

# THE SKIN EFFECT IN PRODUCING WELLS

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## ABSTRACT

What is developed in this paper is the cumulative fluid flow across the resistance existing at the wellbore. This is the skin effect that is encountered that is treated here for both positive and negative skin.

Essentially, what is reproduced are work curves where the cumulative fluid produced vs dimensionless time are related to the different values for the skin effect.

In treating with the negative skin in connection with the cumulative fluid produced, it has been learned that the mathematical model that designates such a skin to exist at the wellbore cannot apply, but what is attained is an effectively increased wellbore radius beyond the well bit size that shows the influence of fracturing or any remedial work performed on the well. Thus, the matrix of the sand is effected to evidence discontinuity in the pressure drop for the fluid flowing toward the well; and for the increased effective wellbore radius to the wellbore proper, the flowing pressure is essentially constant. Thus, the mathematical model indicates an open network can exist in the region of the well if the flow is radial and such is shown by the Lord Kelvin effect in pressure buildup.

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References and illustrations at end of paper.

## INTRODUCTION

The basis for this work is the Laplace transformation<sup>1</sup> published by van Everdingen and Hurst, for which the present paper is an extension of the necessary work curves, incorporating the mathematics for the skin effect. The skin effect is also the joint undertaking of these authors published separately in the literature.<sup>2,3</sup>

It is shown in the first mentioned publication that the cumulative fluid influx into a wellbore is expressed by

$$Q(t) = 2\pi \phi c h r_w^2 \int_0^{t_D} \frac{\Delta P}{dt'_D} Q(t_D - t'_D) dt'_D \dots \dots \dots [1]$$

subject to transient fluid flow or unsteady-state flow, where  $Q[t_D]$ , the mathematical symbol for fluid influx, is a function of the physical parameters in the reservoir or

$$t_D = \frac{kt}{\phi \mu c r_w^2} \dots \dots \dots [2]$$

Employing the superposition theorem for instantaneous pressure drop observed in the production history, the former equation is expressed as

$$Q(t) = 2\pi \phi c h r_w^2 [\Delta P_0 Q(t_D) + \Delta P_1 Q(t_D - t_{D1}) + \Delta P_2 Q(t_D - t_{D2}) + \dots] \dots \dots [3]$$

The values for  $q[t_D]$  have been reported extensively in that publication,<sup>1</sup> and such is expressed by

$$Q(t) = \frac{4}{\pi^2} \int_0^\infty \frac{(1 - e^{-u^2 t_D}) du}{u^3 [J_0^2(u) + \gamma_0^2(u)]} \dots [4]$$

given in Bessel functions of the first and second kind of zero order for radial flow.

The change that now appears in these formulas is due to the skin effect, which can be expressed simply by the cumulative pressure drop as

$$\Delta p = \frac{q(t) \mu}{2\pi kh} [S + P(t_D)] \dots [5]$$

subject to a constant rate of production at the well, with  $P[t_D]$  the cumulative pressure drop for a unit rate of production, which in turn is a function of time and unsteady-state fluid flow.

Its accumulative fluid influx for a positive skin effect applicable in Eq. 1 is now given as

$$Q(t_D) = \frac{4}{\pi^2} \int_0^\infty \frac{(1 - e^{-u^2 t_D}) du}{u^3 \{ [J_0(u) + uS J_1(u)] + [\gamma_0(u) + uS \gamma_1(u)]^2 \}} \dots [6]$$

Thus, Eq. 6 is the identity of Eq. 4 when no skin effect exists, and in itself forms the basis of the work entailed here. What is meticulously reported is for a positive skin; if a negative skin occurs, an additional term appears in Eq. 6 that is a reversal of fluid flow and the mathematical model can no longer apply.

#### ANALYSES

The skin effect as defined in the early papers<sup>2,3</sup> is an impediment that exists around the wellbore and retards the fluid flow into the well. Its graphical picture as shown in the original sketch,<sup>3</sup> Fig. 1, is that immediately at the well there is an incrustment, that these authors defined as the damaging of the well by the drilling operation that offers a lower permeability than the formation proper, and as a result there is a discontinuity in the flow of fluids once this barrier is reached and the greatest pressure drop takes place in the fluids flowing into the well. In a sense this has been regarded as an infinitesimal thickness, and the appendage of a skin effect assigned to identify this phenomenon is comparable to the

film coefficient of heat transfer.

This is one version; others have subscribed partial penetration to the skin effect,<sup>4</sup> some to the second order velocity gas can obtain if turbulence exists at the wellbore. All this may be true and a compounding of these effects; the significant thing is that in interpreting pressure buildup Darcy's law is readily manifested at earliest times by the Lord Kelvin effect of pressure buildup vs logarithm of time, and if one extrapolates back to the time of flow, an additional pressure increase is observed above the flowing pressure that is this skin effect.

Such has been the performance of many of these pressure buildup tests, and with the advent of fracturing and large scale remedial operations on wells, a reversal of these conditions has been observed where the pressure is lower than the flowing pressure in extrapolating back to earliest times. This is the negative skin effect.

Thus, in its implication and meaning, it is recognized that a positive skin terminates at the wellbore because this is its physical limit. A negative skin, however, offers further thought as the mathematical model does not apply for the explicit interpretation of a skin.

Rather than regarding this aspect as a failure or a limitation of analyses in employing the skin, it offers the only alternative that a skin cannot exist under these conditions and the sand matrix is effected further than the physical boundaries for a skin.

The Laplace transform for the influx of fluid that identifies this skin follows from the earlier paper,<sup>1</sup> given by the equation

$$\frac{1}{p^2} = p \bar{P}(p) \bar{Q}(p) \dots [7]$$

where  $\bar{P}[p]$  is the cumulative pressure drop for a unit rate of production, expressed in operational form, with  $p$  the operator for dimensionless time,  $t_D$ .  $\bar{Q}[p]$  is the cumulative production for a fixed terminal pressure, also given as a Laplace transform. It is this latter term we wish to identify when the skin effect exists.

The development of this formula, although shown as a generalized relation for these two boundary problems,<sup>1</sup> can here be specifically identified with the skin effect by treating Eq. 5 for the unit rate of production in Laplace transformations, or

$$\bar{P}_i(p) = \frac{S}{p} + \bar{P}(p) \dots [8]$$

Such introduced in Eq. 7, gives

$$\bar{Q}(p) = \frac{1}{p^2 [S + p \bar{P}(p)]} \dots [9]$$

This is the development of Eq. 6 employing the Mellin's inversion formula, where  $\bar{P}(p)$  is the pressure drop for the unit rate of production of the undamaged or clean sand, with

$$\bar{P}(p) = \frac{K_0(\sqrt{p})}{p^{3/2} K_1(\sqrt{p})} \dots \dots \dots [10]$$

expressed in modified Bessel functions.

An alternate solution to this problem is to use the actual boundary conditions shown in Fig. 1. This is the fixed terminal pressure case. The initial pressure  $P_i$  is also constant and extends throughout the formation to infinity.

If we recognize in Eq. 5 the component pressure drop that is the skin effect, then

$$P_i - P_{wf} = \frac{q \mu S}{2\pi kh} \dots \dots \dots [11]$$

and with the rate of fluid production at the wellbore

$$q = \frac{2\pi kh r_w}{\mu} \left( \frac{\partial P}{\partial r} \right)_{r_w} \dots \dots \dots [12]$$

therefore,

$$P_i - P_{wf} = \left( \frac{\partial P}{\partial r} \right)_{r_w} S \dots \dots \dots [13]$$

and it is the condition that must be met for  $r_w = 1$ .

The cumulative pressure drop from  $P_i$  expressed as a Laplace transformation is given<sup>1</sup> as

$$\bar{\Delta P}(r_1 p) = A K_0(r\sqrt{p}) \dots \dots \dots [14]$$

where A is a constant. Therefore, the pressure in the formation is expressed as

$$\bar{P}(r_1 p) = \frac{P_i}{p} - A K_0(r\sqrt{p}) \dots \dots \dots [15]$$

Its substitution in Eq. 13, yields

$$\frac{(P_i - P_{wf})}{p} - A K_0(\sqrt{p}) = A S \sqrt{p} K_1(\sqrt{p}) \dots \dots \dots [16]$$

for  $r_w = 1$ , and

$$A = \frac{(P_i - P_{wf})}{p} \frac{1}{[K_0(\sqrt{p}) + S \sqrt{p} K_1(\sqrt{p})]} \dots \dots \dots [17]$$

where  $K_0'[z] = -K_1[z]$ . Therefore

$$\bar{P}(r_1 p) = \frac{P_i}{p}$$

$$\frac{(P_i - P_{wf})}{p} \frac{K_0(r\sqrt{p})}{[K_0(\sqrt{p}) + S \sqrt{p} K_1(\sqrt{p})]} \dots \dots \dots [18]$$

This is the pressure distribution in Laplace transformations when a skin effect exists at the wellbore and it is compatible with a solution by Carslaw.<sup>5</sup>

The interpretation of Eq. 12 as the fluid flux just upstream from the skin gives

$$\bar{q} = \frac{2\pi kh}{\mu} \left( \frac{\partial P}{\partial r} \right)_{r_w} = \frac{2\pi kh}{\mu} \frac{(P_i - P_{wf})}{\sqrt{p}} \frac{K_1(\sqrt{p})}{[K_0(\sqrt{p}) + S \sqrt{p} K_1(\sqrt{p})]} \dots \dots \dots [19]$$

Its integration with absolute time, although made necessarily in dimensionless time as expressed through the operator p, and

$$\bar{Q}(p) = \frac{\phi \mu c r_w^2}{k} \frac{\bar{q}}{p} = 2\pi \phi c h r_w^2 \frac{(P_i - P_{wf}) K_1(\sqrt{p})}{p^{3/2} [K_0(\sqrt{p}) + S \sqrt{p} K_1(\sqrt{p})]} \dots \dots \dots [20]$$

This is the form of Eq. 1, or more appropriate, Eq. 3, when only a fixed pressure drop  $P_i - P_{wf}$  prevails at the wellbore. Therefore,  $\bar{Q}(p)$  as expressed independent of these coefficients is given as

$$\bar{Q}(p) = \frac{K_1(\sqrt{p})}{p^{3/2} [K_0(\sqrt{p}) + S \sqrt{p} K_1(\sqrt{p})]} \dots \dots \dots [21]$$

which is Eq. 9, upon substituting in Eq. 10.

#### NUMERICAL INTERPRETATION

The inordinate time it takes to program a calculation to arrive at numerical values, particularly of the complex nature of the Bessel functions with their oscillatory characteristics of the type of Eq. 6, suggests the simplification of Eq. 21 in employing the Mellin's inversion formula.

To make this as adaptable as possible for a computer calculation, it is recognized<sup>1</sup> that if  $t_D$  is large, p is small, since for the fluid

influx into a well  $t_D$  is significantly large in a matter of seconds from the start of production, employing the necessary reservoir parameters.

This application is again referred to the basic paper<sup>1</sup> where Eq. 10 for  $p$  small is given as

$$\bar{P}(p) = \frac{1}{p} \left[ \frac{-\ln p}{2} + (\ln 2 - \gamma) \right], \dots [22]$$

with  $\gamma = 0.57722$ , the Euler's constant. Substituted in Eq. 9, it yields

$$\bar{Q}(p) = \frac{1}{p^2 \left[ s - \frac{\ln p}{2} + (\ln 2 - \gamma) \right]}, \dots [23]$$

now introducing the skin effect as such retards the fluid entry.

Its inversion follows along the "cut" of the negative real axis of the  $[x, y]$  coordinates employing the Mellin's inversion formula

$$Q(t_D) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} e^{\lambda t_D} \bar{Q}(\lambda) d\lambda, [24]$$

where  $\lambda$  is related to these coordinates as  $\lambda = x + iy$ , or in polar coordinates  $\lambda = re^{i\theta}$ , whichever is most suited, and  $\gamma$  is the distance removed on the positive real axis parallel to the  $y$  coordinate that includes all poles observed for  $\bar{Q}[\lambda]$  to the left of this line. The  $\lambda$  mentioned here is the operator  $p$  in Eq. 23.

The discussion of this inversion has now received considerable reference in the literature, and it suffices to report this inversion of Eq. 23 without engaging in mathematics. Thus, the alternate form to Eq. 6, is

$$Q(t_D) = \int_0^\infty \frac{(1 - e^{-u^2 t_D}) du}{u^3 \left\{ [s - \ln u + (\ln 2 - \gamma)]^2 + \frac{\pi^2}{4} \right\}} \dots [25]$$

when  $t_D$  is large and suitable for machine calculation.

For  $u$  close to the zero, an additional equation results from Eq. 25, as that form cannot handle the integration at the origin. This is for  $u = \delta = \sqrt{.04/t_D}$ , which has been learned from experience is appropriate in breaking up this integration. Thus, for Eq. 25, and integrating from 0 to  $\delta$ , gives

$$Q_\delta(t_D) = \int_0^\delta \frac{(1 - e^{-u^2 t_D}) du}{u^3 \left\{ [s - \ln u + (\ln 2 - \gamma)]^2 + \frac{\pi^2}{4} \right\}}$$

$$= \int_0^\delta \frac{t_D du}{u \left\{ [s - \ln u + (\ln 2 - \gamma)]^2 + \frac{\pi^2}{4} \right\}} \dots [26]$$

where  $1 - e^{-u^2 t_D} \cong u^2 t_D$ . By changing the variable and letting  $u = e^{-v}$  the above reduces to

$$Q_\delta(t_D) = t_D \int_{\ln \frac{1}{\delta}}^\infty \frac{dv}{[s + v + (\ln 2 - \gamma)]^2 + \frac{\pi^2}{4}} [27]$$

Furthermore, for  $m = \frac{2}{\pi} [s + v + (\ln 2 - \gamma)]$  this becomes

$$Q_\delta(t_D) = \frac{2}{\pi} t_D \int_{\frac{2}{\pi} (s + \ln \frac{1}{\delta} + \ln 2 - \gamma)}^\infty \frac{dm}{m^2 + 1} [28]$$

that yields

$$Q_\delta(t_D) = \frac{2}{\pi} t_D \tan^{-1} m \Big|_{\frac{2}{\pi} (s + \ln \frac{1}{\delta} + \ln 2 - \gamma)}^\infty$$

$$= \frac{2}{\pi} t_D \left[ \frac{\pi}{2} - \tan^{-1} \frac{2}{\pi} (s + \ln \frac{1}{\delta} + \ln 2 - \gamma) \right] \dots [29]$$

It has been learned that numerically this is the largest part of the contribution of Eq. 25 when the skin effect is positive.

The results obtained for  $Q[t_D]$  from Eqs. 25 and 29 are shown in Table 1 for a positive skin effect extending from 0, 2.5, 5, 10, 15, 25 and 50, listed vs dimensionless time  $t_D$  100 to 10,000,000.

These calculations are shown plotted in Fig. 2 as log of  $Q[t_D]$  vs log  $t_D$  for the specific skin effect identified. As will be observed from this plot, the greater the value of the skin, the larger the impediment to fluid flow into the well, yielding a lowered efflux of fluid issuing from the well for a given time,  $t_D$ .

The check on the validity of these equations and computations can be immediately evidenced to the reader by comparing the fourth and fifth columns of Table 1 in the Laplace transformation<sup>1</sup> with  $S = 0$  in Table 1 cited here to show that the values are almost identical.

Other numerical checks have been made of individual calculations of the computer, but what has been stated suffices to show the reliability of this procedure.

The interest here, however, has been to identify what evidence can be given as to the physical problem when the skin effect is negative as manifested by the pressure buildup for a fractured or remedial treated well. This amounts to  $S$  being negative in these equations, but such is not the complete solution as we must treat with an essential singularity along the positive real axis in the Mellin's inversion formula. This refers back to Eq. 24 and the significance of that formula.

In effect the development of Eq. 6 when treating with the Bessel functions, and then Eq. 25 when dealing with the simple transcendental function of the logarithm of the operator, has been performed by substituting  $\lambda$  in Eq. 24 as  $\lambda = u^2 e^{i\pi}$ , in making the integration along the negative real axis from zero to infinity as indicated for the direction of the cut along the negative real axis shown in Fig. 3.

However, what is also indicated is that for a negative skin effect a pole also resides on the positive real axis illustrated in Fig. 3. This refers back to the Laplace transformation for  $\bar{Q}(p)$ , Eq. 9, when  $S$  is negative, or as observed in Eq. 21.

$$\bar{Q}(p) = \frac{K_1(\sqrt{p})}{p^{3/2} [K_0(\sqrt{p}) - S\sqrt{p} K_1(\sqrt{p})]} \quad [30]$$

As before, the cut extends along the negative real axis, but also observed is that the term in the bracket of the denominator becomes zero when

$$S = \frac{K_0(\sqrt{v})}{\sqrt{v} K_1(\sqrt{v})}, \quad \dots \quad [31]$$

with  $v = p$ , the root when this identity is met.

It does not prevail along the negative real axis, but for positive values for  $v$  these are single valued Bessel functions to reveal poles for the whole range of  $S$  from zero to infinity when substituted in Eq. 31.

Its residual as subscribed in Eq. 24, is

$$Q(t_D) = \frac{e^{v t_D} K_1(\sqrt{v})}{\sqrt{3/2} \left\{ \frac{d[K_0(\sqrt{p}) - S\sqrt{p} K_1(\sqrt{p})]}{dp} \right\}_{p \rightarrow v}} \quad [32]$$

to yield

$$Q(t_D) = \frac{2 e^{v t_D} K_1(\sqrt{v})}{v [-SK_1(\sqrt{v}) - \sqrt{v} SK_1'(\sqrt{v}) + K_0'(\sqrt{v})]}$$

and by the differentiation of these modified Bessel functions, where

$$K_0'(z) = -K_1(z)$$

$$K_1'(z) = -K_0(z) - \frac{K_1(z)}{z} \dots \dots [34]$$

given in the literature,<sup>6</sup> yields

$$Q(t_D) = \frac{2 e^{v t_D} K_1(\sqrt{v})}{v [\sqrt{v} SK_0(\sqrt{v}) - K_1(\sqrt{v})]} \quad [35]$$

Thus, upon substituting the identity Eq. 31, gives

$$Q(t_D) = \frac{2 e^{v t_D}}{v(S^2 - 1)} \dots \dots [36]$$

which upon introducing the limits for  $Q[t_D]$  production in  $t_D$  time, and zero production in time zero, yields

$$Q(t_D) = \frac{2(1 - e^{v t_D})}{v(1 - v S^2)} \dots \dots [37]$$

From the observation of the asymptotic expansions for Eq. 31 as defined by these modified Bessel functions,<sup>6</sup>  $v S^2 < 1$ , except when  $S$  is infinity; thus, the contribution of the essential singularity along the positive real axis is always negative as  $e^{v t_D} \geq 1$ .

These are the results now shown as a plot in Fig. 4, which are the contributions of Eqs. 25, 29 and 37 when  $S$  is negative, with the latter the largest contributory value to give negative efflux, or specifically fluid influx for  $Q[t_D]$ .

In a sense, what is recognized is a situation where some form of pressure maintenance or fluid injection into a well is superimposed above the initial pressure. This occurs when credence is given to a negative skin effect as required in these formulas.

It is necessary to determine exactly what this physical problem is, and whether such is tangible to sustain such a case, since the mathematics makes no distinction as to what is introduced for the skin effect.

This refers back to the physical problem and boundary conditions stated for Fig. 1, but now identified with fluid injection as shown in Fig. 5. This is for fluid injected into a well, encountering a skin effect, with the resulting pressure superimposed above the initial pressure.

As in Eq. 13, the boundary conditions at the skin effect can be stated as

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$$P_{wf} - P_f = - \left| S \right| \left( \frac{\partial P}{\partial r} \right)_{r_w}, \dots \dots \dots [38]$$

giving full recognition that the well pressure will be greater than the interfacial pressure past the skin. Further, the skin itself will be identified for its absolute value of positive skin determined for a damaged well. Here the minus sign is affixed to the right of the equation as the slope of the pressure gradient will be negative for fluid moving into the formation. Thus, the combined effect yields the positive value necessary for  $P_{wf} > P_f$ .

As in the earlier development, the Laplace transformation for pressure in the formation is given as

$$\bar{P}(r, p) = \frac{P_i}{p} + A K_o(r\sqrt{p}) \dots \dots \dots [39]$$

for which A is established from Eq. 38 in substituting  $P_f$ , or

$$\frac{P_{wf} - P_i}{p} - A K_o(\sqrt{p}) = A S K_1(\sqrt{p}) \dots [40]$$

at  $r = r_w$ , and

$$A = \frac{(P_{wf} - P_i)}{p} \frac{1}{[K_o(\sqrt{p}) + S\sqrt{p} K_1(\sqrt{p})]} \dots \dots \dots [41]$$

with the pressure in Eq. 39, given as

$$\bar{P}(r, p) = \frac{P_i}{p} + \frac{(P_{wf} - P_i)}{p} \frac{K_o(r\sqrt{p})}{[K_o(\sqrt{p}) + S\sqrt{p} K_1(\sqrt{p})]} \dots \dots \dots [42]$$

This suffices for the present development as it is recognized from Eq. 18, and now Eq. 42, that this unsteady-state fluid injection, for which  $P_{wf} - P_i$  is fixed, and the work curves shown in Fig. 2 for a positive skin effect apply.

The question is, what gives the form of Eq. 30 that can interject a negative skin in the pressure influx formulas. This goes back to Eq. 38, and to give full cognizance to S for a negative skin, thus recognizing its algebraic sign. Therefore, Eq. 38 becomes

$$P_{wf} - P_f = S \left( \frac{\partial P}{\partial r} \right)_{r_w} \dots \dots \dots [43]$$

If we repeat the steps of the previous development, then we observe that the operational pressure in the formation is

$$\bar{P}(r, p) =$$

$$\frac{P_i}{p} + \frac{(P_{wf} - P_i)}{p} \frac{K_o(r\sqrt{p})}{[K_o(\sqrt{p}) - S\sqrt{p} K_1(\sqrt{p})]} \dots \dots \dots [44]$$

This is the first time we observe the operational form that leads to Eq. 30 for a negative skin.

In order to understand what is implied by a negative skin, it is necessary to make the inversion for Eq. 44. This is given by the pressure distribution relationship

$$P(r, t_D) = P_i + 2 \frac{(P_{wf} - P_i)}{\pi} \int_0^\infty \frac{(1 - e^{-u^2 t_D}) \{ [Y_0(r\sqrt{u})] [J_0(u) - S u Y_1(u)] - J_0(r\sqrt{u}) [Y_0(u) - S u Y_1(u)] \}}{u \{ [J_0(u) - S u Y_1(u)]^2 + [Y_0(u) - S u Y_1(u)]^2 \}} du + 2 \frac{(P_{wf} - P_i) (1 - e^{-t_D}) K_o(r\sqrt{p})}{\sqrt{p} K_1(\sqrt{p}) (1 - v S^2)} \dots \dots \dots [45]$$

with the initial pressure being the static pressure  $P_i$  at time zero.

The second term is the cumulative pressure drop for the superimposed pressure injected from well pressure  $P_{wf}$ , to static pressure  $P_i$ , which in these mathematical formulas show that even with a negative skin, fluid flows into the formation proper. However, the contribution of the pole on the positive real axis of Fig. 3 in this inversion shows that a sink is generated of unlimited extent with time. This is the third term in Eq. 45.

The result is that the entire formula for pressure distribution is projected as if this were an accumulative pressure drop from  $P_i$  for fluid flowing into the well, which is the predominance of this term. This is indicated by the negative values for  $Q[t_D]$  in Fig. 4, of fluid entering the well from the formation. A situation incompatible for the boundary conditions set forth in this problem illustrates the lack of any physical significance that could be attached to a mathematical model for a negative skin effect.

In substance, a pole along the positive real axis generates a source or sink of uncontrollable extent with time. Yet that this pole exists is dictated by mathematical procedure indicated by Eqs. 30 and 31 and proves that a negative skin is untenable.

Yet for pressure buildups after fracturing or remedial treating a well, the numerical interpretation of such tests shows that the skin can be negative in value. By a process of elimination undertaken here one other interpretation exists--that the matrix of a sand can be affected.

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This can be evidenced from Eq. 5, subscribing a negative skin, and writing out the full meaning for  $P[t_D]$  as would apply to a pressure buildup.<sup>3</sup>

$$\Delta P = \frac{q\mu}{4\pi kh} \left[ -2S + \ln \left( \frac{kt}{\phi\mu cr_w^2} \right) + 0.80907 \right], \dots [46]$$

where  $t$  is the time of buildup yielding the Lord Kelvin effect.

If we let  $r_{wa}$  represent the apparent or increased wellbore radius by fracturing beyond the actual wellbore radius drilled,  $r_w$ , then

$$-S = \ln \left( \frac{r_w}{r_{wa}} \right), \dots [47]$$

where  $r_w \leq r_{wa}$  that yields a negative value for  $S$  and its substitution in the above formula, gives

$$\Delta P = \frac{q\mu}{4\pi kh} \left[ \ln \left( \frac{kt}{\phi\mu cr_{wa}^2} \right) + 0.80907 \right], \dots [48]$$

which is also a Lord Kelvin effect. Its physical aspect is illustrated in Fig. 6.

While the effect of a negative skin from the ordinary interpretation of a pressure buildup is to give a lower wellbore pressure than that observed prior to buildup, the configuration of Fig. 6 shows that the matrix of the sand is discontinuous at a distance  $r_{wa}$  to give a uniform pressure  $P_{wf}$  to the wellbore radius  $r_w$ , which is the only means for sustaining fluid flow into the well itself.

Mathematical conversions similar to Eq. 47 have been used in the past to incorporate the skin effect with the wellbore boundary conditions, but what is revealed in this work is that by a process of elimination an enlarged wellbore radius is the only condition that can sustain a negative skin if it is observed upon pressure buildup, which is a measure of the effectiveness of fracturing or other remedial operations, provided of course the Lord Kelvin effect is pronounced in the buildup.

The last assertion made as such is the criterion of what is developed here. This is limited to one system, radial interpretation of fracturing that appears upon many tests as evidenced from pressure buildups. Yet the authors are cognizant that other systems can prevail. In this respect one of us may be joined in such an undertaking with an associate to encompass yet a different system.

Recognizing that an enlarged wellbore is the only basis that can explain a negative skin

effect, the cumulative fluid influx readily follows. This is to treat the enlarged radius  $r_{wa}$  as free of any skin effect, or  $S$  equal to zero, and equating the following relation:

$$r_w^2 Q \left( t_D = \frac{kt}{\phi\mu cr_w^2} \right) = r_{wa}^2 Q \left( t_D = \frac{kt}{\phi\mu cr_{wa}^2} \right) \dots [49]$$

This reduces the effective increased wellbore radius to a unit radius that is mandatory to apply for  $Q[t_D = kt/\phi\mu cr_w^2]$  at the wellbore in Eqs. 1 or 3.

It could be assumed that the effected sand [either fractured or acidized] is already filled with the treating fluid in the immediate wellbore area, and there is an instantaneous relationship of one fluid displacing another in the movement of oil to the well.

The time  $t_D = kt/\phi\mu cr_{wa}^2$  will necessarily be much smaller than the corresponding time,  $t_D = kt/\phi\mu cr_w^2$  for the well radius. Yet Table 1 in the Laplace transformation<sup>1</sup> will readily apply to small dimensionless time to  $t_D = 0.01$ . For less than this value the reader can use  $Q[t_D] = 2 t_D^{1/2}/\pi^{1/2}$ , given in that paper. Such are now reproduced with the positive skin effect shown in Fig. 7. The summary of values for the negative skin are given in Table 2.

To summarize, the basis for this work has been to observe the work curves for cumulative fluid influx, Fig. 2, when a skin effect exists. The fact that we have been taken further afield by the mathematics that appeared upon applying the negative skin effect necessarily made it mandatory to pursue the problem that resolved itself that a negative skin cannot exist except as an enlarged wellbore radius, which is the purpose for the presentation.

The measure of the success of the degree of fracturing by this increased wellbore radius has now been reported to the Corporation Commission of Kansas<sup>7</sup> in the study of the Kansas Hugoton field. It has been learned there in comparing the performance of early completed wells with recently fractured wells that what is now accomplished in fracturing these wells is to restore their gas deliverability somewhere near to the capacity of the earlier wells. This is not surprising as over the years with the wells in operation a large amount of debris from sand deposited, and even connate water, has collected in these wells, and fracturing is restoring these wells nearly to their early performance as observed in making this comparison for the effective increased wellbore radius of the early unfractured wells with the fractured wells.

## ACKNOWLEDGMENT

Appreciation is extended to the management of Union Oil Co. of California for permission to present this paper.

## NOMENCLATURE

$c$  = fluid compressibility,  $Lt^2/m$   
 $h$  = reservoir thickness normal to flow,  $L$   
 $k$  = effective permeability,  $L^2$   
 $\mu$  = viscosity,  $m/Lt$   
 $\phi$  = porosity  
 $P$  = pressure,  $m/Lt^2$   
 $P_f$  = formation pressure,  $m/Lt^2$   
 $P_i$  = initial pressure,  $m/Lt^2$   
 $P_{wf}$  = wellbore pressure,  $m/Lt^2$   
 $\bar{P}$  = dimensionless pressure  
 $Q[t_D]$  = fluid efflux  
 $r_w$  = wellbore radius,  $L$   
 $r_{wa}$  = apparent wellbore radius,  $L$   
 $s$  = skin effect  
 $t$  = time,  $t$   
 $t_D$  = dimensionless time

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Table 1  
FLUID EFFLUX,  $Q(t_p)$   
FOR POSITIVE SKIN EFFECT

$t_p$	0	2.5	5	10	15	25	50
100	42.21	21.11	13.85	8.19	5.81	3.68	1.92
150	58.93	30.38	20.21	12.09	8.62	5.48	2.86
200	74.80	39.37	26.44	15.93	11.40	7.26	3.81
250	90.10	48.16	32.57	19.74	14.16	9.04	4.75
300	104.97	56.81	38.63	23.52	16.90	10.81	5.69
400	133.75	73.75	50.57	31.01	22.36	14.34	7.56
500	161.56	90.35	62.34	38.43	27.77	17.86	9.44
600	188.66	106.67	73.97	45.80	33.16	21.36	11.30
700	215.17	122.78	85.48	53.12	38.52	24.85	13.17
800	241.22	138.71	96.91	60.40	43.86	28.34	15.03
900	266.86	154.48	108.25	67.65	49.19	31.81	16.89
1,000	292.15	170.12	119.53	74.88	54.50	35.28	18.75
1,500	414.65	246.73	175.06	110.64	80.85	52.55	28.02
2,000	532.42	321.41	229.57	145.97	106.97	69.71	37.26
2,500	646.88	394.73	283.35	180.99	132.92	86.80	46.47
3,000	758.84	467.00	336.55	215.77	158.74	102.83	55.68
4,000	977.09	609.10	441.63	284.75	210.06	137.76	74.04
5,000	1,189.65	748.72	545.34	353.13	261.04	171.54	92.35
6,000	1,397.86	886.43	647.98	421.05	311.77	205.20	110.64
7,000	1,602.58	1,022.58	749.76	488.59	362.28	238.77	128.90
8,000	1,804.42	1,157.43	850.80	555.80	412.61	272.27	147.14
9,000	2,003.80	1,291.16	951.21	622.73	462.78	305.69	165.34
10,000	2,201.03	1,423.91	1,051.06	689.41	512.81	339.04	183.54
15,000	3,162.46	2,076.20	1,543.78	1,019.88	761.31	505.10	274.28
20,000	4,094.03	2,714.46	2,028.42	1,346.68	1,007.73	670.22	364.75
25,000	5,004.56	3,342.66	2,507.22	1,670.83	1,252.62	834.65	455.01
30,000	5,898.99	3,963.09	2,981.47	1,992.85	1,496.32	998.55	545.11
40,000	7,651.11	5,185.97	3,919.40	2,632.01	1,980.89	1,325.05	724.92
50,000	9,365.89	6,390.38	4,846.39	3,266.06	2,462.52	1,650.22	904.42
60,000	11,052.02	7,580.46	5,764.85	3,896.10	2,941.82	1,974.33	1,083.40
70,000	12,715.04	8,758.89	6,676.32	4,522.84	3,419.21	2,297.56	1,262.21
80,000	14,358.76	9,927.52	7,581.91	5,146.78	3,894.98	2,620.04	1,440.40
90,000	15,985.98	11,087.72	8,482.41	5,768.30	4,369.33	2,941.87	1,619.18
100,000	17,598.80	12,240.53	9,378.44	6,387.68	4,842.43	3,263.13	1,797.39
150,000	25,495.21	17,917.67	13,805.72	9,459.14	7,193.09	4,862.54	2,686.29
200,000	33,187.72	23,487.99	18,167.60	12,498.96	9,525.12	6,453.31	3,572.53
250,000	40,735.32	28,981.45	22,482.10	15,515.62	11,843.52	8,037.72	4,456.78
300,000	48,171.28	34,415.18	26,759.58	18,514.11	14,151.15	9,617.09	5,339.46
400,000	62,786.78	45,144.25	35,228.32	24,468.50	18,741.12	12,763.95	7,101.11
500,000	77,140.16	55,730.63	43,607.72	30,378.50	23,304.66	15,898.36	8,858.84
600,000	91,291.25	66,206.09	51,917.26	36,253.61	27,847.35	19,022.92	10,613.48
800,000	119,128.17	86,899.33	68,373.57	47,922.05	36,883.72	25,248.79	14,115.40
1,000,000	146,488.71	107,327.02	84,661.12	59,505.26	45,868.86	31,450.29	17,609.59
5,000,000	655,512.76	494,071.46	396,348.54	283,942.46	221,185.19	153,371.44	86,818.12
10,000,000	1,254,253.45	955,495.44	771,533.85	556,933.43	435,694.66	303,517.51	172,597.96

Table 2  
 $Q(t_p)$ , WITH NEGATIVE SKIN EFFECT  
AND INCREASED WELL BORE RADIUS

$t_p, R_w$	S = -2.5 $Q(t_p, R_w)$	S = -5.0 $Q(t_p, R_w)$
100	182.22	1,674.65
150	234.56	2,051.01
200	281.49	2,368.78
250	324.93	2,770.49
300	365.02	3,047.58
400	398.70	3,548.24
500	514.70	3,988.43
600	582.60	4,393.61
700	647.52	4,770.27
800	709.23	5,124.01
900	769.78	5,458.81
1,000	828.84	5,777.77
1,500	1,117.11	7,192.51
2,000	1,267.18	8,441.63
2,500	1,610.07	9,653.75
3,000	1,838.37	10,814.54
4,000	2,290.38	12,727.76
5,000	2,719.34	14,373.57
6,000	3,124.32	15,895.16
7,000	3,519.85	17,274.23
8,000	3,904.44	18,691.63
9,000	4,280.12	20,026.88
10,000	4,649.11	21,293.18
15,000	6,437.71	27,105.95
20,000	8,127.06	32,542.96
25,000	9,767.23	37,441.86
30,000	11,335.03	42,063.45
40,000	14,394.40	50,599.79
50,000	17,314.46	58,567.41
60,000	20,185.85	66,241.42
70,000	22,962.52	73,451.12
80,000	25,701.76	80,375.35
90,000	28,409.01	87,097.38
100,000	30,723.51	93,617.86
150,000	43,886.62	124,177.12
200,000	56,036.66	152,099.81
250,000	67,885.29	179,007.53
300,000	79,621.83	204,494.10
400,000	102,525.03	252,437.50
500,000	124,588.99	297,587.73
600,000	146,169.99	342,794.13
800,000	188,363.44	427,268.15
1,000,000	229,423.68	507,135.13
5,000,000	970,360.46	1,855,421.76
10,000,000	1,819,135.77	3,298,291.45

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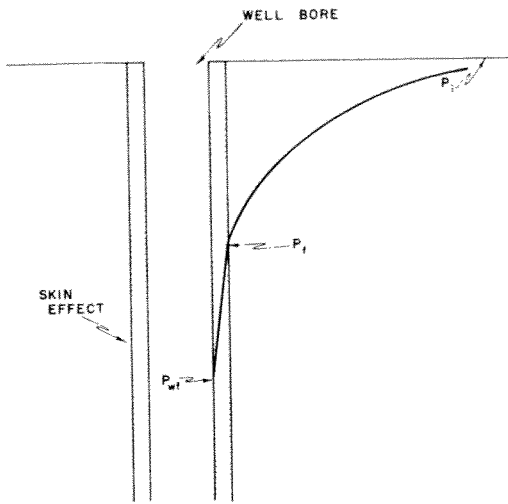


Fig 1 - DAMAGED WELL BORE

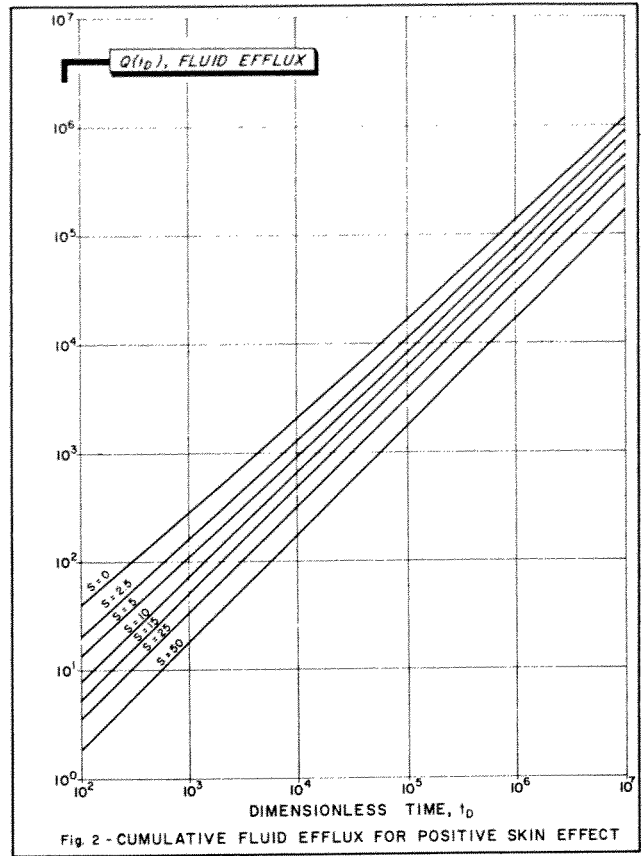


Fig 2 - CUMULATIVE FLUID EFFLUX FOR POSITIVE SKIN EFFECT

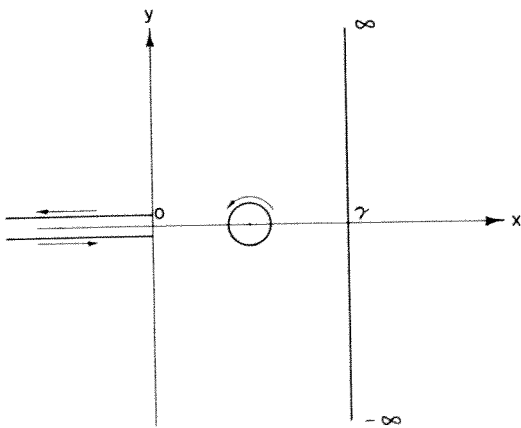


Fig 3 - GRAPHICAL ILLUSTRATION OF 'CUT' IN MELLIN'S INVERSION FORMULA

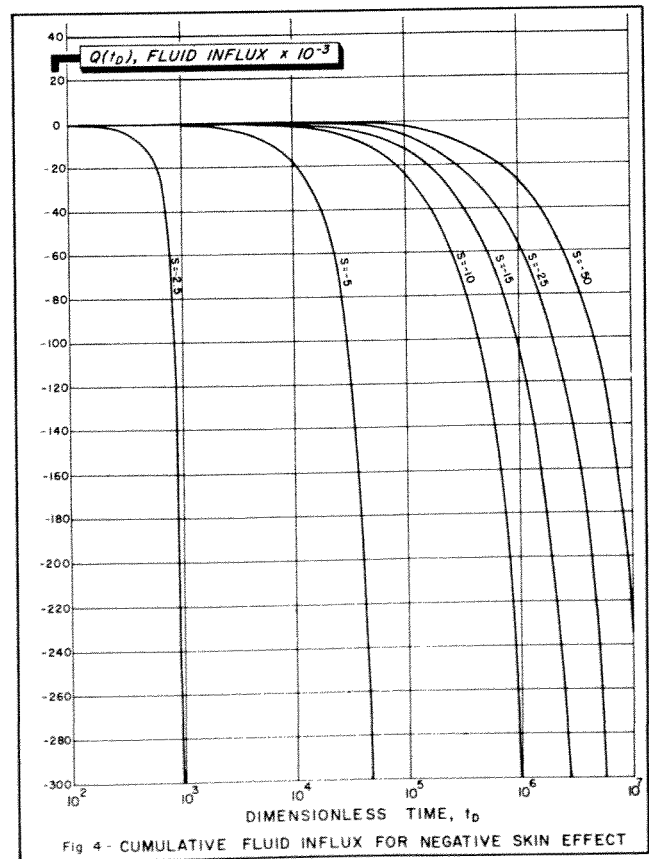


Fig 4 - CUMULATIVE FLUID INFLUX FOR NEGATIVE SKIN EFFECT

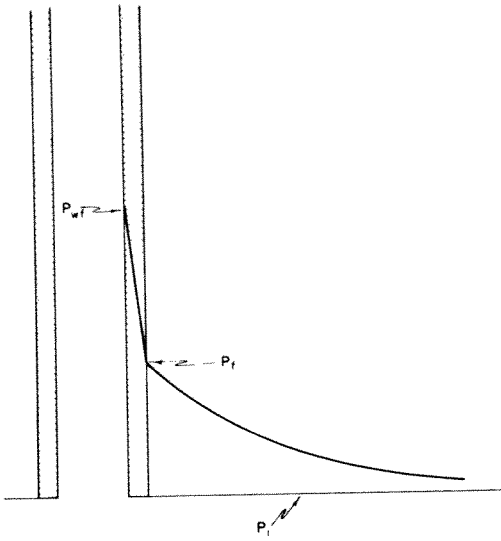


Fig 5 - FLUID INJECTION SUPERIMPOSED ABOVE THE INITIAL PRESSURE

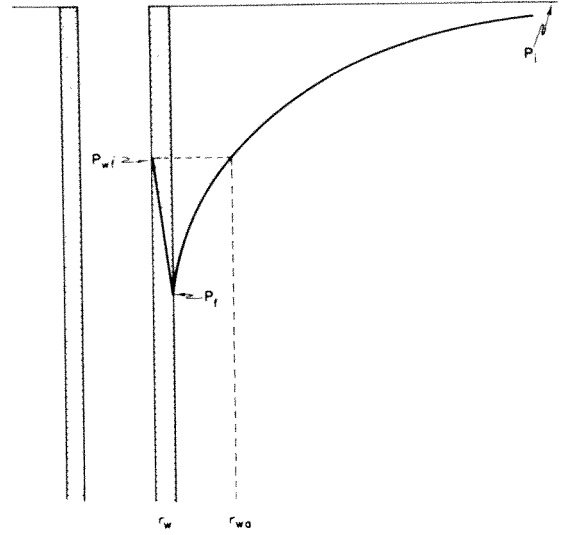


Fig 6 - EFFECTIVE WELL BORE RADIUS FOR NEGATIVE SKIN

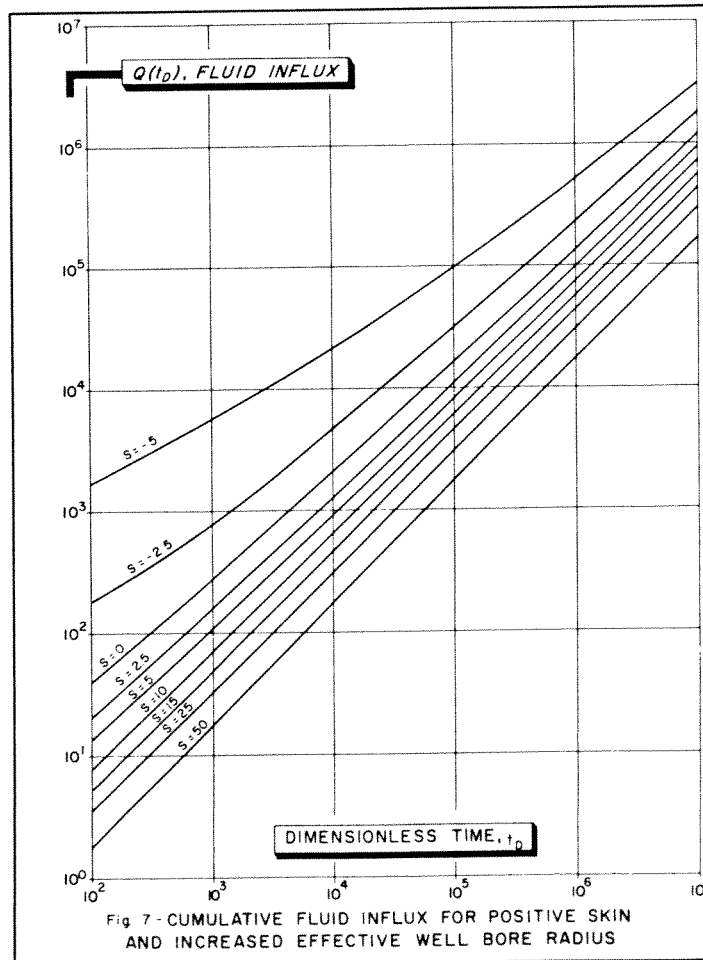


Fig 7 - CUMULATIVE FLUID INFLUX FOR POSITIVE SKIN AND INCREASED EFFECTIVE WELL BORE RADIUS