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Estimation of Natural Gas Reserves

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ABSTRACT

In order that a natural gas reservoir may be properly exploited, the amount of gas contained in the reservoir must be known. Since this quantity, commonly called the reserve, cannot be measured directly during the life of the reservoir, it must be estimated. This paper, which restricts itself to dry gas reservoirs, presents a detailed review of the various reserve estimation methods currently available. Specifically, it reviews the volumetric and pressure decline methods and presents a unified material balance approach designed to deal with reservoirs which exhibit water production and encroachment simultaneously. The latter is capable of dealing with reservoir aquifer systems where the rate of influx is constant, or the rate of influx is a function of time and the influx, either radial or linear, is from aquifers which are either finite or infinite.

INTRODUCTION

As soon as an accumulation of a natural resource is discovered it is necessary to decide whether its exploration is economically feasible. One of the important factors entering into such a decision is the size of the accumulation. In the particular case where petroleum accumulations are involved it has become common practice to refer to the amount of petroleum contained in the accumulation as the reserve.

Although the reserve serves as the basis for the evaluation of assets and the making of exploitation policy decisions for all petroleum reservoirs, an error in the determination of a gas reserve is often more critical than a similar error in the determination of an oil reserve. This stems from the fact that the sale and distribution of natural gas is such as to require producers and distributors to enter into long-term contractual agreements. However, the reserve can be determined with certainty only when the accumulation has been depleted, at which time such knowledge is of historical interest only. Throughout the producing li[fe of] an accumulation, the reserve can be estimate[d] with an accuracy which usually varies direct[ly] as the degree of depletion. Thus when an accumulation is first discovered and an accu[rate] reserve estimate is most desired, it is lea[st] possible to obtain it. Consequently, the es[ti]mation of gas reserves has long been conside[red] a problem of major significance.

In recent years the demand for natural [gas] has increased to the point where a more soph[is]ticated engineering approach to natural gas reservoirs is desirable. Therefore, it seem[s] appropriate at this time to review the vario[us] reserve estimation techniques available and examine them critically with a view towards improvement wherever possible. Since the ma[jor] portion of existing reserves is of the dry-[gas] type this paper will be restricted to reserv[es] of that type.

THE VOLUMETRIC METHOD

One of the oldest known methods of res[erve] estimation has been referred to as the pore space method[1,2] or the volumetric method.[1,4] It consists of determining the total pore vo[lume] available to the gas and, from a knowledge o[f] the gas composition and the reservoir pressu[re] and temperature, calculating the amount of g[as] present. This method may be described in de[tail] by means of a mathematical model. Therefore us consider a differential segment of a rese[r]voir rock, as indicated in Fig. 1. It may b[e] noted that:

$$dV = dxdydh = dAdh \cdot \cdot \cdot \cdot \cdot \cdot$$

= an incremental volume of the porous medium.

If the porosity and fractional liquid saturations are denoted by ϕ and S_1, respectively, then the volume, at resident conditions, of [gas] contained in the incremental volume dV may b[e] expressed as

$$dV_g = \phi (1 - S_1)\ dV \quad [2]$$

Converting this gas volume to reference conditions we may express it as

$$dQ = \frac{T_oP}{P_oTZ} \phi (1 - S_1)\ dV \quad [3]$$

Where dQ is the gas volume in cu ft at reference conditions Po and To, and Zo is assumed equal to unity. This equation may be written in integral form as

$$Q = 43560 \frac{T_o}{P_o} \iint^{Ah} \phi (1 - S_1) \frac{P}{TZ}\ dhdA \quad [4]$$

where Q = reserve, cu ft at reference conditions
A = area of reservoir, acres
h = pay thickness, ft

In theory, Eq. 4 could be integrated if ϕ, S_1, P, T and Z were known functions of position. In practice, one must be content with an estimation of A; estimates of ϕ, S_1 and h at a finite number of points from cuttings, core samples and log analysis; measurements of P and T at bottom-hole conditions in each well; and either the evaluation of Z from PVT analysis on a sample of the gas or an estimate of Z from either a gas analysis or a gas gravity. This equation must therefore be integrated graphically. Both the method of integration and the accuracy of the resulting reserve estimate are dependent upon the nature of the data available. The various possible methods may be described as follows.

Case I

If a field is fully developed and extensive data are available for each well, Eq. 4 may be written as

$$Q = 43560 \frac{T_o}{P_o} \int^A \int^h \alpha_1\ dh\ dA, \quad [5]$$

where

$$\alpha_1 = \phi (1 - S_1) \frac{P}{TZ}$$

Consequently, we may proceed as follows.

1. Evaluate

$$\beta_1 = \int^h \alpha_1\ dh$$

for each well using any appropriate graphical or numerical method.

2. Evaluate

$$\gamma_1 = \int^A \beta_1\ dA$$

by means of an iso-β_1 map and a planimeter.

3. Calculate the reserve Q using the equation

$$Q = 43560 \frac{T_o}{P_o} \gamma_1$$

Case II

If the situation is identical with that in Case I, but it is observed that the reservoir temperature is constant or varies so slightly that it may be considered constant at an average value $\overline{T}$, Eq. 4 may be written

$$Q = 43560 \frac{T_o}{P_o\overline{T}} \int^A \int^h \alpha_2\ dhdA, \quad [6]$$

where

$$\alpha_2 = \phi (1 - S_1) \frac{P}{Z}$$

Consequently, we may proceed as follows.

1. Evaluate

$$\beta_2 = \int^h \alpha_2\ dh$$

for each well using any appropriate technique.

2. Evaluate

$$\gamma_2 = \int^A \beta_2\ dA$$

by means of an iso-β_2 map and a planimeter.

3. Obtain the best possible estimate of $\overline{T}$.

4. Calculate the reserve Q using the equation

$$Q = 43560 \frac{T_o \gamma_2}{P_o\overline{T}}.$$

Case III

If reliable and extensive data are available with regard to ϕ, h, P and A and the reservoir temperature is essentially constant at $\overline{T}$, but little is known concerning S_1, Eq. 4 may be written as

$$Q = 43560 \frac{T_o}{P_o} \frac{[1 - \overline{S_1}]}{\overline{T}} \int^A \int^h \alpha_3\ dhdA, \quad [7]$$

where $\alpha_3 = \frac{\phi P}{Z}$ and $\overline{S_1}$ is an assumed average value of liquid saturation over the

Consequently, we may proceed as follows.

1. Evaluate

$$\beta_3 = \int^{h} \alpha_3 \, dh$$

for each well using any appropriate technique.

2. Evaluate

$$\gamma_3 = \int^{A} \beta_3 \, dA$$

by means of an iso-β_3 map and a planimeter.

3. Obtain the best possible estimates for $\overline{T}$ and $\overline{S_1}$.

4. Calculate the reserve Q by means of the equation

$$Q = 43560 \frac{To}{Po} \frac{(1 - \overline{S_1})}{\overline{T}} \gamma_3$$

Case IV

If in addition to the conditions cited in Case III the Φ data are meager, Eq. 4 may be written

$$Q = 43560 \frac{To}{Po} \frac{[1 - \overline{S_1}]}{\overline{T}} \overline{\Phi} \int^{A}\int^{h} \alpha_4 \, dhdA, \quad [8]$$

where

$$\alpha_4 = \frac{P}{Z}.$$

Consequently, we may proceed as follows.

1. Evaluate for each well

$$\beta_4 = \int^{h} \alpha_4 \, dh .$$

2. Evaluate

$$\gamma_4 = \int^{A} \beta_4 \, dA$$

by means of an iso-γ_4 map.

3. Obtain the best possible estimates for $\overline{T}$, $\overline{S_1}$ and $\overline{\Phi}$.

4. Calculate the reserve Q by means of the equation

$$Q = 43560 \frac{To}{Po} \frac{[1 - \overline{S_1}]}{\overline{T}} \overline{\Phi} \gamma_4 .$$

Case V

If only limited data are available for all parameters other than area and pay thickness,

$$Q = 43560 \frac{To}{Po} \frac{[1 - \overline{S_1}]}{\overline{T}} \overline{\Phi} \frac{\overline{P}}{\overline{Z}} \int^{A} hdA \quad . [9]$$

Consequently, we may proceed as follows.

1. Evaluate

$$\gamma^5 = \int^{A} hdA$$

by means of an isopachous map and a planimeter.

2. Obtain the best possible estimates for the average values of $\overline{T}$, $\overline{S_1}$, $\overline{\Phi}$, $\overline{P}$ and $\overline{Z}$.

3. Calculate the reserve Q by means of the equation

$$Q = 43560 \frac{To}{Po} \frac{[1 - \overline{S_1}]}{\overline{T}} \overline{\Phi} \frac{\overline{P}}{\overline{Z}} \gamma^5 .$$

Case VI

If data are available for one or two wells only, Eq. 4 may be modified to the form

$$\frac{Q}{A} = 43560 \frac{To}{Po} \Phi [1 - S_1] \frac{P}{TZ} h , \quad . . [10]$$

which expresses the reserve per acre. This equation may then be solved for each well using simplifications suggested in Cases I through V depending upon the amount and nature of data available. The results may be averaged in any manner which best utilizes the available knowledge of the regional geology. Although the resulting estimate of the average reserve per acre may be sufficient for making certain decisions, a total reserve may still be required. In such a case Q may be estimated by multiplying the average reserve per acre foot by the best estimate of the total area. However, a total reserve figure obtained in this manner must be used with extreme caution.

THE PRESSURE DECLINE METHOD

A reserve estimation method variously referred to as the rock-pressure decline method,[2] the decline-curve method[1] and the production pressure-decline method,[7] and herein referred to as the pressure decline method has been used since the early part of the twentieth century. This method is also best described by means of a mathematical model, which may be developed as follows.

If there be a gas reservoir such that

V = bulk volume of the reservoir, cu ft,

$\overline{\Phi}$ = average fractional porosity for the entire reservoir,

$\overline{S_1}$= average fractional liquid saturation for the entire reservoir,

$\overline{P}$ = average reservoir pressure,

$\bar{T}$ = average reservoir temperature, and
$\bar{Z}$ = average reservoir gas compressibility,

the gas reserve at the time such average values apply may be expressed in terms of a modified form of Eq. 4 and may be written

$$Q = 43560 \frac{To}{Po} \bar{\phi} (1 - \bar{S}_1) \frac{\bar{P}}{\bar{T}\,\bar{Z}} V \quad . \quad \ldots\ldots [11]$$

In the particular case where the reservoir pore volume available to the gas remains constant throughout the life of the reservoir, that is where V, $\bar{\Phi}$, $\bar{S}_1$ and $\bar{T}$ are all independent of time, this equation may be written

$$Q = K \frac{\bar{P}}{\bar{Z}} , \quad \ldots\ldots [12]$$

where

$$K = 43560 \frac{To}{Po} \bar{\Phi} \frac{[1 - \bar{S}_1]}{\bar{T}} V = \text{constant.}$$

If we now let

Q_i = initial reserve, cu ft at reference conditions,
Q_p = cumulative gas produced, cu ft at reference conditions, and
Q_r = remaining reserve, cu ft at reference conditions,

we may write

$$Q_i = Q_p + Q_r \ldots\ldots [13]$$

on the basis of the law of conservation of matter. The substitution of Eq. 12 into Eq. 13 yields

$$K \left(\frac{\bar{P}}{\bar{Z}}\right)_i = Q_p + K \left(\frac{\bar{P}}{\bar{Z}}\right) \quad \ldots . [14]$$

Since $K \neq 0$ one may divide by K and rearrange this equation to obtain

$$\left(\frac{\bar{P}}{\bar{Z}}\right) = \left(\frac{\bar{P}}{\bar{Z}}\right)_i - \frac{1}{K} Q_p \quad \ldots [15]$$

This is an equation for a straight line with intercept

$$\left(\frac{\bar{P}}{\bar{Z}}\right)_i$$

and slope

$$- \frac{1}{K} .$$

Given sufficient data to establish such a line, we may obtain the initial reserve Q_i by observing that at the condition $\frac{\bar{P}}{\bar{Z}} = 0$, $Q_p = Q_i$.

In practice the procedure is to establish a best fit straight line on a plot of $[\bar{P}/\bar{Z}]$ vs Q_p as indicated in Fig. 2.

If only two pressure points are available, say the initial and one other, the line may be established mathematically rather than graphically. The method consists of rearranging Eq. 15 into the form

$$K = \frac{Q_p}{\left(\frac{\bar{P}}{\bar{Z}}\right)_i - \left(\frac{\bar{P}}{\bar{Z}}\right)} , \quad \ldots\ldots [16]$$

and substituting for K in Eq. 12 to obtain

$$Q_i = Q_p \frac{\left(\frac{\bar{P}}{\bar{Z}}\right)_i}{\left(\frac{\bar{P}}{\bar{Z}}\right)_i - \left(\frac{\bar{P}}{\bar{Z}}\right)} \quad \ldots\ldots [17]$$

This particular technique is sometimes improperly referred to as the equal-pound-loss method. The original equal-pound-loss method made the additional assumption that Z is equal to unity.[1]

THE MATERIAL BALANCE METHOD

Certain reservoirs are such that the pore volume available to gas within them varies throughout their productive lives. This variation may be due to various causes, such as water production, water encroachment, retrograde behavior, liquid hydrocarbon production, and communication with an aquifer in which the pressure is declining rapidly. Although the pressure decline method may not be applied to such reservoirs, the reserve may still be estimated by properly applying the material balance principle.

To demonstrate the method, we may use as an example a reservoir in which the change in pore volume is due to a combination of water production and encroachment. This problem may be expressed mathematically in a fashion similar to that used by Hubbard and Elenbaas,[4] e.g.,

letting V = gas filled pore volume, cu ft at any time,
V_i = initial gas filled pore volume, cu ft,
We = cumulative water influx, cu ft,
Wp = cumulative water produced, bbl,
Bw = water formation volume factor,

we may write a material balance on the available pore space as

$$V = V_i - We + 5.61\, Bw\, Wp . \quad .[18]$$

Likewise, letting

n_i = lb moles of gas initially in place,

n_p = lb moles of gas produced,
n_r = lb moles of gas remaining in place,

we may write a material balance on the gas as

$$n_r = n_i - n_p \qquad [19]$$

If the behavior of the gas may be described by means of modified gas law,

$$144\ PV = ZnRT , \qquad [20]$$

where P = pressure, psia
V = volume, cu ft
Z = gas compressibility
n = lb moles of gas
R = gas constant, ft lb/lb mole °R
T = temperature, °R

then Eq. 19 may be written as

$$144 \frac{PV}{ZRT} = \frac{144Po}{RTo} (Q_i - Q_p), \qquad [21]$$

provided

$$Zo = 1.0 .$$

Therefore,

$$V = \frac{ZPoT}{PTo} (Q_i - Q_p) \qquad [22]$$

Substitution of Eq. 22 into Eq. 18 yields

$$\frac{ZPoT}{PTo} (Q_i - Q_p) = V_i - W_e + 5.61\ BwWp \qquad [23]$$

However,

$$V_i = \frac{Zi\ PoTi\ Qi}{Pi\ To} = \frac{Zi\ Po\ T\ Qi}{Pi\ To}$$

if Ti = T = constant. Therefore, Eq. 23 may be written

$$\frac{Z\ Po\ T}{PTo} (Qi - Qp) = \frac{ZiPoT\ Qi}{PiTo} - We + 5.61\ BwWp , \qquad [24]$$

which may be rearranged to yield

$$Q_p \left(\frac{Z}{P}\right) + 5.61 \frac{To}{PoT}\ BwWp = Q_i \left[\left(\frac{Z}{P}\right) - \left(\frac{Z}{P}\right)_i\right] + \frac{To}{PoT}\ We \qquad [25]$$

If the rate of water influx is constant at t value Q_w, we may write

$$We = Qwt ,$$

where t = time. Consequently, Eq. 25 may be rearranged as

$$\frac{Q_p \left(\frac{Z}{P}\right) + 5.61 \frac{To}{PoT}\ BwWp}{t} = Q_i \frac{\left[\left(\frac{Z}{P}\right) - \left(\frac{Z}{P}\right)_i\right]}{t} + \frac{To}{PoT} Qw$$

provided t > 0. Since this equation describ a straight line, a plot of the left-hand sid

$$\frac{\left[\left(\frac{Z}{P}\right) - \left(\frac{Z}{P}\right)_i\right]}{t}$$

will result in a straight line whose slope equal to the initial reserve Q_i and whose in cept will yield the constant rate Q_w. Alth this method is simple and straightforward, application is limited by the fact that in general the rate of influx varies with time only in rare cases may it be assumed to be constant.

A more realistic approach consists of assuming that water influx may be described means of the constant terminal pressure sol of the diffusivity equation which describes in the aquifer, in which case,

$$We = Cq\ \Delta PQ_t ,$$

as suggested by van Everdingen and Hurst.[10] Better still, if it is assumed that the pres at the reservoir-aquifer interface changes i stepwise fashion, the cumulative influx may described by means of the combination of Eq and the superposition principle. In this ca we may write[4,5]

$$We = Cq \sum_{i=o}^{j-1} \Delta P_i\ Q_{t(j-1)} ,$$

where Cq = a coefficient which is dependent the geometry of the system and rock and fluid properties,

$\Delta P_i = \frac{P_{i-1} - P_{i+1}}{2}$ = the pressure drop term where $P_1 = P_o$,

Q_t = dimensionless water influx quant

$Q_{t[j-1]}$ = value of dimensionless water inf at $t_D = [j - 1]\ t_D$.

into Eq. 25 and divide by $\sum_{i=o}^{j-1} \Delta P_i Q_{t(j-1)}$

so as to obtain

$$\frac{Q_p(\frac{Z}{P}) + 5.61 \frac{To}{PoT} BwWp}{\sum_{i=o}^{j-1} \Delta P_i \; Q_{t(j-1)}} =$$

$$\frac{Q_i \left[(\frac{Z}{P}) - (\frac{Z}{P})_i \right]}{\sum_{i=1}^{j-1} \Delta P_i \; Q_{t(j-1)}} + \frac{ToCq}{PoT}, \quad \ldots [30]$$

which is the equation of a straight line. Consequently a plot of the left-hand side vs

$$\frac{\left[(\frac{Z}{P}) - (\frac{Z}{P})_i \right]}{\sum_{i=1}^{j-1} \Delta P_i \; Q_{t(j-1)}}$$

will result in a straight line whose slope is equal to the initial reserve and whose intercept will yield the coefficient C_q. Here again the application of the method is straightforward provided Q_t is known. Since Q_t is dependent upon geometry and time, its handling is best described as follows.

Case I

If water influx is radial so that the reservoir-aquifer boundary is cylindrical, as indicated in Fig. 3, and the aquifer behaves as if it were infinite, Q_t is a known and tabulated function of t_D,[3,5]

where $t_D = K_t t$ = dimensionless time

$K_t = \frac{0.00633k}{\phi\mu c r_b^2}$ = time normalization coefficient

t = time, days

ϕ = average aquifer fractional porosity

μ = aquifer water viscosity, cp

c = composite compressibility of aquifer and water therein

r_b = inner radius of aquifer, ft.

Furthermore, in this case

$$C_q = 2\pi\phi c h r_b^2 ,$$

where h = aquifer thickness, ft.

Consequently Eq. 30 contains three unknowns, Q_i, C_q, and K_t. Since the suggested plot will yield only the first two, K_t must be known from independent tests. If K_t is not known, as is generally true, one must resort to a trial-and-error solution.

Case II

If the geometry of the reservoir-aquifer system is identical with that of Case I except that the aquifer is finite with outer radius r_e, Q_t is a known and tabulated function of t_D and r_e/r_b.[3,5]

In this case Eq. 30 contains four unknowns, Q_i, C_q, K_t and r_e/r_b. Therefore if K_t and r_e/r_b are known from independent tests, we proceed directly; otherwise a trial-and-error solution must be employed.[4]

Case III

If water influx is linear so that the reservoir-aquifer interface consists of a plane of width b and height h, as shown in Fig. 4, and the aquifer behaves as if it were infinite, Q_t is a known and graphically represented function of t_D,[6]

where $t_D = K_t t$

$K_t = \frac{0.00633k}{\phi\mu C X_c^2}$

X_c = unit length.

Furthermore,

$$C_q = \phi \; chX_c^2 .$$

In this case, Eq. 30 contains three unknowns, Q_i, C_q and K_t. Therefore K_t must be known from independent tests if we are to solve for Q_i and C_q. If K_t is unknown, the method requires the use of trial and error. However, if instead of using the graphical solution for Q_t we go directly to the analytical solution, trial and error may be avoided in the following manner. Since we may write[7,8]

$$We = 2bh\sqrt{\frac{k\phi c}{\pi\mu}} \sum_{i=1}^{n} (P_{i-1} - P_i)\left(\sqrt{t_n - t_{i-1}}\right)$$

$$= C \sum_{i=1}^{n} (P_{i-1} - P_i)\left(\sqrt{t_n - t_{i-1}}\right) \quad \ldots [31]$$

where t_o = o, we may substitute Eq. 31 into Eq.

25 and rearrange to obtain

$$\frac{Q_p\left(\frac{Z}{p}\right) + 5.61\ \frac{To}{PoT}\ BwWp}{\sum_{i=1}^{n}(P_{i-1}-P_i)\left(\sqrt{t_n - t_{i-1}}\right)} = \frac{Q_i\left[\left(\frac{Z}{P}\right) - \left(\frac{Z}{P}\right)_i\right]}{\sum_{i=1}^{n}(P_{i-1}-P_i)\left(\sqrt{t_n-t_{i-1}}\right)} + \frac{ToC}{PoT} \quad \ldots\ldots [32]$$

which may be used in lieu of Eq. 30. The method of application is similar to that previously suggested with the exception that the equation contains only two unknowns, Q_i and C, which may be obtained from the slope and intercept, respectively.

Case IV

If the reservoir-aquifer geometry is identical with that in Case III except that the aquifer is of finite length L, Q_t is a known and graphically represented function of t_D and L.[6] Therefore Eq. 30 contains four unknowns, Q_i, C_q, K_t and L, and we may obtain Q_i and C_q from a linear fit if K_t and L are available from other tests; otherwise, a trial-and-error approach is necessary. As in the previous case the method may be modified by going directly to the analytical solution as follows.

Since the analytical solution may be written[7]

$$We = 2bh\sqrt{\frac{k\phi c}{\pi\mu}}\sum_{i=1}^{n}(P_{i-1}-P_i)\left(\sqrt{t_n-t_{i-1}}\right)\left\{1 + 2\sum_{j=1}^{\infty}(-1)^j \exp\left(\frac{-j^2L^2\phi\mu c}{k(t_n-t_{i-1})}\right)\right\}$$

$$= C\sum_{i=1}^{n}(P_{i-1}-P_i)\sqrt{(t_n-t_{i-1})}\left\{1 + 2\sum_{j=1}^{\infty}(-1)^j \exp\left(\frac{-j^2L^2\phi\mu c}{k(t_n-t_{i-1})}\right)\right\}$$

$$= C\ \{B\}\ , \quad \ldots\ldots [33]$$

we may substitute Eq. 33 into Eq. 25 and rearrange to obtain

$$\frac{Q_p\left(\frac{Z}{p}\right) + 5.61\ \frac{To}{PoT}\ BwWp}{\{B\}} = \frac{Q_i\left[\left(\frac{Z}{P}\right)-\left(\frac{Z}{P}\right)_i\right]}{\{B\}} + \frac{ToC}{PoT}$$

which may be used in a fashion similar to Eq. 32. However, this equation, like Eq. 32 contains four unknowns, Q_i, C, L and the group $\frac{[\phi\mu c]}{k}$. Consequently, to solve for Q_i and C we must know L and $\frac{[\phi\mu c]}{k}$ from other tests or else resort to trial and error.

Although the use of the graphical or analytical solution may appear to be optional the latter offers certain advantages. First the analytical solution provides more insight into, and a better feel for the problem. Secondly, it is more accurate and better lends itself to computer application. Finally, during the early life of such a reservoir-aquifer system the aquifer behaves as if it were infinite, thereby permitting the estimation of Q_i and C. However C is not dependent upon whether the aquifer is finite or infinite but related to the group $\frac{\phi\mu c}{k}$. Therefore this estimate of C may be used later in the life of the reservoir to limit the trial-and-error search on $\frac{\phi\mu c}{k}$.

DISCUSSION

As has been stated in the introduction immediately upon discovery of a gas reservoir a reserve estimate is desirable. Since no production history is available at the time of discovery, the volumetric method of estimation is the only one available. Unfortunately, this method is not very accurate. For even if the field were completely drilled out and extensive data were available on each well, the areal extent of the reservoir would not be known absolutely and the extensive data would represent but a small sample of the reservoir. Furthermore the various integration schemes suggested merely represent a means for averaging the available data. Although these methods seem reasonable and are likely the best one can expect under the circumstances, there is no way of assuring that they will provide the best possible estimate in all cases. This point can best be appreciated when it is realized that, statistically spe the extensive data represent very small non-random samples and that as yet there is no unique way of estimating the average properties of a population from such samples.

As soon as production data are available, both the volumetric and pressure-decline methods may be used. An examination of the pressure-decline method indicates that it is extremely important to obtain an accurate measurement of reservoir pressure, temperature and gas composition prior to the initiation of production, since these data establish the pivotal point of the plot. Further examination of this method reveals that during the early life of a reservoir, the method is rather inaccurate since small errors in the P/Z term result in larger errors in the estimated value of Q_i. Furthermore the method is only applicable if the pore volume available to the gas remains constant throughout the life of a reservoir.

If a reservoir is such that the pore volume available to gas is decreasing, as would be the case if water were encroaching, the resulting pressure-decline curve will generally lie above the true curve and be concave upward. Unfortunately, such behavior is not easily detected until considerable depletion has occurred.

Since the P/Z values, which are to be used in constructing a pressure-decline plot, must be average values for the entire reservoir, care should be exercised in obtaining and averaging such data. Although most estimators prefer to use a volumetric average, here again no one method of averaging can be said to be the method to be used. Consequently, the choice of method should be based on experience. However, the common practice for obtaining the raw data consists of shutting in producing wells one at a time, obtaining their buildups, calculating their shut-in pressures and recording them. These pressures are then averaged by some preselected method. Such pressures are of necessity equal to or less than the true value since no correction has been made for interference. That is, if a well is shut in, but is surrounded by producing wells, the shut-in pressure recorded will be equal to the shut-in pressure which would have been recorded had the surrounding well been shut in, less the pressure drop at that point due to production from the surrounding wells. Since the interference error is non-random, a proper evaluation should either correct for it or show that it is so small that it can be neglected. Such a correction may be evaluated using available solutions to the Darcy-Continuity equation and the superposition principle or by solving the Darcy-Continuity equation numerically for the reservoir in question. Although both methods are straightforward, their description is considered outside the scope of this presentation.

In view of the fact that the material balance method is more involved than is the pressure-decline method, it should be introduced only after considerable depletion has occurred and curvature has been detected on the pressure-decline plot. Although methods similar to that presented herein have been used by Guerrero[8] to treat a linear infinite case, and by Hubbard and Elenbaas[4] to treat a radial finite injection case, the suggested method as a whole must be considered as untried. Therefore it is inadvisable at this time to venture an opinion concerning the appropriatness of the various schemes for particular field cases.

CONCLUSIONS

In conclusion it may be stated that,

1. At time of discovery the only method available is the volumetric method. This method is such that accuracy is not to be expected.
2. During the early life of a field, both the volumetric and pressure-decline methods may be used. The methods are such that neither can be expected to produce accuracy.
3. No effort should be spared to obtain an accurate initial point for the pressure-decline curve.
4. The pressure-decline method should be reapplied periodically with critical examination of random scatter for indications of curvature.
5. Once curvature is observed and water influx is indicated, the various material balance schemes should be applied and the results compared with a view to selecting the scheme most applicable to the situation at hand.

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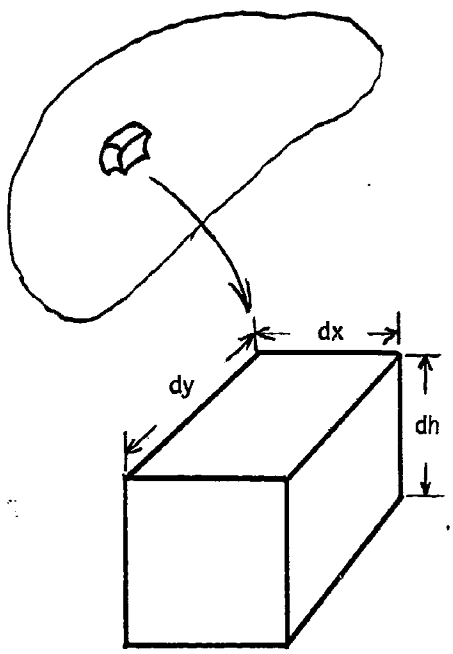


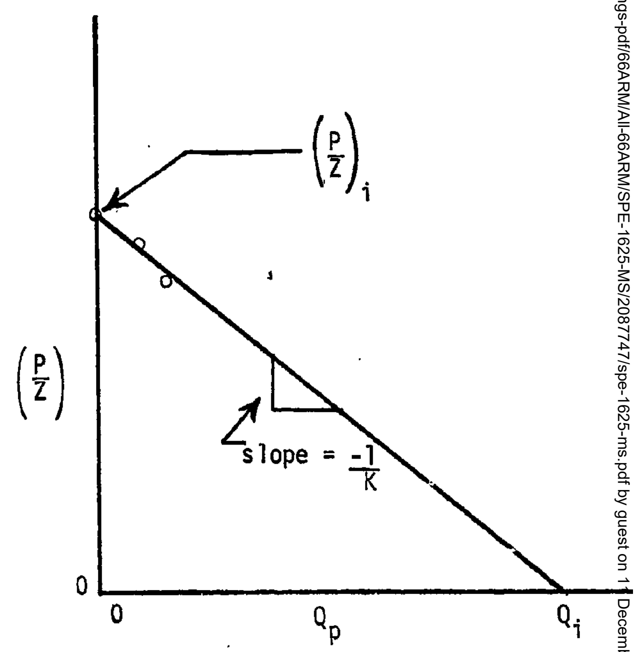
$dV = dx\,dy\,dh$ = incremental volume

ϕ = rock porosity (fraction)

S_ℓ = fractional liquid saturation

$dV_g = dV\phi(1-S_\ell)$ = gas volume

Fig. 1



$$K = 43560\ \frac{T_o}{P_o}\ \frac{\phi(1-S_\ell)\ V}{T}$$

Fig. 2

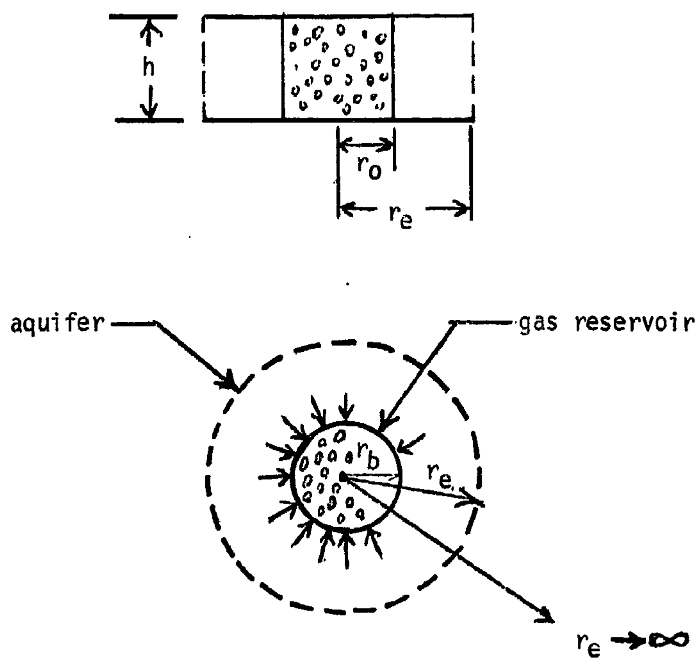

h
r_o
r_e
aquifer
gas reservoir
r_b
r_e
$r_e \to \infty$

Fig. 3

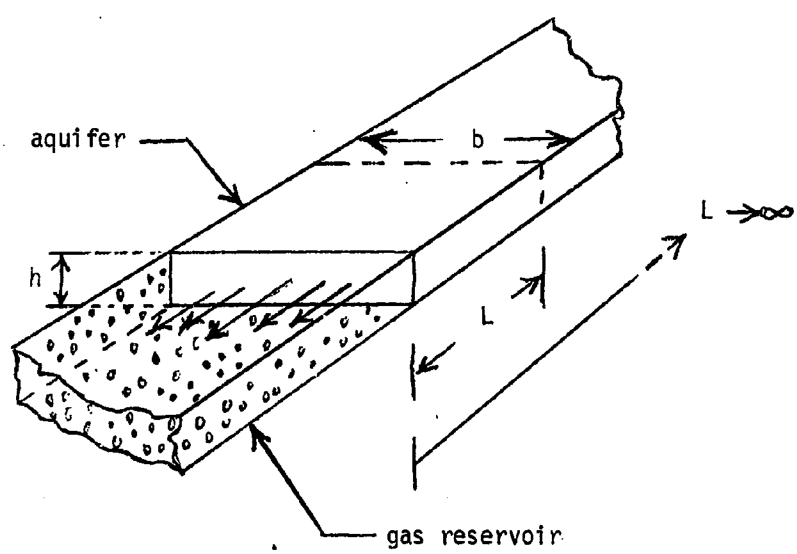

aquifer
b
h
L
L →∞
gas reservoir

Fig. 4