Thermal Pulses with MESA: Resolving the Third Dredge-Up

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ABSTRACT

The Thermally Pulsing- (Super) Asymptotic Giant Branch is a late stage in the evolution of low- and intermediate-mass stars. These stars undergo strong wind mass loss and diverse nucleosynthesis, responsible for enriching the surfaces of Asymptotic Giant Branch stars, hence an understanding of this process is crucial for constraining galactic chemical evolution. Using a custom numerical scheme, we investigate the temporal and spatial resolution required to resolve the third dredge-up in the 1D stellar evolution code MESA. With MESA’s default controls, the third dredge-up efficiency is underestimated by as much as \( \approx 76\% \). In stars that undergo hot third dredge-up \( (M \gtrsim M_\odot) \), the third dredge-up efficiency is overestimated by \( \gtrsim 55\% \). The Thermally Pulsing- (Super) Asymptotic Giant Branch (TP-(S)AGB) evolution is computed for models with initial masses \( 1 \leq M_i/M_\odot \leq 8 \) at Solar metallicity \( (Z = 0.014) \). The minimum initial mass for carbon stars falls in the range \( 1.5 \leq M_i/M_\odot \leq 1.75 \), compatible with observations. The use of MESA for TP-(S)AGB evolution is validated by comparison to the widely used MONASH models which show good agreement in the maximum third dredge-up efficiency at initial masses \( M_i \gtrsim 2M_\odot \). We also compare the third dredge-up efficiency in exhibit weaker third dredge-up episodes.

Key words: stars: AGB and post-AGB - winds - carbon - evolution

1 INTRODUCTION

The Asymptotic Giant Branch (AGB) is a late stage in the evolution of low- and intermediate-mass stars \( (0.8 \leq M/M_\odot \leq 8) \) (Busso et al. 1999; Herwig 2005; Karakas & Lattanzio 2014). AGB stars are cool and luminous with degenerate carbon-oxygen cores and large convective envelopes. The thermally pulsing-AGB (TP-AGB) phase starts when the helium-rich intershell region between the hydrogen and helium burning shells becomes thin \( (M_{\text{intershell}} \lesssim 0.001 M_\odot) \). The helium-burning shell ignites in a degenerate He-shell flash, leading to periodic thermal pulses as a result of alternate hydrogen- and helium-shell burning. The He-shell flash leads to temporary convective mixing in the intershell region known as the pulse-driven convection zone (PDCZ). The PDCZ mixes carbon from partial He-shell burning throughout the intershell region. Little oxygen is produced by partial He-shell burning, however depending upon the location of the bottom of the PDCZ, oxygen may also be mixed from the C-O core into the intershell at this stage \( \text{(Herwig 2000; Lugaro et al. 2003).} \) In the aftermath of the He-shell flash, all layers above the middle of the He-shell expand and cool, causing the bottom of the convective envelope to extend further towards the core. If the He-shell flash releases sufficient energy, the envelope’s lower convective boundary can dredge past the temporarily extinct hydrogen burning shell into the intershell region. This is known as the third dredge-up (TDU) and leads to the mixing of CO-enhanced material into the H-rich envelope. If the envelope is sufficiently enriched in carbon such that the ratio of carbon to oxygen atoms, C/O, exceeds unity the AGB star may be observed as a carbon star.

TDU can also lead to the production of s-process elements via the formation of a \(^{13}\text{C}\) pocket in the intershell region between thermal pulses. Once produced, s-process elements are mixed throughout the intershell region by the PDCZ and then mixed into the stellar envelope during the next TDU event. The detection of technetium \( (\text{Tc}) \), an s-process element with no stable isotopes, in giant star envelopes is a tracer of active s-process nucleosynthesis and TDU in TP-AGB stars. In intermediate-mass AGB stars \( (M \gtrsim 4M_\odot) \), the temperature at the base of the convective envelope is sufficient that the CN cycle operates, converting carbon into nitrogen in the envelope. This is known as hot bottom burning (HBB) and produces nitrogen-rich stars \( \text{(Wood et al. 1983; McSaveney et al. 2007; Pols et al. 2012; Fernández-Trincado et al. 2020).} \) AGB stars experience strong stellar winds, which remove the stellar envelope until thermal pulses cease and the star evolves to the post-AGB. The star contracts as nuclear burning in the hydrogen-shell ceases and the star eventually moves onto the white dwarf (WD) cooling track. The nucleosynthesis products that enrich the envelope, including CNO and s-process elements, are expelled into the interstellar medium by the wind and contribute towards chemical evolution. In particular, AGB stars are thought to be the main contributors to the production of s-process elements \( \text{(Travagli et al. 2001; Straniero et al. 2006; Kobayashi et al. 2020; Prantzos et al. 2020).} \) A detailed review of the evolution of AGB stars is conducted in \( \text{Herwig (2005) whilst a stronger focus on the nucleosynthesis of AGB stars is given in Karakas & Lattanzio (2014).} \)

The upper mass limit of AGB stars with carbon-oxygen cores...
then introduce and calibrate our technique for resolving the TDU in N. Rees et al. \( \sim \) (Doherty et al. 2017), carbon ignites on the early-AGB (EAGB). The required temperature for oxygen burning is not reached, thus AGB-like stars with ONe cores are produced, known as super-AGB (SAGB) stars. SAGB stars undergo a thermally pulsing phase (TP-SAGB) but due to the larger core mass the interpulse period is shortened compared to TP-AGB stars. They undergo hundreds of thermal pulses before the envelope is removed and thermal pulses cease. In this work we are interested in the TDU properties of both AGB and SAGB stars and so we refer to them collectively as (S)AGB stars.

The efficiency of TDU is crucial to our understanding of (S)AGB stars and their chemical contribution to the Universe. As well as mixing nucleosynthesis products into the stellar envelope, TDU limits the overall growth of the core on the TP-(S)AGB. Thus, it impacts the final masses of white dwarf remnants. The efficiency of TDU has mostly been studied using 1D hydrostatic models (e.g., Karakas et al. 2002; Karakas 2014; Cristallo et al. 2015; Ventura et al. 2018). However, much of the physics of convection is uncertain and simplifications are made to fit convection into a 1D framework. The TDU is highly sensitive to different numerical treatments of the convective region (Frost & Lattanzio 1996). The most common approach is to implement mixing-length theory (MLT, Böhm-Vitense 1958) which requires input parameters such as the mixing-length parameter and convective boundary mixing parameters. 2D and 3D hydrodynamical models are starting to constrain the physics of convection, along with calibration of MLT parameters for 1D models (e.g., Freytag et al. 2018; Sonoi et al. 2019; Higl et al. 2021; Goldberg et al. 2022).

We check the TDU is resolved by finding the necessary controls such that its efficiency does not change with increased spatial or temporal resolution. In Section 2 we describe our chosen physics, in particular the wind mass-loss recipe and treatment of convection. We then introduce and calibrate our technique for resolving the TDU in Section 3. In Section 4 we present model results for a range of initial masses.

2 STELLAR MODELLING

In all our models we use the 1D stellar-evolution code MESA (version 15140, Paxton et al. 2011, 2013, 2015, 2018, 2019; Jerjen et al. 2022a). MESA is open-source software used to model many different phases of stellar evolution. This paper, along with future work, focuses on developing tools for TP-(S)AGB evolution to make the production of resolved and complete models available to researchers anywhere in the world. The use of MESA for TP-AGB models was recently validated by comparison to the MONASH code by Cinquegrina et al. (2022).

We start by discussing some choices of important controls. Full MESA controls, including inlists and run_star_extras code, are available on the MESA marketplace. A future publication will cover the controls used to ensure convergence along the TP-(S)AGB as well as a discussion of instabilities that occur near the end of the TP-(S)AGB. Models are evolved using MESA from the pre-main sequence to the onset of the first thermal pulse to provide starting models for the TP-AGB. As the preceding evolution is not a focus of this work we give a brief overview of the chosen controls for the evolution up to the TP-AGB. Pre-main sequence models are produced with a metallicity \( Z = 0.0014 \) and abundances from Asplund et al. (2009). We use the nuclear-reaction network sagb_NeNa_MgA1.net which contains the main reactions to follow the carbon burning in SAGB models as well as the NeNa and MgA1 cycles. We use low-temperature opacity tables computed using the AESOPUS tool (Marigo & Aringer 2009) with Asplund et al. (2009) solar abundances. In particular, we use the AESOPUS_AGSS09.h5 table which is provided in MESA as standard. This provides opacities as a function of CNO enhancement which is vital for the TP-AGB where TDU and HBB greatly impact CNO abundances in the stellar envelope.

2.1 Wind mass loss

Mass loss via a stellar wind is neglected until the AGB to be consistent with Karakas (2014). The mass-loss prescription used on the AGB is the stellar wind of Vassiliadis & Wood (1993). For \( M_t \leq 2.5 M_\odot \),

\[
\log \dot{M} (M_\odot \text{yr}^{-1}) = -11.4 + 0.0123 P / \text{days},
\]

while for \( M_t > 2.5 M_\odot \),

\[
\log \dot{M} (M_\odot \text{yr}^{-1}) = -11.4 + 0.0125 [P / \text{days} - 100(M_t/M_\odot - 2.5)].
\]

At long pulsation periods (\( P \sim 500 - 750 \text{ days} \)) the mass-loss rate is capped by the radiation-pressure-driven wind limit, Eqs. (1)-(7) constitute a semi-empirical formula between Mira pulsation period and mass-loss rate which results in a superwind phase at the limit of the TP-AGB during which mass-loss rates reach \( M \sim 10^{-4} M_\odot \text{yr}^{-1} \). To calculate the Mira pulsation period we use a fit to pulsation models from Trabucchi et al. (2019, Eq. 12) which has bicubic dependences on mass and radius as well as linear dependences on metallicity, helium mass fraction and carbon-to-oxygen ratio. Compared to the period-mass-radius relation of Vassiliadis & Wood (1993, Eq. 4) this relation gives consistently longer pulsation periods for a given initial mass. The difference is small at the lowest masses but widens as the mass increases, changing from a factor of \( \approx 1.03 \) at \( M_t = 1 M_\odot \) to \( \approx 1.05 \) at \( M_t = 8 M_\odot \). This does not impact the mass-loss rate whilst in the superwind phase because it is limited by the radiation-pressure-driven wind limit which does not depend on the pulsation period. However, the longer pulsation period does cause the star to enter the superwind phase earlier, potentially reducing the total number of thermal pulses. In Fig. 1 we compare the pulsation period and resulting mass-loss rate for a \( M_t = 3 M_\odot \) model sequence. Despite the longer pulsation period, by a factor \( \approx 1.2 \), there is only a small difference in the time at the onset of the superwind. However, the higher the initial mass the greater the impact and at initial masses 4, 5, and 7 \( M_\odot \) the difference in the thermal-pulse number at the superwind onset is approximately 1, 3, 6 and 12 respectively. In model sequences with \( M_t \gtrsim 7 M_\odot \) the pulsation period is long enough for non-negligible mass loss on the EAGB which does not occur with the Vassiliadis & Wood (1993, Eq. 4) period-mass-radius relation.

2.2 Treatment of convection

Convection is implemented using the MLT (Böhm-Vitense 1958) formalism of Henyey et al. (1965). The Schwarzschild criterion is used as we find using the Ledoux criterion makes negligible difference on the AGB. For the mixing length parameter, we use the solar

\(^2\) Note there was a typo in Vassiliadis & Wood (1993) and the correct version of Eq. (1) is \( \log \dot{M} (M_\odot \text{yr}^{-1}) = -11.4 + 0.0125 P / \text{days} \).

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1 https://cococubed.com/mesa_market/inlists.html
Resolving the Third Dredge-Up

Figure 1. The pulsation period and wind mass loss rate of a $M_1 = 3 M_\odot$ model computed with the Trabucchi et al. (2019) period-mass-radius relation (orange, solid line) compared to the Vassiliadis & Wood (1993, Eq. 4) period-mass-radius relation (blue, dashed line). Both use the period-mass-loss relation in Eqs. (1)-(?)). The Trabucchi et al. (2019) relation gives consistently higher pulsation periods which cause the superwind phase to begin earlier.

calibration $\alpha_{\text{mht}} = 1.931$ of Cinquegrana & Joyce (2022), which was conducted using the AESOPUS opacities.

As standard, convective boundaries in MESA are located where the sign of the discriminant $y = \nabla_{\text{rad}} - \nabla_{\text{ad}}$, or $y = \nabla_{\text{rad}} - \nabla_{L}$ for the Ledoux criterion, changes, where $\nabla_{\text{rad}}$ and $\nabla_{\text{ad}}$ are the adiabatic and radiative temperature gradients respectively and $\nabla_{L}$ is the Ledoux temperature gradient (Paxton et al. 2013, Eq 11). More recent versions of MESA include new prescriptions for calculating convective boundaries, predictive mixing (Paxton et al. 2018) and convective premixing (Paxton et al. 2019). These schemes notably increase the mass of convective cores on the main sequence (MS) and core helium-burning (CHeB), such that they are more consistent with observational constraints (Gabriel et al. 2014). However, we find these schemes cause convergence problems during TDU and so we disable them in the (S)AGB phase.

In MESA, like many stellar evolution codes, convective boundary mixing is implemented by an overshooting scheme. On the MS we use the convective premixing scheme and step overshooting with overshooting parameter determined by the fit from Jermy et al. (2022), Eq (11) to the convective penetration found in Anders et al. (2022). During CHeB we neglect overshooting but utilise predictive mixing upon recommendation of Ostrowski et al. (2021).

On the (S)AGB we use the exponential diffusive overshoot of Herwig (2000), motivated by two- and three-dimensional hydrodynamical simulations of convection that show an exponentially decaying velocity field beyond the convective boundary (Freytag et al. 1996). A free parameter, $f$, decides the efficiency of the extra diffusive mixing and thus how far it extends beyond the convective boundary. A second free parameter, $f_0$, is chosen such that the switch from convective mixing to overshooting occurs at a distance $f_0 \, \mathcal{H}_P$ into the convection zone from the formal Schwarzschild boundary, where $\mathcal{H}_P$ is the pressure scale height. In this work we always set $f_0$ to be one-tenth of the overshooting parameter $f$. Exponential overshooting is implemented in the carbon-oxygen core with $f = 0.005$ (Jones et al. 2013) on the EAGB, which affects the convective regions that form during carbon burning in SAGB models.

The efficiency of third dredge-up in TP-(S)AGB models depends on the choice of convective boundary mixing. Herwig (2000) discusses how the efficiency of overshooting affects various quantities including the interstellar abundances which consequently affects the production of s-process elements and the abundance and composition of carbon stars. Furthermore, $^{13}\text{C}$ pockets can be formed by the inclusion of overshoot (Herwig 2000; Cristallo et al. 2009), which are crucial for the production of s-process elements (Lugaro et al. 2003; Straniero et al. 2006). Theoretically, such quantities could constrain the efficiency of third dredge-up and the required overshooting parameters. However, it is very difficult to disentangle the effects of overshooting from other parameters such as the wind mass loss rate, mixing-length parameter and opacities. In addition, stellar-evolution codes vary and so a particular choice of overshooting in one code does not necessarily lead to the same results in another code. Nevertheless, using a combination of 1D stellar evolution models, 2- and 3-D hydrodynamical simulations and observations, constraints exist to assist with the choice of overshooting parameters in low- and intermediate-mass AGB stars (Herwig 2000; Lugaro et al. 2003; Herwig 2004a,b; Herwig et al. 2007; Herwig 2008).

Overshooting at the bottom boundary of the PDCZ has two important consequences. First, it alters the interstellar abundances due to the mixing of mainly carbon- and some oxygen-rich material from the core. Secondly, extra mixing leads to a stronger He-shell flash, which leads to more efficient third dredge-up. We apply exponential overshooting at the bottom of the PDCZ, $f_{\text{PDCZ}} = 0.008$, motivated by hydrodynamic He-shell flash convection simulations (Herwig 2008). Overshooting at the top boundary of the PDCZ does not affect the strength of helium burning and so we do not include it. Overshooting at the bottom of the convective envelope (CE) also impacts the depth reached during third dredge-up, and thus the surface enrichment of elements. Following Herwig (2000) we use a CE overshooting parameter $f_{\text{CE}} = 0.016$.

2.3 Hot third dredge-up

We find that HTDU occurs in our models when $M_1 \gtrsim 5 M_\odot$. In models with $M_1 \gtrsim 6 M_\odot$, it can result in a runaway dredge-up which leads to computational difficulties. However, this runaway situation does not occur when there is no convective envelope overshooting and instead stable HTDU occurs (Fig. 4). Jones et al. (2016) discuss that HTDU should stiffen the boundary between radiative and convective regions, due to the entropy barrier produced by hydrogen burning, and therefore lower the efficiency of convective boundary mixing. However, the associated overshooting is unknown and we require detailed hydrodynamical models to investigate. In the absence of a mass-dependent overshooting prescription we turn off convective envelope overshooting during TDU when the temperature at the base of the convective envelope exceeds $4 \times 10^7$ K. This is the temperature
at which Goriely & Siess (2004) found the s-process to be inhibited due to HTDU within a diffusive-convective overshooting scheme. In our models, the core boundary is defined to be the outermost location where the $^1$H mass fraction is less than 0.01 and the $^4$He mass fraction is greater than 0.1. We define the base of the convective envelope to be the bottom of all mixing (either in the overshooting or fully convective regions) above the core boundary as we find some splitting of the convective region in massive models without envelope overshooting.

3 RESOLVING THIRD DREDGE-UP

To ensure that the He-shell flash is correctly resolved, we use the control `delta_lgl_He_limit = 0.01` to keep changes in the logarithm of the helium burning luminosity ($L_{He}$) to less than 0.01 from one model to the next. This forces the timestep to drop as the helium-burning luminosity increases during the flash. This is essential to ensure that the following thermal pulse cycle is correctly resolved as many quantities, including the TDU efficiency, depend on the peak helium burning luminosity in each pulse. However, to resolve the TDU we also require sufficient spatial and temporal resolution during the dredge-up phase. In MESA, there is no specific timestep control that limits the change in core mass due to the motion of convection regions during a dredge-up event. Instead, during this phase the timestep is limited by the `varcontrol_target ($v_t$)`, which is the target value for relative variation in the structure from one model to the next. The relative variation is calculated as the unweighted average over all cells of the relative changes in the logarithms of the density, temperature and radial co-ordinate. The timestep is increased or decreased depending on whether the actual variation is smaller or greater than the $v_t$ which has a default value of $10^{-4}$ (Paxton et al. 2011, Eq 16). We increase the temporal resolution during the TDU by decreasing the $v_t$.

To control the spatial resolution of models, MESA uses an adaptive meshing scheme (Paxton et al. 2011, Sec 6.5). At the start of each timestep, the structure and composition profiles of the model are checked and, if necessary, remeshing splits cells into two of more pieces or merges two or more adjacent cells. There are a number of controls to determine which cells should split or merge based on allowed changes in certain quantities (pressure, temperature, abundances) between adjacent cells. To increase the spatial resolution in the intershell region and primarily around the H-He core boundary we set the control `mesh_logX_species = 'he4'`. The mesh resolution is controlled with the corresponding control `mesh_dlogX_dlogP_extra ($m_X$)`. This increases the resolution in regions where the gradient of the abundance of the chosen species (in this case $^4$He) with respect to pressure is large. $m_X$ has a default value of 1.0 and the smaller the value, the higher the mesh resolution. Halving $m_X$ approximately doubles the number of mesh points in the relevant region.

Increased resolution also increases computation time. To minimise this increase, we use a custom routine which detects the inward movement of the base of the convective envelope and only temporarily increases the resolution during this phase. The routine requires as input $v_t$, $m_X$ during TDU that are smaller than the defaults of $10^{-6}$ and 1.0 respectively. At the end of each timestep, the routine checks if the base of the convective envelope has moved inwards in mass compared to the last timestep. If so, $v_t$ and $m_X$ are decreased by increments of $10^{-6}$ and 0.01 respectively, provided they exceed the target $v_t$ and $m_X$ during TDU. If the convective envelope has moved outwards in mass, $v_t$ and $m_X$ are instead increased by the same increments, provided they are currently less than the default $v_t$ and $m_X$. This ensures the time and spatial resolution is smoothly increased at the start of TDU and smoothly decreased again at the end.

We quantify the extent of TDU with the TDU efficiency parameter,

$$\lambda = \frac{\Delta M_{\text{dredge}}}{\Delta M_{\text{H}}}$$

where $\Delta M_{\text{dredge}}$ is the mass dredged up from the intershell region and $\Delta M_{\text{H}}$ is the increase in core mass during the preceding interpulse period (Izzard et al. 2004, Fig 1). When $\lambda < 1$, the core mass increases over the entire thermal pulse cycle whilst when $\lambda > 1$ the core mass decreases. In this work we test whether TDU is resolved by computing $\lambda$ for various choices of the resolution parameters $v_t$, $m_X$, and $m_{X,\text{TDU}}$. Marigo (2022) used the observed masses of WDs to constrain $\lambda$. This is possible because the growth of the H-exhausted core whilst on the TP-(S)AGB is limited by the number of TDU events and the efficiency of each event. Thus, assuming that the core mass at the onset of thermal pulses and the wind mass-loss rate are known, the final WD mass can be used to constrain the TDU efficiency. Using this uncertainty as a benchmark, we accept TDU to be sufficiently converged when the changes in $\lambda$ compared to our highest resolution run are less than $\sim 5\%$.

In Fig. 2 we compare the TDU efficiency, $\lambda$, for different choices of $v_t$, $m_{X,\text{TDU}}$, and $m_{X}$ at the 15th thermal pulse of an $M_i = 3 \, M_\odot$ star with H-He core boundary (panel a) and the 9th thermal pulse of an $M_i = 5 \, M_\odot$, $Z = 0.014$ model star (panel b). If the resolution is sufficient, the TDU efficiency converges to $\lambda \approx 0.84$ and $\lambda \approx 0.89$ respectively. However, using the default $v_t$, $m_{X,\text{TDU}} = 10^{-4}$ underestimates $\lambda$ by $\approx 42\%$ and $76\%$ respectively. The temporal resolution has the greatest impact whilst the spatial resolution changes $\lambda$ by $\leq 1\%$ if the temporal resolution is sufficient ($v_t$, $m_{X,\text{TDU}} \leq 2 \times 10^{-5}$). To strike a balance between accuracy and computation time we suggest a combination of $v_t = 2 \times 10^{-5}$ and $m_{X,\text{TDU}} = 1.0$. Any further increases in resolution lead to changes in $\lambda$ of $\leq 5\%$. This should also lead to errors in the total mass dredged-up during the TP-(S)AGB of $\leq 5\%$ which is likely small compared to the uncertainties in other chosen physics and within our benchmark set by the error in WD masses.

In Fig. 3 we compare the model evolution with the default MESA temporal resolution and our chosen $v_t$, $m_{X,\text{TDU}} = 2 \times 10^{-5}$ in both TDU events. The increased resolution increases the computation time of the thermal-pulse cycles by approximately 17% and 27% respectively.

3.1 Resolving hot third dredge-up

We also perform a resolution study for the 17th thermal pulse of an $M_i = 7 \, M_\odot$ star with HTDU (panel c, Fig. 2). The TDU efficiency converges to $\lambda \approx 1.01$ with sufficient resolution, but the default resolution model sequence overestimates the TDU efficiency by $\sim 55\%$. The TDU efficiency is not a smooth function of temporal resolution due to the computation difficulties that arise from very efficient TDU leading to hydrogen burning. In low- and intermediate-mass stars we find resolution parameters $v_t$, $m_{X,\text{TDU}} = 2 \times 10^{-5}$ and $m_{X} = 1.0$ are sufficient to resolve $\lambda$ to within $5\%$. With these choices, $\lambda$ differs compared to the highest-resolution run by less than $\sim 3\%$. Thus, we find the same choice of $v_t$, $m_{X,\text{TDU}} = 2 \times 10^{-5}$ and $m_{X} = 1.0$ are suitable for resolving HTDU. A comparison of our imposed resolution with the default MESA controls, $v_t$, $m_{X,\text{TDU}} = 2 \times 10^{-5}$ and $m_{X} = 1.0$, is shown in Fig. 4. The default model experiences stronger hydrogen burning because of its deeper TDU.
evolve as far as the end point of the TP-AGB defined by Vassiliadis & Wood (1993) which requires the star to move away from the AGB by an amount $\Delta \log T_{\text{eff}} = 0.3$. stars with $M_i \geq 1.75 M_\odot$ terminate before this due to instabilities when the envelope mass is reduced (Wagenhuber & Weiss 1994; Lau et al. 2012). These instabilities will be discussed in a later paper but early termination should not affect the maximum TDU efficiency because this decreases as the envelope mass reduces (Straniero et al. 2003; Cristallo et al. 2015; Marigo 2022).

Fig. 5 shows the mass dredged-up, $\Delta M_{\text{dredge}}$ (top) and TDU efficiency, $\lambda$ (bottom), vs the core mass, $M_c$, in all our computed models. Details of the individual models are in Table 1. In low-mass stars, $M_i \leq 2.75 M_\odot$, the core mass must grow to a mass $M_c \gtrsim 0.56 M_\odot$ before TDU starts. Thus, TDU only occurs in models with, $M_i \geq 1.5 M_\odot$. The final C/O ratio is limited by the number and strength of TDU episodes before the envelope is removed by the stellar wind. We find the minimum initial mass to produce a carbon star is in the range $1.5 \sim 1.75 M_\odot$. This is compatible with results from other theoretical models and observations of carbon stars that place the minimum mass in the range $1.4 \sim 2 M_\odot$ (Karaka & Lugero 2016). In particular, Groenewegen et al. (1995) found a minimum initial mass of $\approx 1.5 M_\odot$. In all our models that terminate early, the final C/O ratio is underestimated as the ratio continues to increase in the final thermal pulses. The amount of extra material that would be dredged-up depends on the strength of further TDU episodes as the envelope mass is reduced. Thus, the upper initial stellar mass limit for carbon star production predicted by these models is particularly uncertain. In intermediate- and high-mass stars with HBB ($M_i \gtrsim 4.5 M_\odot$), the final C/O ratio is an interplay between HBB and TDU, the latter of which dominates at the end of the TP-(S)AGB when HBB ceases. Thus intermediate and massive stars may become carbon stars in the last thermal pulses which are not computed in this work (Frost et al. 1998; van Loon et al. 1999).

In our $M_i = 7, 7.5$ and $8 M_\odot$ stars we find an oscillating TDU efficiency between consecutive pulses. This behaviour is seen more clearly in Fig. 6 (b) where $\lambda$ is plotted as a function of the total stellar mass remaining for each initial mass. In the $M_i = 7.5$ and $8 M_\odot$ stars, the oscillation is damped out after a number of pulses however in the $M_i = 7 M_\odot$ star it is still occurring when the model terminates. This oscillation leads to more efficient TDU ($\lambda \gtrsim 1$) than is reached at lower mass.

5 DISCUSSION

We validate our MESA models by comparison to the widely used MONASH models (Karaka 2014; Karaka & Lugero 2016) at solar metallicity. the MONASH models were computed with the period-mass-radius relation of Vassiliadis & Wood (1993), which as discussed in section 2.1 delays the onset of the superwind phase as compared to the fit from Trabucchi et al. (2019). In addition, the treatment of convection is quite different. The MONASH models were not computed with convective overshooting, but instead with an algorithm to search for a neutrally stable point at the border between convective and radiative zones (Lattanzio 1986). An additional difference between the two sets of models is the definition of the H-exhausted core. In our models we use the MESA convention that the core boundary is at the bottom of the H-burning shell where the $^1$H mass fraction, $X = 0.01$. The MONASH models define the core boundary to be the middle of the H-burning shell where $X = 0.35$. However, the mass of the H-burning shell is thin in (S)AGB stars so this disparity should not make a substantial difference to the mass of

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{TDU efficiency, $\lambda$, as a function of the spatial ($m_X, \text{TDU}$) and temporal ($v_t, \text{TDU}$) resolution in the 15th thermal pulse of an $M_i = 3 M_\odot$ star (panel a), the 9th thermal pulse of an $M_i = 5 M_\odot$ star (panel b) and the 17th thermal pulse of an $M_i = 7 M_\odot$ star (panel c). See the text for explanation of the resolution parameters $m_X$ and $v_t$.}
\end{figure}
the H-exhausted core. A detailed comparison between the two stellar evolution codes is given in Cinquegrana et al. (2022).

5.1 Third dredge-up efficiency

In Fig. 6 we compare the TDU efficiency evolution at each initial mass in our MESA models (top) with the MONASH models (middle). We plot the $\lambda$ evolution as a function of both core mass and total mass which is useful to examine how the TDU changes as the envelope mass decreases. At low initial masses, our MESA models have slightly smaller core masses throughout the TP-(S)AGB, whilst at high initial mass, they have higher core masses. For example, the $M_i = 7 \, M_\odot$ MESA model and $M_i = 8 \, M_\odot$ MONASH model have comparable core masses of $\sim 1.05 \, M_\odot$, right on the SAGB boundary. In the MESA models, TDU kicks in at $M_c \gtrsim 0.56 \, M_\odot$. This minimum core mass depends on the amount of convective envelope overshooting used. In the MONASH models, computed without overshooting, TDU requires a more massive core, $M_c \gtrsim 0.6 \, M_\odot$, and for $M_i \leq 0.64 \, M_\odot$ stars require a number of thermal pulses ($\sim 10$–20) before TDU starts. Both sets of models show the same pattern of increasing maximum TDU efficiency with core mass to a peak $\lambda \sim 0.95$–1.0 around $M_c \sim 0.8 \, M_\odot$, followed by a gradual decrease at higher core masses (with the exception of the MESA SAGB models). The MONASH mostly do not show the oscillating TDU efficiency exhibited by the $M_i = 7, 7.5$ and $8 \, M_\odot$ MESA models with the exception of a few thermal pulses in the $8 \, M_\odot$ model. This may suggest that an oscillating TDU behaviour is a feature of SAGB models with core masses $\gtrsim 1.05 \, M_\odot$.

In Fig. 7 we compare the maximum TDU efficiency, $\lambda_{\text{max}}$, reached at each initial mass. The MESA and MONASH models show good agreement in the mass range $2 - 6 \, M_\odot$. Due to the sharp onset of TDU at lower core masses in the MESA models, TDU occurs at lower initial masses ($M_i \gtrsim 1.5 \, M_\odot$) than in the MONASH models ($M_i \gtrsim 2 \, M_\odot$). Due to the observational evidence that carbon stars can form at masses as low as $\sim 1.5 \, M_\odot$, Karakas & Lugaro (2016) also compute 1.5 $M_\odot$ and 1.75 $M_\odot$ models with an additional convective overshoot at the base of the convective envelope (Kamath et al. 2012). These models show efficient dredge-up with $\lambda_{\text{max}} \sim 0.5$, which is consistent with our $M_i = 1.75 \, M_\odot$ model.

We also compare our MESA models to the ATON (Ventura et al. 2018) and FRUITY (Cristallo et al. 2015) models in Fig. 7. At initial masses $\leq 1.5 \, M_\odot$ there is good agreement between MESA, ATON and FRUITY but at all higher masses there is a wide range of TDU efficiencies. The ATON models have the lowest TDU efficiencies with the largest value of $\lambda_{\text{max}} \approx 0.4$ at $M_i = 3 \, M_\odot$ and decreasing to $\approx 0$ at $M_i = 8.5 \, M_\odot$. The FRUITY models reach $\lambda_{\text{max}} = 0.4$–0.6 for $M_i = 2 - 6 \, M_\odot$ but are not computed at higher initial masses. These differ from the MONASH and MESA models which have much more
efficient TDU. The stellar-evolution codes differ in their treatments of convection and mass loss prescriptions. It is likely that both these factors, along with other differences in the evolution codes, result in the range of TDU efficiencies.

It is interesting to note that the MESA and MONASH models, which both use the mass-loss prescription of Vassiliadis & Wood (1993), have much better agreement at $M_i = 1.5 M_\odot$ than between any other set of models. The FRUITY models use a mass-loss-period relation derived from infrared observations of AGB stars (Straniero et al. 2006) whilst ATON models use the Bloecker (1995) prescription for oxygen-rich models and a carbon-star prescription from Wachter et al. (2008). Both these prescriptions lead to non-negligible mass-loss rates on the entire TP-(S)AGB.

In addition, the FRUITY models experience strong mass loss even on the EAGB with the $M_i = 0.6 M_\odot$ model already losing $\approx 1 M_\odot$ before the onset of thermal pulses. In comparison, the Vassiliadis & Wood (1993) prescription used in our MESA and the MONASH models has negligible mass loss until the super wind phase (Straniero et al. 2006, Fig. 6).

To examine the impact of the wind mass loss regime we include the $\lambda$ evolution of the FRUITY models in Fig. 6 (bottom). The curves have a very different characteristic shape compared to the MESA and MONASH models. In the latter (excluding SAGB models), there is negligible mass loss for a number of thermal pulses causing $\lambda$ to increase at constant $M$. When the superwind eventually kicks in, the envelope is quickly lost and few thermal pulses occur with a reduced envelope mass. In contrast, the FRUITY models with a more constant mass loss show a smooth increase and then decrease in $\lambda$ as a function of $M$. Non-negligible mass loss early on the TP-AGB removes some envelope before the thermal pulses build up to high values of $\lambda$ and so this gives lower $\lambda_{\max}$. The MESA SAGB models, which experience the superwind for the majority of their thermal pulses, exhibit a similar behaviour to the lower-mass FRUITY models where $\lambda$ is a function of the total mass, although they terminate due to instabilities before $\lambda$ reduces significantly.

The treatment of convection also impacts the TDU efficiency. As discussed in section 2.2, convective-boundary mixing at the bottom of the PDCZ increases the strength of the He-shell flash and thus leads to more efficient TDU. The efficiency of TDU is also directly impacted by the strength of convective boundary mixing at the bottom of the convective envelope. The ATON models use the full-spectrum of turbulence model to calculate the temperature gradient within regions unstable to convection. Beyond the boundary determined by the Schwarzschild criterion, the velocity of convective eddies decay

### Table 1

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Figure 4. A hot third dredge-up event during the 17th thermal pulse in our $M_i = 7 M_\odot$ star with default MESA resolution controls (solid, orange) and our chosen $v_{t,\, TDU} = 2 \times 10^{-5}$ and $m_{X,\, TDU} = 1.0$ (dashed, blue). See section 3 for an explanation of the resolution controls.

Figure 5. The mass dredged-up, $\Delta M_{\text{dredge}}$ (panel a), during TDU and dredge-up efficiency, $\lambda$ (panel b), vs core mass at each thermal pulse in models with $1 \leq M_i/M_\odot \leq 8$ at $Z = 0.014$.

expontentially with an e-folding distance of $0.002 H_\odot$. The FRUITY models use a time-dependent mixing scheme with convective velocities calculated from MLT. Again, mixing beyond the Schwarzschild boundary occurs via an exponential decay in convective velocity but with an e-folding distance of $0.1 H_\odot$. This value was chosen to maximise the mass of the $^{13}$C pocket produced for s-process nucleosynthesis. This convective-boundary mixing approach differs from the exponential convective overshooting of Herwig (2000), which is used in our MESA models, because their velocity is fixed to decline exponentially beyond the boundary instead of the diffusion coefficient. In addition to the convective boundary mixing, the mixing length parameter is also known to affect the third dredge-up efficiency (Boothroyd & Sackmann 1988).

5.2 Core-mass-luminosity relation

Low-mass TP-AGB stars exhibit a linear relationship between the core mass and the maximum surface luminosity reached during the quiescent interpulse phase (Paczyński 1970). In intermediate and high mass stars, the luminosity is increased due to HBB. However, when HBB ceases because the envelope mass is sufficiently reduced, the linear relation is regained. Besides core mass, the radiated luminosity depends on the used reaction rates for hydrogen burning as well as the efficiency of convection at the base of the convective envelope. In Fig. 8 we compare the core-mass-luminosity relation in our MESA models and the MONASH models. Also shown is the fit from Paczyński (1970) and the fit to the MONASH models from Izzard et al. (2004). At $M_C \gtrsim 0.7 M_\odot$ there is strong agreement between the peak luminosities achieved in the MESA and MONASH models, suggesting that the treatment of HBB is consistent. However, at $M_C \lesssim 0.7 M_\odot$, our MESA models are $\approx 2 - 4 \times 10^3 L_\odot$ more luminous than the MONASH models and the fit from Paczyński (1970), indicating a difference in the CNO reaction rates used. The reaction rates in MESA are from JINA REACLIB (Cyburt et al. 2010) whilst the MONASH models used Harris et al. (1983) and Fowler et al. (1975) for hydrogen burning.

6 CONCLUSIONS

We present a custom routine to increase the spatial and temporal resolution during the third dredge-up, in the 1D stellar evolution code MESA. This was used to conduct a resolution study in models with initial masses 3, 5 and 7 $M_\odot$. Whilst the default MESA spatial resolution was found to be sufficient, the default MESA temporal resolution controls underestimates the TDU efficiency by factors of 42%, 52% and overestimates by 55% respectively. We compute the TP-(S)AGB evolution for models with initial masses $1 \leq M_i/M_\odot \leq 8$ at $Z = 0.014$. We find the onset of TDU for core masses $M_C \gtrsim$
Figure 6. TDU efficiency, $\lambda$, as a function of core mass (left) and total mass (right) in our MESA models (top), the MONASH models (middle) and FRUITY models (bottom). Models with the same initial mass are plotted in the same colour.
Figure 7. The maximum TDU efficiency, $\lambda_{\text{max}}$ reached by each initial mass model for our MESA models (orange squares) compared to the MONASH models (blue circles), FRUITY models (green stars) and ATON models (black diamonds). Also plotted are two MONASH models computed with overshooting (blue triangles).

Figure 8. The core-mass-luminosity relation of our MESA models (blue) compared to the MONASH models (orange) and fits from Izzard et al. (2004) (dot-dashed line) and Paczyński (1970) (dashed line).

0.56 $M_\odot$ and the lower boundary for the formation of carbon stars to be in the range 1.5 – 1.75 $M_\odot$. The models are compared to the widely used MONASH models and show good agreement in the maximum TDU efficiency for initial masses $M_i > 2 M_\odot$. A further comparison to the FRUITY and ATON model sets shows the wide range of TDU efficiencies produced by different evolution codes and chosen physics, as well as the importance of the wind mass-loss prescription on the behaviour of TDU efficiency as a function of envelope mass. The core-mass-luminosity relation is also compared between the MESA and MONASH sets of models and shows good agreement in the treatment of HBB. Comparisons between the MESA and MONASH models show that MESA can be reliably used for the computation of thermal pulses and instabilities are provided for this purpose. Future work in an upcoming paper will address the problem of instabilities and convergence issues on the TP-AGB to provide further tools for the use of MESA in modelling this phase of evolution.

ACKNOWLEDGEMENTS

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DATA AVAILABILITY

MESA is open source software available at https://docs.mesastar.org/en/release-r23.05.1/. The inlists and run_star_extras code used will be available at the MESA marketplace https://cococubed.com/mesa_market/inlists.html upon publication. The MESA output data will be available at https://zenodo.org/communities/binary_c-community upon publication.

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