The Heat Efficiency of Thermal Recovery Processes

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Introduction

Most of the information available on the heat efficiency of hot fluid injection processes, both water and steam, has been obtained from calculated temperature distributions in the pay zone and adjacent formations. The usual approach has been to write the heat balance equations in terms of the temperatures, and then to introduce whatever simplifications are necessary to help obtain an analytical or a numerical solution.

In either case, the assumption has often been made that the vertical thermal conductivity in the flooded interval is infinite, so that the temperatures within the flooded interval are then independent of vertical position. Because it was first made by Lauwerier,[1] and has been used extensively since, this assumption will be referred to as the Lauwerier assumption.

Excellent reviews of the literature on the heat efficiency of hot fluid injection processes have been given by Spillette,[2] Flock *et al.*,[3] and Ramey.[4] Of course, the most significant contributions take into account the effect of a finite vertical thermal conductivity on the vertical temperature profile.

The most general analytic expression for the heat efficiency of a hot fluid injection process is that of Antimirov[5] who considers injection of a heated incompressible fluid into a reservoir through an arbitrary number of wells. The rate of heat injection into the reservoir, as well as the injection temperature, is an arbitrary function of time. The geometry of the horizontal flow is arbitrary, although the reservoir is considered to be of uniform thickness and properties and to be of infinite areal extent. Heat transfer within the reservoir is by horizontal convection and conduction, and by vertical conduction. In the formations adjacent to the flooded interval, heat transfer is by conduction in any direction. Thus, the available expression for the heat efficiency due to hot water injection has been developed for rather general conditions.

This degree of applicability has not been obtained for other thermal recovery processes. By making the Lauwerier assumption we have been able to obtain expressions for the heat efficiency that are essentially independent of the thermal recovery process, be it steam, hot-water, or underground combustion. Clearly, then, the approach followed here is more restrictive than that of Antimirov[5] in that it uses the Lauwerier assumption. At the same time less restrictive assumptions are made about the horizontal heat transfer mechanisms, either in the flooded interval or in the formations adjacent to it.

The main difference between our approach and that of other investigators is that the heat balance in the pay zone is expressed for the pay zone as a whole rather than for a volume element within it. The Lauwerier assumption is introduced after the problem is developed along these lines as far as possible. The details of the development of the heat efficiency are given in the Appendix, while the main features of the model are discussed in the next section. Results and their implications are discussed later in a separate section.

The Model

Certain assumptions have been made to determine

For any rate of heat injection into a reservoir, the fraction of the heat remaining in it is independent of the recovery process, be it steam, hot water, or combustion.

the heat content of a pay zone resulting from injection of a hot fluid into it and heat losses from it into the adjacent formation.

1. The Lauwerier assumption is made: There is perfect vertical heat transfer within the pay zone. This is by far the most critical assumption made in this study.

2. We assume that the reservoir is operated in such a way that essentially all the lost heat is in the formations above and below the pay zone. This would appear to be a reasonable assumption since it is not likely that wells used for hot fluid injection will be located close to the lateral boundaries of an oil reservoir. From a heat balance point of view this means that the reservoir may be considered to be of infinite areal extent.

3. It is assumed that heat losses from the injection and production wells into the overburden (or vice versa) have no significant effect on the heat losses from the pay zone to the overburden, and that all the heat lost to the overburden from the pay zone stays in the overburden.

4. It is assumed that the volumetric heat capacities and vertical thermal conductivities are the same in both the cap rock and base rock. These thermal properties are assumed to have uniform and constant values (although no assumptions need be made about the horizontal thermal conductivity in the cap and base rock).

5. The pay zone is considered to be of uniform thickness and to have a constant and uniform volumetric heat capacity.

It is just as important to recognize items about which no assumptions need be made. No assumptions are made with regard to the horizontal heat transfer mechanisms in the pay zone or the adjacent formations. This heat transfer may be by convection, by conduction, or by both, and can vary with time and distance from the midplane of the pay zone. Natural groundwater motion in the cap rock close to the pay zone is thus possible. The horizontal thermal conductivities may be anisotropic and temperature dependent. Also, no assumptions are made about the type, compressibility, distribution, injection rate, temperature or injection profile of the injected fluids, or about the distribution of the fluids initially within the pay zone.

Heat is injected into the formation through any number of wells, located arbitrarily within the field, as long as not too many of them are located very close to the lateral boundaries of the reservoir. Heat may also be withdrawn via produced fluids. The net heat injection rate into the reservoir, $\dot{Q}(t)$, is the difference between the sum of the rates of heat injection into each injection well in the reservoir minus the sum of the rates of heat production from each production well in the reservoir. This net heat injection rate is always referred to the initial reservoir temperature, and does *not* include heat losses to the formations overlying the pay zone from the injection and production wells. It is the net rate of heat delivered to the pay zone. With this as a definition, it is then clear that heat can also be generated within the formation itself without any loss in generality. That is, the model also applies to underground combustion processes, be they dry, wet, or partially quenched.[6]

In developing the results it has been found useful to consider two types of regions within the pay zone, each having uniform and constant volumetric heat capacities. These two types of regions can be associated, for example, with the burned and nonburned volumes of the pay zone in an underground combustion operation, or with the steam zone and oil zone in a steam injection operation. The regions around the injection wells (called the near regions) need not have the same shape or size.

A schematic diagram of the reservoir under consideration is shown in Fig. 1. Details of how the heat efficiency is obtained are discussed in the Appendix. The results are discussed in the next section.

Results

The heat content of the pay zone as a function of time $H(t)$ due to a net heat injection rate $\dot{Q}(t)$ is given (see Appendix) by

$$H(t) = \int_0^t \dot{Q}(t')\, K(\theta_n \sqrt{t - t'})dt' - F \int_0^t H_f(t') \frac{d}{dt} K(\theta_n \sqrt{t - t'})dt' , \quad (1)$$

where

$$K(z) = e^{z^2} \operatorname{erfc} z , \quad (2)$$

$$\theta_n = \frac{2k_{h2z}}{h\sqrt{\alpha_2}(\rho C)_n} , \quad (3)$$

$$F = \frac{(\rho C)_f - (\rho C)_n}{(\rho C)_f} , \quad (4)$$

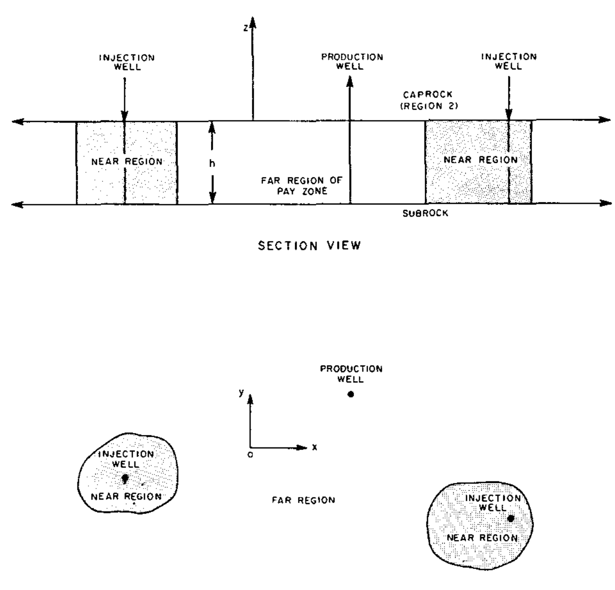

Fig. 1—Schematic cross-section of reservoir.

and

$$H_f(t) = (\rho C)_f \int\int\int_{V_f} T(x, y, z, t)\, dx dy dz \quad \ldots\ldots\ldots \quad (5)$$

Subscripts n and f are used to designate the properties of the near and far regions discussed in the previous section and shown schematically in Fig. 1. $H_f(t)$ is the heat content in the far region of volume $V_f(t)$.

Eq. 1 is equally applicable to hot water injection, to steam injection, or to underground combustion, and reduces to published results for certain cases.

Hot Water Injection

For hot water injection we consider the reservoir to be liquid filled, and it is customary to assume that the effective volumetric specific heat of the pay zone is independent of the liquid saturation distribution. In such a case there is no distinction between the near and far regions. Then $(\rho C)_n = (\rho C)_f = (\rho C)_1$, $F \equiv 0$, $\theta_n = \theta_1$, and the heat content of the pay zone resulting from hot water injection at an arbitrary time rate of heat injection is

$$H(t) = \int_0^t \dot{Q}(t')\, K(\theta_1\sqrt{t - t'})\, dt', \quad . \quad . \quad (6)$$

where the reservoir constant θ_1 is determined using the value of the effective volumetric specific heat corresponding to the liquid-filled pay zone, given by

$$(\rho C)_1 = \rho_r C_r (1 - \phi) + \phi[(1 - S_w)\rho_o C_o + S_w \rho_w C_w] \quad \ldots\ldots \quad (7)$$

The quantity $(\rho C)_1$ is fairly insensitive to variations in the liquid saturations.

For an arbitrary net rate of heat injection the heat efficiency for this process, or fraction of the injected heat present in the reservoir formation, is given by

$$E_h(\theta_1^2 t) = \frac{\int_0^{\theta_1^2 t} \dot{Q}(\tau)\, K(\sqrt{\theta_1^2 t - \tau})\, d\tau}{\int_0^{\theta_1^2 t} \dot{Q}(\tau)\, d\tau}, \quad . \quad . \quad (8)$$

which is identical with that given by Antimirov,[7] but only for incompressible hot fluid injection at constant rate of injection, and for linear and radial flow. To obtain this expression Antimirov also made the Lauwerier assumption, but in addition he develops expressions for the heat efficiency that consider vertical temperature variations.

We find that for constant heat injection rate, not necessarily at constant temperature or total mass rate of injection,

$$E_h(\theta_1^2 t) = \frac{1}{\theta_1^2 t}\left[\frac{2\theta_1^2\sqrt{t}}{\sqrt{\pi}} - 1 + e^{\theta_1^2 t}\operatorname{erfc}(\theta_1^2\sqrt{t})\right] \quad \ldots\ldots\ldots \quad (9)$$

This is identical with the heat efficiency obtainable from Lauwerier's work,[1] which is based on a constant injection rate and temperature. Results of Eq. 9 are plotted on Fig. 2.

Steam Injection

Eq. 1 is valid for steam injection as it stands, with the understanding that the near region corresponds to the steam zone. Mandl and Volek[8] have shown that during the early life of a steam drive, essentially all the heat in the reservoir is in the steam zone. Sometimes, as Flock *et al.*[3] have done, the heat content outside the steam zone is neglected. In either case $H_f(t)$ vanishes, and Eqs. 6 through 9 are applicable to the steam drive process provided that the reservoir constant $\theta_n = \theta_{st}$ is now determined using the value of the effective volumetric specific heat of the steam zone, given by

$$(\rho C)_{st} = (1 - \phi)\rho_r C_r + \phi\left(S_o \rho_o C_o + S_w \rho_w C_w + S_{st}\rho_{st} C_w + S_{st}\frac{\rho_{st} L_V}{T_{st}}\right). \quad . \quad (10)$$

The last term, which is the contribution to the effective specific heat due to the latent heat in the steam zone, can be shown to contribute little to the value of $(\rho C)_{st}$. It follows that the heat efficiency of the hot water process is of the same form as that for steam injection when there is no heat stored in the pay outside the steam zones. This is true for any arbitrary net rate of heat injection as a function of time, and results in a heat efficiency of the form given by Eqs. 8 and 9. It should be noted that although the *form* of Eqs. 8 and 9 is the same, the value of θ is defined in terms of different quantities. For hot water, we use Eq. 7; for steam we use Eq. 10.

Of course the same results are also obtained when both the steam region and the region beyond it have (or may be considered to have) the same value for the effective volumetric specific heats, even when there is heat stored in the pay zone outside the steam bank. Then $F = 0$, and the last term on the right-hand side of Eq. 1 vanishes. Thus Eqs. 8 and 9 are also applicable to steam injection processes when heat is stored outside the steam bank, provided that the effective volumetric specific heat of the pay zone within and outside the steam zone can be represented by the same constant value. This would be the case when differences in the value of θ calculated using Eqs. 7 and 10 are small.

Marx and Langenheim[9] and Ramey[10] have considered a constant effective volumetric specific heat throughout the formation. Eq. 9 has essentially been developed by Marx and Langenheim,[9] but only for the case of constant steam temperature. However, Eq. 9 is also valid for variable steam temperature, and for variable steam quality, as long as the net rate of heat injection is constant. Ramey[10] has developed Eq. 8, but also for steam injection at constant temperature. Again, Eq. 8 is valid for arbitrary steam injection temperature and quality.

Upper and Lower Bounds. It is well known that the effective volumetric specific heat of the steam zone calculated from Eq. 10 is smaller than that of a liquid-filled formation calculated from Eq. 7. That is, the parameter F is generally positive, and of the order of 0.2. Mandl and Volek[8] have shown that the heat content in the pay zone outside the steam region may be important, especially for large times. Although the amount of this heat cannot be determined very readily, it is possible to obtain lower and upper bounds for the heat content in the entire pay zone.

A lower limit for the heat content in the pay zone is readily obtained after we recognize that the second integral in Eq. 1 is negative. It is clear then that a lower limit to Eq. 1 is obtained by setting the contribution due to the last term (which is positive) to zero. This is essentially what has already been done to obtain Eqs. 8 and 9. Thus the minimum heat content in the pay zone and, consequently, the minimum heat efficiency, are given by Eqs. 6 and 8 when θ_{st} is used instead of θ_1.

To obtain the upper bound we resort to the alternative expression for the heat content in the pay zone developed in the Appendix:

$$H(t) = \int_0^t \dot{Q}(t')\, K(\theta_1\sqrt{t-t'})\,dt'$$

$$+ \frac{F}{\xi}\int_0^t H_{st}\,(t')\frac{d}{dt}K(\theta_1\sqrt{t-t'})\,dt' \, , \quad . \quad (11)$$

where H_{st} is the heat content of the steam zone. Since the contribution due to the last term in Eq. 11 is negative, the upper bound for the heat content in the pay zone is obtained by setting this term to zero.

Accordingly, we obtain

$$E_h\,(t) \leq \frac{\int_0^t \dot{Q}(t')\, K(\theta_1\sqrt{t-t'})\,dt'}{\int_0^t \dot{Q}(t')\,dt'} \quad . \quad . \quad (12)$$

Note that the maximum value of the heat efficiency for steam drives (Eq. 12) not only is of the same form but also is identical with that for hot waterflood (Eq. 8). For constant heat injection rate, the lower and upper bounds for the heat efficiency are

$$E_{h\,\min}\,(\theta_{st}{}^2 t) = \frac{1}{\theta_{st}{}^2 t}\left(\frac{2\sqrt{\theta_{st}{}^2 t}}{\sqrt{\pi}} - 1 + e^{\theta_{st}{}^2 t}\,\mathrm{erfc}\,\sqrt{\theta_{st}{}^2 t}\right) , \quad . \quad (13)$$

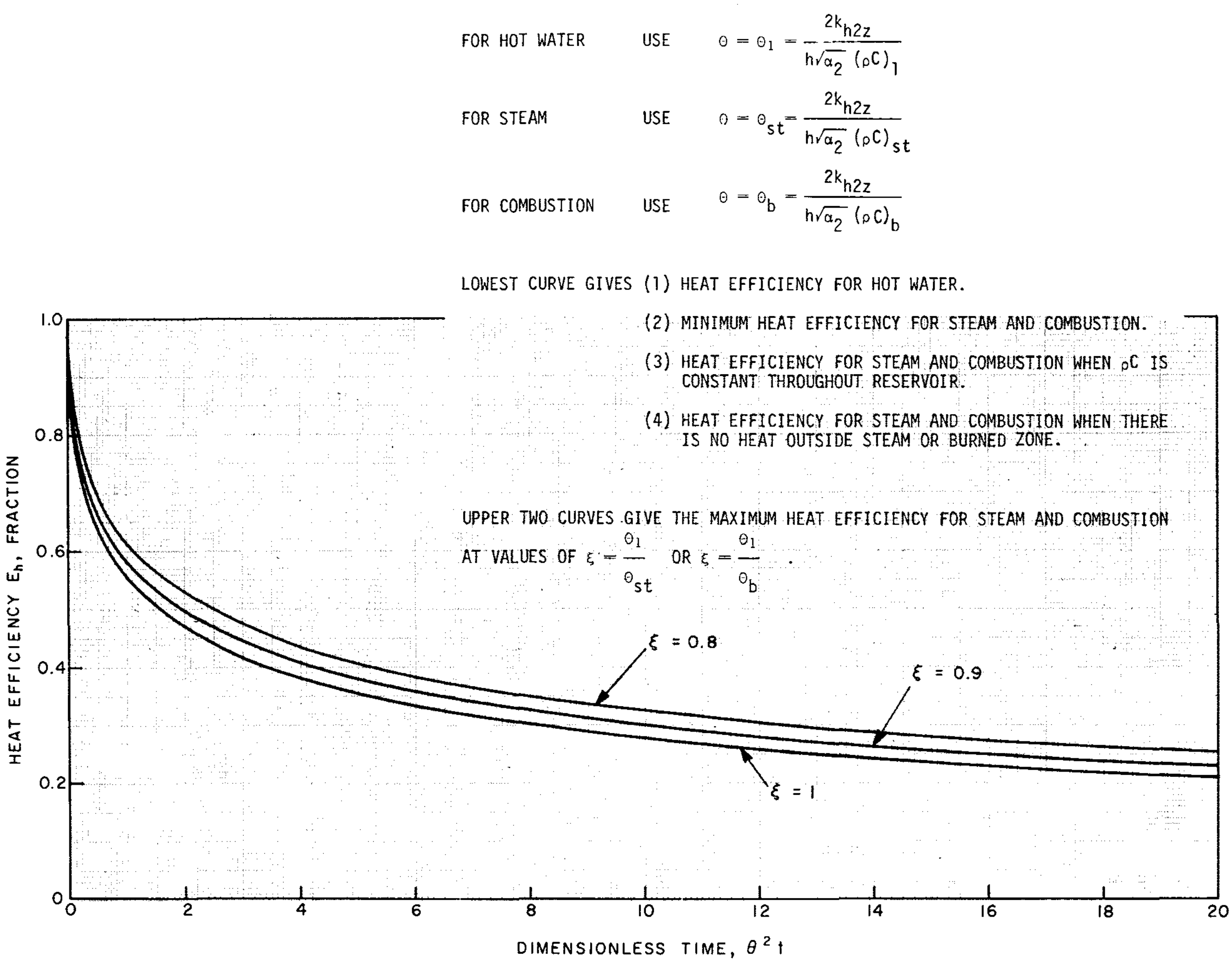

Fig. 2—Heat efficiency as a function of time for a constant net rate of heat injection.

and

$$E_{h\max}(\theta_{st}^2 t) = \frac{1}{\xi^2\theta_{st}^2 t}\left(\frac{2\xi\sqrt{\theta_{st}^2 t}}{\sqrt{\pi}} - 1 + e^{\xi^2\theta_{st}^2 t}\,\mathrm{erfc}\,\xi\sqrt{\theta_{st}^2 t}\right), \quad . \quad (14)$$

where

$$\xi = \frac{(\rho C)_{st}}{(\rho C)_1} = \frac{\theta_1}{\theta_{st}} = 1 - F. \quad . \quad (15)$$

Fig. 2 gives the upper bound for several values of ξ, and the lower bound for the heat efficiency at constant heat injection rate of the steam drive process. Fig. 2 also shows the heat efficiency of the hot water injection process.

Discussion

Significance of Results

For hot water injection the heat efficiency of the process has been found to be dependent only on the rate of heat injection (regardless of the temperature and location of the injection fluid, its injection profile into the pay zone, and its total volume rate of injection) and on the parameter θ_1. This parameter θ_1 depends on the thickness of the pay zone, on the vertical thermal conductivity and volumetric specific heat of the formation adjacent to the pay zone and on the effective volumetric specific heat of the liquid-filled pay zone. Specifically, and this is somewhat surprising, the parameter θ_1 is not dependent upon the horizontal thermal conductivity of the pay zone. More generally, the heat efficiency of the hot water injection process is independent of the horizontal thermal conductivity of both pay zone and adjacent formation.

Of course this conclusion results from our use of the Lauwerier assumption: The temperature in the pay zone is independent of vertical position. This is equivalent to having an infinite vertical thermal conductivity in the pay zone. It is surprising, however, that the heat efficiency of the hot water injection process can be expressed so simply for otherwise so general a set of conditions. The Lauwerier assumption is really the only questionable one.

With regard to steam injection processes, the upper bound for the heat efficiency is exactly the heat efficiency for the hot water injection process. Thus we may conclude that for the same heat injection history and at the same cumulative heat injection, heating by hot water is always more efficient than heating by steam. This is not surprising since the effective volumetric heat capacity of the formation is highest when it is liquid filled.

A minimum bound for the heat efficiency of the steam injection process is of the same form as that for hot water, but with θ_{st} substituted for θ_1. The difference between the maximum and minimum bounds increases as the ratio θ_1/θ_{st} decreases. For reasonable values of θ_1/θ_{st} (say 0.8), and of time, the percent spread between the two limits is at most 15 percent of the mean. The minimum bound gives the right heat efficiency when there is no heat stored in the pay zone outside the steam zone. It should thus be applicable for early times. The work of Mandl and Volek[8] can be used to determine the time at which steam condensate will start to transport heat across the condensation front and into the region outside the steam zone. Conduction effects are more difficult to estimate, but the work of Avdonin[11] may be useful. In any event, the lower bound is a reasonable estimate for the heat efficiency of the steam injection process when the heat content in the pay zone outside the steam zone is small compared with the heat content inside it.

The actual heat efficiency of steam injection processes depends upon the heat injection rate, the parameters θ_1 and θ_{st}, and the volume of the steam zone. Although we have been unable to calculate the actual heat efficiency, the maximum and minimum bounds given in Fig. 2 may be useful in actual practice.

Another interesting point to note is that since the results are applicable for any manner of heat transfer into the reservoir (see Eq. A-1), they are also valid for the combustion process, be it dry, wet, or partially quenched. For the combustion process three regions would need to be defined: the burned zone, a steam zone, and the region downstream of the steam condensation zone. In some cases it is possible to ignore one of the regions. For example, for dry combustion the effect of steam is small and could easily be neglected, thus yielding a two-zone problem analogous to the two-zone steam injection case. For this two-region case, the results developed and discussed for steam injection are exactly applicable to the underground combustion process, with the understanding, of course, that we must use the value of the effective specific heat of the burned volume $(\rho C)_b$ instead of that for the steam zone. Thus, for example, the minimum heat efficiency for the dry underground combustion process is given by

$$E_{h\min}(\theta_b^2 t) = \frac{\int_0^{\theta_b^2 t} \dot{Q}(\tau)\, K\sqrt{\theta_b^2 t - \tau}\, d\tau}{\int_0^{\theta_b^2 t} \dot{Q}(\tau)\, d\tau}, \quad . \quad . \quad . \quad (16)$$

where θ_b is evaluated using the effective specific heat of the burned region

$$(\rho C)_b = (1-\phi)\rho_r C_r \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Results for constant net rate of heat input are shown in Fig. 2, along with similar results for hot water and steam injection processes. Similar reductions are also possible for the wet and quenched combustion processes. In this manner, the heat efficiency of the combustion processes may be estimated from the results discussed above in connection with hot fluid injection methods. Of course the parameter θ_b should reflect conditions appropriate to the particular combustion process in mind. For example, the contribution due to water in the burned zone should be included in Eq. 17 for the wet and partially quenched combustion processes.

As already pointed out by Ramey,[12] Antimirov,[7] and Baker,[13] the heat efficiency of the steam and hot water injection processes are idential for linear and radial flow geometries if the effective volumetric specific heat is constant and is the same for both processes, and when the rate of heat injection is constant with time. Our own results show that the heat efficiency is independent of the flow geometry and of the thermal recovery process, be it steam, hot water, or any type of underground combustion, when the effective volumetric specific heat is constant (or can be considered to be constant) throughout the reservoir and is the same for all processes. This is true for any net rate of heat injection into the reservoir, and is independent of time and space variations in mass and volume injection rates, and in injection temperatures.

Corrections for Vertical Temperature Variations in the Pay Zone

In a crude way, it is possible to compensate for the error introduced by the Lauwerier assumption, especially where gravity segregation effects are not too important. In the case of frontal drives we know that because of heat losses the temperature at the pay zone-overburden contact is no greater than the temperature in the pay zone averaged over its height. The Lauwerier assumption could be generalized to state that the temperature at the pay zone-overburden contact be a certain fraction f of the temperature in the pay zone averaged over its height. When this fraction f is constant in time and space, one can derive heat efficiencies that are again of the same form as those given in the text. The only effect on these expressions is that now $f\theta$ appears wherever θ appeared before. Since f is smaller than 1 for frontal drives, it can be seen from Fig. 2 that the Lauwerier assumption will always give a low value for the heat efficiency. When f is dependent upon space and time, its space and time average will again be less than unity (even for steam injection) so that this conclusion is generally valid for frontal drives. This conclusion is in agreement with the results of Spillette,[2] who has shown that for some specific hot water injection conditions the heat efficiency resulting from the Lauwerier assumption is low, but only by about 15 percent. If such small differences are typical, then it is expected that estimating the parameter f could improve engineering evaluations of thermal projects to only a small extent.

The Lauwerier assumption ($f = 1.0$) leads to somewhat low values of the heat efficiency for steam injection. Corresponding curves can be obtained for other values of f as indicated in the preceding paragraph, but since effective values of f can only be estimated even when temperature profiles are available from field observations, nothing can be said as to whether heat efficiencies estimated in such a way are true lower or upper bounds.

A cruder technique may have some application when gravity segregation plays an important role and heat losses above and below the pay zone differ widely. In extreme cases of thick pay zones and very pronounced segregation, heat losses may essentially be limited either to the overburden (for steam and underground combustion) or to the subrock (for hot water), although the latter extreme case is not likely to occur frequently. In such extreme cases an upper bound to the heat efficiency may be obtained by reducing the cumulative heat losses for the symmetrical case by a half. For example, if the heat efficiency for two-sided heat losses is 0.40, the upper bound for the heat efficiency would be 0.70 when losses are one-sided.

Other Applications

The results are also applicable to heat recuperation schemes applied to hot water, steam, and underground combustion processes. For hot water injection followed by cold water injection the heat efficiency results presented in the report are exact. As can be seen from Eq. 6, stopping heat injection is equivalent to stopping all injection into the reservoir, in which case the heat content of the reservoir can only decrease with time. Thus the heat efficiency of a frontal hot water drive can never be increased by switching to cold water injection. For steam injection, results are only approximate since the effective volumetric specific heat near the injection well is increased slightly upon injection of cold water. In any event, the main effect of cold fluid injection as a heat recuperation process is to change the distribution of heat within the injection interval. Its main advantage is that it provides a deeper penetration of a heated zone for a given amount of heat injected than would be possible by a constant injection of heat.

From the results presented in this study it is clear that widely different horizontal temperature distributions within the injection interval may result in the same heat efficiency. On the other hand, it seems reasonable to suppose that the relative distribution of heat and oil within the formation, rather than the heat efficiency itself, is the main factor influencing the oil production rate of a thermal recovery operation. Thus, although the heat efficiency of a thermal recovery process as defined in the literature often provides a measure of the attractiveness of the thermal recovery operation, it is recognized that the temperature distribution itself plays a significant role in determining the total heat requirements, the rate of oil recovery and the amount of oil produced per unit of thermal energy used. This has been demonstrated by Dietz and Weijdema[6] for the case where water, as well as air, is injected in combustion processes in order to transfer more heat downstream of the combustion front, where the oil is.

The results presented here are independent of the well pattern used in the field. They apply not only to linear and radial flow systems, but also to any other flow geometry. The flow need not be steady. We should keep in mind that when heat is withdrawn from the reservoir via produced fluids, we still can find the heat content of the reservoir from the heat efficiency results by subtracting this produced heat from that which is injected into or generated within the formation.

Summary

To summarize the results of this report it is desirable to define a few terms in order to avoid repetition of long descriptions.

Heat Efficiency — the fraction of the injected heat remaining in the injection interval.

General Conditions — the injection temperature, injection rates, and injection profiles are arbitrary functions of time. The vertical heat conductivity in the pay zone is infinite (the Lauwerier assumption), but otherwise the fluid flow distribution and the heat transfer mechanisms within the injection interval are quite arbitrary. Specifically, horizontal heat conduction is considered in both the injection interval and adjacent formations. An arbitrary well pattern and flow geometry is considered.

Restrictive Assumptions — the injection profile and the injection rate and/or the injection temperature are constant with time. The fluid flow within the injection interval is usually either linear or radial and independent of vertical position. The vertical heat conductivity in the pay zone is infinite (the Lauwerier assumption), and heat transfer in the pay zone is usually by horizontal convection only.

Unless otherwise restricted, the following comments are valid for the general conditions. Please refer to the text for additional explanations and discussion.

1. Hot water injection has the highest thermal efficiency of all the presently employed thermal recovery processes.
2. There is no difference between the heat efficiency obtained for the hot water drive under the general conditions of this report and that obtainable from Lauwerier's work (which is based on more restrictive assumptions).
3. The minimum heat efficiency for a steam drive obtained under general conditions is exactly that obtainable from Marx and Langenheim,[9] although the latter developed their results under relatively restrictive assumptions.
4. When the effective volumetric specific heat is constant throughout a reservoir, the heat efficiency of all thermal recovery processes is of the same *form.* The heat efficiency of every thermal recovery process is *identical* with that of every other if the effective volumetric specific heat of the formation is independent of the recovery process.
5. The upper bound for the heat efficiency of a steam drive is dependent only upon the ratio of the effective heat capacity of the steam zone to that of its downstream region, on the variation of the heat injection rate, and on a dimensionless time. For a constant rate of heat injection, the heat efficiency is independent of the value of the rate of heat injection, and, for practical cases, the difference between upper and lower bounds is then not greater than 10 percent of their mean.
6. The heat efficiency of any process is independent of the horizontal thermal conductivity in the injection interval and adjacent formations.
7. Methods are suggested for correcting heat efficiencies for vertical temperature variations within the pay zone, that is, when the Lauwerier assumption is a poor one.
8. For any thermal recovery process, the Lauwerier assumption always yields low values of calculated heat efficiencies.
9. Generally speaking, vertical temperature asymmetry introduced by gravity effects results in a higher heat efficiency than would be indicated by the Lauwerier assumption.
10. Heat recuperation from the overburden by cold water injection cannot increase the heat efficiency of hot water drives. It merely results in a redistribution of heat, which may be beneficial in improving the oil production rate or the heat requirement.
11. For the general conditions considered in this report, the minimum heat efficiencies of the dry, wet, and partially quenched combustion processes are of the same form as that obtainable from Marx and Langenheim[9] for the steam drive (and for more restrictive assumptions).

Nomenclature

$F = [(\rho C)_f - (\rho C)_n]/(\rho C)_f$
h = height of the pay zone
$H(t)$ = heat stored in the pay zone, Btu
k_h = thermal conductivity, Btu/ft day °F
$K(z) = e^{z^2} \operatorname{erfc} z$
$\dot{Q}(t)$ = time rate of net heat injection, Btu/D
s = Laplace transform variable for time t, day^{-1}
t = time since heat injection started, days
T = temperature of pay zone, °F, above initial reservoir temperature
u_h = heat flux, Btu/sq ft day
V = volume, cu ft
x, y, z = Cartesian coordinates, ft
$\alpha_2 = k_{h2z}/\rho_2 C_2$, thermal diffusivity of cap rock, sq ft/D
$\theta_1 = 2k_{h2z}/[h\sqrt{\alpha_2}\,(\rho C)_1]$
$\theta' = 2k_{h2z}/h\sqrt{\alpha_2}$
$\xi = (\rho C)_n/(\rho C)_f$, for example $(\rho C)_{st}/(\rho C)_1$ or $(\rho C)_b/(\rho C)_1$
(ρC) = effective volumetric specific heat, Btu/cu ft °F

Subscripts and Superscripts

b = properties of the burned zone
max = maximum
min = minimum
n, f = properties of the regions in the pay zone, near to and far from the injection well(s)
o = oil
r = rock
R = reservoir
st = properties of the steam zone or of the steam
w = water
x, y, z = component directions

1 = properties of the liquid-filled part of the pay zone

2 = properties of the formations adjacent to the pay zone

* = Laplace transform

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APPENDIX

Heat Stored in the Reservoir

The net rate of heat input $\dot{Q}(t)$ into a reservoir is equal to the sum of the time rate of change of the heat content of the reservoir and the time rate of change of the total heat losses to cap and base rock. This is expressed as

$$\dot{Q}(t) = \frac{dH(t)}{dt} + 2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u_{hz}(x, y, o, t)\,dxdy\,, \quad \text{(A-1)}$$

when the rate of heat losses to cap and base rock are the same and the reservoir is of uniform thickness and of large areal extent. This expression is valid for any number of wells in the reservoir, and drilled anywhere, provided that $\dot{Q}(t)$ is interpreted as the net rate of heat input into the reservoir itself rather than at the wellhead. That is, Eq. A-1 does not account for any heat losses from the wells to the formation; these would have to be considered separately. Furthermore, since $\dot{Q}(t)$ is the *net* rate of heat input into the reservoir, it is equal to the actual rate of heat injection into the reservoir during a hot fluid injection process reduced by the rate of heat removal from the reservoir by produced fluids. Since Eq. A-1 is a bulk enthalpy balance it is also valid for moving heat sources. It is thus applicable to underground combustion processes.

In underground combustion processes, the quantity $\dot{Q}(t)$ is the rate of heat generation within the reservoir reduced by the rate of heat removal by the produced fluids. In the early phases of a thermal recovery project the rate of heat removal via produced fluids is zero, so that at early times $\dot{Q}(t)$ is just the rate of heat input into the reservoir. In developing Eq. A-1 it is tacitly assumed that the lateral heat losses from the reservoir are negligible compared with those into cap and base rock. This will generally be the case. It should be noted that Eq. A-1 is completely general in the sense that it is valid for any manner of heat transfer into and within the injection interval and for any manner of heat input, injection fluid, injection rate, and injection temperature. It should be noted that the distribution of the net rate of heat input over the various wells in the pay zone is immaterial insofar as the heat balance given by Eq. A-1 is concerned.

In the overburden, we shall assume that the thermal properties are uniform and independent of temperature, and that the vertical heat transfer is by conduction only. Horizontal heat transfer is arbitrary, although in most cases it will also be by conduction only. Accordingly, the heat balance in the overburden can be written as

$$\frac{1}{k_{h2z}}\left(\frac{\partial u_{hx}}{\partial x} + \frac{\partial u_{hy}}{\partial y}\right) + \frac{\partial^2 T_2}{\partial z^2} = \frac{1}{\alpha_2}\frac{\partial T_2}{\partial t}\,. \quad \text{(A-2)}$$

We will assume that the rate of heat transfer from the overburden to the wells is zero. Then we can define a new variable

$$\overline{T}_2(z, t) = \frac{1}{h^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} T_2(x, y, z, t)\,dxdy \quad \text{(A-3)}$$

Introducing Eq. A-3 into Eq. A-2 leads to

$$\frac{\partial^2 \overline{T}_2(z, t)}{\partial z^2} = \frac{1}{\alpha_2}\frac{\partial \overline{T}_2(z, t)}{\partial t} \quad \text{(A-4)}$$

since the horizontal heat fluxes at infinity vanish and those at wells are assumed to be negligible in our model. No assumption whatsoever is made about the horizontal heat transfer mechanism in the overburden. In fact, it is quite general. Since $T_2(x, y, z, t)$ is defined as the temperature in the overburden above an initial steady-state temperature, such as that due to a geothermal gradient, it follows that the Laplace transform of Eq. A-4 is

$$\frac{d^2 \overline{T}_2^* (z, s)}{dz^2} = \frac{s}{\alpha_2} \overline{T}_2^* (z, s) \quad . \quad . \quad . \quad . \quad \text{(A-5)}$$

The solution of Eq. A-5 that vanishes at $+\infty$ is

$$\overline{T}_2^* (z, s) = \overline{T}_2^* (o, s) \exp \left(- \sqrt{\frac{s}{\alpha_2}} \, z \right) . \quad \text{(A-6)}$$

Since the rate of heat losses from pay zone to overburden is by conduction only, it follows that

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u_h{}^*{}_z (x, y, o, s) \, dxdy =$$

$$-k_{h2z} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\partial T_2^*}{\partial z} (x, y, o, s) \, dxdy \, , \quad . \quad \text{(A-7)}$$

and substitution of the results from Eq. A-6 into the right-hand side of Eq. A-7 yields

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u_h{}^*{}_z (x, y, o, s) \, dxdy =$$

$$k_{h2z} \sqrt{\frac{s}{\alpha_2}} \, \overline{T}_2^* (o, s) \, h^2 \quad . \quad . \quad . \quad . \quad . \quad \text{(A-8)}$$

Now we can substitute the last expression into Eq. A-1, after taking the Laplace transform of the latter, to obtain

$$\dot{Q}^*(s) = sH^*(s) + h^3 \, \theta' \sqrt{s} \, \overline{T}_2^* (o, s) \, , \quad . \quad \text{(A-9)}$$

where

$$\theta' = \frac{2k_{h2z}}{h\sqrt{\alpha_2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(A-10)}$$

Now we use the condition that the temperature in the overburden is continuous with the temperature in the pay zone, the latter being assumed independent of vertical position:

$$T_2^* (x, y, o, s) = \frac{2}{h} \int_{-\frac{h}{2}}^{0} T^* (x, y, z, s) \, dz \, . \quad \text{(A-11)}$$

This is the key assumption introduced by Lauwerier[1] and used frequently since then. It then follows that

$$\overline{T}_2^* (o, s) = \frac{2}{h^3} \int_{-\frac{h}{2}}^{0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} T^* (x, y, z, s) \, dxdydz \, . \quad \text{(A-12)}$$

By definition, the heat content of the pay zone is

$$H(t) = 2 \int_{-\frac{h}{2}}^{0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho_R \, C_R \, T \, (x, y, z, t) \, dxdydz \, . \quad \text{(A-13)}$$

Thus, for the case where the volumetric heat capacity of the reservoir may be considered to be constant throughout the reservoir, Eqs. A-9, A-12 and A-13 can be combined to give

$$\dot{Q}^*(s) = (s + \theta\sqrt{s}) \, H^*(s) \quad , \quad . \quad . \quad . \quad \text{(A-14)}$$

where

$$\theta = \frac{2k_{h2z}}{h\sqrt{\alpha_2} \, \rho_R \, C_R} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(A-15)}$$

We also wish to consider the case where the reservoir is divided into two types of regions, each having a uniform value of $\rho_R \, C_R$, which are constant in time. All we need to say at the moment about these two types of regions, which can change in shape and size with time, is that one type is near the injection well(s). Thus the center of volume of the second type must not be near the injection well(s). The properties of the regions near to and far from the injection well(s) will be designated by the subscripts n and f, respectively. Later on we may associate the boundaries between these two types of regions as combustion or steam condensation surfaces. In a more general sense, then, the heat stored in the formation can be written as

$$H(t) = H_n \, (t) + H_f \, (t) \quad , \quad . \quad . \quad . \quad \text{(A-16)}$$

where $H_n \, (t)$ is the total heat stored in the pay zone near the injection well(s), and similarly for $H_f \, (t)$. It is not necessary that the regions near the injection well(s) be connected to each other, or even that they all have the same shape and size. This last expression can also be written as

$$H(t) = 2(\rho C)_n \iiint_{V_n} T(x, y, z, t) dxdydz + H_f \, (t) \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(A-17)}$$

Since the sum of the volumes of the near and far regions is equal to the entire volume of the pay zone, we can write

$$H(t) = 2(\rho C)_n \int_{-\frac{h}{2}}^{0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} T(x, y, z, t) dxdydz - 2(\rho C)_n \iiint_{V_f} T(x, y, z, t) dxdydz + H_f \, (t) \quad . \quad . \quad . \quad \text{(A-18)}$$

$$= 2(\rho C)_n \int_{-\frac{h}{2}}^{0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} T(x, y, z, t) dxdydz + FH_f \, (t) \quad , \quad . \quad . \quad . \quad \text{(A-19)}$$

where

$$F = \frac{(\rho C)_f - (\rho C)_n}{(\rho C)_f} \quad , \quad . \quad . \quad . \quad . \quad \text{(A-20)}$$

and

$$H_f(t) = 2(\rho C)_f \iiint_{V_f} T(x, y, z, t)\,dxdydz. \quad \text{(A-21)}$$

Using Eq. A-12, Eq. A-19 reduces to

$$H(t) = 2(\rho C)_n h^3 \overline{T}_2(o, t) + FH_f(t). \quad \text{(A-22)}$$

Alternatively, one could have written Eq. A-17 as

$$H(t) = 2(\rho C)_f \iiint_{V_f} T(x, y, z, t)\,dxdydz + H_n(t), \quad \text{(A-23)}$$

to obtain

$$H(t) = 2(\rho C)_f h^3 \overline{T}_2(o, t) - \frac{F}{\xi} H_n(t), \quad \text{(A-24)}$$

where

$$\frac{F}{\xi} = \frac{(\rho C)_f - (\rho C)_n}{(\rho C)_n}. \quad \text{(A-25)}$$

Application of the Laplace transform to Eqs. A-22 and A-24, and elimination of $\overline{T}_2{}^*(o, s)$ from Eq. A-9 give the two equivalent expressions

$$\dot{Q}^*(s) = (s + \theta_n\sqrt{s})\, H^*(s) - F\theta_n\sqrt{s}\, H_f{}^*(s), \quad \text{(A-26)}$$

and

$$\dot{Q}^*(s) = (s + \theta_f\sqrt{s})\, H^*(s) + \frac{F}{\xi}\theta_f\sqrt{s}\, H_n{}^*(s), \quad \text{(A-27)}$$

where

$$\theta_n = \frac{2k_{h2z}}{h\sqrt{\alpha_2}\,(\rho C)_n}. \quad \text{(A-28)}$$

$$\theta_f = \frac{2k_{h2z}}{h\sqrt{\alpha_2}\,(\rho C)_f}. \quad \text{(A-29)}$$

The solutions to Eqs. A-18 and A-19, which are entirely equivalent, are

$$H(t) = \int_0^t \dot{Q}(t')\, K(\theta_n\sqrt{t - t'})\,dt' - F\int_0^t H_f(t')\frac{d}{dt}K(\theta_n\sqrt{t - t'})\,dt' \quad \text{(A-30)}$$

and

$$H(t) = \int_0^t \dot{Q}(t')\, K(\theta_f\sqrt{t - t'})\,dt' + \frac{F}{\xi}\int_0^t H_n(t')\frac{d}{dt}K(\theta_f\sqrt{t - t'})\,dt'. \quad \text{A-31)}$$

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