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Application of Pseudo-Steady-State Flow to Pressure-Buildup Analysis

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SUMMARY

This report develops a method of analyzing pressure buildup data in a finite-acting reservoir and demonstrates the method with exemplifying calculations. It is believed that this method is, in most cases, superior to methods currently in use. Many engineers currently attempt to apply the Horner[3] method of analysis,* which is based entirely upon infinite-acting reservoir equations, to wells which have obviously been affected by the drainage limits of the reservoir and thus are finite-acting. Furthermore, the methods presently available for buildup analysis in a finite-acting reservoir are not as simple as the Horner method to apply and require a knowledge of the reservoir size, effective compressibility, effective formation thickness, porosity and fluid viscosity if the skin factor and average pressure are to be evaluated. In contrast, the method presented herein is very similar to the Horner method in that it requires similar pressure plots and uses similar equations; also, the formation capacity, the average drainage area pressure, and the skin factor can all be determined with a knowledge of only the fluid viscosity and the $[r_e/r_w]$ ratio. Even the Horner solution does not provide the skin factor with such limited data.

A solution based on such limited data is made possible by using the change in reservoir pressure prior to shut-in to determine the product of effective compressibility, porosity and effective formation thickness.

The application of pseudo-steady-state flow to pressure-buildup analysis is made practical by an analytic derivation of the time when pseudo-steady-state flow begins. The development of these concepts, with a review of the basic pseudo-steady-state technology, is presented herein.

COMPARISON OF PSEUDO-STEADY-STATE FLOW WITH OTHER FLOW REGIMES

Reservoir fluid flow may be generally classified as steady-state, unsteady-state and pseudo-steady-state. This last classification, with which this report is concerned, has also been referred to as "stabilized", or as "steady state in a bounded drainage area".[2] This type of reservoir flow occurs much more frequently than steady-state flow or unsteady-state flow with an expanding drainage radius.

Pseudo-steady-state flow is defined as flow under conditions where the flow rate and

*The Horner method employs a semilog plot of the well pressure vs $[T + t]/t$ where T is the total producing time and t is the shut-in time. See Appendix for reference.

References and illustrations at end of paper.

pressure gradients are constant but absolute reservoir pressure varies with time. Equations describing pseudo-steady-state flow appear to be more realistic than those describing steady-state and more generally useful than those developed for unsteady-state flow.

In steady-state flow, the pressure and the flow rate throughout the reservoir remain constant with time. This means that the mass of fluid entering the reservoir must equal the mass of fluid withdrawn from the reservoir. Except for very active water drives and gas-cap drives, this assumption is not valid, so that the steady-state equations yield only qualitative answers.

Unsteady-state flow takes place as a result of fluid expansion, and thus the rate and pressure do change with time. These conditions are much more realistic than steady-state conditions, but the solutions to unsteady-state flow equations are generally complex. The development of certain dimensionless solutions has made it possible to apply unsteady-state flow equations to a few reservoir problems, but the solutions that can be used by most engineers are very limited.

Although pseudo-steady-state flow has not been used frequently, it has been recognized for some time. In wells produced at constant rates, it has been observed that after a certain time, the change in well pressure with time becomes constant. Hurst and van Everdingen[8] showed that the pressure throughout the reservoir also changes at a constant rate. The pressure history in the reservoir would then be as shown in Fig. 1.

Note that from t_2 to t_5 [Fig. 1], the pressure distribution in the reservoir remains the same, which means that dp/dr at a particular radius is independent of time. By Darcy's law,[1] which is a steady-state equation,

$$q = \frac{1.127\ kA}{\mu} \frac{dp}{dx} \quad \ldots\ldots\ldots\ldots [1]$$

One might conclude from Eq. 1 and Fig. 1 that steady-state flow is taking place, since the pressure gradient dp_w/dr and the flow rate at the well q_w, are maintained constant. However, the absolute pressure p is not constant with time, and thus the reservoir flow is actually unsteady-state -- hence, the term "pseudo-steady state".

Recognition and application of this type of flow appears to be receiving increased attention in recent years. The application of pseudo-steady-state fundamentals to pressure-buildup analyses appears to have some advantages over methods previously published for finite reservoirs.[5,6]

PSEUDO-STEADY-STATE FLOW EQUATIONS

Assuming for the moment that pseudo-steady-state flow does exist, we can derive simple flow equations from Darcy's equation.[2] The flow of fluid in pseudo-steady state is the result of fluid expansion due to declining pressures in the reservoir, and pressure decline per unit of time is uniform throughout the reservoir. It follows, then, that for radial flow of fluid with constant compressibility, the flow rate at any radius will be proportional to the amount of reservoir fluid outside the subject radius. Since radial areas and volumes are directly proportional to r^2, the rate of flow at any radius, in terms of that radius, the external radius, and the well rate, will be

$$q_r = q_w \left(\frac{r_e^2 - r^2}{r_e^2}\right) \quad \ldots\ldots\ldots [2]$$

This value and $A = 2\pi rh$ can be substituted in Darcy's equation. Integrating this expression between the well radius r_w and the radius of interest r gives the pseudo-steady-state radial flow equation:

$$q_w = \frac{(1.127)\ 2\pi kh\ (p_r - p_w)}{\mu \left[\ln\left(\frac{r}{r_w}\right) - \frac{r^2}{2r_e^2} + \frac{r_w^2}{2r_e^2}\right]}, \quad \ldots [3]$$

where p_r is the pressure at the radius, r. The term $r_w^2/2r_e^2$ is normally insignificant. Thus, we drop this term to obtain

$$q_w = \frac{7.08\ kh\ (p_r - p_w)}{\mu \left[\ln\left(\frac{r}{r_w}\right) - \frac{r^2}{2r_e^2}\right]} \quad \ldots\ldots [4]$$

Or, defining the equation for $r = r_e$ and $p_r = p_e$,

$$q_w = \frac{7.08\ k\ h\ (p_e - p_w)}{\mu \left[\ln\left(\frac{r_e}{r_w}\right) - \frac{1}{2}\right]} \quad \ldots\ldots [4A]$$

Similarly, for linear flow,

$$q_\ell = q_w \left(\frac{L - \ell}{L}\right), \quad \ldots\ldots\ldots\ldots [5]$$

where L is the total length of the reservoir system and ℓ is the distance from the producing point. Substituting into Darcy's equation and integrating, we obtain

$$q_w = \frac{2.254\ kA}{\mu}\ \frac{(p_L - p_w)}{L} \quad . \ . \ . \ . \ . \ . \ [6]$$

Note that for a given pressure drop, the flow rate is greater than would be estimated by the steady-state equations. In the case of linear flow, the flow rate is twice that indicated by the steady-state equation. According to Darcy's law, the pressure loss in a reservoir is directly proportional to the fluid velocity q/A. The average velocity in pseudo-steady state is less than q_w/A, since the velocity at the outer boundary is zero; thus, the pressure drop is smaller for a given rate at the well.

THEORETICAL JUSTIFICATION OF PSEUDO-STEADY-STATE FLOW

We will now show that for long producing times and constant producing rates, the change in well pressure with time is the same for unsteady-state and pseudo-steady-state flow, and thus that pseudo-steady-state flow conditions do prevail for long times in reservoirs containing fluids of constant compressibility. As stated previously, Hurst and van Everdingen first[8] recognized these reservoir conditions in 1949.[8]

For pseudo-steady-state flow to occur, the reservoir must be produced at a constant rate. Thus, for fluids of constant compressibility, the B/D producing rate in terms of the average reservoir pressure is

$$q = \frac{\pi r_e^2\ \phi\ h\ c\ \Delta p_{avg}}{5.615\ \Delta t}, \ . \ . \ . \ . \ . \ . \ . \ [7]$$

and the change in the average reservoir pressure with time is

$$\frac{dp_{avg}}{dt} = \frac{1.8\ q}{r_e^2\ \phi\ h\ c} \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ [8]$$

With unsteady-state flow at a constant rate over long times, this same change in pressure takes place at the well. Hurst and van Everdingen[8] have shown that the well pressure under unsteady-state conditions is

$$p_w = p_i - \frac{0.141\ q\ \mu}{kh}\ P_{(t_D)}, \ . \ . \ . \ . \ [9]$$

where $P_{[t_D]}$ is the Hurst-van Everdingen[8] pressure function explicitly defined as

$$P_{(t_D)} = \frac{2(t_D + \frac{1}{4})}{r_D^2 - 1} - \frac{3r_D^4 - 4r_D^4 \ln r_D - 2r_D^2 - 1}{4\ (r_D^2 - 1)^2} + 2 \sum_{n=1}^{n=\infty} \frac{e^{-B_n^2 t_D}\ J_1^2 (B_n r_D)}{B_n^2 \left[J_1^2 (B_n r_D) - J_1^2 (B_n) \right]},$$
$$. \ [10]$$

where $r_D = r_e/r_w$, J_1 is a Bessel function and B_n is a series of constants. For our case $P_{[t_D]}$ represents the psi of pressure drop at the well radius per unit of "dimensionless" rate [0.141 qμ/kh] at some dimensionless time t_D, where

$$t = \frac{\phi\ \mu\ c\ r_w^2\ t_D}{6.33\ k} \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ [11]$$

Then from Eqs. 9 and 11,

$$\frac{dp_w}{dt} = \frac{(0.141\ \frac{q\mu}{kh})\ dP_{tD}/dt_D}{\left(\frac{\phi\mu c r_w^2}{6.33\ k}\right)} \ . \ . \ . \ . \ . \ [12]$$

When t_D is large as compared with r_D, the Bessel function term of Eq. 10 becomes insignificant.[4] It will be shown in the next section that for $t_D > r_D^2/4$,

$$P_{(t_D)} = \frac{2\ (t_D + \frac{1}{4})}{r_D^2 - 1} - \frac{3r_D^4 - 4r_D^4\ \ln r_D - 2r_D^2 - 1}{4\ (r_D^2 - 1)^2}$$
$$. \ [13]$$

Differentiating Eq. 13 with respect to t_D gives

$$\frac{d\ P_{(t_D)}}{d\ t_D} = \frac{2}{r_D^2 - 1} \ . \ . \ . \ . \ . \ . \ . \ . \ . \ [14]$$

For realistic values of r_D, the "1" is insignificant and

$$\frac{dP_{(t_D)}}{d\,t_D} = \frac{2}{r_D^{\;2}} \quad \ldots\ldots\ldots\ldots \quad [15]$$

Substituting this value into Eq. 12, we obtain

$$\frac{dp_w}{dt} = \frac{(0.141\,\frac{q\mu}{kh})}{\left(\frac{\phi\,\mu\,c r_w^2}{6.32\ k}\right)}\left(\frac{2}{r_D^{\;2}}\right), \quad \ldots\ldots \quad [16]$$

and simplifying,

$$\frac{dp_w}{dt} = \frac{1.8\ q}{r_e^{\;2}\,\phi\ h\ c} \quad \ldots\ldots\ldots\ldots \quad [17]$$

Thus, from Eqs. 8 and 17, we see that for large times and a constant rate of production,

$$\left(\frac{dp_w}{dt}\right)_{\text{unsteady state}} = \left(\frac{dp_{avg}}{dt}\right)_{\text{pseudo-steady state}} .$$

It is apparent, then, that pseudo-steady-state flow conditions do prevail at large times for reservoirs containing fluids of constant compressibility. It can also be seen that whenever Eq. 13 defines the function of $P_{[t_D]}$, pseudo-steady-state flow prevails.

PSEUDO-STEADY-STATE FLOW TIME LIMIT

Having established that the applicability of Eq. 13 denotes pseudo-steady-state flow conditions, we can now establish the time when pseudo-steady-state flow begins by evaluating the time when Eq. 13 branches from the infinite-acting $P_{[t_D]}$ curve.

Fig. 2 shows the $P_{[t_D]}$ data published by Hurst and van Everdingen in their 1949 AIME paper.[8] The circled points on this figure were calculated by Eq. 13 for $r_D = 10$. It will be seen that the particular $P_{[t_D]}$ function curve for a finite reservoir can be closely approximated by calculating $P_{[t_D]}$ values for various t_D values until they become approximately tangent to the infinite reservoir $P_{[t_D]}$ curve. For t_D values greater than about 100, the infinite reservoir curve is

$$P_{(t_D)} = \frac{1}{2}\,(\ln t_D + 0.809) \quad \ldots\ldots \quad [18]$$

Fig. 3 shows $P_{[t_D]}$ functions for a particular reservoir size, $r_D = 1980$, calculated by Eqs. 18 and 13. We know that $P_{[t_D]}$ by Eq. 18 is applicable for relatively small t_D values and and $P_{[t_D]}$ values by Eq. 13 are applicable at relatively large values of t_D. Thus, the point where these curves are effectively joined would be expected to be the point where they have the least separation or where $P_{[t_D]\text{by Eq. 13}} - P_{[t_D]\text{by Eq. 18}}$ = Minimum,

or when

$$\left[\frac{2\,(t_D + \frac{1}{4})}{r_D^{\;2} - 1} - \frac{3r_D^{\;2} - 4r_D^{\;4}\ \ln r_D - 2r_D^{\;2} - 1}{4\,(r_D^{\;2} - 1)^2}\right] - \left[\frac{1}{2}\,(\ln t_D + 0.809)\right]$$

$$= \text{Minimum.} \quad \ldots\ldots\ldots\ldots \quad [19]$$

When the change in the difference Eq. 19 with respect to dimensionless time equals zero,

$$d\left[\frac{2\,(t_D + \frac{1}{4})}{r_D^{\;2} - 1} - \frac{3r_D^{\;4} - 4r_D^{\;4}\ \ln r_D - 2r_D^{\;2} - 1}{4\,(r_D^{\;2} - 1)^2} - \frac{1}{2}\,(\ln t_D + 0.809)\right] \Big/ dt_D = 0,$$

$$\ldots\ldots\ldots\ldots\ldots\ldots \quad [20]$$

$$\frac{2}{r_D^2 - 1} - \frac{1}{2t_d} = 0,$$

and

$$(t_D)_c = \frac{r_D^{\;2} - 1}{4}. \quad \ldots\ldots\ldots\ldots \quad [21]$$

Thus, for $r_D > 10$, the one in the numerator is negligible and the critical dimensionless time when pseudo-steady state begins is $[t_D]_c = \frac{r_D^2}{4}$; or expanding the critical time in days,

$$\frac{t_c\ 6.33k}{\phi\mu c\ r_w^2} = \frac{r_e^2/r_w^2}{4},$$

or,

$$t_c = \frac{\phi\mu c\ r_e^2}{25.3\ k} \quad \ldots\ldots\ldots\ldots\ [22]$$

The time in Eq. 22 is nearly equal to the time derived by Craft and Hawkins[2] to create a logarithmic pressure distribution out to any transient drainage radius r_e,

$$t_c = \frac{0.040\mu c\ \phi\ r_e^2}{k} \quad \ldots\ldots\ldots\ [23]$$

This equation is widely used to estimate the time it takes for a reservoir disturbance [pressure or rate] to be "felt" at a particular radius. Just what is meant by "felt" is not clear since it is known that a disturbance travels through the reservoir with the speed of sound although the pressure change is not immediately measurable. The term "felt" is then meaningless unless some pressure change per unit of dimensionless rate, $\left[\Delta p \middle/ \left(\frac{0.141\ q\mu}{kh}\right)\right]$ is used to define the magnitude of the term.

Actually, the nature of the derivation used for Eqs. 21 and 22 causes us to define t_c as the time when the pressure in a well producing at a constant rate is no longer governed by infinite reservoir equations. At radii larger than the well radius [with the same r_e], the outer boundary should affect the pressure at some lesser time since the "effect" of the boundary must travel from the boundary to the well.

PSEUDO-STEADY-STATE PRESSURE BUILDUP ANALYSIS

Recognition of the time limit on radial pseudo-steady-state flow provides an accurate, well-defined limit to the application of infinite-acting reservoir behavior. Also, recognition of pseudo-steady state permits some reservoir techniques previously restricted to infinite-acting reservoirs to be applied to finite-acting reservoirs. The standard buildup pressure analysis technique employs equations which are derived for infinite-acting reservoirs; i.e., the reservoir analyzed must be acting as an infinite reservoir at both start and end of the buildup if the equations are to be theoretically valid.[3]

Consider a well producing under pseudo-steady state [$t > t_c$, see Eq. 22]. In such a case, there will be a straight line relationship between time and the well pressure prior to shut-in [Fig. 4]. Therefore, this pressure trend can be extrapolated, as shown by the dashed line in Fig. 4, to indicate the pressures which would have resulted if the well had not been shut in. The Δp^* shown is then the pressure change resulting from the negative rate caused by shutting in the well. This Δp^* can be stated as

$$\Delta p^* = \frac{0.141\ q\mu}{kh}\left(P_{(t_D)} + S\right), \quad \ldots\ [24]$$

where S is the skin factor representing damage around the well and $P_{[t_D]} = 1/2\ [\ln t_D + 0.809]$ for infinite reservoir action. That is, t_D measured from shut-in time must be greater than 100 [this occurs in a matter of seconds in most wells] and less than t_c. Then

$$\Delta p^* = \frac{0.0705\ q\mu}{kh}\left[\ln t + \ln\left(\frac{6.33\ k}{\phi\ \mu c\ r_w^2}\right) + 0.809 + 2S\right].$$

Or, changing the equation to base 10 logs,

$$\Delta p^* = \frac{0.1625\ q\mu}{kh}\left[\log t + \log\frac{6.33\ k}{\phi\ \mu c\ r_w^2} + \frac{0.809}{2.3} + \frac{2S}{2.3}\right] \quad \ldots\ldots\ [25]$$

Thus, Δp^* is proportional to log t and the slope m of such a plot is

$$m = \frac{0.1625\ q\mu}{kh} \quad \ldots\ldots\ldots\ [25A]$$

The slope of this plot is then the same as the slope of the conventional plot of log [t/T+t] vs well pressure for an infinite-acting reservoir where T is the producing time before shut-in and t is the shut-in time.

Furthermore, Eq. 25 can be solved for the skin factor S in terms of the slope, $m = 0.1625\ q\mu/kh$, and Δp at t = 1 hour:

$$S = 1.15\ \frac{(\Delta p)_{1\ hr}}{m} -$$

$$- 1.15 \log \frac{q}{10.4\ m\ c\ \phi\ h\ r_w^2} \quad . . . . [26]$$

It will be noted that this equation is the same as the API equation for evaluation of the skin factor from a pressure buildup in an infinite-acting reservoir[7] except that $[\Delta p^*]_{1hr}$ replaces $[p_{1hr} - p_{wf}]$.

Skin factors are often difficult to evaluate because the term, $q/c\ \phi\ h\ r_w^2$ is unknown. However, the change in the well pressure with time during the flowing period $[dp_w/dt]_q$ can be substituted for most of this group according to Eq. 17 to obtain

$$S = 1.15 \frac{(\Delta p^*)_{1\ hr}}{m}$$

$$- 1.15 \log \frac{(dp_w/dt)_q}{18.7\ m\ (r_w^2/r_e^2)} \quad . . . [26A]$$

Thus, evaluation of the damage requires a knowledge of only r_w/r_e.

After calculating the skin factor S the pressure drop due to the skin can be calculated [see Eq. 24].

$$(\Delta p)_{skin} = \frac{0.141\ q\mu}{kh} S. \quad . . . . [27]$$

Or, in terms of m, $\Delta p_{skin} = [m/1.15]S$.

After the well has been shut in for a time equal to t_c, the rate of pressure change at the well caused by -q [shutting in the well] will, by Eq. 17, be

$$\left(\frac{d\ (\Delta p^*)}{dt}\right)_{-q} = - \frac{1.8\ q}{r_e^2\ \phi hc} .$$

Since the well was in pseudo-steady state and producing at a rate q at time of shut-in, the rate of pressure change without shut-in would be:

$$\left(\frac{dp_w}{dt}\right)_q = \frac{1.8\ q}{r_e^2\ \phi hc} .$$

Thus, the total change in pressure after the well is shut in for a time equal to or greater than $[t_D]_c$ is

$$\left(\frac{dp_w}{dt}\right)_{total} = \left(\frac{dp_w}{dt}\right)_q + \left(\frac{d(\Delta p^*)}{dt}\right)_{-q} .$$

Or, substituting

$$\left(\frac{dp_w}{dt}\right)_{total} = \frac{1.8\ q}{r_e^2 \phi hc} - \frac{1.8\ q}{r_e^2 \phi hc} ,$$

then

$$\left(\frac{dp_w}{dt}\right)_{total} = 0.$$

If there is no change in the pressure with time, the well has reached the static pressure. Then the static pressure can be calculated by extrapolating the Δp^* vs log t plot to t_c and calculating the static pressure on the basis of $[\Delta p^*]_{t_c}$.

Thus, the static pressure will equal the pressure at time of shut-in p_{wf}, plus the pressure buildup due to the [-q], less the pressure decline that would have been experienced during a time interval t_c, if the well had not been shut in. Or

$$p_s = p_{wf} + (\Delta p^*)_{t_c} - \left(\frac{dp_w}{dt}\right)_q (t_c).$$

Substituting for $[dp_w/dt]_q$ by Eq. 17 and for t_c by Eq. 22, we obtain

$$p_s = p_{wf} + (\Delta p^*)_{t_c} - \left(\frac{1.8\ q}{r_e^2 \phi hc}\right) \left(\frac{\phi\mu\ c r_e^2}{25.3\ k}\right),$$

or

$$p_s = p_{wf} + (\Delta p^*)_{t_c} - \frac{1.8\ q\mu}{25.3\ kh} .$$

Now substituting m for $0.1625\ q\mu/kh$, we obtain

$$p_s = p_{wf} + (\Delta p^*)_{t_c} - 0.438\ m. \quad . . . [28]$$

In order to use Eq. 28, we must know t_c for the evaluation of $[\Delta p^*]_{t_c}$. The equation for t_c, Eq. 22, can be rewritten by multiplying both sides of the equation by h and regrouping the constants to obtain

$$t_c = \frac{(\phi\ c\ h\ r_e^2)}{25.3\ (kh/\mu)} .$$

Now substituting for [ϕ c h r_e^2] and [kh/μ] according to Eqs. 17 and 25A, respectively, we obtain

$$t_c = \frac{.438\ m}{(dp_w/dt)_q} \quad . \quad . \quad . \quad [29]$$

So, to calculate the static pressure from Eq. 28:

1. Observe the flowing well pressure p_{wf} prior to shut-in.
2. Calculate m from the slope of Δp^* vs log t.
3. Calculate t_c according to Eq. 29.
4. Read $[\Delta p^*]_{t_c}$ from the Δp^* vs log t plot or an extrapolation of the plot.
5. Calculate $\bar{p}$ by Eq. 28.

The application of this method can be better explained by the following example:

Example Problem* Solution

Given: A well drilled in a field with uniform 40-acre spacing has produced a prorated 280 STB/D for 10 days; the well is then shut in for a pressure-buildup survey. In the five days prior to shut-in, the flowing tubing-head pressure declined about 24 psi/day. The GOR has been constant during production. The following data are available:

Estimated Reservoir Data

Oil formation volume factor, B_o = 1.31
Oil viscosity, μ_o = 2.0
Well radius, r_w = 0.333 ft
Net pay thickness, h = 40 ft

Pressure Survey Data

Time Shut In [hrs]	Pressure [psia] [at Time of Shut-In]
0	1123
2	2290
4	2514
8	2584
12	2612
16	2632
20	2643
24	2650
30	2658

Determine:

1. How rapidly the reservoir deviates from an infinite-acting reservoir.
2. The value of the porosity -- compressibility product [ϕc].
3. The reservoir permeability.
4. How pseudo-steady- and unsteady-state analyses compare.
5. The skin factor.
6. The average reservoir pressure in the drainage area.

*Problem data equivalent to Example 6.7, p. 323, in Ref. 2.

Solution:

[1] To obtain some concept of how rapidly a "reservoir" of this size deviates from infinite-acting, we will prepare a plot of the $P_{[t_D]}$ function vs dimensionless time t_D. To find the reservoir's outer boundary r_e consider the given well spacing of 40 acres. Rectangularly spaced wells on a 40-acre pattern are about 1,320 ft apart. We will then estimate the drainage radius at 660 ft. This is clearly a minimum value for this well spacing. On this basis,

$$r_d = r_e/r_w = (660/0.333) = 1980.$$

We would then expect this reservoir to act as infinite until [from Eq. 21]

$$(t_D)_c = \frac{r_D^2 - 1}{4} = \frac{(1980)^2}{4} = 0.980 \times 10^6. \quad . \quad . \quad . \quad [21]$$

The $P_{[t_D]}$ function for infinite-acting flow at this dimensionless time is [from Eq. 18]

$$P_{(t_D)} = 1/2\ (\ln t_D + 0.809)$$

$$= 1/2 \left[\ln (0.980 \times 10^6) + 0.809\right]$$

$$= 7.294.$$

For $t_D < 0.98 \times 10^6$, the $P_{[t_D]}$ function can be calculated by Eq. 18, and for $t_D > 0.98 \times 10^6$, the pseudo-steady-state Eq. 13 will apply. For example, when $t_D = 10^5$,

$$P_{(t_D)=10^5} = 1/2\ (\ln t_D + 0.809)$$

$$= 1/2\ (\ln 10^5 + 0.809)$$

$$= 6.154. \quad . \quad . \quad . \quad [18]$$

In employing Eq. 13,

$$P_{(t_D)} = \frac{2(t_D + 1/4)}{r_D^2 - 1} - \frac{3r_D^4 - 4r_D^4 \ln r_D - 2r_D^2 - 1}{4(r_D^2 - 1)^2},$$

to calculate $P_{[t_D]}$ functions, we find that for r_D = 1,980, the r_D^2 and "1" values are insignificant in the second term when compared with the value of r_D^4. Also, the "1" and 1/4 values in the first term are insignificant when compared with t_D and r_D^2. Thus, Eq. 13 simplifies to

$$P_{(t_D)} = \frac{2t_D}{r_D^2} - \frac{3}{4} + \ln r_D . \quad . \quad . \quad . \quad . \quad . \quad [13A]$$

This simplified form can be used for most practical r_D and t_D values. For r_D = 1980, Eq. 13A becomes

$$P_{(t_D)} = \frac{2t_D}{(1980)^2} - \frac{3}{4} + \ln 1980$$

$$= \frac{t_D}{(1.96 \times 10^6)} + 6.83.$$

For $t_D = 10^7$,

$$P_{(t_D = 10^7)} = \frac{10^7}{1.96 \times 10^6} + 6.83$$

$$= 11.93 \text{ (see footnote)}.$$

Note: It is of interest that the pressure function $P_{[t_D]}$ calculated at the critical dimensionless time $[t_D]_c = 0.98 \times 10^6$, by the pseudo-steady-state Eq. 13;

$$P_{(t_D = 0.98 \times 10^6)} = \frac{t_D}{(1.96 \times 10^6)} + 6.83$$

$$= \frac{.98 \times 10^6}{1.96 \times 10^6} + 6.83 = 7.33,$$

is 0.04 larger than the $P_{t_D = 0.98 \times 10^6}$ value calculated by the infinite acting Eq. 18. It can be shown that for all practical size reservoirs at the critical time $[t_D]_c = r_D^2/4$, the P_{t_D} functions calculated by Eqs. 13 and 18 will always differ by about 0.04.

Continued calculation of the P_{t_D} vs t_D functions results in the plot of Fig. 3.

$$P_{t_D = 2.0 \times 10^6} = \frac{2.0 \times 10^6}{1.96 \times 10^6} + 6.83 = 7.850,$$

$$P_{t_D = 3.5 \times 10^6} = \frac{3.5 \times 10^6}{1.96 \times 10^6} + 6.83 = 8.616,$$

$$P_{t_D = 7.0 \times 10^6} = \frac{7.0 \times 10^6}{1.96 \times 10^6} + 6.83 = 10.401,$$

etc.

Fig. 3 indicates the rapidity with which the reservoir pressures deviate from infinite acting.

[2] A knowledge of ϕc can be obtained from the change in the well pressure with time before shut-in if the reservoir is in pseudo-steady state before shut-in [which is evaluated in (3) below]. The tubing-head flowing pressure may be used for this purpose because the flow rate and fluid composition [GOR] are constant. Thus, the static pressure differences and friction losses in the well are constant. Clearly, bottom-hole pressure measurements would be more accurate, but they are less likely to be available. Calculating ϕc from Eq. 17:

$$\left(\frac{dp_w}{dt}\right)_q = \frac{1.8\, q}{r_e^2 \phi h c}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [17]$$

$$24 = \frac{(1.8)\ (280)\ (1.31)}{(660)^2\ (40)\ (\phi c)}, \text{ and solving,}$$

$$\phi c = 1.578 \times 10^{-6}.$$

[3] The effective permeability can be evaluated only if we know whether the well is in pseudo-steady-state [as assumed in (2) above] or unsteady-state flow at time of shut-in. Calculation of t_c as a function of the permeability will permit us to guess as to whether an unsteady-state or pseudo-steady-state analysis should be used. Applying Eq. 22,

$$t_c = \frac{\phi \mu c\, r_e^2}{25.3\, k}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [22]$$

$$t_c = \frac{(1.578 \times 10^{-6})\ (2.0)\ (4.356 \times 10^5)}{25.3\, k},$$

or

$$t_c = \frac{0.0543}{k} \text{ days.}$$

Thus, because the well has produced for 10 days, it will be in pseudo-steady state if

$$k > \frac{0.0543}{10} = 0.00543 \text{ Darcys, or } 5.43 \text{ md.}$$

It then appears that with a producing rate of 280 STB/D from 40 ft of pay, the effective permeability will be greater than 5.43 md. Thus, we will proceed with the pseudo-steady-state analysis of the permeability. Assuming the well was in pseudo-steady state at the time of shut-in and that the pressure was dropping at the rate of 24 psi/day or 1 psi/hour, we calculate the pressure change due to shutting the well in, Δp^*, by subtracting from the observed pressure the pressure that would result had the well continued producing.

$$\Delta p^* = p_{observed} - \left[p_{wf} - t\left(\frac{dp_w}{dt}\right)_q\right]$$

[1] Shut-In Time [Hours]	[2] Observed Pressure [psia]	[3] Pressure if Production Were Continued [1,123] - [Col. 1]	[4] Δp^* Due to Shut-In [Col. 2]-[Col. 3]
0	1,123	1,123	0
2	2,290	1,121	1,169
4	2,514	1,119	1,395
8	2,584	1,115	1,469
12	2,612	1,111	1,501
16	2,632	1,107	1,525
20	2,643	1,103	1,540
24	2,650	1,099	1,551
30	2,658	1,093	1,565

Δp^* is plotted vs the log of shut-in time in Fig. 5. The slope of this line is about 145 psi/cycle.

Thus, k can be calculated from the slope of the plot,

$$m = \frac{0.1625\, q\mu}{kh} .$$

By substitution,

$$145 = \frac{(0.1625)\ (280)\ (1.31)\ (2.0)}{k\ (40)}$$

and

$$k_o = 0.0206 \text{ Darcys or } 20.6 \text{ md.}$$

We showed previously that the permeability must be greater than 5.43 md for the well to be in pseudo-steady state in 10 days [the producing time before shut-in]. The reservoir actually started producing in pseudo-steady state when

$$t_c = \left(\frac{.00543}{.02060}\right)(10) = 2.64 \text{ days or } 63.4 \text{ hrs.}$$

Note that this conclusion could also be reached without having any known reservoir parameters by assuming pseudo-steady-state flow and calculating t_c from Eq. 29,

$$t_c = \frac{.438\ m}{(dp_w/dt)_q} = \frac{(.438)\ (145)}{24} = 2.64 \text{ days.}$$

Thus, the reservoir permeability to oil is 20.6 md as calculated.

[4] To compare the pseudo-steady- and unsteady-state analyses of this problem, we have shown in Fig. 5 the conventional unsteady-state plot of the log $[t/(T+1)]$ vs the well pressure p_w where t is the shut-in time and T is the total producing time prior to shut-in. This plot has a shorter straight-line data fit than the Δp^* vs log t curve and indicates that both prior to and during the shut-in time, the outer boundary [or some other reservoir obstruction] was affecting the well pressure.

[5] The skin factor can be calculated from Eq. 26A,

$$S = 1.15\,\frac{(\Delta p^*)_{1\ hr}}{m} - 1.15 \log \frac{(dp_w/dt)_q}{18.7\ m\ (r_w^2/r_e^2)} .$$

Substituting,

$$S = 1.15\,\frac{1350}{145} - 1.15 \log \frac{24}{(18.7)(145)\left[(.333)^2/(660)^2\right]}$$

$$= 10.71 - 1.15 \log (3.47)(10^4)$$

$$= 5.22.$$

A positive skin factor indicates well damage has occurred.

[6] The average reservoir pressure in the drainage area can now be calculated by Eq. 28,

$$p_s = p_{wf} + (\Delta p^*)_{t_c} - 0.438\ m.$$

t_c was previously calculated as 63.4 hrs. So from Fig. 5, $[\Delta p^*]t_c$ = 1615 psia, and using the slope of the curve as previously calculated,

$$p_s = 1123 + 1615 - 0.439\ (145)$$

$$= 2674\ \text{psia.}$$

Note that this value gives a reasonable extrapolation to the plot of p_w vs log $[t/(T + t)]$ for t = ∞ or $[t/(T + t)] = 1.0$.

NOMENCLATURE

A = cross-sectional area, sq ft
c = effective compressibility of reservoir fluids, psi^{-1}
h = reservoir thickness, ft
e = mathematical constant, 2.7
J_1 = Bessel function
k = permeability, darcys
ℓ = distance from producing face, ft
L = length of linear system, ft
n = mathematical counter
p = pressure, psia
p_{avg} = weighted average pressure, psia
p_e = pressure at internal reservoir radius r_e, psia
p_i = initial reservoir pressure, psia
p_2 = pressure at outer reservoir distance L, psia
p_r = pressure at radius r, psia
p_w = pressure at well radius r_w, psia
$P_{[t_D]}$ = Hurst-van Everdingen pressure function, psi/dimensionless rate
q = flow rate, reservoir B/D
q_l = flow rate at distance l from producing face, reservoir B/D
q_r = flow rate at radius r, reservoir B/D
q_w = flow rate at well radius r_w, reservoir B/D
r = radius, ft
r_D = dimensionless reservoir size
r_e = external reservoir boundary, ft
r_w = well radius, ft
S = skin factor, psi/"dimensionless" rate
t = producing time or shut-in time, days
t_c = critical real time equivalent to $[t_D]_c$, days
t_D = dimensionless time
$[t_D]_c$ = critical dimensionless time when pressure at r_w quits acting as part of an infinite reservoir
T = total producing time until well shut-in, days
μ = viscosity, cp
B_n = Bessel functions root
ϕ = porosity, fraction

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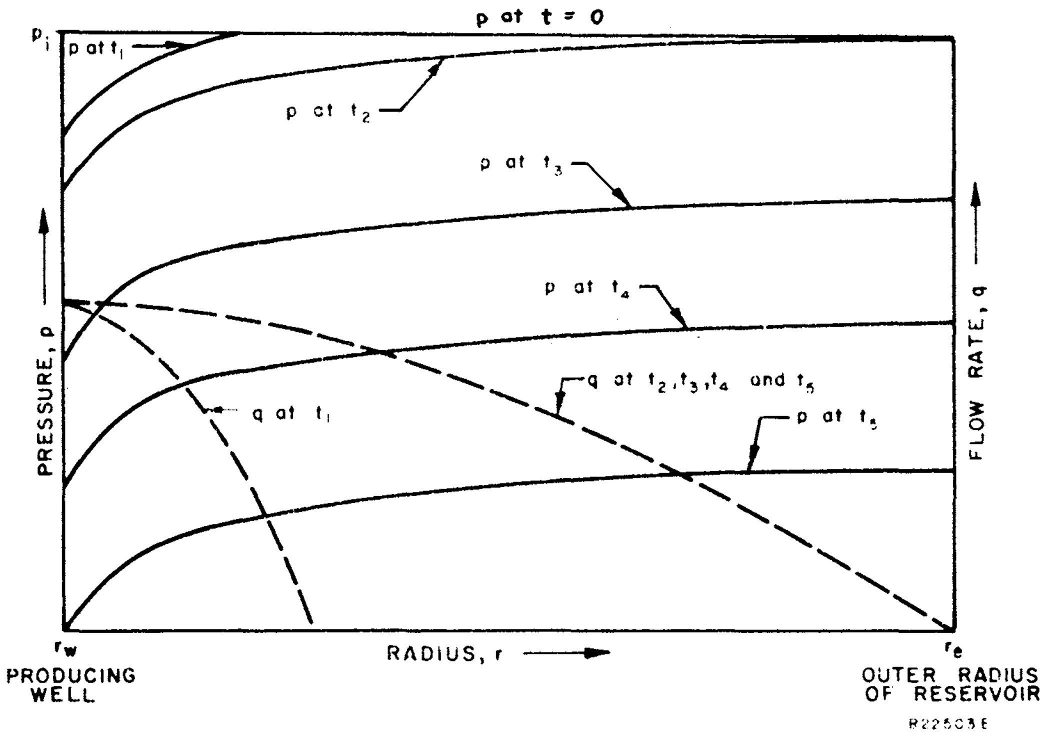

FIG. 1 PRESSURE AND RATE VS DISTANCE FROM WELL BORE FOR UNSTEADY-STATE RADIAL FLOW WITH CONSTANT PRODUCING RATE-PSEUDOSTEADY-STATE FROM t_2 TO t_5

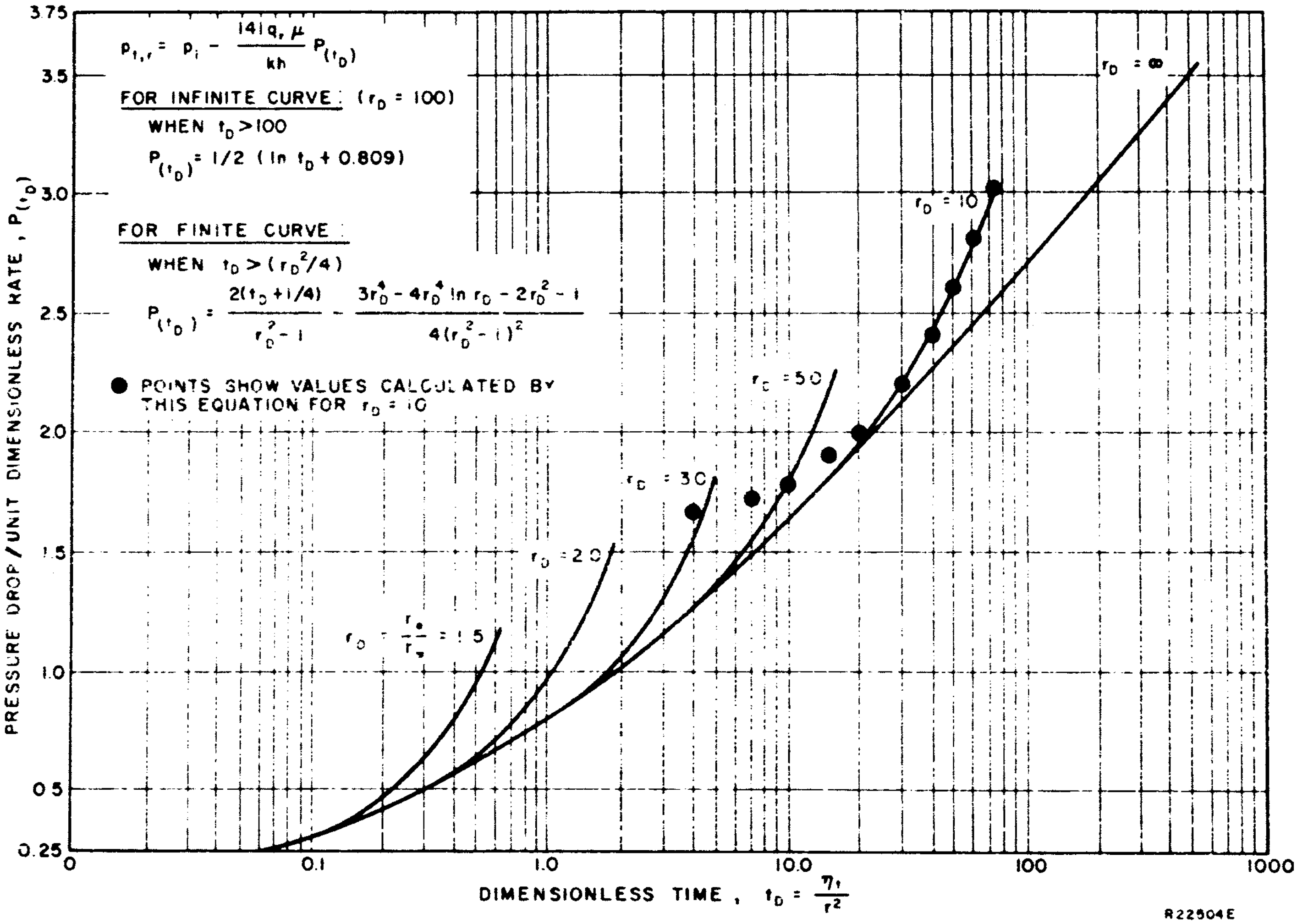

FIG. 2. PRESSURE BEHAVIOR AT A PRODUCING POINT FOR THE RADIAL UNSTEADY-STATE, CONSTANT TERMINAL RATE CASE

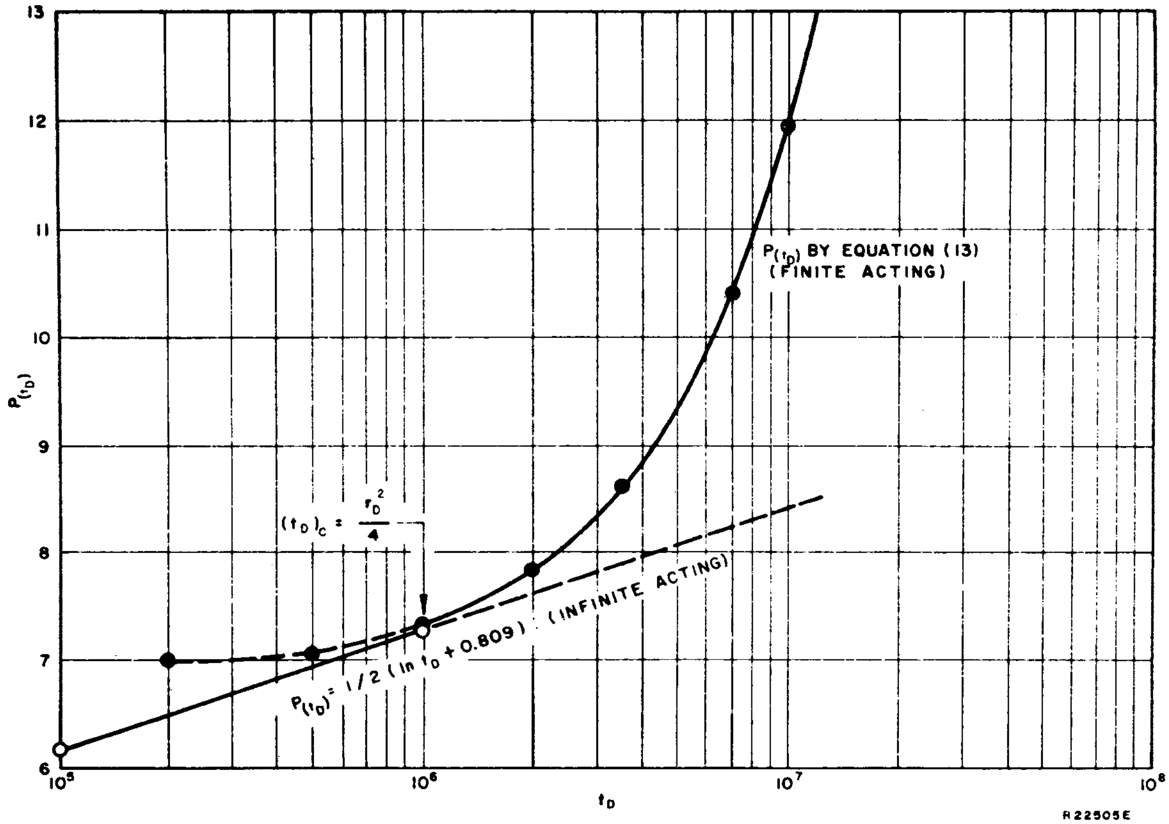

FIG. 3. $P_{(t_D)}$ VS t_D FOR r_D = 1980

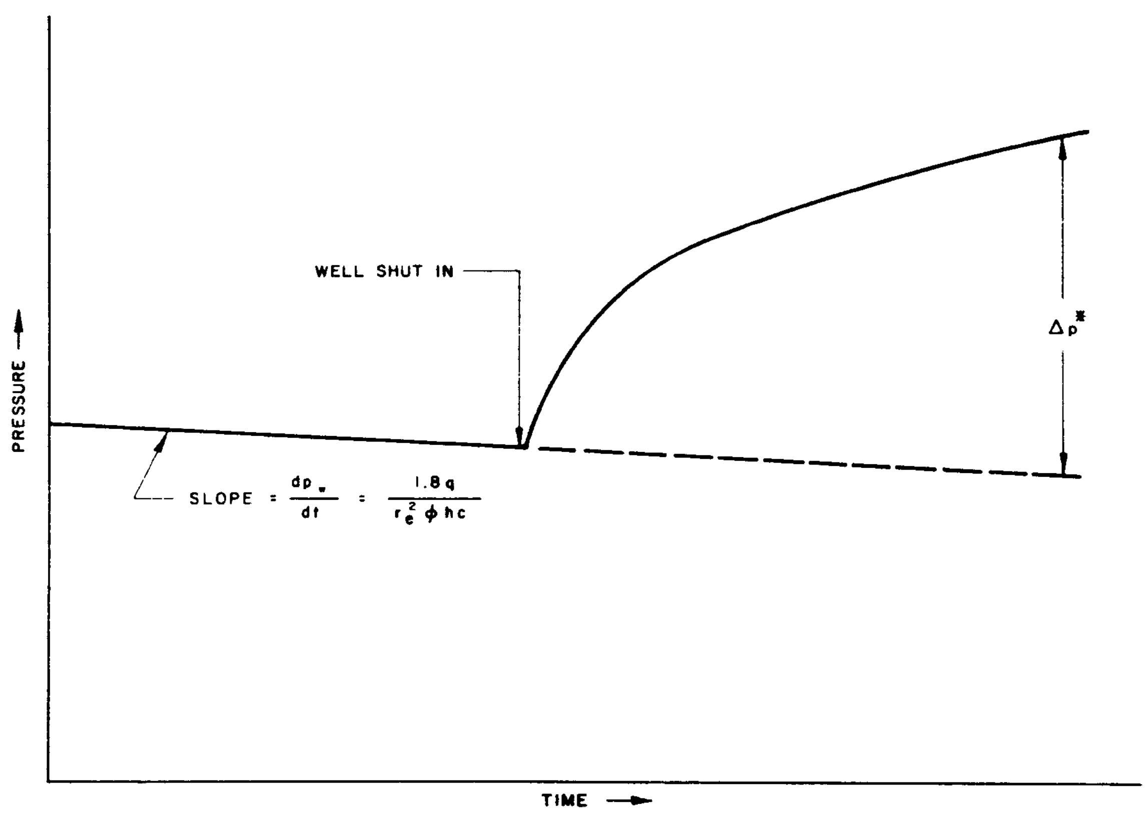

FIG. 4. PRESSURE BUILDUP OF WELL PRODUCING IN PSEUDO-STEADY STATE

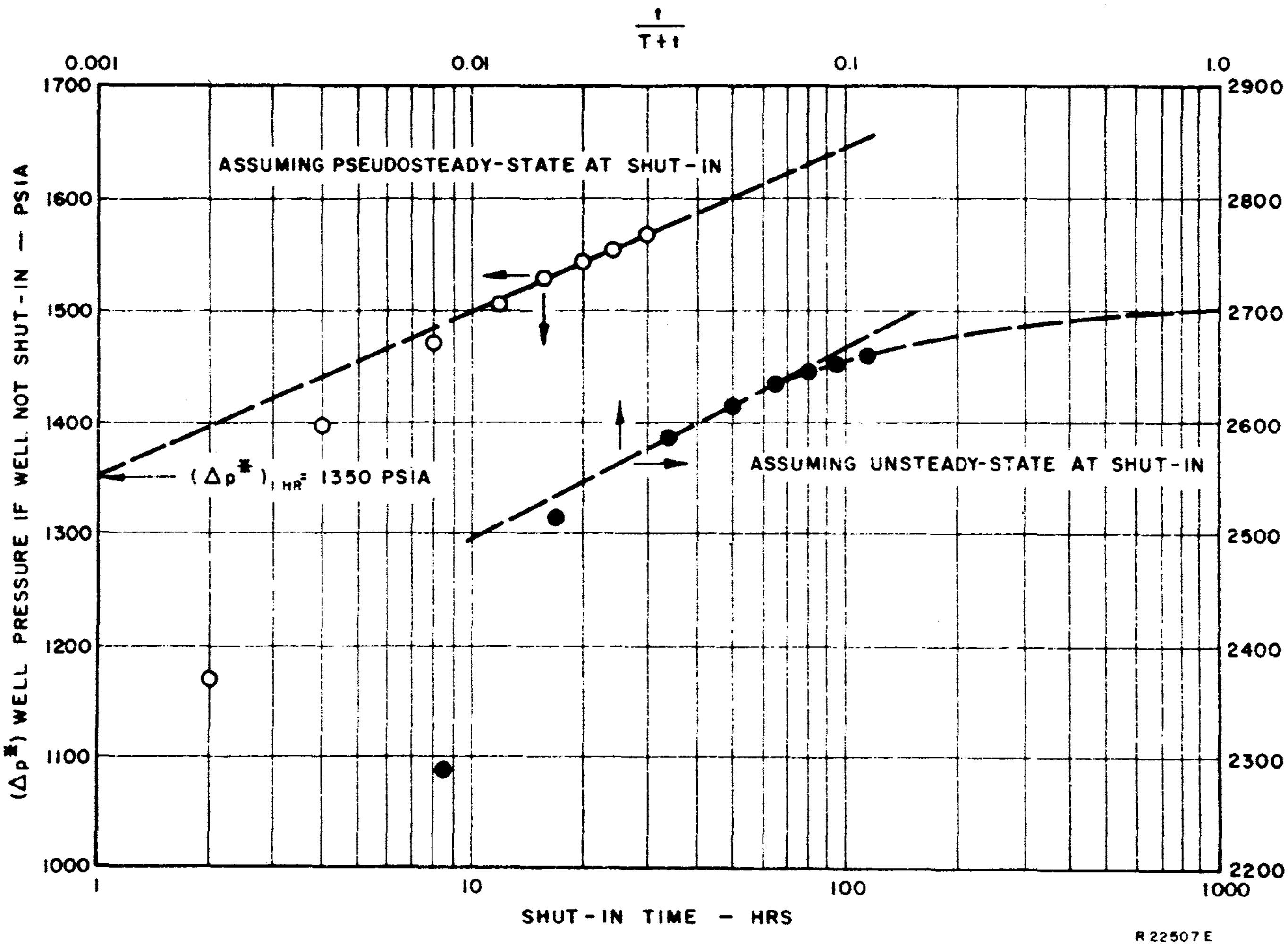

FIG. 5 COMPARISON OF PSEUDOSTEADY-STATE AND UNSTEADY-STATE PLOTS OF PRESSURE BUILDUP DATA