compounded monthly. Tables available are limited to 100 periods.

$$\text{Amount of 1 } (\tfrac{1}{2}\%\text{—240 periods}) = (1.005)^{240} = (1.005)^{80\times3}$$

$$\text{Amount of 1 } (\tfrac{1}{2}\%\text{— 80 periods}) = 1.49033857 \text{ (from table)}$$

$$\text{Amount of 1 } (\tfrac{1}{2}\%\text{—240 periods}) = (1.49033857)^3 = 3.3102045$$

$$\text{Amount} = \$1{,}000 \times 3.312045 = \$3{,}310.20.$$

2. Find the payment on a \$10,000 loan at 5 per cent nominal interest rate compounded monthly for 25 years from interest tables that only show 100 periods.

$$j = 0.05 \qquad p = 12 \qquad n = 25$$

$$\text{PW of 1 } (5/12\%\text{—100 periods}) = 0.65988155 \text{ (from table)}$$

$$\text{PW of 1 } (5/12\%\text{—300 periods}) = (0.65988155)^3 = 0.2873412$$

$$\text{Partial Payment} = \frac{j/p}{1-\left(1+\frac{j}{p}\right)^{-pn}} = \frac{.004167}{1-(0.65988155)^3}$$

$$= 0.0058459004$$

$$\text{Monthly Payment} = 0.0058459 \times 10{,}000 = \$58.46.$$

Use of Exponential Functions to Solve Interest Problems

Tables of the exponential functions e^x and e^{-x} can be used in place of continuous-interest tables. They also can be substituted for other types of tables to obtain approximate answers when other tables are not available.

1. Find the amount to which \$1,000 will grow in 20 years at 6 per cent nominal interest rate compounded

TABLE 4

Year	Income	PW of 1 30%	PW at 30%	PW of 1 35%	PW at 35%	PW of 1 40%	PW at 40%
1	53,000	.7692	40,800	.7407	39,200	.7143	37,800
2	43,000	.5917	25,400	.5487	23,600	.5102	21,900
3	36,000	.4552	16,400	.4064	14,600	.3644	13,100
4	26,000	.3501	9,100	.3011	7,800	.2603	6,800
5	18,000	.2693	4,800	.2230	4,000	.1859	3,300
6	10,000	.2070	2,100	.1652	1,700	.1328	1,300
Total			98,600		90,900		84,200

monthly using exponential tables.

$$j_{(p=12)} = 0.06;\ (1+i) = \left(1+\frac{0.06}{12}\right)^{12}$$

$$= (1.005)^{12} = 1.0617 \text{ (slide rule)}$$

$$j_{(p=\infty)} = \ln(1+i) = \ln(1.0617) = 0.05986$$

$$e^{jn} = e^{0.05986 \times 0.2} = e^{1.1972}$$

$$\left.\begin{array}{l} e^{1.19} = 3.2871 \\ e^{1.20} = 3.3201 \end{array}\right\} e^{1.1972} = 3.3109 \text{ (by interpolation)}$$

$$\text{Amount} = \$1,000 \times 3.3109 = \$3,310.90$$

$$\text{(Actual value} = \$3,310.20)$$

$$j_{(p=12)} = 0.06 \approx j_{(p=\infty)}$$

$$\text{Amount of } 1 = e^{jn} = e^{0.06 \times 20} = e^{1.2} = 3.3201$$

$$\text{Amount} = \$3.320.10 \text{ (approximately).}$$

2. Find the approximate payment on a $10,000 loan at 5 per cent nominal interest rate compounded monthly for 25 years using exponential tables.

$$j_{(p=\infty)} \approx 0.05$$

$$\text{Partial payment} = \frac{j}{1-e^{-jn}} = \frac{0.05}{1-e^{-1.25}} = \frac{0.05}{0.7135}$$

$$= 0.070077$$

$$\text{Payment} = \$700.77/\text{year} = \$58.40/\text{month}$$

$$\text{(Actual value} = \$58.46/\text{month).}$$

Use of Conversion Factors to Solve Interest Problems

Conversion factors can be used to correct conventional interest tables for monthly receipt of income or for continuous receipt of income. Use of conversion factors greatly extends the utility of conventional interest tables.

1. A company pays $100,000 cash for the following income, after taxes.

Year	Income	Year	Income
1	$53,000	4	$26,000
2	43,000	5	18,000
3	36,000	6	10,000

What is the effective rate of return on the investment: (1) if the income is assumed to be received at the year end, (2) if the income is assumed to be received at midyear, (3) if it is assumed to be received continuously?

TABLE 5

Year	Income	PW of 1 33%	PW at 33%	PW of 1 34%	PW at 34%
1	53,000	.8517	45,100	.8477	44,900
2	43,000	.6123	26,300	.6034	25,900
3	36,000	.4402	15,800	.4295	15,500
4	26,000	.3165	8,200	.3057	7,900
5	18,000	.2275	4,100	.2176	3,900
6	10,000	.1636	1,600	.1549	1,500
Total	186,000		101,100		99,600

What is the nominal rate of return if (4) the income is assumed to be received continuously and is discounted continuously?

Income Received at Year End

A plot of discounted income vs per cent indicates a rate of return = 29.2 per cent. (Effective rate — income received at year end.)

Income Received at Midyear

$$(PW)_{\text{midyear}} \ @\ 30\% = (1.30)^{1/2} \times 98,600 = 1.14\ (98,600) = \$112,200$$

$$(PW)_{\text{midyear}} \ @\ 35\% = (1.35)^{1/2} \times 90,900 = 1.162\ (90,900) = \$105,700$$

$$(PW)_{\text{midyear}} \ @\ 40\% = (1.40)^{1/2} \times 84,200 = 1.183\ (84,200) = \$99,700$$

By interpolation: i = rate of return = 39.75% (Effective rate — income received at midyear.)

Income Received Continuously

$$(PW)_{\text{continuous}} \ @\ 35\% = \frac{0.35}{\ln(1.35)}(90,900) = 1.1663\ (90,900) = \$106,000$$

$$(PW)_{\text{continuous}} \ @\ 40\% = \frac{0.40}{\ln 1.40}(84,200) = 1.1888\ (84,200) = \$100,100$$

i = rate of return = 40.0+% (Effective rate — income received continuously.)

Income Received and Discounted Continuously

$$j = \ln(1+i) = \ln 1.4 = 0.33647.$$

Rate of return = 33.6% (Nominal rate — income received continuously.)

Use continuous interest factors as shown in Table 5.

Rate of return = 33.7% (Nominal rate — income received continuously.)

Note that factors obtained from conventional interest tables were used with appropriate conversion factors to solve the last three problems. When used with proper conversion factors, the conventional interest equations or tables offer considerable versatility. ★★★

EDITOR'S NOTE: A PICTURE AND BIOGRAPHICAL SKETCH OF P. L. ESSLEY, JR., WERE PUBLISHED IN THE JAN., 1965 ISSUE OF JOURNAL OF PETROLEUM TECHNOLOGY.

Further Discussion of a Paper Published in
Journal of Petroleum Technology, Aug., 1965

The Difference Between Nominal and Effective Interest Tables And Nominal and Effective Rates of Return

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DISCUSSION

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I agree with P. L. Essley's conclusions about the various compound interest assumptions. But I do not feel that he has made a strong enough appeal for the virtues of the assumption of interest compounded annually at mid-year. It is significant to note that the assumption of interest compounded annually at mid-year has been employed for many years to evaluate investments in oil properties. The assumption of *continuously compounded* interest has received a great deal of engineering attention since its introduction in the 1950's. The latter assumption has been used as the basis for simplified rate of return calculations. As shown by Essley, the continuously compounded rate of return is unfortunately not equal to the annual-at-mid-year rate of return. To justify the simplified rate of return computation, it has been necessary to show the continuously compounded interest assumption to be more appropriate. A considerable difference of opinion about compound interest assumptions has therefore erupted over arguments used to justify the continuously compounded interest assumption.

Essley's paper would seem to settle the difference of opinion. However, he has considered mainly the relationship between compound interest assumptions rather than this difference of opinion. The effective annual interest rate is desirable for use in rate of return computations to indicate the real earning power of annual income, in per cent per year. Reference to the effective annual rate of return, for example, shows exactly the average interest rate that will be earned by the investment each year for the life of the project. Reference to a nominal rate of return, as in the case of the continuously compounded interest assumption, shows only the average interest rate earned by the investment (not in per cent per year but, in this case, in per cent compounded continuously). The effective annual rate of return, then, is a more manageable parameter that gives a better idea of the actual earning power of the investment. There is some difference between compounding annually at mid-year and at year-end. The effective annual interest rate must, therefore, be based on one or the other type of annual compounding. The objectionable feature of the year-end assumption is the implication that income is received once a year at year-end.

Essley shows that the continuously compounded interest assumption, or the monthly compounded interest assumption (when converted to the effective annual interest rate), are essentially equivalent to the mid-year assumption. And in such cases it is implied that income is received continuously or monthly with interest compounded annually at year-end. The same also applies when interest compounded quarterly or semi-annually is converted to the effective annual interest rate; i.e., it is implied that income is received quarterly or semi-annually with interest compounded annually at year-end. Thus, except in the case of the year-end assumption—which is not considered a good assumption for the reason already specified—all of the common compound interest assumptions revert back essentially to the assumption of interest compounded annually at mid-year when converted to the effective annual interest rate. Since the error involved in the mid-year assumption is insignificant, and since it does not require conversion to the effective annual interest rate, it is concluded that the mid-year is superior to *all* other compound interest assumptions. Other assumptions may produce equally reliable results if converted back into terms of the mid-year assumption, according to the conversion factors devised by Essley.

It is not by coincidence that the assumption of interest compounded annually at mid-year prevails throughout the petroleum industry. A check with a major consulting firm revealed that all of the oil banks with which it deals accept the mid-year assumption. The consulting firm reported that it based all of its valuation reports for the past eight years on the mid-year assumption. Furthermore, a spot check of several sources revealed no known company or bank that has standardized the assumption of continuously compounded interest.

While it is not the purpose of this discussion to dispute Essley's ideas, another idea presented in his paper warrants further discussion. He has shown in his discussion and Fig. 2 the error involved in assuming uniform income where production declines at a constant monthly decline rate. A constant monthly decline as used by Essley, produces an odd series of concave downward curves when plotted on semi-log or log-log paper, and it is felt that such a curve is not really indicative of a production decline curve. I would therefore take issue with his implication that the assumption that income is received continuously and uniformly is equivalent to the assumption that income is received monthly and declines at the approximate rate of 40 per cent a year. On the contrary, if the mid-year assumption is used, it has been shown that income received and compounded continuously and uniformly is essentially equivalent to the assumption that the average annual income is compounded annually at year-end with income received according to any schedule up to six-month intervals. The production or income decline is taken care of by compounding the average annual income. These characteristics require that a correction be added to the mid-year assumption to account for fiscal or calendar year schedules rather than *years after investment.* This is no real handicap, however, since we are rarely interested in such insignificant corrections.

In summary, some companies must employ the continuously compounded or other compound interest assumptions. But the annual-at-mid-year assumption, which has been widely accepted in the petroleum industry for many years, has never been replaced by a better compound interest assumption.

AUTHOR'S REPLY TO D. F. GARIES

D. F. Garies states that "the 'mid-year' is superior to all other compound interest assumptions". He does not believe that I made a strong enough appeal for the virtues of mid-year discounting. I made no appeal. My intent was to show the relationship between various type interest tables, not to argue their merits.

In SPE 1140 I showed that for interest calculations the assumption that income is received at the mid-point of a period is approximately the same as assuming that income is received continuously or monthly. Garies does not dispute this contention, but in fact amplifies it. He states "... except for the year-end assumption ... all the common compound interest assumptions revert back essentially to the assumption of interest compounded annually at mid-year when converted to the effective annual interest rate". This is another way of saying that the other methods are essentially equivalent to the mid-year method. Yet from his statement, which implies equivalence of various discounting methods, Garies concludes that one method (mid-year discounting) is superior.

Garies prefers the use of an effective interest rate i to the use of a nominal interest rate j. He states that effective annual interest rate i represents *real* earning power of an annual income in per cent per year, whereas nominal annual interest rate j represents only an "average interest rate (not in per cent per year but in ... per cent compounded continuously)". He concludes that an effective annual rate of return is more manageable and gives a better idea of the actual earning power of an investment. He fails to show how an effective rate of return is more "manageable" than a nominal rate of return and he does not define *real* or *actual* earning power.

He takes issue with my statement that "the assumption that income is received continuously and uniformly is equivalent to the assumption that income is received monthly and declines at an approximate rate of 40 per cent per year". He apparently objects to this because I used constant percentage decline in my derivation which he says is not indicative of a production decline curve. The use of any other type decline would not appreciably alter Fig. 2 and would not change my statement. I was attempting to show that the correction for non-uniform receipt of income throughout a year is quite small and can be ignored. Garies, in effect, says the same thing in his "on the contrary" sentence.

Garies has focused attention on an existing controversy over the relative merits of various type discount factors. In my opinion, the controversy is based more upon preconceived opinions and biased assumptions than on logic. Both sides have logical arguments and can supposedly prove their claims. They usually do this by comparing the various methods to some standard and show that their method more closely approaches this standard. The trouble is that each side picks a different standard and the controversy degenerates into arguments over basic assumptions and the meaning of such abstract terms as *real earning power.*

Interest equations and tables are used in the industry to evaluate investment possibilities and to compare investment alternatives. Any discount method will give results equivalent to any other method if used consistently by all groups within a company. Different methods will give different numbers but not different relative values. By this, I mean that if project A indicates a higher rate of return, or a higher discounted net worth, than project B using one type discount table, it will also indicate a higher value using any other type discount table. The problem arises when different methods are used by different companies, or by different groups within a company. In such cases, results are not comparable unless they are converted to an equivalent basis. The purpose of my paper was to show a basis for such a conversion.

The nominal interest rate j and the effective interest rate i are directly related to each other. I cannot see that one has a definite superiority over the other. They are equally manageable in ease of computation. The best argument that Garies makes for the use of mid-year interest factors is that more people use them than any other type. I agree, and would like to see the industry adopt a standard. Mid-year interest and discount tables would serve this purpose as well as any other. There is enough confusion concerning the use of interest tables that standardization of nomenclature, symbols and a way to handle the receipt of income throughout a year would be a worthwhile accomplishment of an industry committee. ★★★