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EQUATIONS OF MOTION FOR A LINEAR MISCIBLE DISPLACEMENT

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ABSTRACT

Equations for the motion of miscible fluids are considered which are based on a single hypothesis, namely, the fractional flow of the invading fluid depends in a known way on its saturation and saturation gradient. They are similar in form to the equations of the Buckley-Leverett theory.

The differential equation satisfied by the fractional flow, considered as a function of the saturation and the time, is derived. A similar equation may also be obtained in the Buckley-Leverett theory for immiscible displacements. This equation, because of its simple form, is susceptible to well-known numerical methods and makes the computation of the history of a displacement a reasonable project.

A comparison of the implications of the equations with some of the published data is made. While it seems possible to reconcile some of the apparently incompatible interpretations which have been given on the ground that the techniques employed affect the nature of the motions, several differences still remain.

For a horizontal drive, in which gravity segregation enters, it is shown that a motion may exist in which the planes on which the saturation of the invading fluid is constant are inclined at a constant angle with the direction of motion.

References and illustrations at end of paper

INTRODUCTION

Many investigations have been made into the behavior of a fluid while being displaced from a porous medium by a miscible fluid.[1-7] On only one occasion,[4] however, have equations been suggested as descriptive of the motion. For this reason an account is given here of the consequences which follow from some simple hypotheses about the movement of miscible fluids. The equations are similar in form to those of the Buckley-Leverett theory for immiscible displacements and differ from them only in detail. Consequently, the method outlined here for discussing them is equally applicable to the Buckley-Leverett theory and provides a simple way of explaining and computing the formation or nonformation of a stabilized zone in that theory due to the interaction of gravity and capillary forces.

The interpretation ascribed to the many experiments on miscible flooding does not, at first sight, present a consistent picture. The dependence of the length of the mixing zone on rate, viscosity and pore structure is explained in a different way by the various authors. With this in mind, a general type of equation is examined here and a comparison with some of the available data is made. In this way it may be possible to reconcile apparently divergent opinions.

We suppose that the displacement takes place in a vertical column with fluid injected at the top and removed from the bottom. The porous medium is homogeneous and isotropic and bounded by impermeable walls so that the motion is as unidirectional as possible. The rate of displacement is maintained constant.

It is imagined that the structure of the porous medium produces a mixing action so the particles originally lying in the same cross section occupy different cross sections at a later time. This mixing is distinct from that caused by molecular diffusion in the sense that it occurs only when the fluid is in motion. Thus, if we start with the two miscible fluids separated by a horizontal plane we find them, later, mixed in a certain region. In a cross section in this region some areas are occupied by the one fluid and some by the other fluid while due to molecular diffusion the boundary between these areas is somewhat diffuse.

In some experimental procedures, the fluids are passed through a series of cores, or cycled repeatedly through the same core. The effect of this cycling is to introduce additional mixing between the fluids in different parts of the same cross section. Hence, the degree of mixing within a cross section depends, somewhat, on the procedure followed. We might imagine the fluids uniformly mixed as one extreme or completely separate like immiscible fluids as the other extreme.

In this article, a particular degree of mixing is not assumed. Instead we consider the flux of the invading fluid across a cross section. About it we make a broad hypotheses: the fractional flow of invading fluid crossing any cross section is determined by the saturation of the invading fluid in the cross section and its gradient. This, together with the equations of material balance, determines the motion. Darcy's equations are not used explicitly, because it is assumed that the external pressure is automatically adjusted to maintain a constant total volume flow.

The particular hypothesis made is frankly an adaptation of the fractional flow equation of the Buckley-Leverett theory. The suggestion advanced here is that an equation of this type may account for the facts and that the method of dealing with it makes its implications susceptible to numerical computation.

DEFINITIONS AND HYPOTHESES

In a thin slice between two neighboring cross sections let the fraction of the fluid volume which is occupied by particles of the displacing fluid be denoted by S. This is called the saturation and lies between zero and unity.

The total flow rate is denoted by U and measures the total volume of fluid per unit area per unit time which crosses any cross section. The fraction of this flow which consists of the invading phases is denoted by $\underline{f}$.

The distance from the inlet to a cross section is denoted by $\underline{x}$ and the time by $\underline{t}$. Then S and $\underline{f}$ may be regarded as functions of $\underline{x}$ and $\underline{t}$.

Assuming that the invading phase is incompressible we obtain the equation of material balance

$$U \frac{\partial f}{\partial x} + \phi \frac{\partial S}{\partial t} = 0 \qquad \text{Eq. 1}$$

where ϕ denotes the porosity.

Assuming that the displaced phase is also incompressible it follows that the combined fluids are incompressible. Hence, the total flow rate must be the same at all cross sections and U depends, at most, on $\underline{t}$. We suppose that the external pressure is adjusted so as to maintain U constant.

The fundamental hypothesis which we make is contained in the equation

$$f = A[S] - B[S] \frac{\partial S}{\partial x} \qquad \text{Eq. 2}$$

where A[S] and B[S] are considered as known functions of S. This expresses in a simple way, the idea that the saturation and saturation gradient determine what the fractional flow must be. It is analogous to the fractional flow equation of the Buckley-Leverett theory.

The function A[S] depends on how uniform we imagine the distribution of fluid within any cross section to be. If the fluids are completely mixed then A[S] is equal to S. If the fluids are completely separated then A[S] may resemble the corresponding function in the Buckley-Leverett theory and depend on the viscosities of the fluids. In any case A[S] lies between zero and unity for all S.

The function B[S] represents the mixing caused by the size and interconnection of pores. When S vanishes the fractional flow must also vanish, whatever the saturation gradient may be, and hence B[0] must be zero. Similarly B[1] must equal zero. For other values of S, B[S] is supposed positive. It has the dimensions of a length and by its shape distinguishes one porous strucure from another.

Eq. 1 and 2 are sufficient to determine the motion. The problem is to solve them in a practical manner no matter what functions may be taken for A and B.

METHOD OF SOLUTION

The result of eliminating f between Eq. 1 and 2 is a cumbersome equation for the saturation, which is not easily managed. It proves convenient then to change from x and t as independent variables to S and t. Performing this interchange, Eq. 1 becomes

$$\frac{\partial x}{\partial t} = \frac{U}{\phi}\frac{\partial f}{\partial S} \qquad \text{Eq. 3}$$

By the same change, Eq. 2 may be written

$$\frac{\partial x}{\partial S} = -\frac{B}{f-A} \qquad \text{Eq. 4}$$

Between Eq. 3 and 4 we may eliminate x and find that f[S,t] satisfies

$$\frac{\partial f}{\partial t} = \frac{U[f-A]^2}{\phi B}\frac{\partial^2 f}{\partial S^2} \qquad \text{Eq. 5}$$

This is the final equation that we require. It is a nonlinear equation but is similar to the equation for heat conduction. In fact, the numerical methods of solution for the equation for heat conduction, may be applied here with success.

To this equation we must add two boundary conditions and one initial condition before a unique solution is determined. Restricting consideration to the motion before breakthrough we take

$$\left.\begin{aligned} f &= 0, \text{ when } S = 0 \\ f &= 1, \text{ when } S = 1 \end{aligned}\right] \text{ for all t.} \qquad \text{Eq. 6}$$

The initial condition is that f[S,0] is a known function. As we shall see, the solution of Eq. 5 subject to condition Eq. 6, is easily visualized and may be computed with reasonable accuracy on a desk calculator. Having determined f at any instant the corresponding saturation profile may be calculated from Eq. 2 by a single integration.

NATURE OF THE SOLUTION

Fig. 1 represents the saturation profile along the core at the initial instant. From it we compute the saturation gradient for any value of the saturation. Hence, using Eq. 2, we know the value of f for all S initially. This is represented in Fig. 2 by the solid curve.

From this initial shape the f - curve, in Fig. 2, develops as determined by Eq. 5 and conditions Eq. 6. The boundary conditions imply that its end points are fixed.

From the initial curvature of the f - curve we see that $\partial^2 f/\partial S^2$ is negative for all values of S. Hence, the value of $\partial f/\partial t$, as given by Eq. 5, is negative for all S and f decreases. Thus, each point on the f - curve moves downwards except the end points which are fixed. The general effect, therefore, of Eq. 5 is to straighten the f - curve.

A steady state is reached ultimately when $\partial f/\partial t$ approaches zero. From Eq. 5 we see that this happens either when f approaches A or when $\partial^2 f/\partial S^2$ approaches zero. Which actually happens depends on the function A.

Two possibilities are illustrated in Fig. 2. If A is given by the A_1 - curve then clearly f approaches A before $\partial^2 f/\partial S^2$ approaches zero. From Eq. 2 we see that $\partial S/Sx$ approaches zero in this case. If, on the other hand, A is given by the A_2 - curve then clearly $\partial^2 f/\partial S^2$ approaches zero first. In this case f becomes equal to S ultimately, and by Eq. 2, the saturation profile has a certain shape. That is, the A_2 - curve represents conditions under which stabilization occurs; the A_1 - curve gives a motion in which the saturation profile continually flattens.

If the A - curve is partly concave upward and partly concave downward then stabilization occurs for only a limited range of S.

COMPARISON WITH EXPERIMENT

If we assume that A and B are independent of the total rate U, then the saturation profile depends only on the distance moved by the fluids and not on the rate. This point has been examined by experiment. In References 5 and 6 no effect due to rate is observed. In Reference 4 lower rates give less mixing whereas in Reference 7 it is noted that higher rates give less mixing. In Reference 1 it is reported that an optimum rate exists for least amount of mixing.

In Reference 6 it is reported that the mixing zone increases to a finite length; in reference 5 the mixing zone length increases indefinitely.

It is apparent that these divergent findings can be reconciled only by taking account of the different conditions of performance. Thus in Reference 6 cores are joined in series by "small tubing" to form a long system whereas in Reference 5 the fluid is cycled repeatedly through the same core. Assuming that this difference alters the effective function A it is possible to explain why a stabilized zone forms in one case and not in the other.

MOTION OF A SLUG

The motion of a miscible slug is of considerable practical interest and some observations are reported in Reference 6. The solution of the equations considered here may be illustrated graphically.

The solid curve in Fig. 3 represents the initial concentration distribution of the slug along the core. Using Eq. 2 we can construct the corresponding initial f - curve as indicated by the loop in Fig. 4. If the initial saturation distribution of the slug is symmetric, any vertical secant PQ in Fig. 4 is divided equally by the A - curve at M. For simplicity, let us suppose that A[S] = S, so that the fluid in any cross section is considered a homogeneous mixture.

The f - curve changes as determined by Eq. 5 and evidently the general effect is that the loop shrinks. If the A - curve is a straight line then it follows from the form of Eq. 5 that the secant PQ is bisected at M at all times. This means that the saturation distribution in the core remains symmetrical. If the A - curve is not straight a bias will develop in the distribution. The maximum concentration in the slug is given by the point where the f - curve intersects the A - curve and as the loop shrinks this diminishes.

The rate at which the concentration maximum decreases is of practical importance. This may be examined analytically in a special case. If we take for B a function which is constant for S between 0 and 1, then a solution of Eq. 5 may be written

$$f = S + S\left(\frac{\phi B}{Ut}\right)^{\frac{1}{2}}\left[-\log S[t/T]^{\frac{1}{2}}\right]^{\frac{1}{2}} \qquad \text{Eq. 7}$$

where the initial instant is given by $t = T$.

The initial saturation distribution corresponding to this solution is a Gaussian distribution with standard deviation given by σ where

$$\sigma^2 = \frac{TUB}{\phi} \qquad \text{Eq. 8}$$

The maximum saturation initially is unity. The maximum saturation at any later time is given by

$$S_{max} = \left(\frac{T}{t}\right)^{\frac{1}{2}} \qquad \text{Eq. 9}$$

Eliminating T we may write

$$\frac{\sigma}{L} = B^{\frac{1}{2}} L^{-\frac{1}{2}} \quad S_{max} \qquad \text{Eq. 10}$$

where $L = UT/\phi$ is roughly the distance traveled by the slug in time t.

If we imagine that σ measures the slug size then this expression is equivalent to that given in Reference 5. It gives the slug size initially required so that the maximum concentration after traversing a distance L has not fallen below S_{max}.

EFFECT OF GRAVITY ON A HORIZONTAL DISPLACEMENT

Let us suppose that the phases mix thoroughly so that we may regard the core as filled with a compound fluid whose viscosity and density is determined by the saturation at any point. In a horizontal displacement the motion is complicated by the fact that the fluid may move partly in a horizontal direction and partly in a vertical direction. It is possible to determine, however, whether a motion can exist in which the motion is entirely in the horizontal direction.

If there is no vertical motion then the pressure gradient in the vertical y - direction is due solely to gravity and we have

$$\frac{\partial p}{\partial y} = -\rho g \qquad \text{Eq. 11}$$

where the density ρ depends on the saturation S.

Darcy's equation applied to the horizontal motion gives

$$\frac{\partial p}{\partial x} = -\frac{\mu}{K} U \qquad \text{Eq. 12}$$

where the viscosity μ depends on the saturation S and where K denotes the absolute permeability.

Eliminating the pressure p we obtain

$$g \frac{d\rho}{dS} \frac{\partial S}{\partial x} = \frac{U}{K} \frac{d\mu}{dS} \frac{\partial S}{\partial y} \qquad \text{Eq. 13}$$

If the displacing fluid differs by a small amount from the displaced fluid in viscosity and density it is reasonable to suppose that the viscosity and density of a mixture depends linearly on the saturation. With this assumption, we may write

$$\rho = \rho_1 S + \rho_2[1-S]$$

$$\mu = \mu_1 S + \mu_2[1-S] \qquad \text{Eq. 14}$$

where subscripts 1 and 2 indicate the displacing phase and the displaced phase respectively.

Eq. 13 now reads

$$\frac{\partial S}{\partial x} - \frac{U[\mu_1-\mu_2]}{Kg[\rho_1-\rho_2]} \frac{\partial S}{\partial y} = 0 \qquad \text{Eq. 15}$$

From this it follows that S is constant along the lines

$$y + \frac{U[\mu_1-\mu_2]}{Kg[\rho_1-\rho_2]}\, x = \text{constant.} \qquad \text{Eq. 16}$$

Hence a displacement in which each element of fluid moves horizontally with the same flow rate is possible if the lines on which the saturation is constant are given by Eq. 16. Since this is a steady state solution it seems possible that starting from any initial condition the saturation distribution tends to coincide with this condition ultimately.

CONCLUSIONS

1. Eq. 5, in the text, allows the fractional flow at any time to be computed from known initial and boundary conditions by numerical methods familiar in connection with the conduction of heat.

2. Some experimental data agree with the theory if complete mixing at any cross section is assumed; other data indicate that the mixing cannot be considered complete. The difference may be caused by the experimental techniques.

3. In horizontal drives, with complete mixing, there is a solution in which the constant saturation planes are inclined at a constant angle with the horizon.

NOMENCLATURE

- S = saturation of invading phase in a cross section
- f = fractional flow of invading phase
- U = total flow rate
- ρ = density
- μ = viscosity
- K = absolute permeability
- ϕ = porosity
- p = pressure
- $A[S]$, $B[S]$ = known functions
- x = distance from inlet
- t = time

Subscripts 1 and 2 refer to the invading and displaced phases respectively.

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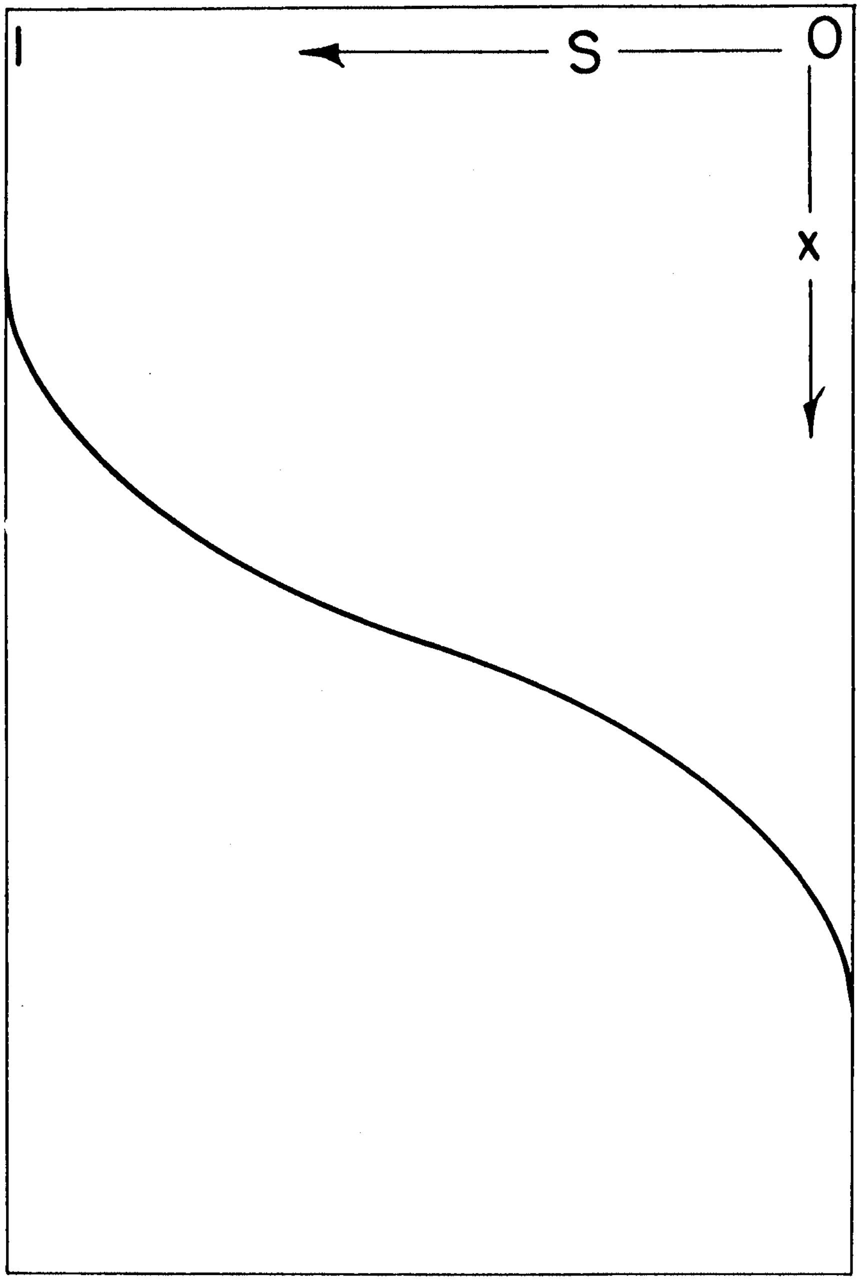

Fig. 1

SATURATION PROFILE IN CORE INITIALLY

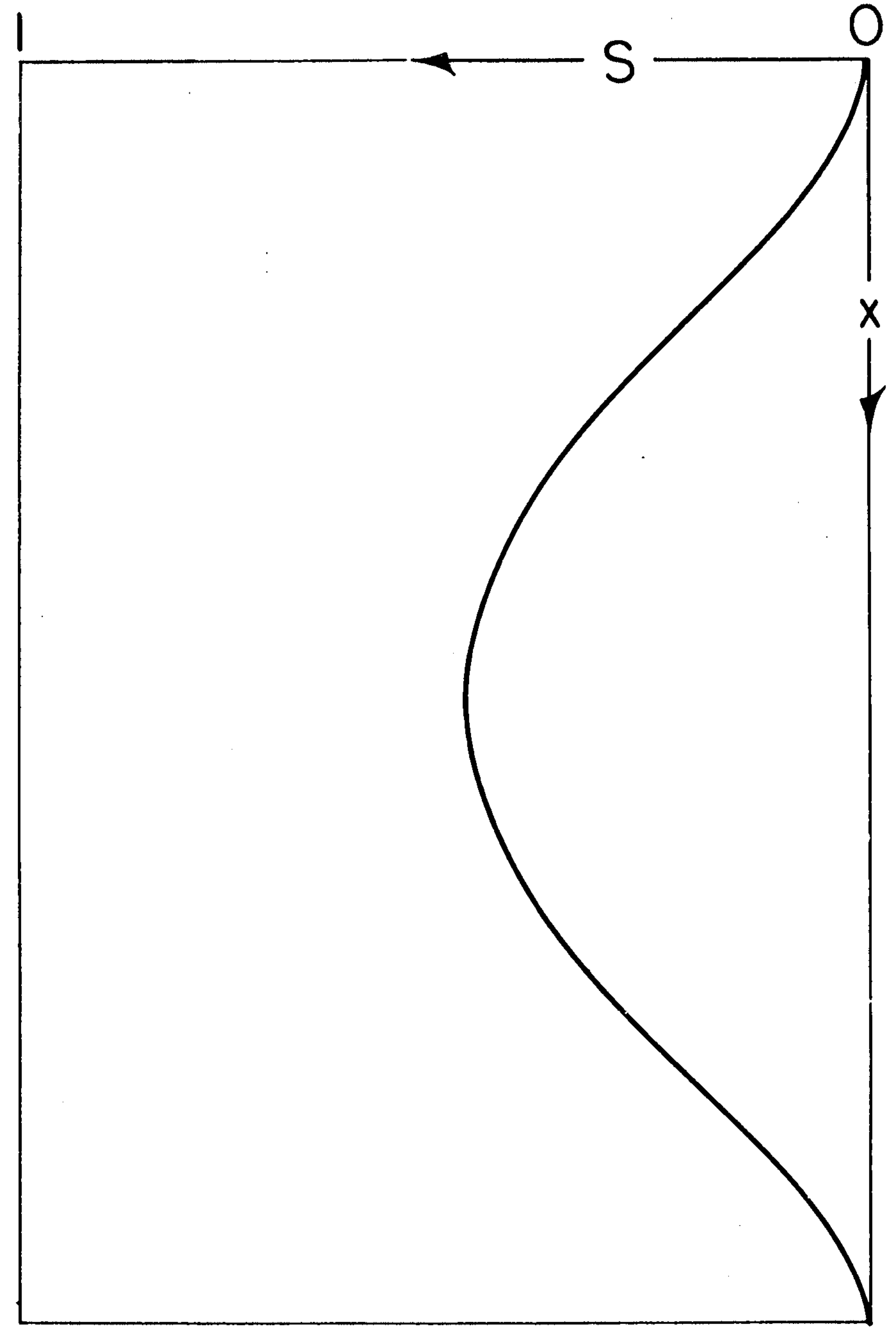

Fig. 3

SATURATION PROFILE OF SLUG INITIALLY

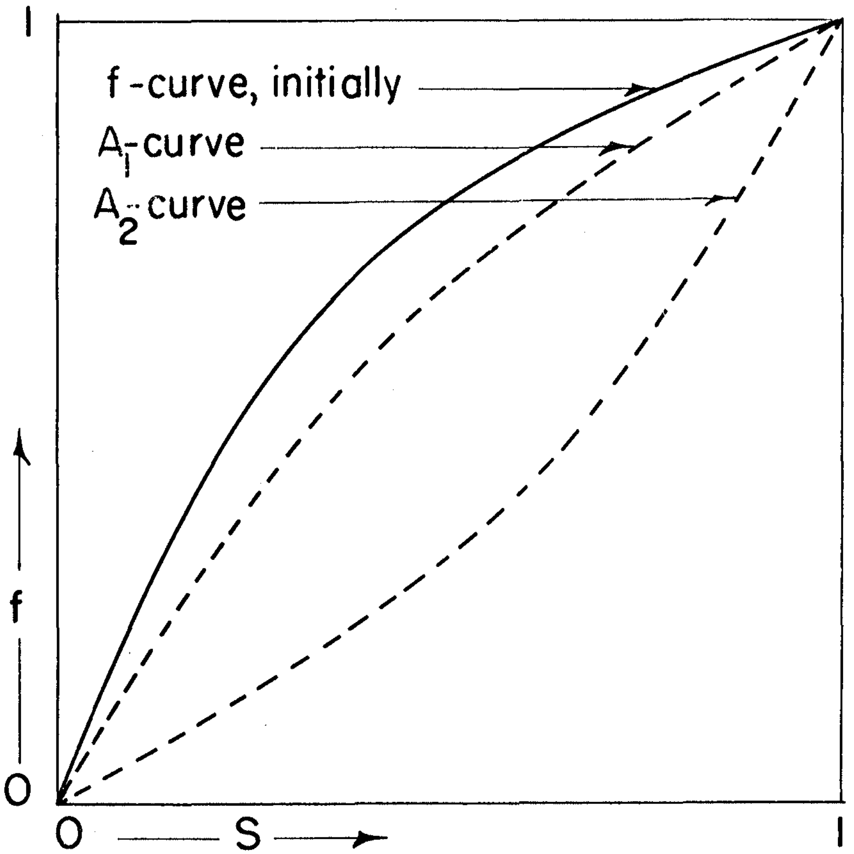

Fig. 2

THE FRACTIONAL FLOW CURVE IN RELATION WITH THE A - CURVE

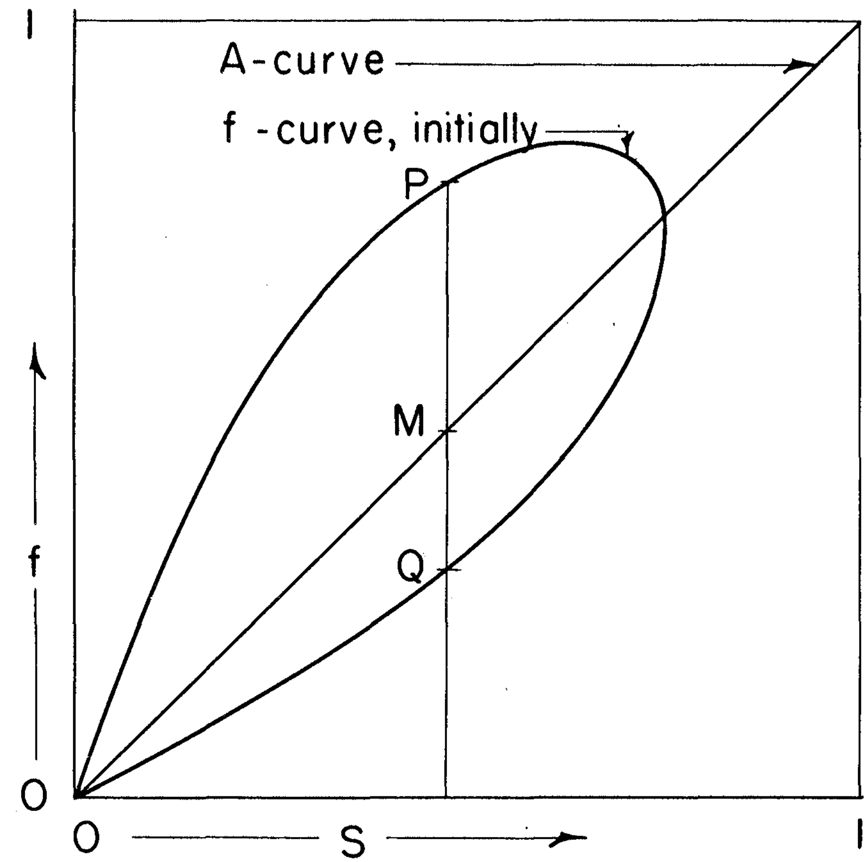

Fig. 4

FRACTIONAL FLOW CURVE FOR SLUG