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Water Drive Gas Reservoirs: Uncertainty in Reserves Evaluation From Past History

G. L. CHIERICI
G. PIZZI
G. M. CIUCCI
MEMBERS AIME

AGIP-DIREZIONE MINERARIA
MILANO, ITALY

ABSTRACT

The use of the material balance equation to estimate the volume of hydrocarbons originally present in a reservoir, whose producing mechanism is partly due to water drive, has been discussed in the literature by several authors. There is no general agreement upon the possibility of obtaining reliable results by this method. Gas reservoirs in contact with an active aquifer are considered in this paper.

Theoretical considerations based on the cybernetic principle of uncertainty (which states that the internal structure of a system cannot be uniquely determined from its observed external behavior) lead to the conclusion that the volume of gas originally present in a reservoir of this type cannot be uniquely determined from its past history. The range of values which encompasses the actual value of the reserves varies from case to case and must be determined by either numerical or analogical methods.

Results obtained for six gas fields are reported. All these fields were produced with small fluctuations in their production rates, as is common practice for gas reservoirs; no gas storage fields were considered. Results obtained show that reserves values in a range of 1 to 2, associated with appropriate aquifers, allow the matching of the reservoir past history with mean-square deviations less than the experimental errors involved in pressure and production measurements. Similar results have been found in several other partial water drive gas reservoirs. From these results it is concluded that gas reserves cannot be uniquely determined from the past performance of partial water drive reservoirs, at least in cases where the reservoir has been submitted to small fluctuations in the production rates, and pressure data of normal accuracy are available.

INTRODUCTION

A number of authors have analyzed the problem of estimating the reserves originally present in a partial water drive reservoir from its past pressure-production performance. Literature which deals with this subject can be grouped, according to their conclusions, as follows.

1. A reliable value for the reserves can be obtained even if reservoir data (pressure and cumulative production) are affected by errors within the normal range.[1-6]

2. A reliable value for the reserves can be obtained only if reservoir data are very accurate[7-9] or if past production performance has been subjected to abrupt variations in the production rate.[10]

3. No unique value for the reserves can be obtained from reservoir past production performance. This conclusion has been based upon theoretical considerations[11] and verified in several field cases.[12,13]

The purpose of this paper, which deals only with partial water drive gas reservoirs, is to test the above conclusions against actual field cases. Some theoretical considerations on this problem are also presented.

THEORETICAL CONSIDERATIONS

As the behavior of a gas reservoir communicating with an aquifer depends on both the aquifer and the reservoir characteristics, the physical system to be studied is the combined gas reservoir plus aquifer. The information which is available for studying the performance of such a system is the well production rates and bottom-hole pressures, all given as functions of time.

The external behavior of the reservoir-aquifer system is therefore described by $2n$ input variables (G_{p_i}, W_{p_i}) and n output variables (p_i), n being the number of wells in the reservoir.

In reservoir engineering it is common practice to consider the reservoir as a whole, disregarding the internal distribution of pressures and of producing wells. This practice is equivalent to substituting the above multivariable system with a single-variable system, where the average reservoir pressure is the only output variable and the cumulative production $G_p(t)$ and $W_p(t)$ are the input variables. The internal structure of such a system, defined by initial gas reserves G, aquifer shape and dimensions, boundary conditions and petrophysical parameters distribution throughout the aquifer, is unknown. Therefore, from a cybernetic point of view the system is a "blackbox".[14-17]

It has been demonstrated[17] that for a blackbox the indetermination principle holds. Accordingly, the number of different internal structures (or set of parameters) which can account for the observed external behavior is infinite. As a consequence, the initial reserves cannot be uniquely determined from the reservoir past performance.

When $W_e(t)$ is known, the determination of G from

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[1]References given at end of paper.

Discussion of this and all following technical papers is invited. Discussion in writing (three copies) may be sent to the office of the *Journal of Petroleum Technology.* Any discussion offered after Dec. 31, 1967, should be in the form of a new paper. No discussion should exceed 10 percent of the manuscript being discussed.

the material balance equation is straightforward. This is an ideal case since it is usually not possible to accurately determine $W_e(t)$ from the gas-water contact advance which can be observed through the wells; the reason lies in the uncertainty affecting the microscopic displacement efficiency and conformance factor values. The range of indetermination in the initial gas reserves value depends on the specific case under consideration, and its estimation requires finding those reservoir-aquifer systems which are able to reproduce the reservoir past performance when subjected to the actual past production history. This has been done for a number of Italian gas fields producing by partial water drive. The computation method used and the experimental results obtained are summarized.

COMPUTATION METHOD

A set of reserves values encompassing the volumetrically computed one was selected for each reservoir. For each of these reserves values an aquifer was found that permits reproduction of the actual reservoir past history with the minimum root mean-square deviation in the pressure data. In accomplishing this work, use was made of a simulator and a trial-and-error procedure. The simulator used[18,19] is a multi-pool analyzer consisting of an RC network (simulating the aquifer) coupled to an analog computer which simulates each gas pool as a whole.

Use of an analog simulator is most convenient when heterogeneous aquifers of any shape or multi-pool problems are to be handled. Also, in the homogeneous radial aquifer cases referred to in this paper, a digital technique could have been used.

The pressure history obtained from the analog simulator can be considered the exact history of the simulated reservoir-aquifer system, due to the small magnitude of experimental errors involved. If the deviation between the actual pressure history of the reservoir and the calculated exact history is within the experimental error affecting reservoir pressure measurements, one can state that the behavior of the simulated reservoir-aquifer system is an exact replica of the actual reservoir behavior. In particular, the assumed value for the reserves is consistent with the past history of the actual reservoir.

To obtain an objective evaluation of the deviation between the two histories to be compared, the past performance of the actual reservoir has been defined through the use of the experimental points only, without attempting any interpolation.

Selection of the aquifers to be associated to the assumed reserves values has been purposely limited to homogeneous aquifers of circular shape. This limitation has been adopted to take the same position as the field engineer who, for simplicity and tradition, is inclined to consider homogeneous and circular (or linear) aquifers only, for which the Q_{tD} and p_{tD} functions are available from the literature. Of course, heterogeneous aquifers of any configuration can be handled very easily by the simulator.

For each simulated reservoir-aquifer system matching the reservoir actual history, the characteristic constants (C, K_t, r_{eD}) of the aquifer were derived from the RC network. These constants and the corresponding reserves value were then entered into a digital computer and the exact pressure history numerically calculated in the usual way[10] (digital computation having the advantage of better accuracy). Data presented in this paper are those obtained from the digital computation.

The deviation of each exact history from the reservoir actual history was expressed quantitatively by means of the root mean-square deviation σ in the pressure data, and also by the percentage average deviation (PAD).[10]

$$\mathrm{PAD} = \frac{\Sigma\,|p_{res.} - p_{comp.}|}{\Sigma\, p_{res.}} \cdot 100 \quad . \; . \; . \; . \; . \quad (1)$$

A comparison between PAD and the percentage error affecting the experimental measurements of the reservoir average pressure gives a straightforward indication of how good the matching is. Taking into account the several factors affecting the computation accuracy of reservoir average pressure (instruments accuracy and errors involved in averaging methods), PAD values of less than 1 percent can be considered an excellent match between the actual and calculated pressure histories. The PAD values obtained from the many reservoir-aquifer systems examined for studying the reservoirs described in this paper are almost always less than 1 percent. In many cases, PAD values could have been further reduced if the selection of possible aquifers had been extended to heterogeneous ones, or if the reservoir initial pressure value had been taken slightly different from that given by the actual reservoir history.

EXPERIMENTAL RESULTS

The range of indetermination which affects the value of gas reserves evaluated from the reservoir past history was experimentally ascertained on many gas fields of the Po River Valley. The results presented in this paper refer to five of these reservoirs (A-1, BM-A, C-C, R-A and S-F). For each reservoir, the basic data and pressure production past history are reported in Table 1. The thermodynamic characteristics of each reservoir gas are given in Table 2. At the time this study was made, all the reservoirs had been producing for several years and a significant percentage of their initial gas reserves had been produced. Each reservoir is in contact with an aquifer. In all cases the presence of a water drive mechanism is indicated by the water invasion of some peripheral wells; nevertheless, the cumulative water production was negligible.

A Gulf Coast gas field, whose data were available from the literature,[4] was also studied. The reason for studying this field arose from the fact that results obtained by the application of an especially accurate statistical method for estimating the reserves were available.[4]

FIELD A-1

The range of initial reserves values which has been explored for Field A-1 is from 141.3 to 353.1 Bscf (Table 3). All the calculated past histories are equivalent as they reproduce the actual past history with PAD values less than 1 percent, except for Case 7 (PAD = 1.08 percent). Therefore, the actual reservoir-aquifer system and all the proposed systems are equivalent. From available information on the volume of water encroached into the actual reservoir, it can be inferred only that G values less than 210 Bscf are unlikely. For one case (G = 264.9 Bscf) the possibility was examined to simulate the past history with several aquifers of different characteristics.

Six different aquifers were equivalent; the relative C and K_t values are plotted, together with the corresponding PAD values, in Fig. 1. From the interpretation of the experimental points shown in this figure it is evident that an infinite number of aquifers can be combined with the above G value, all giving the same past history. Therefore, in this specific case it would be impossible to uniquely define the characteristics of the aquifer even if reserves value is known. This result follows from the indetermination principle, the aquifer being a blackbox whose external behavior only is known.

To evaluate the resolving power of line-fitting methods described in the literature, the one by Hubbard *et al.*[7] has been applied to this reservoir, using two pairs of corresponding values among those found by the analog simulator for K_t and r_{eD}. In both cases the experimental points lie with good accuracy on a straight line (Fig. 2) and the corresponding PAD values are less than the experimental error affecting the measurements of the reservoir average pressure. The two resulting G values are nevertheless quite different (170.4 and 254.8 Bscf, respectively).

However, this method was originally developed for application to gas reservoirs converted to gas storage. These reservoirs are subjected to very strong fluctuations in their production rates; therefore, the resulting pressure response contains much more information about the system. There

TABLE 1—BASIC DATA AND PAST PERFORMANCE OF RESERVOIRS EXAMINED

Reservoir	A - 1		BM - A		C - C		R - A		S - F	
Initial res. vol. calc. (Bscf)	328.7		33.87		126.4		168.4		176.6	
Range assumed Initial res. (Bscf)	141.3 to 353.1		24.72 to 53.00		88.3 to 176.6		105.9 to 158.9		123.6 to 176.6	
Cum. prod. when study was run, % Initial vol. res.	33.6		49.9		57.7		44.5		58.5	
Time (months)	Cumulative Gas Produced G_p (Bscf)	Average Pressure p (psia)	Cumulative Gas Produced G_p (Bscf)	Average Pressure p (psia)	Cumulative Gas Produced G_p (Bscf)	Average Pressure p (psia)	Cumulative Gas Produced G_p (Bscf)	Average Pressure p (psia)	Cumulative Gas Produced G_p (Bscf)	Average Pressure p (psia)
0	0.0000	2,418	0.0000	1,927	0.0000	1,818	0.0000	2,630	0.0000	2,233
2	0.8361	—	0.2666	—	0.6922	—	0.1862	—	0.4664	—
4	3.0159	2,364	0.6672	1,897	1.3508	—	0.3725	—	1.0768	2,229
6	4.9369	2,367	1.1783	—	1.8010	1,781	0.6699	—	1.3072	2,219
8	6.2935	2,365	1.7626	—	3.1535	—	0.9920	—	1.3072	2,225
10	7.8446	2,350	2.1717	—	4.1935	—	1.3233	—	1.7907	—
12	10.0671	2,335	2.4314	1,883	5.2247	—	1.6387	—	2.6891	—
14	10.9672	2,335	2.6274	—	6.2594	—	2.0196	—	3.7689	—
16	12.5068	2,327	3.0311	1,862	7.2923	1,751	2.7017	—	5.1777	—
18	14.2483	—	3.5872	—	8.3959	—	3.9231	—	6.4195	2,169
20	16.4686	—	4.1687	1,832	9.5083	—	5.0600	—	7.3272	2,153
22	19.1569	2,269	4.6375	1,831	10.6348	—	6.1849	—	7.7217	2,145
24	21.8601	2,254	4.9043	1,826	11.7543	—	7.3049	—	8.3740	2,142
26	25.0694	2,237	5.1545	—	12.8190	1,712	8.4935	—	9.8462	—
28	27.3912	2,213	5.5171	1,822	13.7071	—	10.9257	—	11.4127	—
30	30.6637	—	6.1030	1,805	14.5970	—	13.6516	—	12.8441	—
32	33.9971	—	6.7792	—	15.4940	—	14.9303	—	14.2204	—
34	36.9639	—	7.2769	1,795	16.3804	—	15.1784	—	14.8703	2,057
36	39.7172	2,156	7.5702	1,784	17.2632	—	15.4385	—	16.4894	2,044
38	41.5226	2,143	8.0219	—	18.1461	—	16.5508	—	18.3611	2,023
40	44.4413	—	8.6017	1,765	19.0191	1,678	18.4808	—	20.2019	—
42	47.1744	—	9.3227	—	19.9930	—	21.5384	—	22.1236	—
44	49.9532	—	10.1055	—	20.9730	—	23.9335	—	23.8098	—
46	52.8945	2,098	10.6241	1,711	21.9424	—	25.9430	—	25.1021	1,934
48	55.8469	—	11.0437	—	22.9170	—	26.9793	—	27.4847	1,940
50	58.0177	2,091	11.4503	1,700	23.8899	—	29.0273	—	30.2339	1,897
52	61.4001	—	11.9256	—	24.8734	1,643	31.4206	—	33.4714	—
54	64.5613	—	12.4467	—	25.8446	—	34.1037	—	36.6410	—
56	67.3282	2,034	12.9793	—	26.8298	—	35.3486	2,114	39.7014	1,775
58	70.2294	—	13.4397	—	27.8009	—	35.6304	2,108	41.0112	—
60	72.8696	2,014	13.8768	1,680	28.7756	—	35.7354	2,108	44.6297	—
62	75.7672	1,996	14.1546	—	29.7397	—	36.0765	—	48.7076	—
64	78.8404	—	14.6017	1,668	30.8115	1,621	37.7987	—	52.6157	—
66	81.8056	1,980	15.0466	1,668	32.1128	1,599	39.0974	—	54.4177	—
68	84.7276	—	15.3608	1,657	33.5006	—	39.7663	2,067	55.5025	1,599
70	87.7205	—	15.7766	1,641	34.8973	—	40.0428	2,065	56.5722	—
72	90.6385	—	16.1680	1,639	36.2961	1,579	40.2941	2,067	58.9644	1,556
74	93.6779	—	16.2068	1,678	37.4505	—	41.7141	—	62.9243	1,513
76	96.2496	1,913	16.4751	1,661	38.6106	—	43.5249	—	66.0817	—
78	99.1307	—	16.9106	—	39.7660	1,566	45.5659	—	67.6350	1,465
80	102.1177	—			40.8618	—	46.3925	1,977	70.1428	1,415
82	104.9584	—			42.3945	—	46.4783	—	71.1077	1,408
84	107.8226	—			44.0676	1,543	47.1400	—	74.3630	1,363
86	110.3559	1,853			45.0783	1,535	48.9278	—	77.4440	—
88					46.4432	—	51.5351	—	79.7133	—
90					47.9158	—	54.0549	—	81.7708	—
92					49.4008	1,508	55.9902	—	83.6547	—
94					50.9140	—	57.7358	—	84.8277	1,250
96					52.4589	1,498	58.6393	1,815	87.1868	—
98					53.7684	—	60.8932	—	89.7962	—
100					55.2798	—	63.1990	—	92.2207	—
102					56.7782	1,479	65.7134	—	94.0234	1,115
104					58.3553	—	68.0458	1,651	95.4639	—
106					59.9819	—	69.3764	—	96.3466	1,109
108					61.5527	1,444	69.8445	1,637	97.2763	—
110					62.8942	—	72.2631	1,607	98.9844	—
112					64.4735	—	74.8727	—	100.7267	—
114					66.0736	1,411			101.5752	—
116					67.6362	1,402			101.9083	1,057
118					69.0424	—			103.3246	—
120					70.3684	—				
122					70.9727	—				
124					71.9099	—				
126					72.8644	1,401				

TABLE 2—BASIC DATA FOR GASES OF RESERVOIRS EXAMINED

Reservoir Parameters	A-1	BM-A	C-C	R-A	S-F
Reference temperature (°F)	59	59	59	59	59
Reference pressure (psia)	14.69	14.69	14.69	14.69	14.69
Reservoir temperature (°F)	109.8	100.6	98.8	130.8	123.3
	Compressibility factors at reservoir temperature $\left(z = \frac{pV}{nRT} \right)$				
Pressure (psia)					
2702	—	—	—	0.8827	—
2560	0.8475	—	—	0.8812	—
2418	0.8454	—	—	0.8802	—
2276	0.8478	—	—	0.8811	0.8760
2134	0.8504	—	—	0.8816	0.8775
1991	0.8517	0.8544	—	0.8830	0.8797
1849	0.8562	0.8598	0.8474	0.8859	0.8828
1707	0.8612	0.8635	0.8534	0.8914	0.8876
1565	0.8680	0.8695	0.8594	0.8967	0.8928
1422	0.8764	0.8782	0.8676	0.9020	0.8990
1280	0.8840	0.8864	0.8771	0.9093	0.9065
1138	0.8916	0.8940	0.8861	0.9166	0.9144
996	0.9023	0.9065	0.8956	0.9246	0.9235
853	0.9136	0.9162	0.9073	0.9334	0.9327

is no doubt that, in these cases, the Hubbard *et al.*[7] method has a resolving power higher than that shown for reservoirs under primary depletion, where the production rate is fairly constant.

FIELD BM-A

The range of initial reserves values explored for Field BM-A were from 24.7 to 53.0 Bscf (Table 3, Fig. 3). The results show that, for this example also, the correct value for the initial reserves cannot be determined from the past history: the range of indetermination is at least from 24.7 to 53.0 Bscf. Moreover, Fig. 3 shows that infinite G values in the range studied, coupled with aquifers having the

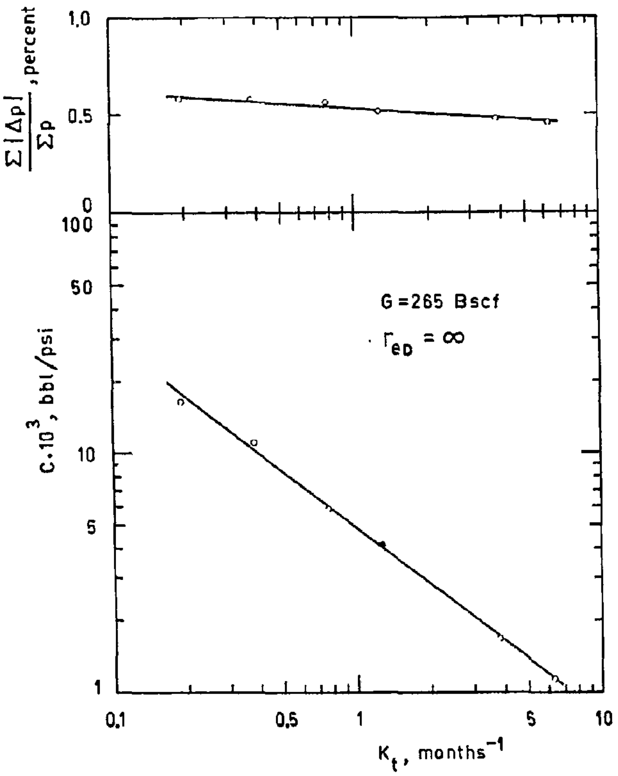

FIG. 1—RESERVOIR A-1: RELATIONSHIP BETWEEN C AND K_t PARAMETERS OF EQUIVALENT AQUIFERS.

TABLE 3—SUMMARY OF RESULTS OBTAINED

Reservoir	Case	Assumed Initial Reserves, G (Bscf)	Aquifer Characteristics C (bbl/psi)	Aquifer Characteristics K_t (month^{-1})	r_{eD}	Deviation Between Actual & Calc. Pressure History $\frac{\Sigma\|\Delta p\|}{\Sigma p}$ (percent)	Deviation Between Actual & Calc. Pressure History σ (psi)	Percent. in. res. vol. invaded at end of past hist.
A-1	1	141.26	14,339.0	0.532	7.0	0.358	9.2	71.1
	2	176.57	10,037.5	0.723	9.0	0.284	8.9	51.1
	3	211.88	5,576.3	1.277	12.0	0.287	7.4	37.2
	4	247.20	5,018.7	1.065	∞	0.473	12.5	26.6
	5	282.51	3,584.8	1.420	∞	0.650	17.1	19.9
	6	317.83	5,282.7	0.639	∞	0.872	22.5	13.9
	7	353.14	2,334.7	1.277	∞	1.082	27.2	9.3
	8	264.86	1,115.2	6.388	∞	0.461	12.5	23.2
	9	264.86	1,672.8	3.833	∞	0.479	12.7	22.9
	10	264.86	4,182.1	1.277	∞	0.517	14.0	23.3
	11	264.86	5,904.0	0.766	∞	0.562	14.8	22.5
	12	264.86	10,037.5	0.383	∞	0.574	14.7	22.5
	13	264.86	16,729.0	0.191	∞	0.582	15.2	22.5
BM-A	1	24.72	9,121.3	0.3968	4.46	0.327	7.6	62.8
	2	31.78	8,181.7	0.3968	4.46	0.466	10.5	44.6
	3	38.85	6,776.9	0.3968	4.46	0.588	12.9	32.5
	4	45.91	6,236.1	0.3968	4.46	0.397	10.1	24.8
	5	52.97	5,647.5	0.3968	4.46	0.371	9.5	19.3
C-C	1	88.29	4,844.1	2.129	14.0	1.053	20.8	77.9
	2	105.94	4,285.2	2.129	14.0	0.607	12.0	59.5
	3	123.60	6,190.1	0.958	∞	0.423	8.3	46.2
	4	141.26	5,571.0	0.958	∞	0.308	6.5	36.0
	5	158.91	5,064.3	0.958	∞	0.384	8.2	28.5
	6	176.57	4,204.2	0.958	∞	0.356	7.2	21.5
R-A	1	105.94	49,135.0	0.0107	50.0	0.829	19.2	48.7
	2	123.60	37,289.0	0.0113	50.0	0.561	13.3	32.4
	3	141.26	3,692.2	0.2680	5.0	0.328	8.2	19.5
	4	158.91	5,319.4	0.3570	2.5	0.441	11.6	8.9
S-F	1	123.60	1,326.5	3.690	14.0	0.819	17.9	64.5
	2	141.26	1,580.4	3.690	9.0	1.273	28.4	39.9
	3	158.91	1,510.5	3.690	6.0	0.507	12.0	17.2
	4	176.57	1,510.5	3.690	4.0	0.664	15.2	6.8
GC	1	123.60	1,909.3	0.533	∞	0.531	31.5	14.0
	2	141.26	1,771.4	0.533	∞	0.421	24.6	11.5
	3	180.00	1,468.0	0.533	∞	0.248	17.3	7.5
	4	211.88	1,218.2	0.533	∞	0.271	20.2	5.3
	5	247.20	941.8	0.533	∞	0.410	27.3	3.5
	6	180.00	1,468.0	0.566	∞	0.264	19.1	7.7

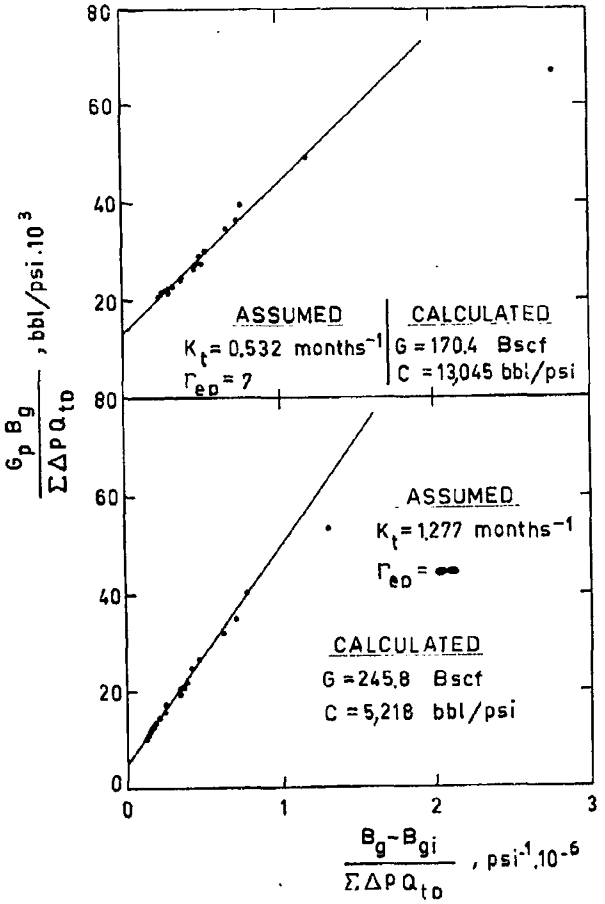

FIG. 2—RESERVOIR A-1: RESULTS OBTAINED BY HUBBARD *et al.*[7] METHOD.

same K_t and r_{eD} and differing by C, can reproduce the past history with the percentage deviation in the pressure values being smaller than the error affecting the measurements of the average reservoir pressure. The information available on the volume of water which actually entered the reservoir did not permit a better definition of the possible value of the initial reserves.

FIELD C-C

The range of initial reserves values which has been explored for Field C-C is from 88.3 to 176.6 Bscf (Table 3). It is also impossible in this example to ascertain from the past history which is the value pertaining to the reserves, since the range of uncertainty is at least from 88.3 to 176.6 Bscf. Information available on the volume of water encroached into the reservoir leads to exclude values below 100 Bscf; there is no other possible means to narrow the range of indetermination.

The line-fitting method described by Havlena *et al.*[5,6] and Pirson[2] has been applied to this reservoir to evaluate the resolving power of the same method. For K_t and r_{eD}, two pairs of corresponding values were taken among those already found by the analog simulator. In both cases the experimental points lie on a straight line (Fig. 4) as required by the method, but the reserves values which result therefrom are different (72.4 and 160.7 Bscf, respectively).

FIELD R-A

At the time this study was made, about 72 percent of the initial reserves had been recovered. Of the 21 wells which were originally in production, 11 had already been shut down because of water invasion and six others were showing evidence that they were nearly water-invaded. This information indicates that the reservoir was close to depletion; this is a situation where the evaluation of the reserves is obviously of no interest to anybody. For this reason it was preferred to examine the situation that would have faced the reservoir engineer some time earlier in the life of the reservoir.

At the time chosen, the reservoir had been producing for 9.3 years, and 44.5 percent of initial reserves had been produced. At this time four peripheral wells had been invaded by water encroaching from the edge aquifer and four other peripheral wells were nearly water-invaded. No reliable pressure data were available for the first 4.5 years of production, which obviously makes evaluating the initial gas reserves from the reservoir past history more difficult.

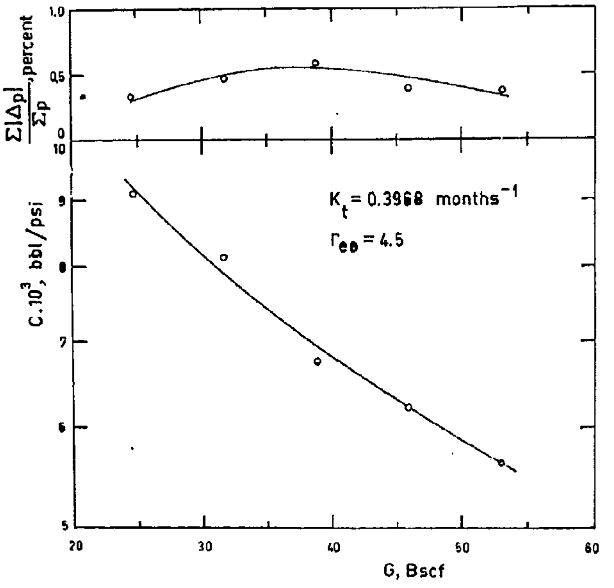

FIG. 3—RESERVOIR BM-A: RELATIONSHIP BETWEEN G AND C VALUES OF RESERVOIR-AQUIFER SYSTEMS REPRODUCING ACTUAL RESERVOIR PAST PERFORMANCE WITHIN THE STANDARD ACCURACY IN PRESSURE MEASUREMENTS.

The range of initial reserves values which were explored is from 105.9 to 158.9 Bscf. Higher values were not considered because they would correspond to a reservoir with a very small water drive, which is in contrast with the actual reservoir behavior as manifested by the water invasion of the peripheral wells (Table 3). Once again it is apparent that initial reserves cannot be uniquely determined from the past history; the range of indetermination is from 105.9 to 158.9 Bscf.

The reservoir-aquifer systems found to be equivalent as far as past history is concerned were tested to determine whether they remained equivalent with respect to the future pressure behavior resulting from a given production schedule. The response of these systems to the actual production history following the period of time previously considered was studied. Results obtained for the two cases $G = 105.9$ and 158.9 Bscf are summarized in Fig. 5. From this figure, it is apparent that the future pressure behavior is somewhat different for the two cases. Therefore, in the example considered, reservoir-aquifer systems which are equivalent as far as the past history is concerned are no longer equivalent in their extrapolation to the future.

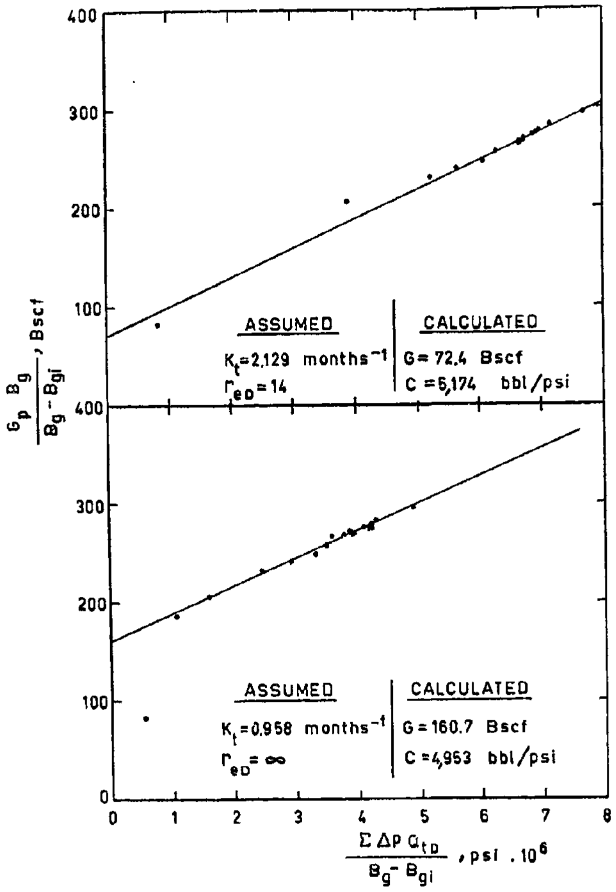

FIG. 4—RESERVOIR C-C: RESULTS OBTAINED BY HAVLENA *et al.*[6] AND PIRSON[2] METHODS.

To obtain an evaluation on the resolving power of the line-fitting method described by McEwen,[4] this method has been applied to the reservoir in question. Two sets of values for K_t, C/G and r_{eD} were chosen among those found by the analog simulator. The results obtained are presented in Fig. 6, which shows the experimental points lying on a straight line as is required by the method. Moreover, SSD_n values are very small, confirming the validity of the values taken for K_t, C/G and r_{eD}. SSD_n values obtained for the other cases not shown in Fig. 6 are of the same order of magnitude; in any case, they do not allow specification by statistical methods of which reservoir-aquifer system best reproduces the behavior of the actual system. Resulting G values are very different (105.9 and 159.4 Bscf, respectively). It is concluded that the applied method is not valuable, in this case, to uniquely determine the G value.

FIELD S-F

The range of initial reserves values explored for Field S-F was from 123.6 to 176.6 Bscf; higher values were not considered because they would correspond to a reservoir with an insignificant water drive, in contrast with the water invasion observed in some peripheral wells (Table 3). Also in this case it can be realized that an exact value for the initial reserves cannot be determined on the basis of the past history (range of indetermination is from 123.6 to 176.6 Bscf). The case for $G = 141.3$ Bscf shows a higher value of PAD; it was ascertained by the simulator that this PAD value can be reduced, assuming a heterogeneous aquifer with kh decreasing along the radius.

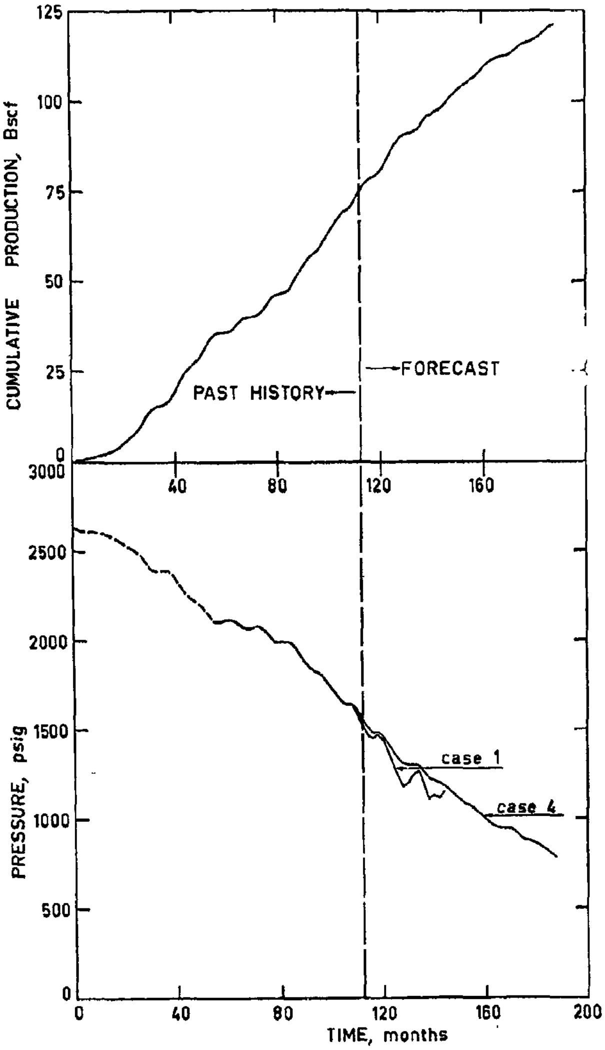

FIG. 5—PAST PERFORMANCE AND PREDICTED BEHAVIOR OF RESERVOIR R-A.

Information available on the volume of water which actually entered the reservoir does not help in defining the range of indetermination. Two additional wells drilled in the central part of the reservoir established that the advance of the water table is very irregular and does not allow an accurate evaluation of W_e.

To obtain an evaluation of the resolving power of the line-fitting methods described in the literature, the one by Hubbard *et al.*[7] has been applied to this reservoir, assuming for K_t and r_{eD} the extreme values found by trial-and-error with the help of the simulator. While in one case (Fig. 7) the experimental points lie on a straight line as required by the method, in the other case the fitting of the experimental points by means of a straight line is uncertain. In both cases the value of PAD is less than the experimental error affecting the measurements of the average reservoir pressure. The reserves values so determined are almost equal (143 and 165 Bscf, respectively); it can be concluded that, in this case, the line-fitting method used has a good resolving power.

FIELD GC

This study was extended to a Gulf Coast (GC) gas field whose data are available from the literature,[4] together with the results obtained by the application of a particularly accurate statistical method. In the reference the most probable G and K_t values were determined by a line-fitting

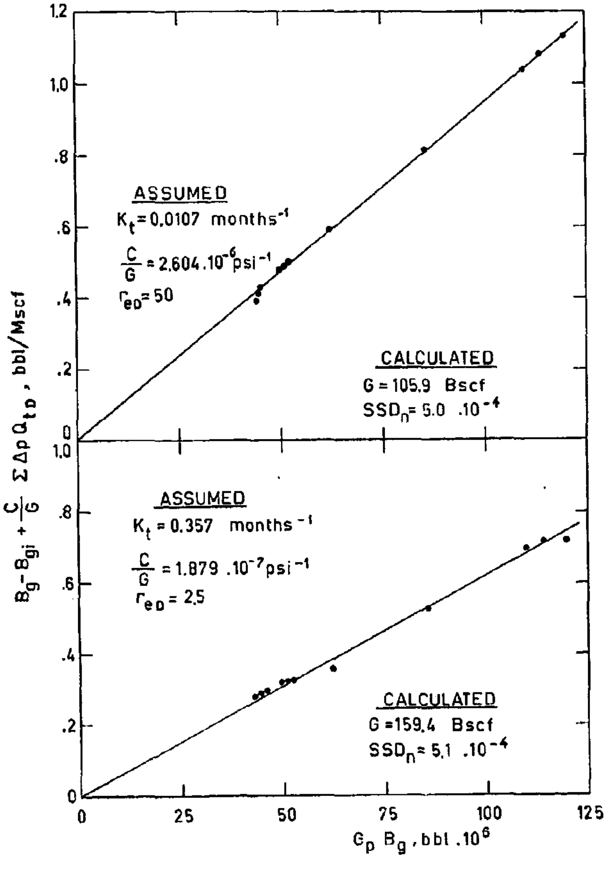

FIG. 6—RESERVOIR R-A: RESULTS OBTAINED BY MCEWEN[4] METHOD.

method assuming $r_{eD} = \infty$ and a constant value for C/G. The above optimum K_t value was assigned to all the aquifers considered in this study. Reserves values between 123.6 and 247.2 Bscf were then assumed and the corresponding C values which allowed the matching of the actual reservoir past history were determined by the simulator.

Results obtained are reported in Table 3 and summarized in Fig. 8. In all cases examined, PAD values are less than the error usually affecting the measurements of the reservoir average pressure; all the reservoir-aquifer systems presented are therefore equivalent to the actual system, within the experimental errors in pressure measurements. It can be concluded that the initial reserves cannot be uniquely determined.

The line-fitting method described in the original work[1] was applied to the cases studied with the simulator to evaluate its resolving power. To this end, K_t, C/G and r_{eD} values obtained by the simulator were used to calculate, by the above method, the corresponding values for G and SSD_n.

Results obtained are reported in Fig. 8 where the SSD_n parameter and the root mean-square deviation σ are given as functions of G. As indicated by the SSD_n vs G diagram, the most probable value for the initial reserves is 180 Bscf. While this conclusion is certainly valid from the point of view of probability, its physical meaning becomes somewhat obscure when one observes that, for all the cases presented, σ value is very low and certainly less than the standard deviation affecting reservoir pressure measurements. A final conclusion is that all the reservoir-aquifer systems reported can reproduce the actual reservoir past history within the limits of the experimental errors.

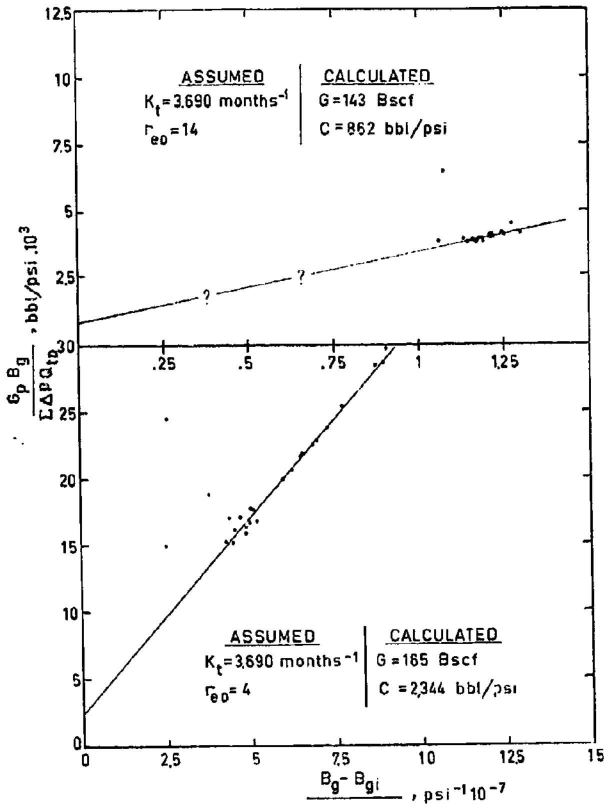

FIG. 7—RESERVOIR S-F: RESULTS OBTAINED BY HUBBARD *et al.*[7] METHOD.

CONCLUSIONS

1. It has been ascertained that different gas reservoir-aquifer systems can show the same pressure performance in response to a given production schedule. Therefore, it is not possible to determine the initial reserves of a water drive gas reservoir from its pressure-production history. This is in agreement with the principle of uncertainty, which states that the internal structure of a system cannot be uniquely determined from its external behavior.

2. The range of indetermination of the initial reserves value must be found for each specific case. This can be accomplished by finding the reservoir-aquifer systems which are able to reproduce the past pressure behavior of the actual reservoir with a root mean-square deviation smaller than the error affecting the values of the average reservoir pressure.

The results presented in this paper, pertaining to fields in an advanced stage of their productive life, show that the range of indetermination is as large as 1 to 2. It can be reasonably inferred that the indetermination would be larger when the reserves evaluation is made at an earlier stage of the productive life. The experimental work reported in this paper was purposely limited to homogeneous circular aquifers. Should the work be extended to heterogeneous aquifers of whatever shape, a wider range of indetermination might result.

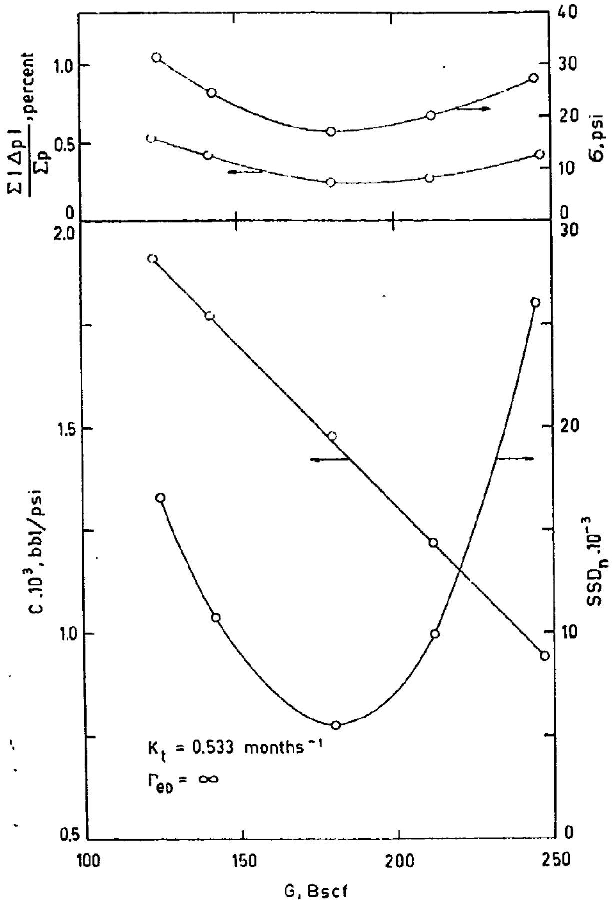

FIG. 8—RESERVOIR GC: RELATIONSHIP BETWEEN G AND C VALUES OF RESERVOIR-AQUIFER SYSTEMS MATCHING ACTUAL RESERVOIR PAST PERFORMANCE (CORRESPONDING VALUES OF SSD_n, PAD AND σ ARE SHOWN).

All the reservoirs which were studied had been subjected to a rather smooth production history, which is quite normal in depleting a gas field. The larger are the fluctuations in the field production rate, the narrower are the ranges of indetermination in the initial reserves. In the extreme case of a gas reservoir being converted to gas storage, the indetermination in the initial gas reserves value usually becomes very small. Moreover, the range of indetermination could be further narrowed if the reservoir past history is defined by accurate average pressure values.

Of course, the initial reserves are no longer indetermined when information is available on the cumulative volume of encroached water in the reservoir. Unfortunately, such information is usually only available (with the accuracy required by the reservoir engineer) late in the producing life of the reservoir when knowing the initial reserves value is of little interest.

NOMENCLATURE

K_t = proportionality constant between dimensionless and real time ($t_D = K_t t$)

PAD = percentage average deviation = $\frac{\sum|p_{res.} - p_{comp.}|}{\sum p_{res.}} \cdot 100$

SSD_n = normalized value of the sum of the squares of deviations from the best straight line[4]

σ = root mean-square deviation in pressure values

SUBSCRIPTS

comp. = computed value for the simulated reservoir-aquifer system

j = jth well

res. = actual reservoir value

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