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Some Reheological Considerations Concerning Rock Behavior

By

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ABSTRACT

It is well known that rocks behave in a viscoelastic manner. Therefore, the theory of elasticity does not provide valid answers to problems of rock behavior. Various rheological models have been developed conforming to rock deformation. Most important of these is the Kelvin solid model. The model involves a coefficient of solid viscosity, which appears to have the makings of a future criterion for rock fracture.

The paper describes the evaluation of the Kelvin model and presents a few analytical developments concerning rock's viscosity from measurement in the mine stopes and the structural volume flow of rocks. Some experimental results obtained in the laboratory are presented. These results indicate the advisability of using a Kelvin model and give an idea of the types of failure mechanisms operating in the rocks.

In light of these and other developments, the directional properties of rocks are discussed. The possibilities of exploitation of these properties in drilling in preferential directions, reorientation of mine workings, and oil well fracturing are briefly mentioned.

INTRODUCTION

It is well known that rocks behave in a viscoelastic manner.. Among rheological models applicable to rock behavior, the most important is that of the Kelvin solid, although the Maxwell and Burger models are also applicable to some extent. One of the two constituents of a Kelvin solid deals with time-dependent behavior of rocks and involves the coefficient of structural viscosity. This solid-viscous parameter appears to be a future potential criterion for structural design involving rocks and rock-like materials. The parameter has been experimentally determined in the laboratory, using a number of independent techniques.[2,3] This paper is a step further in this direction and consists of theoretical development for determination of structural viscosity from mine stope measurements. Some data obtained in the laboratory are used to calculate some values of the coefficient of solid structural viscosity. The above are th[e] used in formulating the comments concerning exploitation of the directional properties of rocks in drilling reorientation of mine workings and oil well fracturing.

References and illustrations at end of paper.

ANALYTICAL CONSIDERATIONS

In analytical development presented her[e] it is assumed that the rocks behave like a Kelvin solid, represented by the equation:

$$\sigma = E\varepsilon + \eta \dot{\varepsilon} \quad \ldots\ldots\ldots (1)$$

The solution of Eq. 1 is given by:

$$\varepsilon = e^{-E/\eta_s \cdot t}\left(\varepsilon_0 + \frac{1}{\eta_s}\int_0^t \sigma e^{E/\eta_s \cdot t} \cdot dt\right) \quad . [2]$$

If σ is constant, Eq. 2 reduces to:

$$\varepsilon = \frac{\sigma}{E} + \left(\varepsilon_0 - \frac{\sigma}{E}\right) e^{-E/\eta_s \cdot t} \quad \ldots\ldots [3]$$

For $\sigma = E\varepsilon_0$, the above gives $\varepsilon = \varepsilon_0$, i.e., strain remains constant. For $\sigma > E\varepsilon_0$, the strain increases; for $\sigma < E\varepsilon_0$, it decreases reaching σ/E asymptotically in both cases.

If $\varepsilon_0 = 0$, i.e., if σ acts on an unstrained body, then:

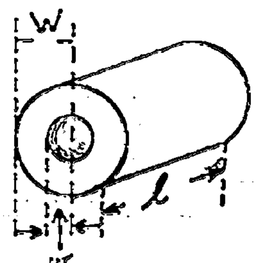
$$\varepsilon = \frac{\sigma}{E}\left(1 - e^{-E/\eta_s \cdot t}\right) \quad \ldots\ldots [4]$$

The interpretation of Eq. 4 is that the elastic strain σ/E is delayed and reaches its value in infinite time. The delayed elastic strain recovery will, therefore, take place according to:

$$\varepsilon = \varepsilon_c e^{-E/\eta_s \cdot t} \quad \ldots (\sigma = 0) \quad \ldots\ldots [5]$$

The phenomenon associated with the above equation is termed "relaxation".

Solid Structural Viscosity and Volume Flow



The solid viscous parameter, as seen from the above, is an important structural parameter, if time-dependent behavior of rocks is considered. Let us consider the flow permitted by Kelvin viscosity and examine an anticipated state of transition in the roof of a mine stope, dipping of the horizontal in a longitudinal direction.

Assume a small cylindrical element [other geometrical configurations can also be assumed] of rock material of unit length l [see inset] in the roof of a mine stope. If this element lies lengthwise in the direction of load, then let Δp be the pressure differential, acting on the element.

For a region w from the center of the section, the base area acted upon by Δp is πw^2. Therefore, the driving force is given by:

$$-\pi w^2 \Delta p \quad \text{[right to left, hence negative]} \quad \ldots\ldots [6]$$

If σ is the viscous resistance per unit area acting on the sides of the cylinder of area $2\pi wl$, then the total viscous resistance is given by:

$$2\pi w l \sigma \quad \ldots\ldots [7]$$

For equilibrium,

$$-\pi w^2 \cdot \Delta p + 2\pi w l \cdot \sigma = 0$$

and therefore,

$$\sigma = \frac{w \cdot \Delta p}{2l} \quad \ldots\ldots [8]$$

Now consider Eq. 1, which can be put in the form:

$$\eta_s \dot{\varepsilon} = \sigma - E\varepsilon.$$

Considering $\dot{\varepsilon}$ to be a velocity gradient, as in shear flow, the above equation can be rewritten as follows:

$$\eta_s \frac{dv}{dw} = \frac{w \cdot \Delta p}{2l} - E\varepsilon \quad \ldots\ldots [9]$$

In case of the cylindrical element being considered, the rock behavior is time-dependent and the important element is the viscous element. On the basis of the concept of a Newtonian liquid, therefore, the rock volume flow within the element resembles that of a rotational paraboloid with focus [o,c]. The equation of such a conic in the rectangular coordinate system is given by:

$$x^2 = 4Cy, \quad \ldots\ldots [10]$$

where C is the distance of focus of the parabola from the origin of the coordinate system. If W is situated along x-axis, then the strain ε becomes manifest along the y-axis. Eq. 10 can therefore be written as:

$$w^2 = 4C$$

or

$$\varepsilon = \frac{w^2}{4C} = Aw^2, \quad \ldots\ldots [11]$$

where $A = 1/4\,C$.

The rheological equation [Eq. 9] can therefore be written as:

$$\eta_s \frac{dv}{dw} = \frac{w \cdot \Delta p}{2l} - EAw^2$$

i.e.,

$$\eta_s dv = \frac{\Delta p \cdot w}{2l} \cdot dw - EAw^2 dw \quad \ldots\ldots [12]$$

which, by integration, yields:

$$v = \frac{1}{\eta_s}\left[\frac{\Delta p \cdot w^2}{4l} - \frac{EAw^3}{3}\right]$$

$$+ C_0, \quad \ldots\ldots\ldots\ldots [13]$$

where C_0 is the constant of integration. At the boundary, $w = W$ and $V = o$. Therefore,

$$C_0 = \frac{1}{\eta_s}\left[\frac{EAW^3}{3} - \frac{\Delta p \cdot W^2}{4l}\right]$$

and Eq. 13 becomes:

$$-v = \frac{1}{\eta_s}\left[\frac{\Delta p}{4l}(W^2 - w^2) - \frac{EA}{3}(W^3 - w^3)\right] \quad \ldots [14]$$

For the velocity v_o at the center of the circular section, $w = o$, and the above equation becomes:

$$v_o = \frac{\Delta p \cdot W^2}{4l\eta_s} - \frac{EAW^3}{3\eta_s} \quad \ldots\ldots\ldots\ldots [15]$$

Using the above relationship for v_o, two further derivations may be made, one to compute the coefficient of structural viscosity and the other two to show the flow equation of a Kelvin solid under constant load.

1. In terms of the inset above, v_o can be written as $\frac{dl}{dt}$, that is, Eq. 15 can now be written as:

$$\frac{dl}{dt} = \frac{\Delta p \cdot W^2}{4l\eta_s} - \frac{EAW^3}{3\eta_s} \quad \ldots\ldots\ldots\ldots [16]$$

On integration, the above equation yields

$$-\frac{3\eta_s l}{EAW^3} + \frac{9\Delta p \cdot \eta_s}{4E^2A^2W^4} \ln\left(\frac{\Delta p \cdot W^2}{4\eta_s} - \frac{EAW^3 l}{3\eta_s}\right) = t + C_1, \quad \ldots [17]$$

where C_1 is the constant of integration. At $t = o$, $l = l_o$ and

$$C_1 = -\frac{3\eta_s l_o}{EAW^3} + \frac{9\Delta p \cdot \eta_s}{4E^2A^2W^4} \ln\left(\frac{\Delta p \cdot W^2}{4\eta_s} - \frac{EAW^3 l_o}{3\eta_s}\right)$$

$$\ldots\ldots\ldots\ldots [18]$$

and the final result is given by

$$\frac{3\eta_s}{EAW^3}(l - l_o) + \frac{9\Delta p \cdot \eta_s}{4E^2A^2W^4} \cdot \ln\left(\frac{3\Delta p \cdot W^2 - 4EAW^3 l}{3\Delta p \cdot W^2 - 4EAW^3 l_o}\right) = t$$

$$\ldots\ldots\ldots\ldots [19]$$

2. Starting with equation

$$\eta\dot{\varepsilon} = \sigma - E\varepsilon$$

the equation for the rock volume flow Q can be obtained. Considering the section in the inset above, it is seen that

$$dQ = w^2 \pi\, dv. \quad \ldots\ldots\ldots [20]$$

In view of the rheological equation [Eq. 12], the above yields:

$$Q = \frac{\pi}{\eta}\int w^2\left(\frac{w \cdot \Delta p}{2l} - EAw^2\right) dw$$

or,

$$Q = \left[\frac{\pi \Delta p \cdot w^4}{8l\eta} - \frac{\pi EAW^5}{5\eta}\right] + C_2$$

where C_2 is the constant of integration. At the boundary, Q = o and w = W. Therefore,

$$C_2 = \frac{\pi EAW^3}{5\eta} - \frac{\pi \Delta p \cdot W^4}{8\ell\eta} .$$

Hence,

$$Q = \left[\frac{\pi EA}{5\eta}(W^3 - w^3) - \frac{\pi \Delta p}{8\ell\eta}(W^4 - w^4)\right] . \ldots . [22]$$

The flow across the section can be obtained by putting w = o in the above equation, i.e.,

$$Q = \pi/\eta \left[\frac{EAW^3}{5} - \frac{\Delta p \cdot W^4}{8\ell}\right] . [23]$$

Expressions 19 and 23 can be simplified if the basic rheological model is changed from a Kelvin solid to a Newtonian liquid. If we were to consider only the time-dependent performance of a Kelvin structure, then the second term on the right-hand side of Eq. 1 becomes very important.

The first term is so insignificant, it could be ignored. Doing this, Emery[4] arrived at an equation that replaces Eq. 19; i.e.,

$$\eta = \frac{\Delta p \cdot W^2 \cdot t}{2(\ell^2 - \ell_0^2)} \ldots \ldots . [24]$$

It should be recognized that this is an approximation only. In comparison, Eq. 19 is a better approximation, especially for the determination of the coefficient of solid structural viscosity.

It can be seen that, depending on a rheological model many equations of solid viscosity can be derived.

Eq. 23 can be similarly simplified, provided only the second term on the right-hand side of Eq. 1 is important. Assuming that Eq. 20 is still valid and that Eq. 12 can be rewritten as:

$$\eta_s dv = \frac{\Delta p \cdot w}{2\ell} \cdot dw , . [25]$$

$$Q = \frac{\pi \cdot \Delta p}{2\ell\eta} \int w^3 dw$$

or

$$Q = \frac{\pi \cdot \Delta p \cdot w^4}{8\ell\eta} + C_3 . [26]$$

At the boundary, Q = o and w = W, and:

$$C_3 = -\frac{\pi \Delta p W^4}{8\ell\eta} ;$$

hence

$$Q = -\frac{\pi \cdot \Delta p}{8\ell\eta}(W^4 - w^4),$$

or

$$Q = \frac{\pi \cdot \Delta p}{8\ell\eta}(W^4 - w^4) . [27]$$

by properly considering the signs in the physical experiment.

The flow across the section is obtained by putting w = o in the above equation; i.e.,

$$Q = \frac{\pi \cdot \Delta p \cdot W^4}{8\ell\eta} \ldots \ldots . [28]$$

Solid flow equations of the types of Eqs. 23 and 28 can be developed for a given material, depending on the rheological model that conforms to the structural behavior of that material.

Volume Structural Viscosity

Full consideration of the volume viscosity is not within the scope of this presentation. However, a few general comments are warranted.

Eqs. 23 and 28 indicate that the coefficient of solid viscosity has a profound effect on the total volume flow. This coefficient has previously been assumed as the "true", "relaxation" or "structural" coefficient of solid viscosity.[2] The application of this coefficient is restricted in that it does not represent the composite resistance to volume flow, although it is a reasonably approximate criterion to be used in structural design.

To consider the three-dimensional behavior of a rheological material, such as a Kelvin solid, a volume rheological equation can be set up. Reiner[1] has arrived at the following:

$$-\sigma = K\varepsilon_v + \eta_{vs}\dot{\varepsilon}_v \; , \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad [29]$$

where K is the bulk modulus

ε_v is the volumetric strain

$\dot{\varepsilon}_v$ is the volumetric strain gradient.

A number of conclusions can be drawn from an equation of this type and the curious reader is referred to the excellent work Reiner has done in this field.

RESULTS

The results presented here were obtained in the laboratory using the following techniques: [1] relaxation of shear strains on rock surfaces, using photoelasticity, [2] uniaxial compressive loading in conjunction with Eq. 24, and [3] compressive loading, using multiple-beam interferometry.

Relaxation

The relaxation of naturally induced rock strains is a well established phenomenon and has been used to estimate the delayed elastic component of strain.[5] Photoelastic gauges were used on three mutually orthogonal faces of each sample to determine the variation of shear strain with time [see Refs. 2 and 6] for the details of experimental procedure]. Typical results are given in Figs. 1 and 2.

Fig. 3 shows the details of sample preparation from 12-in. x 6-in. x 3-in. limestone blocks and the associated nomenclature.

These results pertain to limestone samples, obtained from Queenston area in the Niagara Peninsula in Southern Ontario, Canada. Geological studies conducted on some representative thin sections have revealed that the material is almost entirely carbonate and the chief constituents are[2]

1. Rod-like fragments. These are fragments of organic origin, possibly crinoid stems, fragments of algae and polyzoa. The rods vary in dimensions and are oriented parallel to the bedding plane. Scattered through the rods is a good deal of very fine material. These impurity grains were probably released in the decomposition and diagenetic recrystallization of the organic material.

2. A matrix of granular calcite. This matrix constitutes about 50 per cent of the material rock. The granular matrix tends to be cloudy but is devoid of the fine impurities of 1. above. It seems to have been formed from the carbonates, important in cementing detrital reef fragments, called "flour".

3. The limestone has not undergone intensive penetrative deformation in situ; the twin lamellae are rare and recrystallization obliterating the original texture is not significant.

4. Calcite grains are of average size, 0.145 mm, in each of the three dimensions -- approximating to spherical grains. The average sizes of the less competent grains of the organic material are 0.245 mm, 0.445 mm and 0.645 mm on Faces 3, 1 and 2.

Fig. 4 is a photomicrograph at 80x, showing the grain distribution in the plane normal to the bedding plane, representing the transverse section of the grain size associated with the organic material.

Figs. 1 and 2 indicate a quasi-periodic behavior of shear strain with time in relaxation, a result not unexpected in view of Eq. If we consider this behavior together with the granular structure of the limestone used, and the fact that the material has not undergone intensive penetrative deformation in situ, the differences between the experimental and expected variations of strain with time can be understood.

Further examination of the shear strain-vs-time curves indicates that the variation is definitely directional. It also means that there are planes in rocks in or along which failure may prefer to occur. This observation has been satisfactorily substantiated by vari workers in rocks mechanics.[2,4]

Experimental Results From Eq. 24

As remarked earlier, Eq. 24, when used fo determination of the coefficient of solid viscosity from measurements either in the fiel or laboratory, is only an approximate relationship. However, the values of the viscosity parameter obtained from some uniaxial compression tests on a time-dependent basis can be related to load in the laboratory to establish possible failure mechanisms operative in granular materials.

Tests were made on a number of samples of Queenston limestone and the details of this work are given elsewhere.[2,6] Some typical results are presented in Figs. 5 through 8.

These and other similar results[2] indicate an initial behavior conforming to that of a Maxwell liquid followed by a transition to an approximate Kelvin solid behavior. [A Maxwell liquid results when a Hooke solid and a Newtonian liquid are combined in series. See Ref. 1.] This, in part, is substantiated by some results obtained by Friedman.[8] The resul stand to reason in view of the specific granu nature of the test material.

In Queenston limestone, two types of grains are present -- competent calcite grains and less competent organic grains. It is reasonable to assume that the organic material will be the first to undergo deformation. This deformation will continue until the more competent calcite grains have come in contact with each other, when a transition takes place. This is exactly what the results in Figs. 5 to 8, inclusive, indicate.

In a major part of the work on experimental determination of viscosity of rocks[2] reported elsewhere,[6] it has been demonstrated that the various solid viscosity parameters have a directional preference in rocks. Moreover, solid viscosity is intimately related to failure mechanisms operative in granular materials.[2] These two aspects make the solid viscosity parameter an important factor in structural design involving rocks.

Results from Compressive Loading by Using Multiple-Beam Interferometry

Multiple-beam interferometry is an inexpensive technique recently applied to the study of intergranular and intragranular deformation of rocks.[3,9] Two distinct fringe systems are available for such application: the Fizeau fringe system and the system of the fringes of equal chromatic order. These systems are capable of measuring small deformation down to 5Å and 2.7Å, respectively. At the same time, a fringe pattern or an interferogram, when photographed, represents the microtopography of a surface. The fringes are analogous to the geographical contour lines with a contour interval equal to half the wavelength of light used.

In testing a sample in conjunction with interferometry, a rock surface is polished and coated with a reflective material. An interferogram is then photographically recorded. After loading the sample in a desired fashion, the changing fringe patterns can be recorded at regular time intervals. If an initial pattern is subtracted from a final pattern, the resulting pattern is an optical contour map of surface deformation. Details of the technique are presented elsewhere.[3,10]

The results given here pertain to one sample of Queenston limestone only and were obtained by using the multiple-beam Fizeau fringe system.

Fig. 9 represents an initial Fizeau interferogram, obtained from a polished and coated sample surface of Queenston limestone. The surface formed part of the cross-sectional area of a 3/4-in. diameter core.

After recording the initial interferogram, the surface was subjected to a radial load of 30,000 psi. After keeping the sample under this load for 2.5 hours, another interferogram, shown in Fig. 10, was recorded. This last pattern represents a surface microtopography differing from that represented by Fig. 9.

It should be noted that, although ordinary magnification associated with Fig. 10 is 240 times, the magnification in depth or pertaining to deformation in the up and down direction is much higher. This is demonstrated by the following calculation:

Average interfringe spacing = 5 mm
Interfringe spacing in terms of $\lambda = \lambda/2$

For light used $[\lambda = 5461\text{Å}]$, $\lambda/2 = 2730\text{Å} = 2.73 \times 10^{-4}$ mm.

Therefore, Magnification in Depth =

$$\frac{5}{2.73 \times 10^{-4}} = 18{,}300\text{x}.$$

If properly interpreted in accordance with the theory of interferometry, the interferograms presented here can be used to determine some directional properties of rocks and to arrive at the failure mechanisms operative in rocks on a small scale. This information can then be extended to apply to large rock masses with due modifications. The following are some of the points concerning the use of these interferograms:

1. The initial interferogram can be subtracted from the first one to give an optical contour map representing surface deformation.
2. Sections could be drawn along any direction on these interferograms for a test. It is possible to measure accurately the elastic as well as anelastic components of deformation.
3. From the small hills and valleys produced on sample surface by loading, it is possible to determine the principal strain vectors.

It is not within the scope of this paper to substantiate the points made above by actual calculation. The curious reader is referred to the work done recently where these statements are properly substantiated.[3] In this investigation some of the properties measured and calculated are the coefficients of structural relaxation and true viscosity, the material volume flow and the solid structural volume viscosity.

Some of these calculations are based on equations similar to Eqs. 5, 19, 23 and 29 of this paper.

DIRECTIONAL PROPERTIES OF ROCKS

The classical theory of elasticity is based on the assumption that materials are perfectly elastic, homogeneous and isotropic. However, actual materials do not match these assumptions, and therefore possess directional physical properties. These directional physical properties can be determined, as has been demonstrated by the considerable amount of work that has been done in this field.[2,6,7,11,12,13] A survey of this work is not within the scope of this presentation. However, it can be concluded from the results of such investigations that the directional properties of rocks can be exploited in areas such as drilling, mine planning and layout, oilwell fracturing, and rock bolting.

Some signs of exploitation of the directional properties of rocks in drilling, mine planning and layout, oilwell fracturing and rock bolting have already appeared in the literature.[13-18]

All directional properties should, of course, be modified in the light of a suitable rheological model. Depending on the nature of a design problem, the rock behavior can either be time-dependent or independent of time. This factor should be taken into consideration before the directional rock properties are determined. It is obvious from Eq. 1 and its solution that the time element is very important.

DISCUSSION

The analytical considerations and the experimental data clearly point to the importance of directional properties. Rheological considerations are important whether or not the time element is considered. The experimental results presented here also point to the granular nature of rocks.

An important physical attribute of granular materials is the existence of the planes of weakness or the planes of preferred shear that are so clearly depicted by the interferogram of Fig. 10. The existence of these planes can also be indirectly visualized from the results of Figs. 1 and 2. It is possible to determine these planes of weakness on rock surfaces.[15,19] In a practical problem these planes can be plotted in three dimensions to give the general trend of shear. This can be done on a statistical basis. Consequently, a design could be based on such information. The same is true of the other directional properties of rocks, such as strength, solid viscosity, elastic moduli, etc.

CONCLUSIONS

1. Rock possesses directional properties that can be exploited to solve practical problems involving rock masses.

2. Rheological considerations combined with the directional attributes of rocks help toward understanding the failure mechanisms i rocks.

3. The coefficient of solid-viscosity i an important parameter that may be a more realistic criterion in structural design involving rocks and rock-like materials.

4. Planes of preferred shear exist in granular materials and can be determined for use in solving practical problems in rock mechanics.

NOMENCLATURE

A = constant
η = coefficient of viscosity
η_s = coefficient of structural viscosity
η_{vs} = coefficient of volume structural viscosity
C = y coordinate of the focus of a para or paraboloid
C_0, C_1, C_2, C_3 = constants of integration
E = Young's modulus of elasticity
ε = strain, unit strain
ε_0 = initial strain
$\dot{\varepsilon}$ = strain gradient
ε_v = volumetric strain
$\dot{\varepsilon}_v$ = volumetric strain gradient
K = bulk modulus
l = length
l_0 = initial length
Δp = stress differential
Q = volume flow
t = time
v = velocity
v_0 = initial velocity
w = elemental width, also elemental diam eter
W = width, also diameter
x = coordinate direction along x-axis
y = coordinate direction along y-axis

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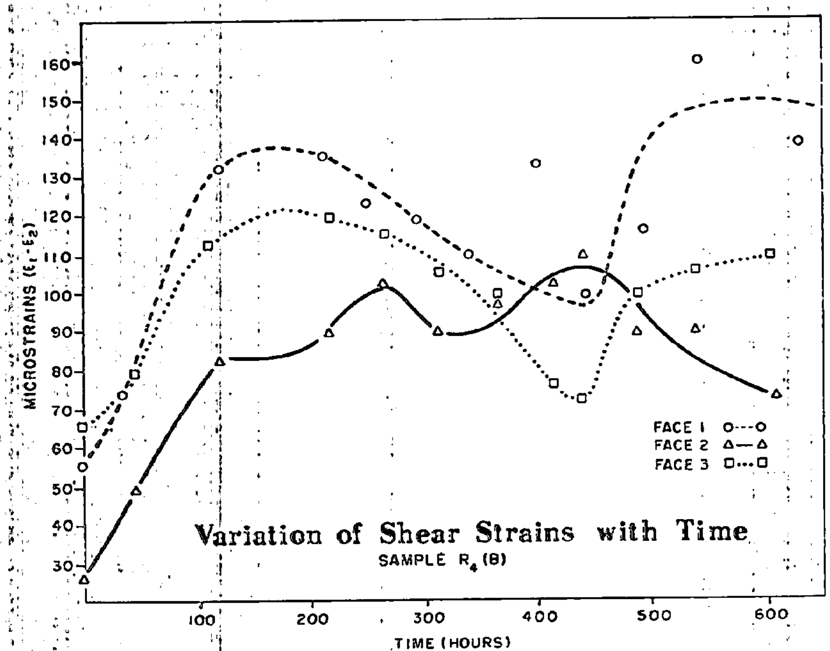

Fig. 1

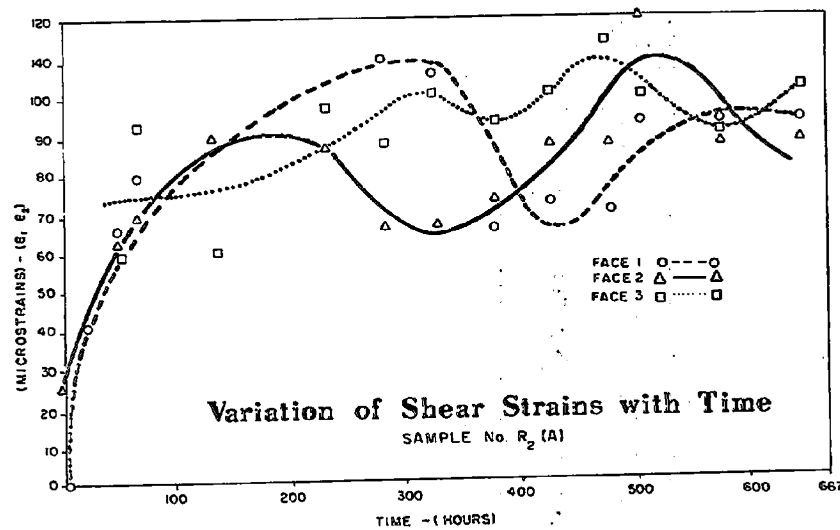

Fig. 2

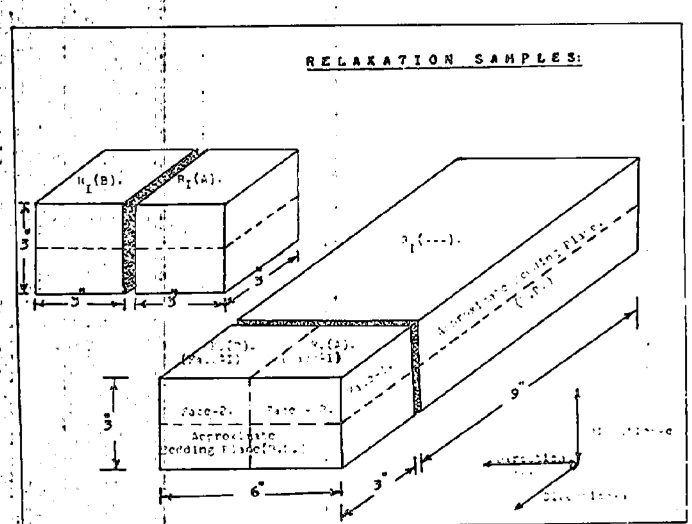

Fig. 4 - Photomicrograph, 80X

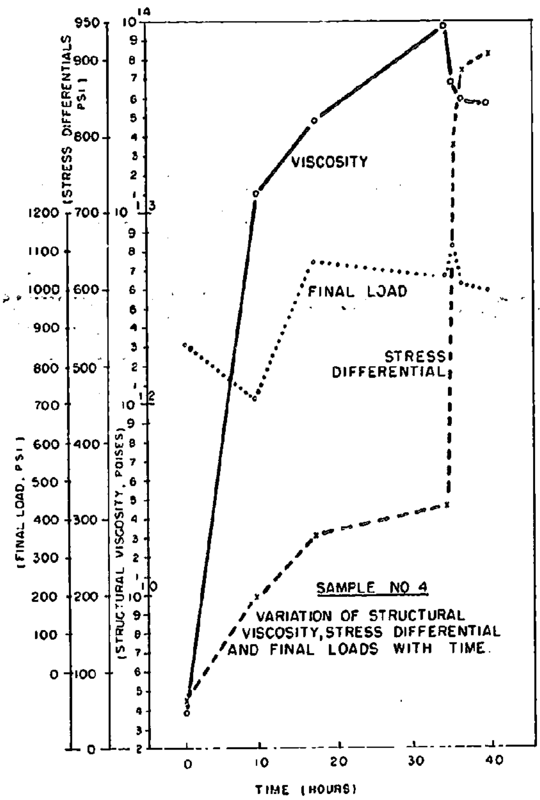

Fig. 5

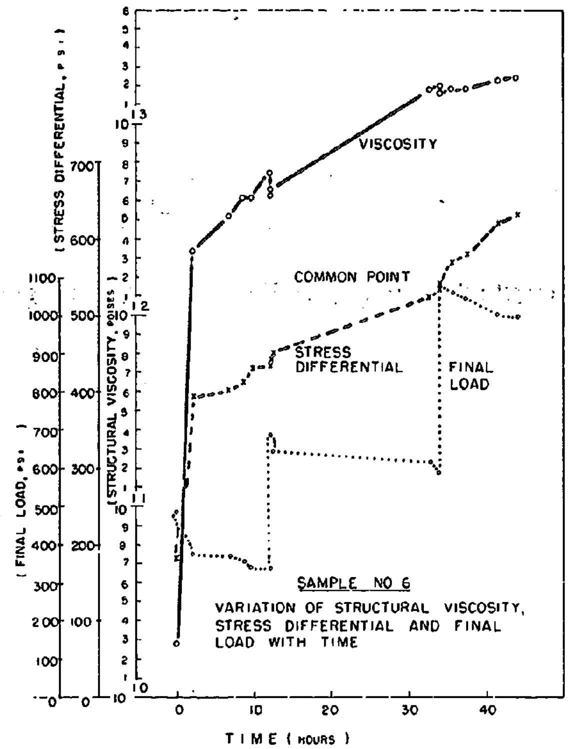

Fig. 6

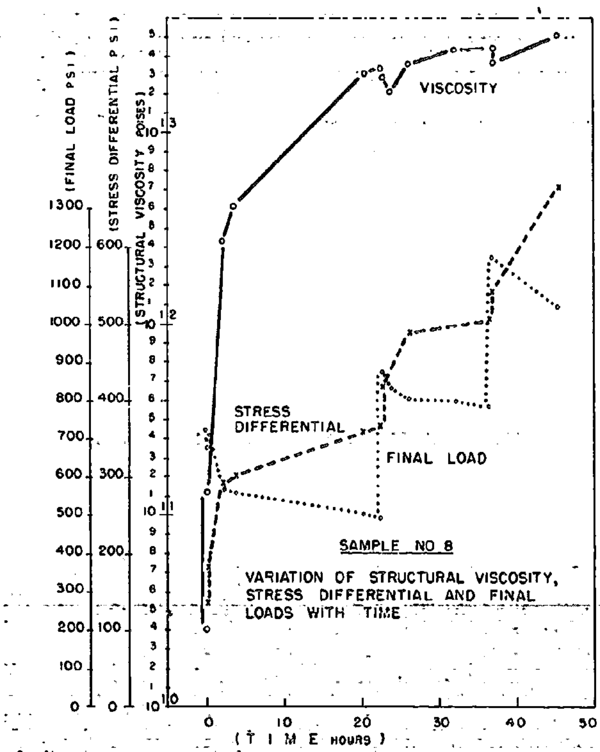

Fig. 7

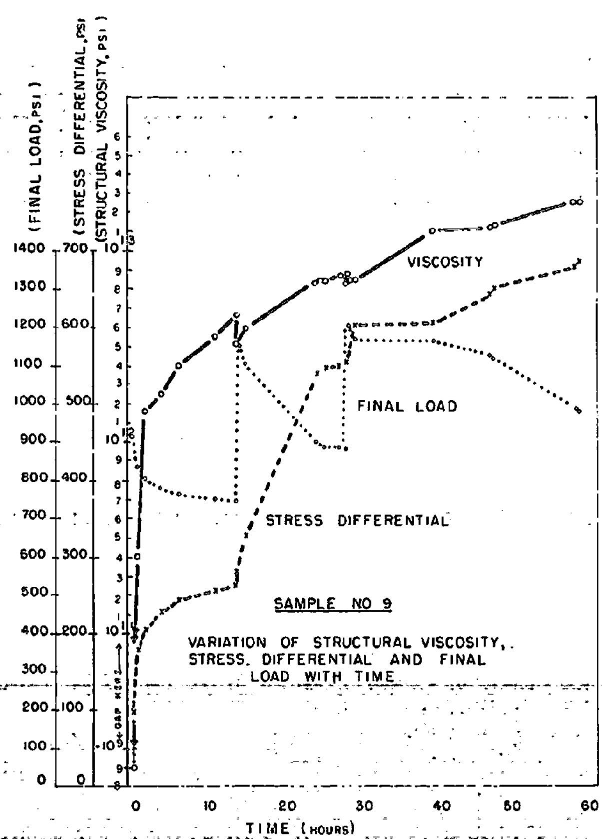

Fig. 8

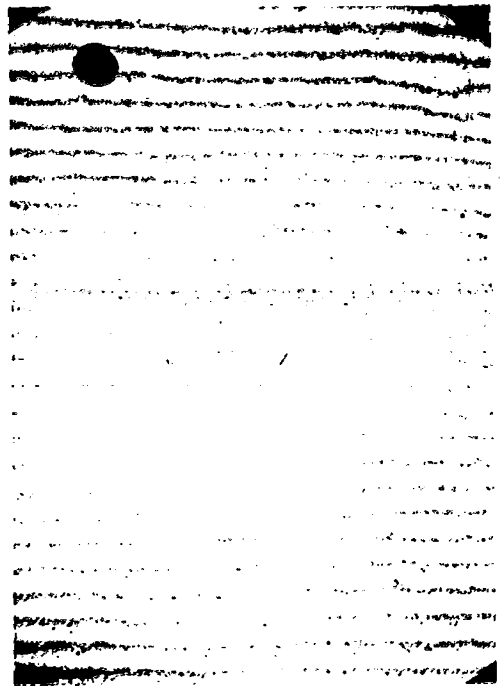

Fig. 9 - Fizeau Interferogram
Queenstown Dolomite (Undeformed)
Coated with 1000A C-I-M
Adox Negative Mag. 10 X
Photo Mag. 120 at 5461A Radiation

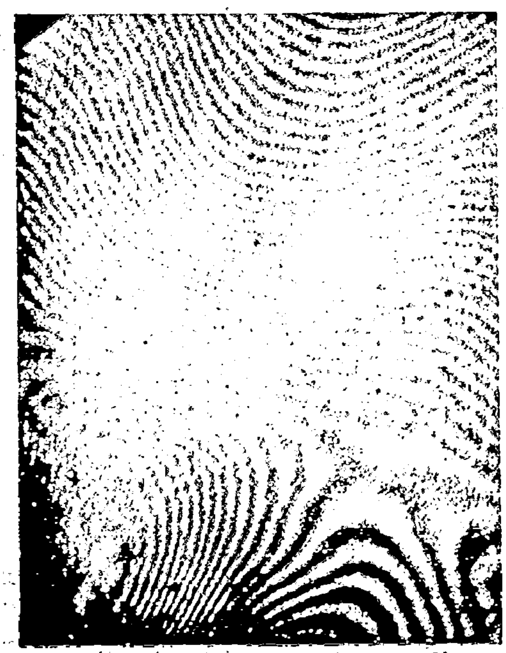

Fig. 10 - Fizeau Interferogram
Queenstown Dolomite (Deformed)
Adox Negative Mag. 20 X
Photo Mag. 240 at 5461A Radiation