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THE REDUCTION OF PRESSURE GRADIENTS IN OIL PIPELINES BY THE ADDITION OF WATER:

NUMERICAL ANALYSIS OF STRATIFIED FLOW*

M.E. Charles and P.J. Redberger

Research Council of Alberta
Edmonton
Canada

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ABSTRACT

The addition of water to crude oil pipelines appears to be an important method for decreasing the pressure gradient for a given oil throughput. Although the concentric oil-in-water flow pattern provides the greatest possible reduction, the general case, in which the liquids are stratified as a result of the oil and water having different densities, is also characterized by reductions in pressure gradient.

To evaluate the magnitude of the pressure gradient reduction to be expected in stratified flow systems, the Navier-Stokes equations were solved by a numerical procedure utilizing a digital computer, for the laminar stratified flow of two Newtonian liquids - oil and water - in a circular pipe. Liquid flow rates were obtained for the flow of five hypothetical oils ranging in viscosity from 4 to 1500 centipoises in the presence of water, by computing the oil and water velocity profiles for a series of arbitrary oil-water interface positions. It was found that the maximum values for the pressure gradient reduction factor ranged from 1.12 to 1.31 for the five oils and occurred at water contents ranging from 12 to 93 per cent respectively.

The computed reduction factors were considerably lower than experimental values and this appears to indicate that wave motion and mixing at the oil-water interface produces a further and very significant reduction in the pressure gradient.

INTRODUCTION

The most convenient and usually the most economical method of transporting petroleum overland is by pipeline. In this mode of transportation the power required to move the oil is a function of the oil viscosity which, for different crude oils, may vary over a wide range. The flow of more viscous oils under pipeline conditions tends to be laminar and in this state the pressure gradient necessary to move oil at a given throughput is proportional to the viscosity of the oil. With highly viscous crude oils several methods are available for reducing the pressure gradient necessary to maintain a given throughput. The oil viscosity may be reduced by raising the temperature of the oil or by adding a diluent, or the resistance to the flow of the oil may be reduced by the addition of a less viscous immiscible liquid. A discussion of the relative merits of these methods in a particular case has been given by Leach (1).

The resistance to flow in either laminar or turbulent flow results for the most part from the friction at the pipe wall and if the viscous liquid at the pipe wall is replaced by a liquid of much lower viscosity the resistance to flow is considerably reduced. The addition of water to crude oil pipelines can therefore be an attractive method of reducing pressure gradient.

Maximum pressure gradient reduction is obtained with a concentric oil-in-water flow pattern. This condition is possible if the densities of the oil and water are approximately equal. The concentric flow of oil and water has been analysed theoretically by Russell and Charles (2) for the case when both the oil and the water are in laminar motion, by Chernikin (3) for the case when the oil is in laminar motion and the water is turbulent. Yuster (4), Odeh (5) and Baker (6) have considered the concentric flow model in idealised capillary systems. Concentric flow was shown to be stable under certain conditions of flow by the experimental work of Charles, Govier and Hodgson (7) in which the densities of the water and oil were equal. The patents by Clark and Shapiro (8) and Chilton and Handley (9), were also concerned with the application of the concentric flow system. Isaacs and Speed (10) and Chernikin (3) indicated that when the density differential is sufficient to produce stratification of the oil and water it is possible that concentric flow may be established if a rotational motion is imparted to the flowing liquids by means of a rifle on the inside of the pipe.

If no attempt is made to equalize the density differential or produce a rifling effect, the oil and water will tend to stratify and the water will be in contact with the oil on a horizontal interface. This type of flow is probably the most important to be considered because of its simple and widespread application.

Looman (11) was probably the first to suggest the use of water as a bottom layer in oil pipelines to reduce the pressure gradient necessary to convey oil at a given rate.

The effect of the oil viscosity in stratified flow between parallel plates has been predicted by Russell and Charles (2) in a theoretical study. The analysis dealt with Newtonian fluids in laminar motion and the maximum pressure gradient reduction factor, i.e. the ratio of the pressure gradient for the oil flowing alone to the pressure gradient for the same oil throughput in the presence of water, was predicted to approach a constant value of approximately 4 at high oil viscosities.

Dumitrescu and Stanescu (12) presented a general analytical method for the treatment of stratified laminar flow in a conduit of any shape and have indicated solutions for the special cases of a circular pipe and a rectangular conduit half full of each liquid. Teletov (13) has also indicated a highly complex analytical method for obtaining velocity profiles for stratified flow in a circular conduit. Neither Dumitrescu and Stanescu nor Teletov obtained flow rate equations or pressure gradient reduction factors.

Russell, Hodgson and Govier (14) investigated the general flow characteristics of the simultaneous horizontal flow of a light oil and water and noted small reductions in pressure gradient for flow in the stratified regime. Charles (15) has reported pressure gradients for the stratified flow of a viscous crude oil and water in a 1-inch laboratory pipeline as well as in a 2.45-inch experimental field pipeline. The results obtained with the 2.45-inch line are reproduced in Figure 1 where the pressure gradient reduction factors are plotted as a function of the percentage of water in the flowing stream. Reductions in pressure gradient by factors of more than 10 were recorded for water percentages ranging from 30 to 50 per cent. Oil viscosities ranged from 124 to 910 centipoise.

The object of the present investigation was to evaluate the effect on the velocity profile of the introduction of water as a lower layer in oil pipelines, and to predict the magnitude of the pressure gradient reduction in terms of the crude oil viscosity and the water flow rate.

The analysis is restricted to Newtonian fluids in laminar motion with a smooth horizontal interface. It is assumed that the velocities of the two phases are equal at the interface and zero at the pipe wall, and that the shear forces exerted on each phase at the interface are equal.

THEORY

The Navier-Stokes equations govern the three-dimensional flow of a fluid. When the flow is undirectional and laminar, and the fluids incompressible, under steady state conditions the Navier-Stokes equations reduce to the single equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{g_c}{\mu} \cdot \frac{dp}{dZ} \qquad (1)$$

which is an elliptic differential equation for which it is possible to specify the conditions at all points along a closed boundary. There is a simple analytical solution of this equation for the flow of a single fluid in a circular pipe. However in the case of stratified flow, although Dumitrescu and Stanescu (12) and Teletov (13) have indicated general analytical approaches, and the former have given the solution for the special case when the interface passes through the axis of the pipe, the general analytical solution for any interface position is exceedingly complex. Iterative and analogue procedures are available as substitutes for analytical methods and a numerical technique was therefore used to give an approximate solution to the differential equation (1). Usually in such a process the number of repetitive calculations is very large and several investigators, including Frankel (16) and Radd and Tek (17), have suggested and demonstrated the use of digital computers to shorten the computation time. The general approach of Radd and Tek was modified to suit the present problem and entailed the replacement of the differential equation (1) by the finite difference equation

$$U'_{i,j} = \frac{1}{4}\left[U_{i+1,j} + U_{i-1,j} + U_{i,j+1} + U_{i,j-1} + \frac{h^2 g_c}{\mu}\frac{dp}{dz}\right] \quad (2)$$

and the superimposition of a grid with a square mesh on the pipe cross-section. A grid which is too coarse to obtain a reasonably accurate solution of the present problem but which will be useful in explaining the method, is shown superimposed on the pipe cross-section in Figure 2 where the relationship between adjacent grid points is illustrated. The finite difference equation is applied to each point in turn and the new velocity, $U'_{i,j}$, calculated from the four neighboring points, is a better approximation to the true velocity at that point. Only one-half of the pipe cross-section is illustrated because the system is symmetrical about the vertical diameter. The position of the pipe wall is approximated by the grid points labelled zero. The finer the mesh, the more accurate is the approximation to the solution of equation (1).

The use of equation (2) gives slow convergence to the final velocities and the extrapolated Liebmann method (18) which utilizes and over-relaxation factor β, was used to produce faster convergence. In the extrapolated Liebmann method the finite difference equation (2) becomes:

$$U'_{i,j} = U_{i,j} + \beta\left[\frac{1}{4}(U_{i+1,j} + U_{i-1,j} + U_{i,j+1} + U_{i,j-1} + C) - U_{i,j}\right] \qquad (3)$$

where

$$1 \leq \beta < 2 \text{ and } C = \frac{h^2 g_c}{\mu} \frac{dp}{dz}$$

The over-relaxation factor, β, was calculated from the equation given by Frankel (15):

$$\beta = 2 - \left[2\pi^2\left(\frac{1}{m^2} + \frac{1}{n^2}\right)\right]^{1/2} \qquad (4)$$

The relaxation constant, C, in Equation (3) contains the fluid viscosity and hence has different values when Equation (3) is applied to the oil and water layers. Equation (3) governs the flow within the oil and water layers but does not apply at the oil-water interface. At the interface the shear forces exerted on each liquid are equal and for Newtonian liquids it follows that:

$$\frac{\left(\frac{\partial U_o}{\partial y}\right)_{int}}{\left(\frac{\partial U_w}{\partial y}\right)_{int}} = \frac{\mu_w}{\mu_o} \qquad (5)$$

in which the velocity gradients are evaluated at the interface. Equation (5) may be introduced into the iteration procedure in at least two ways:

(i) The interface is coincident with a row of grid points and the differential equation (5) is replaced by the finite difference equation:

$$U'_{i,j} = \left(\frac{\mu_o}{\mu_o + \mu_w}\right) U_{i,j-1} + \left(\frac{\mu_w}{\mu_o + \mu_w}\right) U_{i,j+1} \qquad (6)$$

which is used instead of Equation (3) at the grid points which lie at the interface. The relationship of the adjacent grid points and the interface position is illustrated in Figure 3.

This method suffers from the obvious limitation that iterations are performed in one dimension only for the interface grid points.

(ii) The interface lies equidistant between two adjacent rows of grid points. The method is illustrated in Figure 4. Temporary interface points are assigned and the finite difference equation:

$$U'_{i,j'} = \left(\frac{\mu_o}{\mu_o + \mu_w}\right) U_{i,j-1} + \left(\frac{\mu_w}{\mu_o + \mu_w}\right) U_{i,j} \qquad (7)$$

which again corresponds to the differential equation (5) is used to calculate the interface velocity values which are subsequently used in the finite difference equation (3) modified, as suggested by Round, Newton and Redberger (19), to take into account the variation in grid size brought about by placing the interface between two grid rows. This modified finite difference equation, written for a point i, j in the grid row below the interface is

$$U'_{i,j} = U_{i,j} + \beta\left[\frac{1}{6}\left\{U_{i+1,j} + U_{i-1,j} + U_{i,j+1} + \frac{1}{3}\left(U_{i,j+1} + 8U_{i,j'}\right) + C\right\} - U_{i,j}\right] \qquad (8)$$

This method has the advantage over method (i) in that iterations are performed in two-dimensions closer to the interface and hence should yield more accurate results.

Both methods (i) and (ii) were investigated and the relative accuracy of the two methods will be indicated.

In the iterative procedure the grid points were treated successively and new velocity values were calculated. Sweeps through the grid were continued until the differences between the old and the new velocity values at each point were within a predetermined arbitrary tolerance, i.e. $\left|U'_{i,j} - U_{i,j}\right| < e$.

COMPUTATION

In general, in an iterative procedure of the type used in the present study, the smaller the mesh size is, the greater is the accuracy of the computed result and the longer is the computation time.

The mesh size used in this case represented a compromise between the accuracy of the result and the time required for computation. Initial test runs were made on a Royal McBee LGP-30 computer and three mesh size ratios, namely 1/16, 1/32, and 1/48, were investigated. (The mesh size ratio of the grid shown in Figure 2 is 1/8.) Accuracy was gauged by comparing the computed velocity profile for a pipe flowing full of a single liquid with the theoretical profile, and the computed cross-sectional area of the approximate pipe boundary with the theoretical area of the pipe. The mesh size ratio of 1/16 resulted in errors of 3 per cent for the maximum velocity and 4 per cent for the pipe area compared with approximately one per cent errors given by the ratio of 1/32. The ratio of 1/48 gave results very little better than that of 1/32 but required a much longer computation time and thus a mesh size ratio of 1/32 was chosen for the main investigation. Approximately 100 sweeps through the grid were necessary to achieve the above accuracies in the velocity values. It was found that the computation time was 4.5 min. per sweep on the LGP-30 computer and consequently an IBM 704 was used for the complete solutions because it gave the shorter computational time of 0.6 sec. per sweep.

Flow systems were studied for oils of viscosity 4, 20, 150, 450, and 1500 centipoise flowing above water with a viscosity of 0.896 centipoise. For each oil viscosity six interface positions were investigated, more interface positions being in the lower half of the pipe than in the upper so that the effect of combining relatively small flow rates of water with the oil could be evaluated. For each interface position the computer printed out the velocity values at each grid point as well as the oil and water flow rates which were calculated by numerical integration.

RESULTS

The analytical and numerical values of the velocities on the axis of the pipe were compared for the special case when the interface coincided with the horizontal diameter. The equations of Dumitrescu and Staniscu (12) are easily solved for this particular point and the analytical velocity value is given by:

$$U = \frac{D^2 g_c}{8} \cdot \frac{dp}{dz} \cdot \frac{\mu_w}{\mu_o + \mu_w} \tag{9}$$

Numerical velocity values were calculated using both of the methods outlined for incorporating the interface condition into the iterative method. The corresponding velocities given by the analytical and numerical methods are given in Table I for the five oil viscosities.

It was apparent that method (ii), in which the interface was placed between two grid rows, gave the more accurate results and this method was subsequently used throughout the computation.

The effect on the oil phase of the addition of water to form a lower layer in oil pipelines is illustrated by the form of the velocity profiles on the vertical pipe diameter. The profiles obtained at each interface position for each oil viscosity are basically similar, and as examples, profiles are given in Figure 5 for an oil viscosity of 150 centipoise. The pressure gradient is the same for each profile, and the interface positions and percentages of water in the flowing stream are indicated. The parabolic profile for the oil flowing alone under the same pressure gradient is included for comparison. The effect of the addition of the water is to sharply extend the profile in the oil phase for a given pressure gradient.

The general shape of the velocity profile obtained by the addition of water to the flowing stream is further illlustrated in Figure 6 where horizontal profile planes are superimposed in the plan view and vertical profile planes are superimposed in the elevation to give a three-dimensional representation of the profile. The oil viscosity is 150 centipoise and the water flow rate constitutes 45.8 per cent of the total flow for the profile shown.

The effect of the oil viscosity on the shape of the velocity profile for a fixed interface position is illustrated in Figure 7. The profiles are again drawn for a constant pressure gradient. Although the interface position is the same for each oil viscosity the relative flow rates of the two liquids differ considerably. For an oil viscosity of 4 centipoise, water constitutes 39.8 per cent of the total flow, while for an oil viscosity of 1500 centipoise water constitutes 99.0 per cent of the total flow.

In pipeline design, interest may focus on the oil flow rate which may be obtained for a given pressure gradient, or alternatively, the pressure gradient which is necessary to provide a given oil throughput. Hence the results of the present investigation could be reported either in terms of the increase in oil flow rate for a given pressure gradient obtained by addition of water, or alternatively, in terms of the reduction in pressure gradient for a given oil flow rate obtained by the addition of water. In practice the oil throughput is usually known and accordingly the present results are reported in terms of the pressure gradient reduction factor which is defined as the ratio of the pressure gradient for the oil flowing alone to the pressure gradient for the same oil throughput in the presence of water.

The pressure gradient reduction factors calculated from the results of the numerical analysis are plotted against the interface position with the oil viscosity as parameter in Figure 8. The curves all have the same general shape; with the interface at the bottom of the pipe, i.e. the pipe flowing full of oil, the pressure gradient reduction factor is unity for each oil viscosity and increases to a maximum on the addition of water and then falls off to zero as the interface rises to the top of the pipe on the further addition of water. The greater the oil viscosity, the greater is the rate of increase of the reduction factor with increasing water percentage, although for the higher oil viscosities the curves are very close together.

The pressure gradient reduction factors are also plotted against the percentage of water in the flowing stream in Figure 9. Curves are shown only for oil viscosities of 4, 150 and 1,500 centipoise to avoid confusion. The percentage of water needed to bring about the maximum beneficiation varies from 12 per cent for a 4-centipoise oil to 93 per cent for a 1500-centipoise oil. For all viscosities the reduction factor decreases very rapidly as the water percentage approaches 100.

The maximum pressure gradient reduction factors are shown in Figure 10 as a function of oil viscosity and range up to about 1.31 for the viscosity values investigated. It is evident that for oil viscosities greater than about 100 centipoise the reduction factor is approximately constant. Also in Figure 10 the maximum pressure gradient reduction factors are compared with those predicted by Russell and Charles (2) for concentric flow in a circular pipe and stratified flow between parallel plates. For concentric flow the maximum pressure gradient reduction factor is directly proportional to the oil viscosity whereas for stratified flow between parallel plates the maximum reduction factor approaches a value of approximately 4. It is apparent, therefore, that concentric flow, provided it may be established, is very much more successful in reducing pressure gradients.

A further analysis of the experimental pressure drop data obtained by Russell et al (14) for the stratified flow of an 18 centipoise mineral oil with water indicates that for the regimes of flow in which both phases were apparently in laminar motion, pressure gradient reduction factors of approximately 1.2 were noted when water constituted approximately 10 per cent of the total flow. These values compare very closely with the value of 1.21 obtained for the 20 centipoise oil and 10 per cent water by the numerical analysis.

However, the experimental pressure gradient reduction factors reported by Charles (15) for the stratified flow of a crude oil and water in a 2.45-inch diameter pipeline and reproduced in Figure 1 differ considerably from the results obtained in the numerical analysis. Up to a water percentage of about 20 the experimental reduction factors are little different from unity but at water percentages in excess of 20, pressure gradient reduction factors greater than 10 were recorded over a wide range of oil flow rates. In the experimental tests the oil was almost certainly in laminar flow and the water in turbulent flow. The discrepancy between the experimental reduction factors and the results of the numerical analysis is probably accounted for by the turbulent water layer producing wave action and mixing of the oil and water at the interface which would effectively give a gradual change in viscosity across the boundary between the phases and reduce the sharpness of the discontinuity in the velocity profile. Part of the oil phase would then be in the relatively fast moving water phase with a consequent very substantial increase in the pressure gradient reduction factor.

CONCLUSIONS

1. The use of a digital computer has been demonstrated for obtaining velocity profiles and flow rates of two liquids flowing laminarly and stratified within a circular conduit. The method may be applied to any number of liquid layers flowing in a conduit having any geometry.

2. A reduction is predicted in the pressure gradient necessary to maintain a given oil throughput by introducing water as a bottom layer into a pipeline carrying oil. The range of oil viscosity investigated was from 4 to 1500 centipoise and the maximum pressure gradient reduction factors ranged from 1.12 to 1.31 respectively.

3. The pressure gradient reduction factors of the present analysis are considerably less than certain experimental values for stratified flow. This appears to indicate that wave motion and mixing of the two phases at the interface due to turbulence in the water layer appears to give a further and very significant reduction in pressure gradient.

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NOMENCLATURE

C = relaxation constant = $\frac{h^2 g_c}{\mu}\frac{dp}{dz}$, ft. per sec.

D = inside diameter of pipe, ft.

e = tolerance, ft. per sec.

g_c = dimensional conversion factor, $lb._M$ ft. per $lb._F$ $sec.^2$

h = length of side of grid square, ft.

m = number of columns in grid.

n = number of rows of grid.

$\frac{dp}{dz}$ = pressure gradient, $lb._F$ per $ft.^2$ per ft.

s = ratio of distance from top of pipe/diameter of pipe, dimensionless

t = ratio of distance from centre of pipe/radius of pipe, dimensionless.

$U_{i,j}$ = general point velocity, ft. per sec.

$U'_{i,j}$ = newly calculated general point velocity, ft. per sec.

β = over-relaxation factor, dimensionless.

μ = liquid viscosity, $lb._M$ per ft. sec.

Subscripts

i = grid column number ($1 \leq i \leq m$)

j = grid row number ($1 \leq j \leq n$)

j' = temporary grid row number defined by Equation (6)

o = oil

w = water

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CAPTIONS FOR FIGURES

Figure 1. Experimental results for pressure gradient reduction factor as a function of percentage of water in the flowing stream, as reported by Charles (15) for a heavy crude oil.

Figure 2. Illustrative grid on half of cross-sectional area of a circular pipe carrying stratified oil and water.

Figure 3. The position of the interface in relations to the grid points when the interface lies in a row of grid points.

Figure 4. The position of the interface in relation to the grid points when the interface lies between two adjacent rows of grid points.

Figure 5. Typical velocity profiles on the vertical pipe diameter for the 150 centipoise oil flowing above water compared with the parabolic profile for the oil flowing alone. The pressure gradient is the same profile and the lower parts of the three profiles are identical within the limits of the figure.

Figure 6. Plan view and elevation of superimposed profile planes to illustrate a three-dimensional profile for an oil viscosity of 150 centipoise when water constitutes 45.8 per cent of the total flow.

Figure 7. The effect of the oil viscosity on the shape of the velocity profiles for a fixed interface position. The pressure gradient is the same for each profile and the lower parts of the profiles are identical within the limits of the figure.

Figure 8. Pressure gradient reduction factors calculated by the numerical procedure as a function of interface position.

Figure 9. Pressure gradient reduction factors calculated by the numerical procedure as a function of percentage of water in the flowing stream.

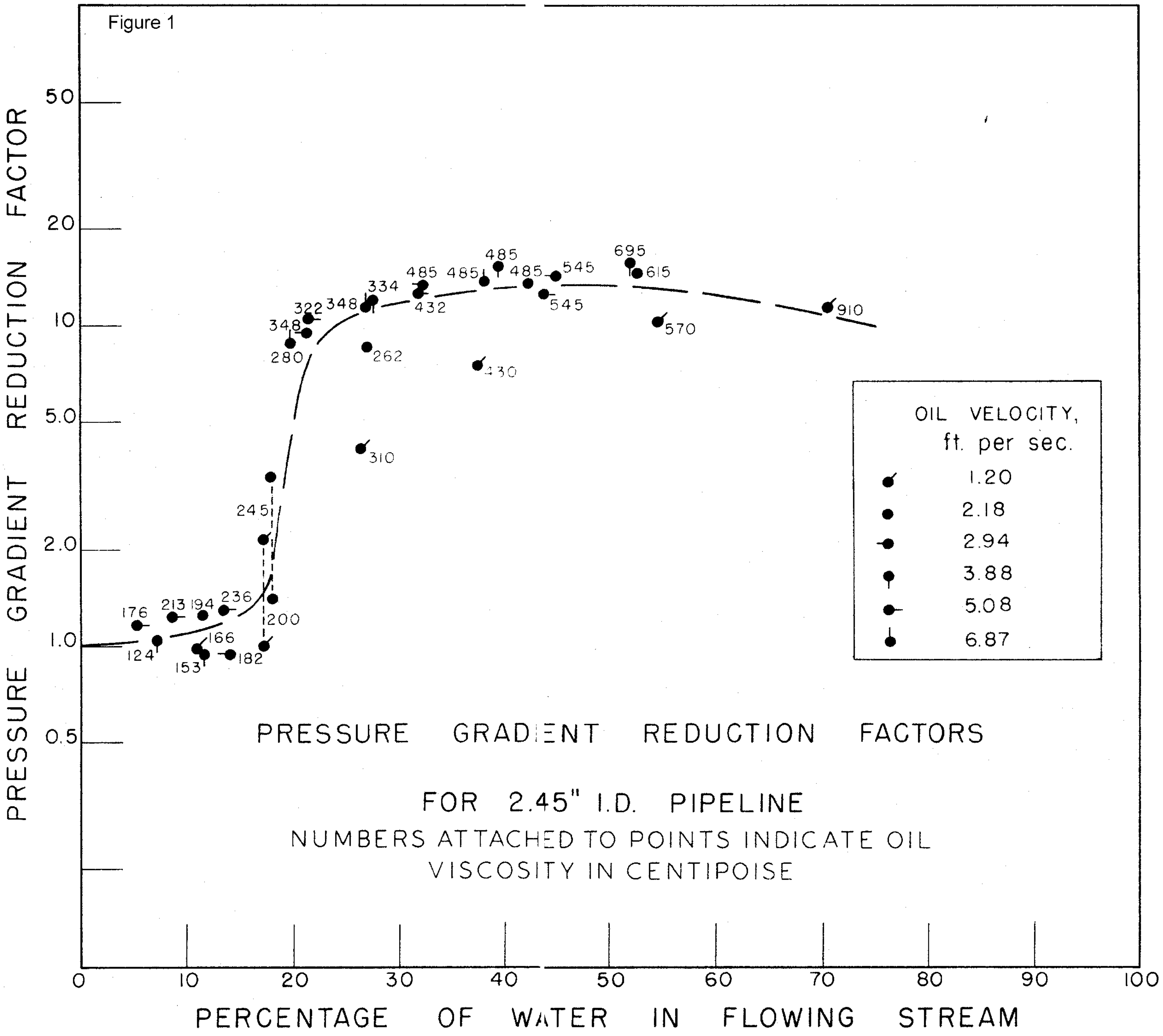
Figure 10. Comparison of maximum pressure gradient reduction factors for stratified flow in a circular pipe, stratified flow between parallel plates and concentric flow in a circular pipe.

TABLE I

COMPARISON OF ANALYTICAL AND NUMERICAL VELOCITY VALUES

Viscosity of oil	Velocity calculated for centre point from analytical equation	Computed velocity values			
		Interface method (i)		Interface method (ii)	
		Velocity predicted	Per cent error	Velocity predicted	Per cent error
Centipoise	Ft. per sec.	Ft. per sec.	-	Ft. per sec.	-
4	5.10	4.77	6.5	5.08	0.39
20	1.19	1.11	6.7	1.18	0.84
150	0.166	0.154	7.2	0.164	1.2
450	0.0557	0.0513	7.9	0.0547	1.8
1500	0.0167	0.0152	9.0	0.0163	2.4

Figure 1

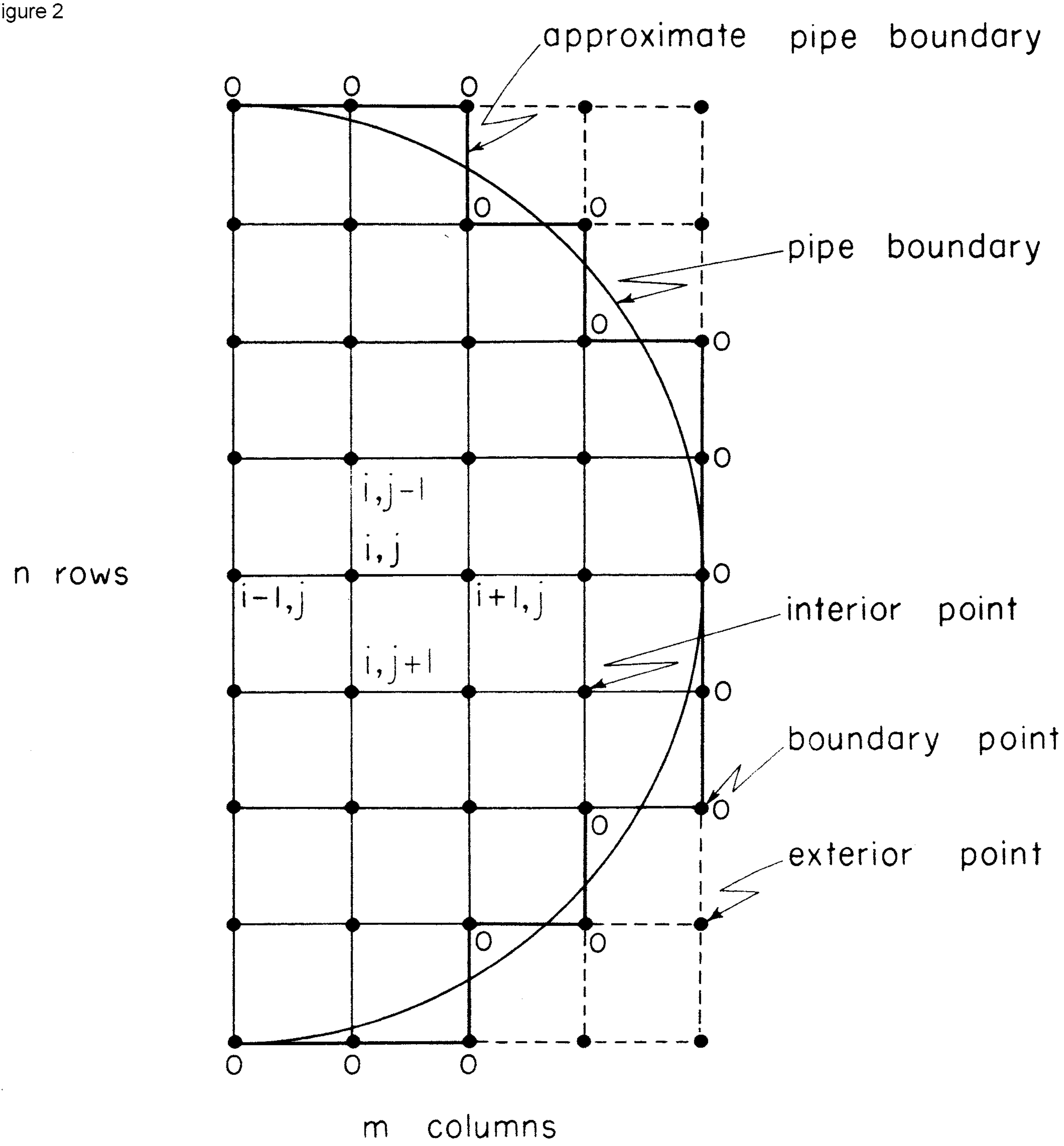


PRESSURE GRADIENT REDUCTION FACTORS

FOR 2.45" I.D. PIPELINE

NUMBERS ATTACHED TO POINTS INDICATE OIL VISCOSITY IN CENTIPOISE

Figure 2


approximate pipe boundary
pipe boundary
interior point
boundary point
exterior point
n rows
m columns
i, j-1
i, j
i-1, j
i+1, j
i, j+1
0

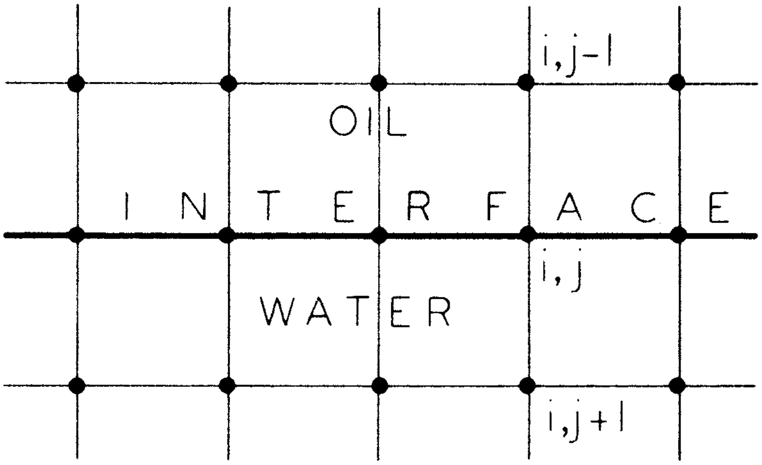


FIGURE 3

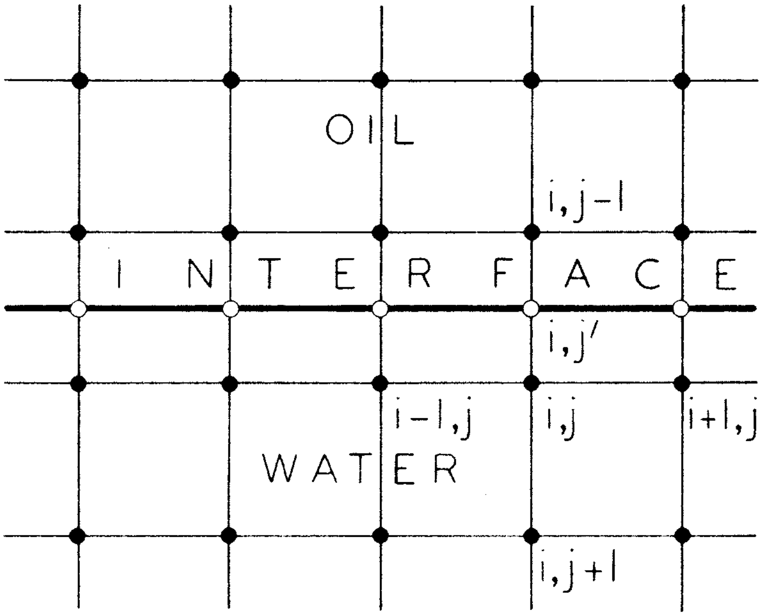


FIGURE 4

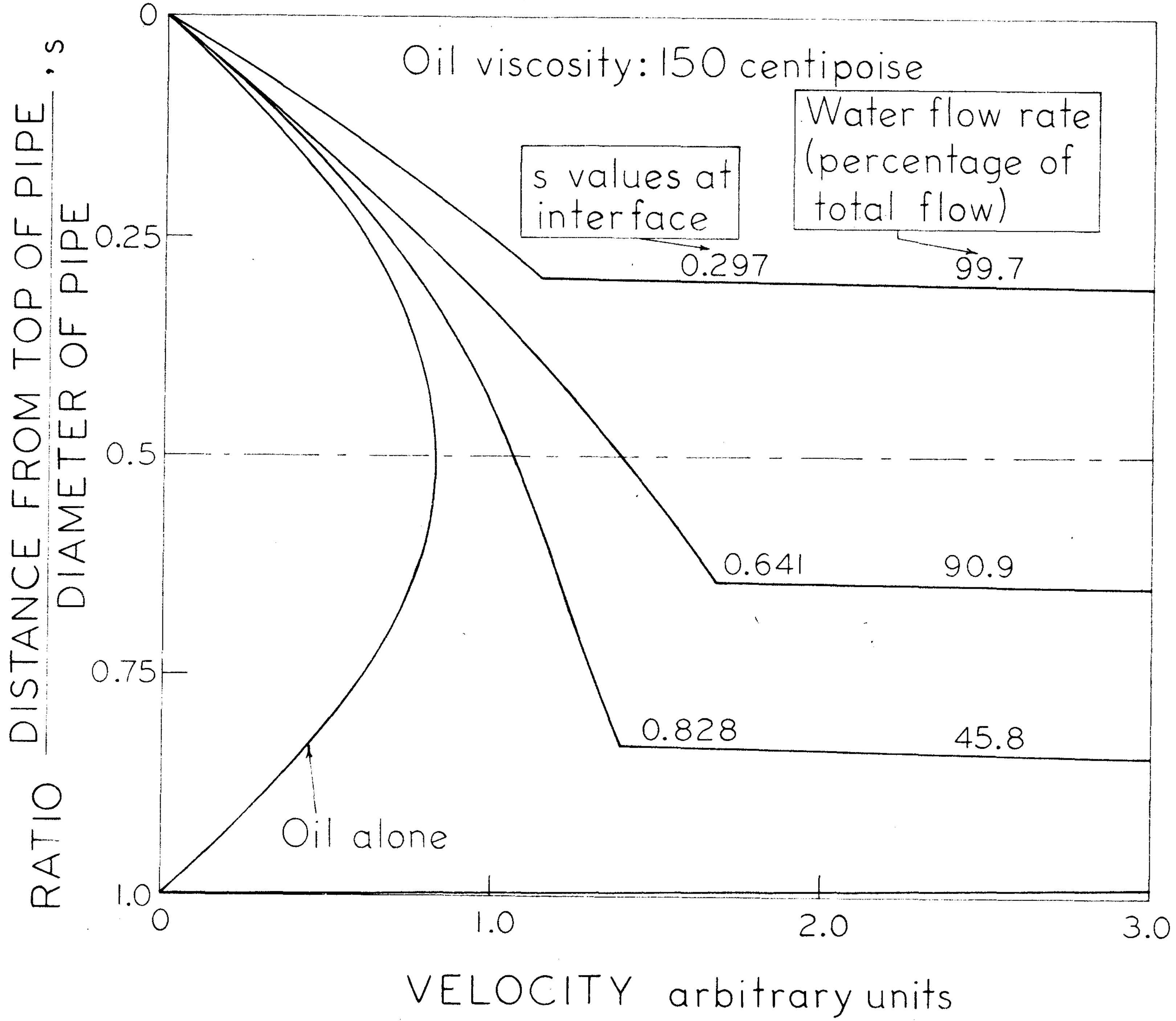
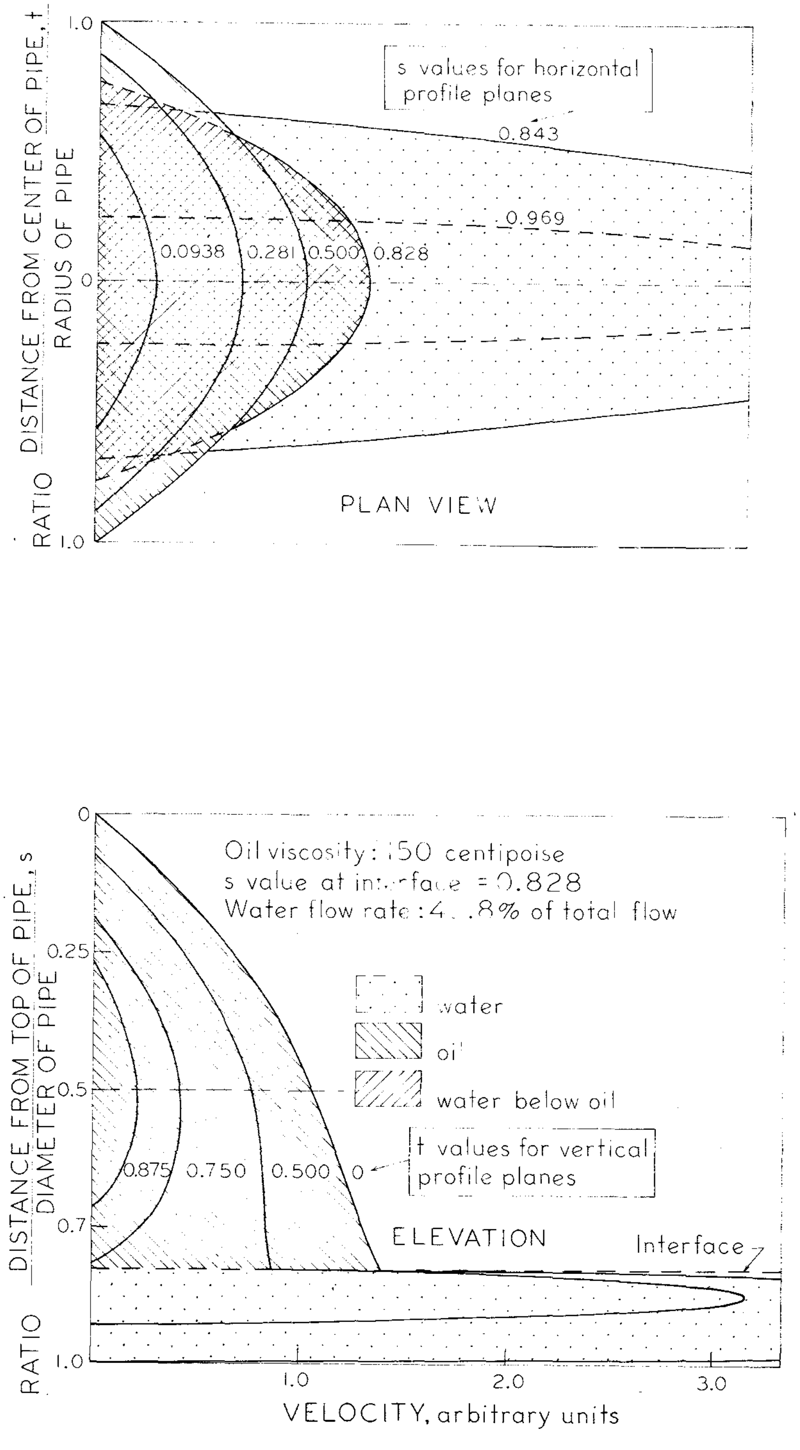

Oil viscosity: 150 centipoise
s values at interface
Water flow rate (percentage of total flow)
0.297
99.7
0.641
90.9
0.828
45.8
Oil alone
RATIO DISTANCE FROM TOP OF PIPE / DIAMETER OF PIPE, s
0
0.25
0.5
0.75
1.0
0
1.0
2.0
3.0
VELOCITY arbitrary units

FIGURE 5

Figure 6


RATIO DISTANCE FROM CENTER OF PIPE, t / RADIUS OF PIPE
1.0
0
1.0
s values for horizontal profile planes
0.843
0.969
0.0938
0.281
0.500
0.828
PLAN VIEW
RATIO DISTANCE FROM TOP OF PIPE, s / DIAMETER OF PIPE
0
0.25
0.5
0.7
1.0
Oil viscosity: 150 centipoise
s value at interface = 0.828
Water flow rate: 4[illegible].8% of total flow
water
oil
water below oil
t values for vertical profile planes
0.875
0.750
0.500
0
ELEVATION
Interface
1.0
2.0
3.0
VELOCITY, arbitrary units

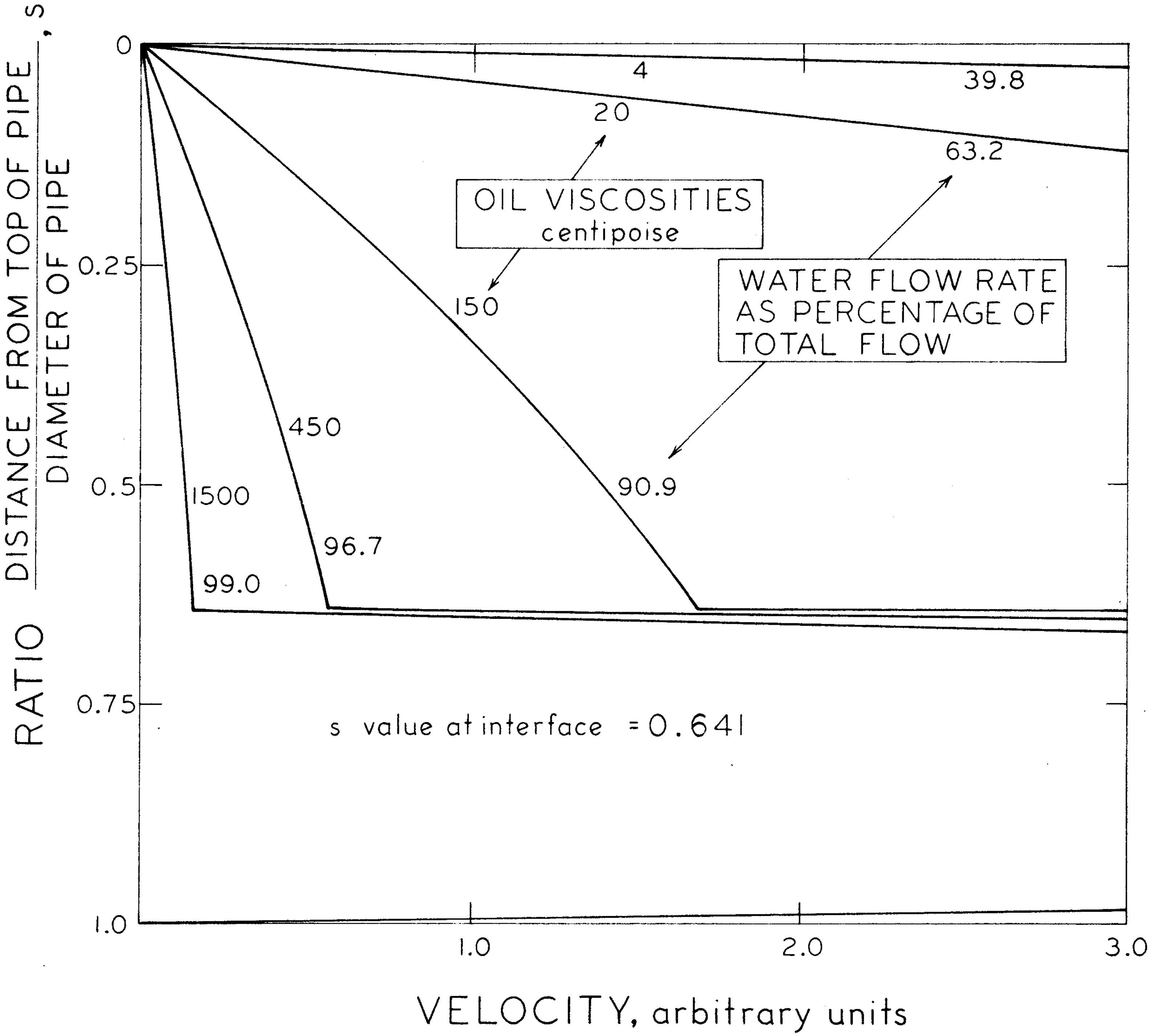

RATIO DISTANCE FROM TOP OF PIPE / DIAMETER OF PIPE, s
0
0.25
0.5
0.75
1.0
1.0
2.0
3.0
VELOCITY, arbitrary units
OIL VISCOSITIES centipoise
WATER FLOW RATE AS PERCENTAGE OF TOTAL FLOW
4
39.8
20
63.2
150
90.9
450
96.7
1500
99.0
s value at interface = 0.641

FIGURE 7

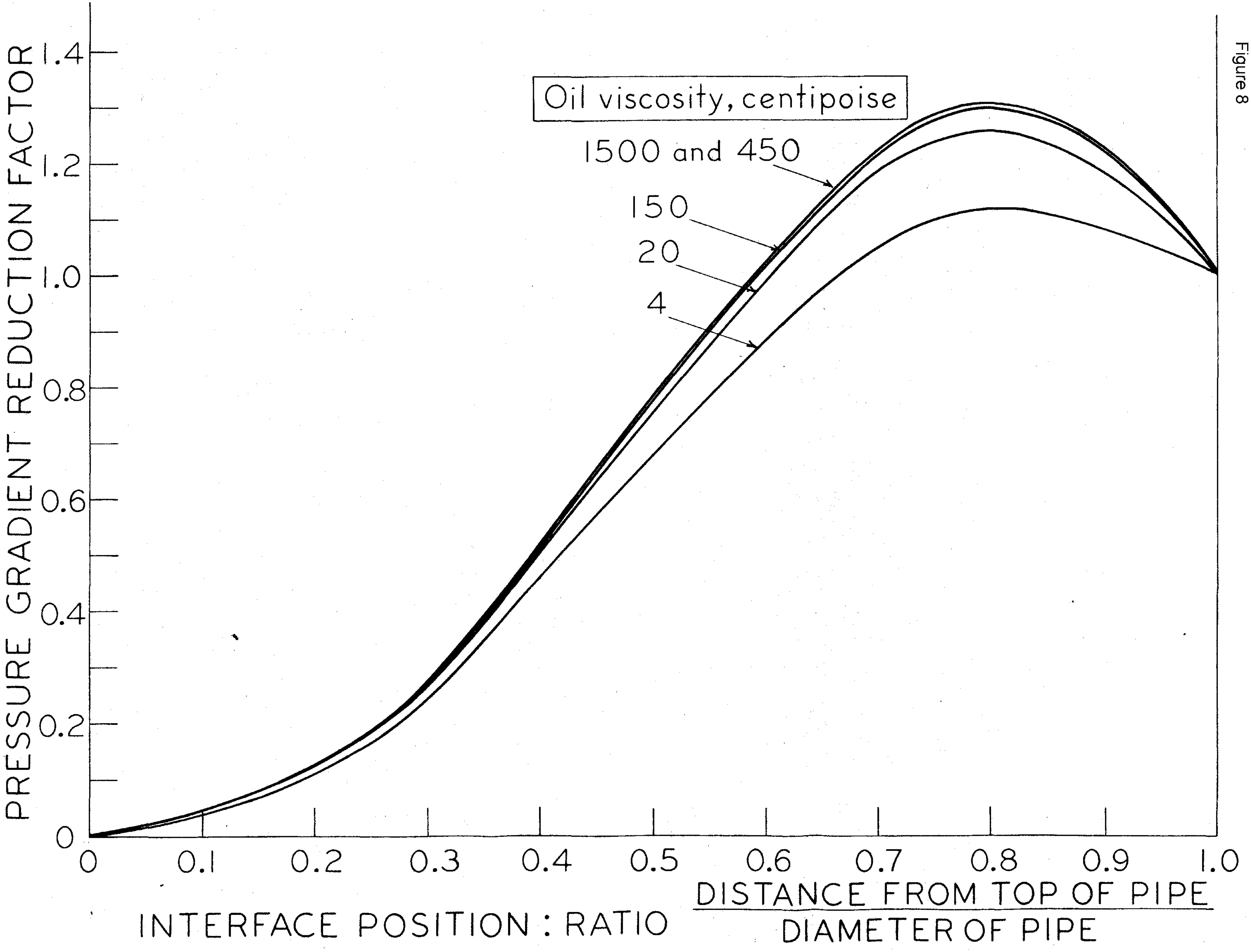

PRESSURE GRADIENT REDUCTION FACTOR
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
Oil viscosity, centipoise
1500 and 450
150
20
4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
INTERFACE POSITION : RATIO DISTANCE FROM TOP OF PIPE / DIAMETER OF PIPE

Figure 8

Figure 9

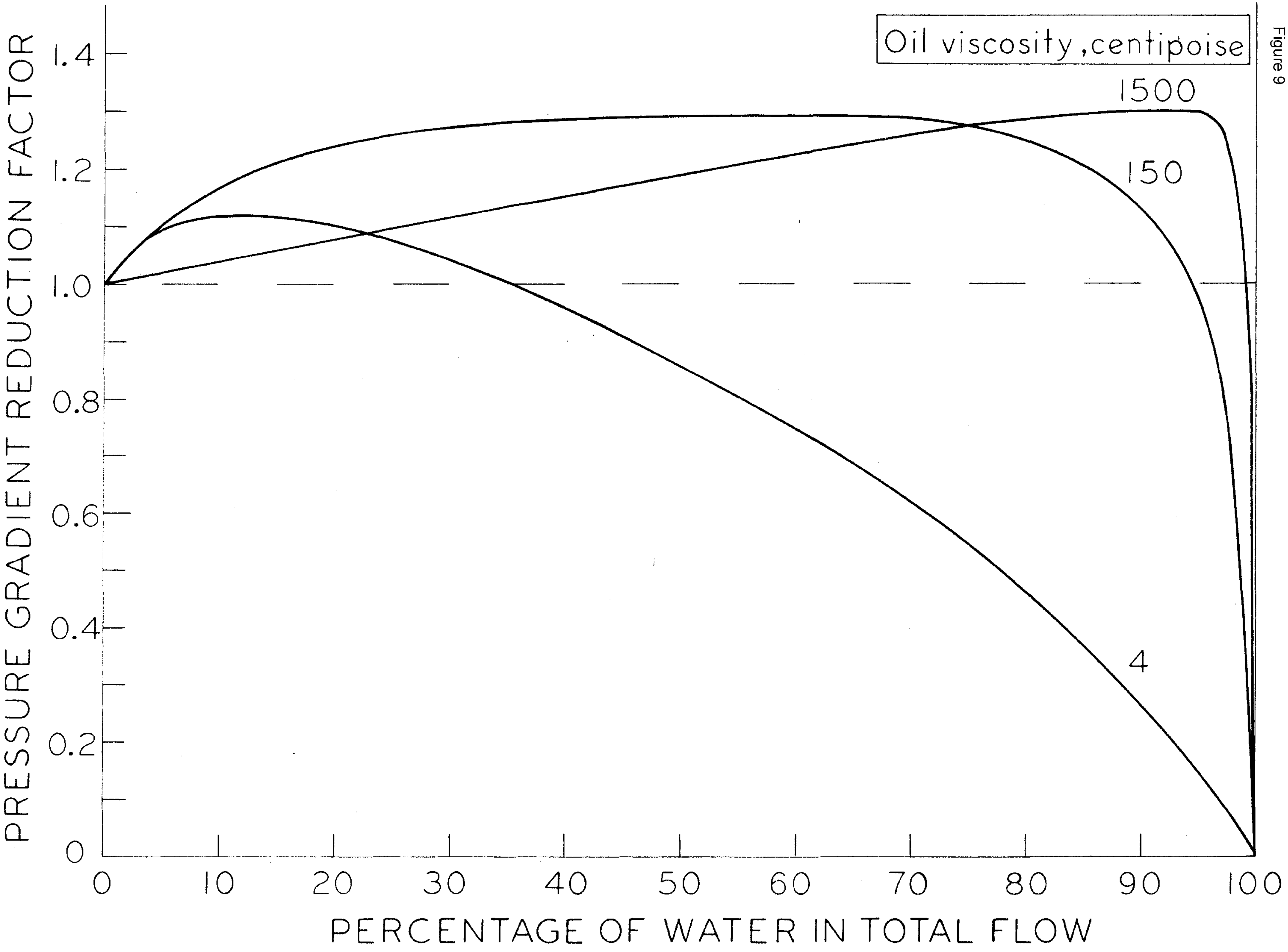
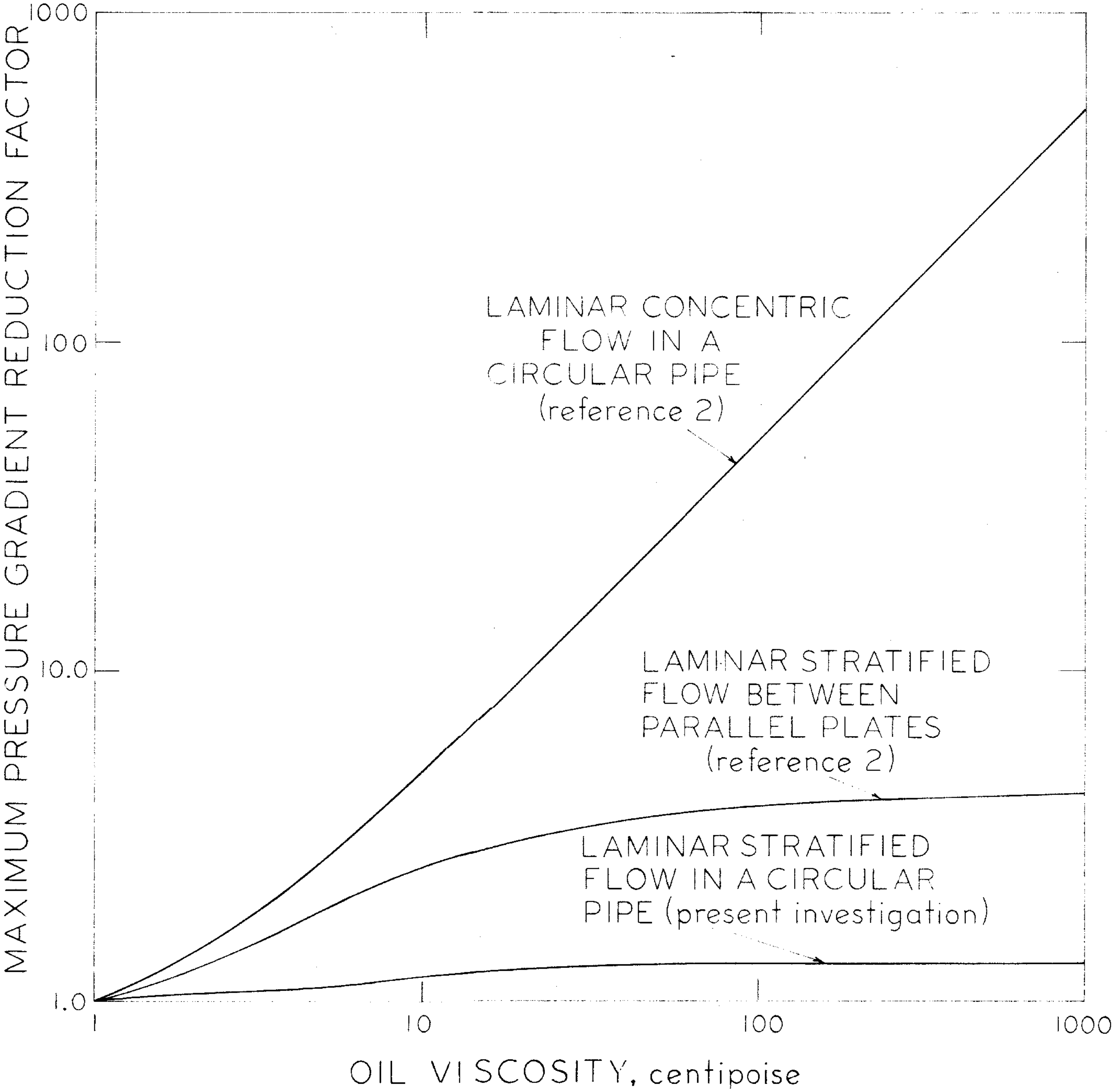

Oil viscosity, centipoise
1500
150
4
PRESSURE GRADIENT REDUCTION FACTOR
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
60
70
80
90
100
PERCENTAGE OF WATER IN TOTAL FLOW

Figure 10



MAXIMUM PRESSURE GRADIENT REDUCTION FACTOR
1000
100
10.0
1.0
1
10
100
1000
OIL VISCOSITY, centipoise
LAMINAR CONCENTRIC FLOW IN A CIRCULAR PIPE (reference 2)
LAMINAR STRATIFIED FLOW BETWEEN PARALLEL PLATES (reference 2)
LAMINAR STRATIFIED FLOW IN A CIRCULAR PIPE (present investigation)