Use of Data on the Build-up of Bottom-hole Pressures

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In preparing a well for pumping, observations are often made of the fluid level in the well bore or bottom-hole pressures at various times before equilibrium has set in. From a qualitative point of view one may immediately infer that if the rise in the bottom-hole pressure or fluid level is rapid the permeability of the sand about the well bore is large, and converse conclusions may be drawn if the rise is slow. However, a more quantitative estimate will give additional information of value, as will be shown in this paper.

To derive such an estimate we may proceed as follows. Let h be the fluid height, above the sand face, at time t, and let γ_0 be the average density of the fluid that enters the bore. The bottom-hole pressure, for a fluid height h will therefore be $p = \gamma_0 g h$, neglecting, of course, the friction drop in the well bore, while the fluid is rising, as may be justifiably done for most pumping wells.

If a is the free area of the open-flow string—assumed to be uniform—the rate of production from the sand during the rise of fluid will be:

$$Q = a\frac{\partial h}{\partial t} = \frac{a}{\gamma_0 g}\frac{\partial p}{dt} = f(p) \qquad [1]^1$$

where the last part of the equation merely indicates that, in general, the production rate Q must be considered as a function of the back pressure, and is determined by it alone. In fact, equation 1 shows that if the rate of rise of the pressure or fluid level is determined as a function of the height of rise, or p, for a number of values of the latter, this functional relation between Q and p will be immediately given.

Conversely, if we know this functional relation, the rate of increase of p or h may be predicted, or, by observing the rate of rise of p or h some of the constants of the sand may be determined. Thus we may approximate the production from a well in a pumping state by that of a dead

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[1] Equation 1 implies that the increasing bottom-hole pressure is due only to the rising fluid head. It will, therefore, break down in the case of flowing wells after shutting in, where the compression of the gas in the well bore will give an additional time-varying contribution to the bottom-hole pressures.

fluid, and neglect the effect of the gas on the law of flow[2]. We shall have then:

$$Q = c(p_e - p) \quad [2]$$

where c is a constant depending on the dimensions of the system (sand thickness, well radius, "external" radius) and is proportional to the sand permeability, and p_e is the reservoir pressure. With this form for Q, the integral of eq. 1 takes the form:

$$p = p_i + (p_e - p_i)(1 - e^{-\gamma_0 gct/a}) \quad [3]$$

or

$$h = h_i + (h_e - h_i)(1 - e^{-\gamma_0 gct/a}) \quad [4]$$

where, since $p = p_e$ and $h = h_e$ at $t = \infty$, p_e and h_e correspond to the actual closed-in bottom-hole pressure and the final equilibrium fluid level, respectively, and p_i and h_i are the initial values of the bottom-hole pressure or fluid height; i.e., those at the instant chosen to represent $t = 0$.

Rewriting equations 3 and 4 in the forms:

$$-\frac{\gamma_0 gct}{a} = \log\frac{p_e - p}{p_e - p_i} = \log\frac{h_e - h}{h_e - h_i}, \quad [5][3]$$

we see that if the time is observed for the rise in the pressure or fluid height to the values p or h, and if the final equilibrium pressure or fluid height p_e, h_e is known, as well as the area of the hole a and the fluid density, equation 5 will give the value of c as:

$$c = \frac{a}{\gamma_0 gt}\log\frac{p_e - p_i}{p_e - p} = \frac{a}{\gamma_0 gt}\log\frac{h_e - h_i}{h_e - h} \quad [6]$$

And if c is known, the production capacity of the well for any back pressure follows from equation 2. Thus the maximum pumping capacity would be ($p = 0$):

$$Q = cp_e = \frac{ap_e}{\gamma_0 gt}\log\frac{p_e - p_i}{p_e - p} = \frac{ah_e}{t}\log\frac{h_e - h_i}{h_e - h} \quad [7]$$

In the latter form one does not need to know the fluid density. However, it should be mentioned that the observations of the pressure p or height h should be made for values that are not very close to the equilibrium

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[2] In fact, if one should extrapolate to consolidated sands the results recently established for the flow of gas-liquid mixtures through unconsolidated sands [R. D. Wyckoff and H. G. Botset, and M. Muskat and M. W. Meres: *Physics* (1936) **7**, 325, 346] it would appear that equation 2 may be a good approximation even if the oil and gas should flow as a "live" fluid, except during the first stages of the pressure build-up when the liquid saturation about the well bore may show an appreciable increase. The constant c will then, of course, no longer be given rigorously by equation 8.

[3] All logs are to the base e, or natural logs.

values p_e, h_e, or else small errors in the measurement of p or h will give a relatively large error in the value of c and in Q.

If the times are observed for more than one pressure or fluid height, they should give the same value of c as computed by equation 6 provided equation 2 is correct. Or, the data may be plotted on semilogarithmic paper with t on the cartesian scale and $p_e - p$ or $h_e - h$ on the logarithmic scale; the points should then lie on a straight line, the slope of which will give the value $\gamma_0 gc/a$.

If equation 2 is found to hold—i.e., if the above plot does give a straight line—the constant c may be resolved theoretically into the form:

$$c = (2\pi kb/\mu)/\log r_e/r_w \qquad [8]^4$$

where k is the sand permeability, b its thickness, μ the viscosity of the oil, r_w the well radius and r_e the external radius at which the pressure is p_e, under flowing conditions. If, then, c is determined by the above method and the sand thickness and radii r_w, r_e are known, the sand permeability may also be computed.

A deviation of the plot of $p_e - p$ or $h_e - h$ against t on semilogarithmic paper from a straight line may be due to one of two reasons. Either the flow is not like that of a dead liquid, so that equation 2 is not valid, or the value of p_e or h_e, from which is subtracted the observed p or h, is incorrect. A choice between these alternatives may be made by attempting to straighten out the curve by changing the value of p_e or h_e. If this is unsuccessful, the incorrectness of equation 2 is established. If, however, a straight line is obtained, equation 2 will not only be verified but the correct value of p_e or h_e will at the same time be obtained.

In fact, this adjustment of p_e or h_e so as to make the plot of $p_e - p$ or $h_e - h$ versus t a straight line, on semilogarithmic paper, affords both a rapid and very sensitive method for determining the true reservoir pressure. For if the sand be "tight" it may take several days or more for the reservoir pressure to be built up at the bottom of the well bore, so that the cost of determining the reservoir pressure will be prohibitive. If, however, bottom-hole pressures or fluid heights be recorded simply for a pressure-differential variation of some twentyfold over a 10 to 20-hr. interval, during the period of appreciable rate of increase, the above method will permit the determination of p_e or h_e without the necessity of waiting for equilibrium to be established. At the same time the value so obtained will be very accurate, as errors in the assumed value of p_e as low as 5 lb. will cause an easily detected deviation of the plot from strict linearity over a pressure range such as 40 to 800 pounds.

[4] It is assumed here that the well completely penetrates the producing sand. If this is not true, and the actual well penetration is known, a correction factor must be applied. This will be found in the paper by R. D. Wyckoff, H. G. Botset, M. Muskat, and D. W. Reed: Amer. Assn. Petr. Geol. (1934) **18**, 161, Fig. 7.

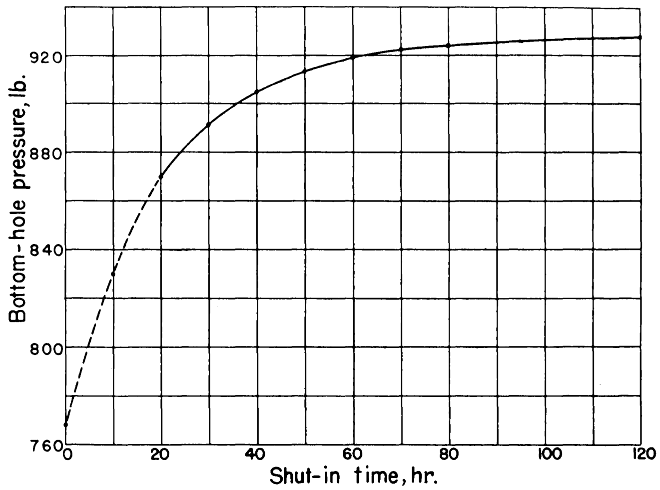

FIG. 1.—AVERAGE BOTTOM-HOLE PRESSURE RISE FOR 20 WELLS IN THE JUDKINS FIELD, ECTOR COUNTY, TEXAS, MARCH 1935.

The author is grateful to Mr. G. A. Pool, Chairman of the Judkins Pool Engineering Committee, for permission to publish this curve.

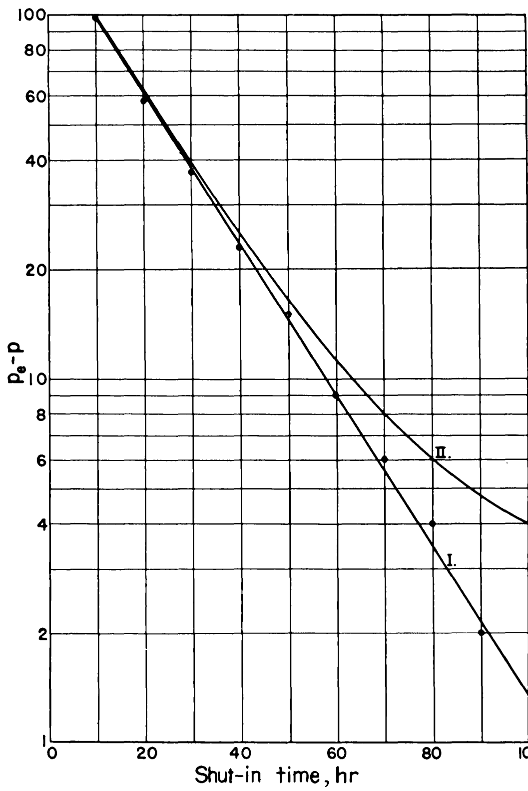

FIG. 2.—DATA OF FIG. 1 REPLOTTED ON A SEMILOGARITHMIC SCALE.

$p_e - p$ = reservoir pressure − bottom-hole pressure. Curve I: p_e = 928 lb. curve II: p_e = 930 lb.

Although the author has not had available suitable field data with which to test satisfactorily the above theoretical results, the manner of the determination of the reservoir pressure may be illustrated by an application of the above outlined procedure to the data shown in Fig. 1, giving the average rate of increase in the bottom-hole pressures of 20 wells in the Judkins field, Texas. When plotted on semilogarithmic paper, with the reservoir pressure p_e taken as 928 lb., these data give the straight line, curve I, of Fig. 2. If the data had referred to a single well, this linearity would have been evidence of the general validity of the above theory for that case, and not essentially fortuitous, as actually it is. Furthermore, the sensitivity of this test to the value 928 lb. for p_e is strikingly shown by comparison of curve I with curve II, in which the same data are plotted, but with $p_e = 930$ pounds.

It is hoped that with a knowledge of the possible applications of such field data as used above, it will be found feasible to collect at least enough to determine their value from a practical point of view[5].

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[5] A study of this kind is now being conducted by E. W. Kemler and G. A. Pool, of the Gulf Oil Corporation.