Photometric study of the open cluster – II. Stellar population and dynamical evolution in NGC 559

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ABSTRACT

We present UBVRI photometry of stars in the field of the intermediate-age open cluster NGC 559. By determining the stellar membership probabilities derived through a photometric and kinematic study of the cluster, we identify the 22 most probable cluster members. These are used to obtain robust cluster parameters. The mean proper motion of the cluster is $\mu_x = -3.29 \pm 0.35$, $\mu_y = -1.24 \pm 0.28$ mas yr$^{-1}$. The radial distribution of the stellar surface density gives a cluster radius of 4.5 ± 0.2 arcmin (3.2 ± 0.2 pc). By fitting solar metallicity stellar isochrones to the colour–colour and colour–magnitude diagrams, we find a uniform cluster reddening of $E(B-V) = 0.82 \pm 0.02$. The cluster has an age of $224 \pm 25$ Myr and is at a distance of $2.43 \pm 0.23$ kpc. From the optical and near-infrared two-colour diagrams, we obtain colour excesses in the direction of the cluster $E(V-K) = 2.14 \pm 0.02$, $E(J-K) = 0.37 \pm 0.01$ and $E(B-V) = 0.76 \pm 0.04$. A total-to-selective extinction of $R_V = 3.5 \pm 0.1$ is found in the direction of the cluster which is marginally higher than the normal value. We derive the luminosity function and the mass function for the cluster main sequence. The mass function slope is found to be $-2.12 \pm 0.31$. We find evidence of mass segregation in this dynamically relaxed cluster.

Key words: techniques: photometric – stars: formation – stars: luminosity function, mass function.

1 INTRODUCTION

Systematic photometric studies of Galactic open star clusters (OCs) offer unique opportunities to understand large-scale star formation processes in the Galaxy and in Galactic clusters (Lada & Lada 2003). The precise knowledge of cluster parameters such as age, distance, reddening and chemical composition as well as knowledge of the stellar population distribution and the cluster mass function (MF) at the time of star formation play a key role in understanding the star formation history. The importance of photometric studies of OCs lies in the colour–colour and colour–magnitude diagrams (CMDs) derived through multiband photometric observations. Since most of the OCs are embedded in the Galactic disc and are likely to be affected by field star contamination, it is essential to discriminate between members and non-members of the clusters. The amount of field star contamination depends on the location of the cluster. It is necessary to perform a detailed membership analysis of the stars found within the observed field for a robust investigation of cluster properties (Carraro et al. 2008; Yadav et al. 2008). For most of the OCs, kinematical data are unavailable. However, recent all-sky proper motion catalogues (e.g. Roeser, Demleitner & Schilbach 2010; Zacharias et al. 2013) provide clues to determine cluster membership. Together with a photometric study of the cluster, it becomes possible to draw some conclusions regarding the dynamical evolution of the cluster.

At ARIES, Nainital, we have been carrying out a long-term observational programme to search and characterize the variable stars in Galactic OCs using various 1–2 m class telescopes in India. The advantage of having such observations is that they can also be used to study the physical properties of the clusters and their stellar and dynamical evolution. In Joshi et al. (2012), we performed a photometric study of the intermediate-age open cluster, NGC 6866, which also included a search for variable stars in the cluster. The results presented here for NGC 559 are a continuation of our efforts to understand star formation in some unstudied or poorly studied young- and intermediate-age open clusters.

NGC 559 (RA = 01h29m35s, Dec. = +63° 18′ 14″; l = 127°2, b = +0°75) is a moderately populated and heavily reddened intermediate-age open cluster, classified as type II2m by Trumpler (1930) and II2m by Ruprecht (1966). It is located in the direction of the second Galactic quadrant in the vicinity of the Perseus and Local arms (Russell, Adami & Georgelin 2007). Photoelectric photometry of the cluster was obtained by Lindoff (1969)
and Jenniss & Helfer (1975), while Grubissich (1975) provided photographic photometry of cluster stars. A subsequent investigation using CCD photometry was carried out by Ann & Lee (2002, hereafter AL02) and Maciejewski & Niedzieski (2007, hereafter MN07). However, a complete *UBVRI* study is still lacking. Moreover, there has not been any systematic attempt to identify cluster members in the field of this cluster.

The main focus of the present study is to accurately determine the fundamental parameters of NGC 559 by identifying cluster members using photometric and kinematic criteria. The outline of the paper is as follows. A photometric study of the cluster is presented in Section 2. The cluster properties are discussed in Section 3 and fundamental parameters are derived in Section 4. The dynamical study of the cluster is presented in Section 5. Finally, we discuss the results in Section 6.

## 2 Photometric Study of the Cluster

### 2.1 Observations and Calibration

Johnson–Cousins *UBVRI* photometry of stars in the field of NGC 559 was obtained on 2010 November 30 using the 1 m Sampurnanand telescope at Nainital, India. The telescope is equipped with a 2k × 2k CCD camera which covers a ~13 arcmin × 13 arcmin field of view. We acquired two frames each in *U*, *B*, *V*, *R* and *I* filters with exposure times of 300, 300, 200, 100 and 60 s in respective passbands, respectively, at a typical airmass of about 1.3. On the same night we also observed two Landolt’s standard fields: SA95 and PG0231+051 (Landolt 1992) at different airmasses. The usual image processing procedures were performed which included bias subtraction, flat-fielding and cosmic ray removal. We used the IRAF software package for this purpose.

Photometry of the frames was performed using the DAOPHOT II profile fitting software (Stetson 1987). Details of the photometric calibration obtained on this night are given in Joshi et al. (2012). Transformation coefficients for the standard stars were determined as follows:

\[ u = U + 8.16 \pm 0.01 - (0.05 \pm 0.01)(U - B) + (0.55 \pm 0.02)X \]

\[ b = B + 5.81 \pm 0.02 - (0.01 \pm 0.02)(B - V) + (0.29 \pm 0.03)X \]

\[ v = V + 5.43 \pm 0.01 - (0.08 \pm 0.01)(B - V) + (0.15 \pm 0.01)X \]

\[ r = R + 5.23 \pm 0.01 - (0.09 \pm 0.02)(V - R) + (0.09 \pm 0.02)X \]

\[ i = I + 5.63 \pm 0.02 + (0.01 \pm 0.01)(R - I) + (0.07 \pm 0.02)X \]

where *u*, *b*, *v*, *r* and *i* are the aperture instrumental magnitudes, *U*, *B*, *V*, *R* and *I* are the standard magnitudes and *X* is the airmass. The difference between the calibrated magnitudes derived from the above transformation equations and the Landolt (1992) magnitudes is plotted in Fig. 1. The standard deviations of these measurements are estimated to be 0.04, 0.05, 0.03, 0.03 and 0.03 mag for the *U*, *B*, *V*, *R* and *I* filters, respectively. The above transformation coefficients were used to convert instrumental magnitudes to the standard system. The average internal photometric error per magnitude bin in all the filters on the night of standardization is listed in Table 1. This shows that photometric errors become large (>0.1 mag) for stars fainter than *V* ≈ 20 mag. To standardize the data on remaining nights, differential photometry was performed using a linear fit between the standard and instrumental magnitudes on each night, assuming that most of the stars are non-variable.

### 2.2 Completeness of the Data

It is necessary to determine the completeness of the data as it is not always possible to detect every star in the CCD frame, particularly the faintest stars. The completeness factor (CF) is required in order to derive the luminosity function (LF) and the MF of the cluster as well as to estimate the stellar density distribution. The ADDSTAR routine in DAOPHOT was used to determine CF. This involves adding randomly selected artificial stars with different, but known, magnitudes and positions to the original frames. We added about 10–15 per cent of the actually detected stars, so that the...
crowding characteristics of the original image is almost unchanged. We added simulated stars to all bands in such a way that they have similar geometric locations. We varied the brightness of the artificial star depending on its location relative to the main sequence (MS) in the V band. We constructed five frames for each passband and re-processed them with the same procedure as used in the original frames. The average ratio of the number of stars recovered to the number of simulated stars in the different magnitude bins gives the CF as a function of magnitude. The CF in all five passbands for both cluster and field regions is given in Table 2. From the table, one can see that the completeness decreases towards the fainter stars because of the increased crowding caused by the large number of low-mass stars.

### 2.3 Astrometry

In order to transform CCD pixel coordinates to celestial coordinates, we used the online digitized European Southern Observatory catalogue included in the SKYCAT software as an absolute astrometric reference frame. A linear astrometric solution was derived for the V-filter reference frame by matching positions of 63 well-isolated, bright stars in the USNOA2.0 catalogue. The ccmap and cctran routines in IRAF were used to find a transformation equation which gives the celestial coordinates (α, δ) as a function of the pixel coordinates (X, Y). The resulting celestial coordinates have standard deviations of 0.1 arcsec in both right ascension and declination.

A finding chart for stars in NGC 559 is shown in Fig. 2. We do not see any significant concentration of stars at the centre which suggests that cluster is loosely bound.

### 2.4 Comparison with previous photometry

Photoelectric and photographic observations of NGC 559 have been carried out by Lindoff (1969) and Grubissich (1975), respectively. Photographic magnitudes contain relatively large errors, while photoelectric magnitudes are mostly confined to stars brighter than V ~ 15; hence, we did not compare them with our photometry in the present study. CCD photometry in the UBVRI bands is discussed in AL02, but these data have not been published. Recently, MN07 performed a wide-field CCD survey of a few clusters using a 90/180 cm Schmidt–Cassegrain telescope equipped with an SBIG camera. This survey also includes NGC 559, for which BV data are presented, but only for stars brighter than about 18 mag.

We found 1112 stars in the MN07 catalogue which are included in our study. However, there are only 687 stars in common for which both B and V magnitudes are available. We have cross-identified stars in the two catalogues on the assumption that stars are correctly matched if the difference in position is less than 1 arcsec. On this basis, we found 505 stars in common which have similar B and V magnitudes within 0.5 mag. A comparison of B magnitudes and (B − V) colours between the two catalogues is shown in Fig. 3. The mean difference and standard deviation in each magnitude bin are given in Table 3. This shows that our B measurements are in fair agreement with those given in the MN07 catalogue. However, there is a systematic difference in (B − V) colours between the two catalogues.

### 2.5 A complete UBVRIJHK proper motion catalogue

We have compiled a photometric catalogue of 2393 stars in the field of NGC 559. The catalogue contains 515, 1288, 2177, 2352 and 2221 stars measured in the UBVRI bands, respectively. Near-infrared magnitudes for point sources around NGC 559 have also been obtained from the Two Micron All-Sky Survey (2MASS;
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The 2MASS provides photometry in the 
(1.25 µm), H (1.65 µm) and K (2.17 µm) bands up to a limiting magnitude of 18.5 in the field of NGC 559, while the USNO-B1.0 and 2MASS all-sky surveys provide photometric magnitudes for 917 stars in the field of NGC 559, of which 906 are identified in our catalogue within 1 arcsec of their positions. The K' magnitudes in our catalogue of NGC 559, of which 960 in Table 3, were derived from the reported magnitudes in the columns 1 to 3. The magnitudes in Table 4 are given in the order of increasing V magnitude. Column 1 gives identification number and columns 2 and 3 give right ascension and declination for J(2000). From columns 4 to 11, we provide photometric magnitudes and corresponding error in the UBVRIHK passbands. Columns 12 and 13 give proper motion in x- and y-directions.

Table 4. Photometric catalogue of 2293 stars detected in the field of cluster NGC 559. The error in magnitudes includes the internal photometric error in the measurement. Table is sorted in the order of increasing V magnitude. Column 1 gives identification number and columns 2 and 3 give right ascension and declination for J(2000). From columns 4 to 11, we provide photometric magnitudes and corresponding error in the UBVRIHK passbands. Columns 12 and 13 give proper motion in x- and y-directions.

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*Table 3. Differences in (B−V) between MN17 and present study. The standard deviation in the difference for each magnitude bin is also given in the parentheses.*
and the cluster radius was estimated to be $4.5 \pm 0.2$ arcmin. Our radius estimate is the same as that determined by AL02. The inner and outer rings in Fig. 2 represent the core and cluster regions, respectively.

We noticed that the core radii derived from bright stars are smaller than those which include stars up to $V = 20$. This suggests that (i) the core and cluster radii derived using the RDP are only approximate or (ii) there is mass segregation due to the dynamical evolution of the cluster. In the latter case, it would seem that bright massive stars move towards the cluster centre, while faint low-mass stars move away from the cluster centre. A similar trend has been noticed by Lee, Kang & Ann (2013) in their investigation of the clusters NGC 1245 and NGC 2506. A detailed study on the dynamical evolution is presented in Section 5.

### 3.2 Colour–magnitude diagram

The identification of the cluster MS in the CMDs allows a model-dependent mass, radius and distance for each star to be determined. To draw the CMD, we used the area within cluster radius (4.5 arcmin) as the ‘cluster region’ and an equal area outside the cluster radius of 5.6 arcmin as the ‘field region’. In the left-hand panels of Fig. 5, we constructed calibrated ($B - V$) and ($V - I$) versus $V$ diagrams of NGC 559 using the stars falling in the cluster region. A similar diagram for the stars in the field region is shown in the middle panels of the same figure.

Since stars in the cluster region are contaminated by the field star population, we adopted a statistical approach to remove the field star contamination. This method is based on a comparison of the cluster and field CMDs. We removed all cluster stars in the ($V - I)/V$ CMD which fall within a grid cell of ($V, V - I) = (\pm 0.25, \pm 0.125)$ of the field star CMD. A similar removal process was done for the ($B - V)/V$ CMD with a grid of $(V, B - V) = (\pm 0.25, \pm 0.10)$. We iterated the procedure for each star lying on the CMDs of the field region. We were finally left with 462 stars in the ($V - I)/V$ CMD and 341 stars in the $(B - V)/V$ CMD. We found more stars in the $(B - V)/V$ CMD because our photometry goes deeper in the $V$ and $I$ bands than in the $B$ band. The statistically cleaned cluster CMDs are shown in the right-hand panels of Fig. 5. The spatial distribution of stars extracted after the statistical subtraction shows that the inner region is dominated by giant and upper-MS stars, whereas the outer region is dominated by low-mass stars. The lack of stars in some pockets is quite evident in the cleaned CMDs. These kinds of gaps in MS are not unusual and have been found in many clusters (see details in Rachford & Canterna 2000). AL02 also noticed a gap at $M_V \sim 3.5$ mag ($m_v \sim 18.1$) in the cluster NGC 559 similar to the one seen in the present study. This suggests that these gaps could be due to a real lack of cluster members in some magnitude bins.

### 3.3 Mean proper motion

Recently, Roesper et al. (2010) provided a catalogue which lists stellar coordinates with an accuracy of 80–300 mas and absolute proper motion with an accuracy of 4–10 mas yr$^{-1}$ for about 900 million stars. A cross-match of these stars with our catalogue using a matching criterion of 1 arcsec resulted in 1824 stars in common. In Table 4, we provide proper motions of these stars along the RA and Dec. directions and their respective errors. Fig. 6 shows the proper motion distribution in the RA–Dec. plane.

To determine the mean proper motion of the cluster, we considered those 341 stars which fall in both the cleaned ($V - I)/V$ and ($B - V)/V$ CMDs. Among them, 307 stars were found within

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2 http://obswww.unige.ch/webda/
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Figure 5. The $(B - V)/V$ and $(V - I)/V$ CMDs for stars in the cluster region (left-hand panels) and field region (middle panels). The right-hand panels show the same for the stars in the cluster field after statistical subtraction of field stars. The most probable cluster members (see Table 5) are shown by large filled circles. The solid line represents the best-fitting isochrone to the cluster MS for $\log(\text{Age}) = 8.35$ (see Section 4.3 for details).

1 arcsec of the Roeser et al. (2010) catalogue positions. We determined the mean and $\sigma$ values of the proper motion in both RA and Dec. directions and rejected those stars which fall outside $3\sigma$ in both the directions. We iterated this procedure until all values fall within $3\sigma$ of the mean. We were finally left with 229 stars which were used to determine the mean proper motion of the cluster NGC 559. These stars are shown by filled circles in Fig. 6. The mean proper motion of the cluster determined in this way is

$$\bar{\mu}_x = -3.29 \pm 0.35 \text{ mas yr}^{-1}; \quad \bar{\mu}_y = -1.24 \pm 0.28 \text{ mas yr}^{-1}$$

where the uncertainties are standard deviations. A similar matching criterion using the UCAC4 catalogue (Zacharias et al. 2013) has given only 167 stars though it provides proper motion with higher accuracy. A 3$\sigma$-clipping analysis done on the proper motions left 145 stars which resulted in a mean proper motion of $\bar{\mu}_x = -4.45 \pm 0.49$ and $\bar{\mu}_y = 1.65 \pm 0.37$ mas yr$^{-1}$ in RA and Dec. directions, respectively. The proper motion for the cluster NGC 559 estimated using two different catalogues is therefore in close agreement within their given uncertainties. From the radial-velocity measurements of 24 stars computed from the data of the Tycho-2 catalogue, Lokin & Beshenov (2003) estimated a proper motion of $\bar{\mu}_x = -1.59 \pm 0.41$ and $\bar{\mu}_y = -0.52 \pm 0.46$ mas yr$^{-1}$ for the cluster NGC 559, which is lower than the present estimates.

3.4 Probable cluster members

Open clusters are mostly located within the densely populated Galactic plane and often contaminated with large numbers of field stars belonging to the disc population. It is therefore essential to discriminate between members and non-members in order to obtain correct cluster parameters. To identify the most likely cluster members in NGC 559, we first derive different membership probabilities for each star in the cluster field based on their spatial distribution, position in the CMD and proper motions.

3.4.1 Spatial probability

The spatial probability, $P_{sp}$, is a function of the angular distance of the star from the cluster centre, $r$, and is given by

$$P_{sp} = 1 - \frac{r}{r_c}.$$
where \(r_c\) is the angular radius of the cluster. Using \(r_c = 4.5\ \text{arcmin}\) derived in Section 3.1, we determined \(P_{sp}\) for all the 960 stars falling within the cluster radius. For \(r \geq r_c\), we assign \(P_{sp} = 0\). We found 176 stars within the core region of the cluster for which \(P_{sp} \geq 0.67\).

### 3.4.2 Statistical probability

We determine statistical probability which is based on a comparison of the cluster CMD with that of the field CMD, as discussed in Section 4.3. In this method, we removed all the stars in the \((B - V)\) CMD of the cluster field which fall within a grid cell of \((V, B - V) = (\pm 0.25, \pm 0.10)\), in the field CMD. After iterating the procedure for each star lying on the CMD of the field region, we found 341 stars for which we assigned statistical probabilities \(P_{st} = 1\). For the remaining stars, we assigned \(P_{st} = 0\).

### 3.4.3 Kinematic probability

The kinematic probability, \(P_k\), is defined as the deviation in the proper motion of stars in both RA and Dec. directions with respect to the mean proper motion of the cluster.

Using the method given by Kharchenko et al. (2004), we determined \(P_k\) for each star using

\[
P_k = \exp \left( -0.25 \left[ (\mu_x - \mu_{x,0})^2 / \sigma_{x,0}^2 + (\mu_y - \mu_{y,0})^2 / \sigma_{y,0}^2 \right] \right),
\]

where \(\sigma_{x,0}^2 = \sigma_{x,0}^2 + \sigma_{x,0}^2\) and \(\sigma_{y,0}^2 = \sigma_{y,0}^2 + \sigma_{y,0}^2\). The mean proper motion of the cluster NGC 559 is taken from our analysis carried out in Section 3.3. We found 1824 stars for which \(P_k\) could be estimated using the Roeser et al. (2010) catalogue.

To identify the most likely members in the cluster NGC 559, we considered those stars which lie in the core region of the cluster \((P_{sp} \geq 0.67)\), fall within the cleaned CMD \((P_{st} = 1.0)\) and have proper motion within \(1\sigma\) of the mean proper motion \((P_k \geq 0.60)\). We identified 22 such stars in our catalogue which fulfil above criteria. These criteria are conservative in the sense that they confer membership status to the selected stars, but it does not mean that other stars are non-members. The positions of these stars along with their magnitude and colours are given in Table 5. To determine robust cluster parameters for NGC 559, these stars were preferentially used in our analysis as explained in the following section.

### 4 CLUSTER PARAMETERS

#### 4.1 Reddening law and two-colour diagrams

Though the normal reddening law, \(R_V = \frac{A_V}{E(B-V)} = 3.1\), is valid for lines of sight that do not pass through dense clouds (Sneden et al. 1978), clusters associated with gas and dust or behind the dusty Galactic spiral arms may give a different value of \(R_V\). To investigate the nature of the reddening law, Chini & Wargau (1990) showed that the two-colour diagrams (TCDs) of the form \((\lambda - V)/\(B - V\)) can be used, where \(\lambda\) is any broad-band filter. The slope of the TCD distinguishes normal extinction produced by grains in the diffuse interstellar medium from that caused by abnormal dust grains (Pandey, Ogura & Sekiguchi 2000). We studied the reddening law in the cluster NGC 559 by drawing the \((\lambda - V)/(B - V)\) diagrams for the \(\lambda = R, I, J, H, K\) and \(J\) bands as shown in Fig. 7. The slope, \(m_{\text{-cluster}}\), was determined by fitting a linear relation in the TCD for the stars in the cluster region and a best fit determined after a 3\(\sigma\)-clipping iteration. The estimated values of \(m_{\text{cluster}}\) for all five colours are given in Table 6 along with their normal values. To derive the value of total-to-selective extinction \(R_{\text{cluster}}\) in the direction of NGC 559, we used the approximate relation (cf. Neckel & Chini 1981)

\[
R_{\text{cluster}} = \frac{m_{\text{cluster}}}{m_{\text{normal}}} \times R_{\text{normal}}.
\]

Using \(R_{\text{normal}} = 3.1\), we estimated \(R_{\text{cluster}}\) in different passbands to be \(3.1 < R_{\text{cluster}} < 3.5\) which is marginally higher than the normal
Figure 7. The \((\lambda - V)/(B - V)\) TCD for the stars within cluster region. The most probable cluster members are shown by large filled circles. The continuous lines represent the slope determined through least-squares linear fit.

Table 6. The slopes of the \((\lambda - V)/(B - V)\) diagrams in the direction of the cluster NGC 559. Normal value in the same colour is given in the parentheses.

<table>
<thead>
<tr>
<th>((R - V)/(B - V))</th>
<th>((J - V)/(B - V))</th>
<th>((I - V)/(B - V))</th>
<th>((H - V)/(B - V))</th>
<th>((K - V)/(B - V))</th>
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<tbody>
<tr>
<td>0.62 ± 0.01</td>
<td>1.21 ± 0.01</td>
<td>1.95 ± 0.02</td>
<td>2.50 ± 0.02</td>
<td>2.63 ± 0.02</td>
</tr>
<tr>
<td>(0.55)</td>
<td>(1.10)</td>
<td>(1.96)</td>
<td>(2.42)</td>
<td>(2.60)</td>
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value. The reddening law in the direction of the cluster is found to be normal at longer wavelengths but anomalous at shorter wavelengths.

4.2 Reddening determination: \((U - B)\) versus \((B - V)\) TCD

The reddening, \(E(B - V)\), in the cluster region is normally determined using the \((U - B)/(B - V)\) TCD. Out of 2393 stars in our catalogue, we found only 501 stars for which all the \(U, B\) and \(V\) magnitudes are available. Among them, we considered only 275 stars within the cluster which have a \(U\)-band photometric error less than 0.05. The resulting TCD is shown in Fig. 8. As mentioned in the previous section, the normal reddening law is not applicable at shorter wavelengths. Therefore, we have fitted intrinsic zero-age main-sequence (ZAMS) isochrones of solar metallicity (Marigo et al. 2008) to the observed MS stars by shifting \(E(B - V)\) and \(E(U - B)\) along different values of the reddening vector \(E((U - B)/(B - V))\). A visual inspection shows that the best fit is achieved for \(E((U - B)/(B - V)) = 0.84 ± 0.01\). This gives a mean reddening of \(E(B - V) = 0.82 ± 0.02\) in the direction of NGC 559 as shown by a solid line in Fig. 8. In determining the reddening, we used only stars having colours corresponding to spectral classes earlier than A0 because stars having later spectral types are more affected by metallicity and background contamination (Hoyle, Shanks & Tanvir 2003). The colour excess obtained in the present study is in good agreement with the value \(E(B - V) = 0.81 ± 0.05\) given by AL02, but higher than \(0.68^{+0.11}_{-0.12}\) obtained by MN07. Using the Johnson & Morgan (1953) \(Q\)-method for stars earlier than A0 \([B - V] < 0.84\), we determined the reddening of each star. The reddening distributions of these stars show that reddening is uniform over the whole cluster.

Considering \(R_{\text{normal}} = 3.1\), we estimated a higher value of \(R_{\text{cluster}} = 3.6\) for ultraviolet wavelengths. This further suggests an anomalous reddening law at shorter wavelengths in the direction of NGC 559. Chini & Wargau (1990) pointed out that both larger and smaller size grains may increase \(R_{\text{cluster}}\). However, some of the recent studies (e.g. Whittet et al. 2001; Pandey et al. 2008, and references therein) suggest that a value of \(R_{\text{cluster}}\) higher than the normal is indicative of the presence of larger dust grains. As NGC 559 is situated behind the Perseus arm, a high reddening and anomalous reddening law are not surprising.

4.3 Distance and age determination

The distance and age of NGC 559 can be estimated by visual fitting of theoretical isochrones to the MS. For this purpose, we used \((B - V)/V\) and \((V - I)/V\) CMDs shown in the right-hand panels of Fig. 5. The stars show a broad but clearly distinct MS in the CMD. The width is mainly caused by cluster binaries and field stars. There are a few stars scattered towards the red side of the CMDs. We suspect that these may be foreground field stars which have remained due to incomplete subtraction of the field star contamination. We presume most of them belong to the Perseus spiral arm. In order to obtain the most reliable estimates of the cluster parameters, we identified those stars in the cleaned CMDs which lie inside the core region and have proper motions within \(1\sigma\) of the mean proper motion of the cluster. These stars are shown by the blue filled circles in Fig. 5. We used stellar evolutionary isochrones published by the...
Padova group\(^3\) (Marigo et al. 2008) to estimate the cluster age and distance. We fixed the reddening to the value estimated in Section 4.2. A simultaneous best fit was made of the isochrones in the bluest envelope of the \((B-V)/V\) and \((V-I)/V\) CMDs, corrected for a mean reddening of \(E(B-V) = 0.82\) and \(E(V-I) = 1.12\) assuming \(E(V-I)/E(B-V) = 1.37\) (Schlegel, Finkbeiner \& Davis 1998). This gives an age of \(\log (\text{Age}) = 8.35 \pm 0.05\) and an apparent distance modulus of \((m-M) = 14.80 \pm 0.05\) for NGC 559. The errors in age and distance are strongly influenced by a few blue and red supergiants in the CMDs.

As we have seen in Sections 4.1 and 4.2, the total-to-selective extinction in the optical region varies from 3.4 to 3.6. We adopted a mean value of \(R_V = 3.5 \pm 0.1\) as the total-to-selective extinction in the direction of NGC 559. Assuming a total extinction of \(A_V = R_V \times E(B-V)\), the reddening-free distance modulus is estimated as \((V_0 - M_V) = 11.93 \pm 0.20\), which corresponds to a distance of 2.43 \pm 0.23 kpc for NGC 559. The linear diameter of the cluster is estimated to be 6.4 \pm 0.4 pc. Since the cluster lies very close to the Galactic plane, a large foreground extinction of about \(E(B-V) = 0.56\) is expected in that direction (Schlegel et al. 1998; Joshi 2005).

The position of NGC 559 in Galactic coordinates is \(l = 127.2^\circ\), \(b = +0.75^\circ\). Assuming that the Sun is at a distance of 8.5 kpc from the Galactic Centre, the Galactocentric rectangular coordinates of NGC 559 are \(X \sim 1.88\) kpc, \(Y \sim 1.44\) kpc, \(Z \sim +30.9\) pc and a Galactocentric distance of \(\sim 10.1\) kpc for the cluster. This places NGC 559 just outside the Perseus spiral arm. The distance of the cluster from the Galactic plane is smaller than the typical scaleheight of the thin disc \((\sim 75\) pc). This is in agreement with Joshi (2007), who found that most of the OCs younger than about 300 Myr lie somewhere within \(\pm 100\) pc of the Galactic plane.

### 4.4 Interstellar extinction in the near-infrared

To determine interstellar extinction in the near-IR, we used 370 stars for which \(VJK\) magnitudes were available in our catalogue. The \((V-K)/(J-K)\) diagram is shown in Fig. 9. We used the normal reddening law for the infrared colours, as given in Table 6, and shifted the stars along the reddening vector \(E(J-K)/E(V-K) = 0.173\) using solar metallicity isochrones given by Marigo et al. (2008). The best fit to points in the \((V-K)/(J-K)\) diagram gives a colour excess of \(E(V-K) = 2.14 \pm 0.02\) and \(E(J-K) = 0.37 \pm 0.01\) by minimizing \(\chi^2\). The theoretical isochrone shifted by the above values is shown by the solid line in Fig. 9. Using the Whittet \& van Breda (1980) relation for \(R_K = 1.1E(V-K)/E(B-V)\), which is insensitive to the reddening law, we obtained \(E(B-V) = 0.76 \pm 0.04\) for the reddening in NGC 559. This is close to \(E(B-V) = 0.82 \pm 0.02\) determined using the \((U-B)/(B-V)\) TCD. The agreement between two complementary methods suggests that our values are robust.

The fundamental parameters derived for NGC 559 in this study are summarized in Table 7.

### 4.5 Comparison to previous results

NGC 559 has been studied in the past by various authors. Lindoff (1969) found it to be a very old cluster with an age of about 1000 Myr, while Jennens \& Helfer (1975) estimated the age at only 100 Myr. Both studies used photoelectric photometry. Grubissich (1975), Lynga (1987), AL02 and MN07 all estimated the cluster age at \(\log (\text{Age}) = 8.7 \pm 0.1\). In this paper, we used only the most probable cluster members to estimate \(\log (\text{Age}) = 8.35 \pm 0.05\).

The distance of the cluster is estimated to be about 1.3 kpc (Lindoff 1969), 6.3 kpc (Jennens \& Helfer 1975) and 1.15 kpc (Lynga 1987). The recent CCD study by AL02 and MN07 determined a distance of 2.3 \pm 0.3 and 2.17 \pm 0.26 kpc, respectively. The latter value is close to the distance of 2.43 \pm 0.22 kpc determined in the present study. Previous estimates of reddening, \(E(B-V)\), are about 0.45 (Lindoff 1969), 0.62 \pm 0.17 (Jennens \& Helfer 1975), 0.54 (Lynga 1987) and 0.68 \pm 0.11 (MN07). However, AL02 obtained a higher value of \(E(B-V) = 0.81 \pm 0.05\), which is in good agreement with our value of 0.82 \pm 0.02.

\(^3\) http://pleiadi.pd.astro.it/
5 DYNAMICAL STUDY OF THE CLUSTER

The dynamical properties of the cluster can be studied by determining the LFs and MFs of the cluster members.

5.1 Luminosity function

The LF is the total number of cluster members in different magnitude bins. After correcting for the data completeness to both cluster and field regions, the number of probable cluster members was obtained by subtracting the contribution of field stars from stars in the cluster region. The estimated numbers of stars in each magnitude bin for both the cluster \( N_C \) and field regions \( N_F \) are given in Table 8. To determine the photometric LFs in the \( (V-I)/V \) and \( (B-V)/V \) CMDs, we subtracted \( N_F \) from \( N_C \), and resultant probable members \( N_P \) are given in the columns 4 and 7 of Table 8, respectively.

5.2 Mass function

The initial mass function (IMF) is defined as the distribution of stellar masses per unit volume in a star formation event. Along with the star formation rate, the IMF determines the subsequent evolution of clusters (Kroupa 2002). Since the direct determination of the IMF is not possible due to the dynamical evolution of stellar systems, we derive the MF, which is the relative number of stars per unit mass and can be expressed by a power law \( N(\log M) \propto M^\Gamma \).

The slope, \( \Gamma \), of the MF can be determined from

\[
\Gamma = \frac{d \log N(\log m)}{d \log m},
\]

where \( N(\log m) \) is the number of stars per unit logarithmic mass. The masses of probable cluster members can be determined by comparing observed magnitudes with those predicted by a stellar evolutionary model if the age, reddening, distance and metallicity are known.

As seen in Fig. 5, the \( (V-I)/V \) CMD goes deeper than the \( (B-V)/V \) CMD, so we used the former to determine the MF of the cluster. The main factors that limit the accuracy of the MF are incompleteness and field star contamination. While the central region of the cluster may be affected by data incompleteness, the outer region is more likely to be affected by field star contamination. After statistically correcting for the field star contamination, we determined the MF in three regions, i.e. the core region \((r \leq 1.3 \text{ arcmin})\), the corona \((1.3 < r \leq 4.5 \text{ arcmin})\) and the whole cluster region \((r \leq 4.5 \text{ arcmin})\). The MF determined for the cluster region is given in Table 9. Fig. 10 shows the MF in the cluster fitted for the MS stars with masses \( 0.8 \leq M/M_\odot < 3.7 \). The error bars were calculated assuming Poisson statistics. In determining the slope, we have considered only those data points which are shown by filled circles in Fig. 10. The slope of the MF \((\Gamma)\) in the mass range \( 1.0 \leq M/M_\odot < 3.7 \) in each region is calculated using a least-squares method and shown by the solid line in the figure. Table 10 summarizes the MF slopes in the cluster for all three regions.

For the mass range \( 0.4 < M/M_\odot < 10 \), the classical value derived by Salpeter (1955) for the MF slope is \(-1.35\). The MF slope in the

![Image](https://example.com/image.png)

<table>
<thead>
<tr>
<th>Region</th>
<th>( \Gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core ((r \leq 1.3 \text{ arcmin}))</td>
<td>(-1.64 \pm 0.62)</td>
</tr>
<tr>
<td>Corona ((1.3 &lt; r \leq 4.5 \text{ arcmin}))</td>
<td>(-2.14 \pm 0.30)</td>
</tr>
<tr>
<td>Cluster ((r \leq 4.5 \text{ arcmin}))</td>
<td>(-2.12 \pm 0.31)</td>
</tr>
</tbody>
</table>
core region is in agreement with the Salpeter MF slope within the given uncertainty, but it is steeper for the corona and cluster regions. This suggests a preferential distribution of relatively massive stars towards the central region of the cluster. When we determined the MF slopes for two extreme age limits of the cluster considering the uncertainty in our age determinations, we found that the MF slope is slightly dependent on the age of the cluster and varies by a maximum of ~20 per cent.

It is worth pointing out that the mass range for probable MS stars in this cluster is quite small. It is possible that some of the low-mass stars may have escaped from the cluster as a result of stellar encounters between stars of different masses. On the other hand, the initially massive stellar members of the cluster have now evolved and may possibly be white dwarfs or have undergone supernova explosions. Very deep photometry will be required to detect white dwarfs or supernova remnants, if present.

### 5.3 Mass segregation

There is ample proof of mass segregation in star clusters, i.e. a tendency of higher mass stars to approach towards the inner region and lower mass stars towards the outer region of the cluster. This appears to be a result of equipartition of energy through stellar encounters (e.g. Mathieu & Latham 1986; Sagart et al. 1988; Pandey et al. 2001). To understand if mass segregation is an imprint of the star formation process in the cluster and/or a result of dynamical evolution, we determined the dynamical relaxation time, $T_E$. This is the time in which individual stars in the cluster exchange energies and their velocity distribution approaches the Maxwellian equilibrium. It can be expressed as

$$ T_E = \frac{8.9 \times 10^5 (N R_h^3/\bar{m})^{1/2}}{\log(0.4N)} $$

where $T_E$ is in Myr, $N$ is the total number of cluster members, $R_h$ is the radius (in parsecs) containing half of the cluster mass and $\bar{m}$ is mean mass of the cluster members in solar units (cf. Spitzer & Hart 1971). We estimated a total of 202 MS stars in the mass range $0.8 \leq M/M_\odot < 3.7$. The total mass of the cluster is obtained by subtracting the total stellar mass in the field region from the cluster region. This results in a total mass of ~344 M$_\odot$ for NGC 559, which gives an average mass of ~1.7 M$_\odot$/per star. The contribution of the low-mass stellar population is critical for constraining the total cluster mass, which is crucial in understanding the dynamical evolution and the long-term survival of a cluster (e.g. de Grijs & Parmentier 2007, and references therein). We cannot rule out the possibility of poor subtraction of field stars from the cluster or an observing bias against detecting low-mass stars which might result in underestimating the total mass of the cluster. Therefore, the present value may be taken as a lower limit for the cluster mass, while the estimated mean stellar mass can be taken as an upper limit.

It can be seen that the half-radius of the cluster, $R_h$, plays an important role in the determination of the dynamical relaxation time, $T_E$. Unfortunately, this quantity is unknown for most clusters and is generally taken as half of the total cluster radius. Nevertheless, we can estimate $R_h$ by taking advantage of the statistical removal of field stars from the field region and knowledge of the approximate stellar masses using stellar isochrones. The value of $R_h$ determined in this way is ~2.3 pc, which is ~70 per cent of the cluster radius. An $R_h$ value larger than half of the cluster radius suggests that the inner region has a deficiency of massive stars which have now evolved. We estimated the dynamical relaxation time $T_E = 19.2$ Myr for NGC 559. However, cluster members fainter than the limiting V magnitude of our observations result in a decrease of $N$ and an increase of $\bar{m}$, leading to an underestimation of $T_E$. Therefore, $T_E$ obtained in this way should be regarded as a lower limit. The values used in the estimation of $T_E$ are summarized in Table 11. $T_E$ determined in the present study is much smaller than the present age of about 224 Myr. We conclude, therefore, that NGC 559 is a dynamically relaxed cluster.

### 6 CONCLUSION AND SUMMARY

We present results of an ongoing photometric survey in order to determine the structure, and astrophysical and dynamic evolution parameters of the intermediate-age Galactic cluster NGC 559. We present a comprehensive $UBVRIJHK$ proper motion catalogue for 2393 stars down to about $V = 21.4$ mag observed in an ~13 arcmin × 13 arcmin field centred on the cluster. Fundamental parameters, such as core and cluster radius, reddening $E(B-V)$, age, distance modulus and mean proper motion, were obtained using optical and near-IR photometry and proper motions. We analysed the cluster membership using criteria based on distance from the cluster centre, position in the CMD and proper motions. The membership probabilities of all stars in the field of the cluster are presented. We found 22 stars which are the most probable cluster members. Our study indicates a distance of $2.43 \pm 0.23$ kpc, a diameter of $6.4 \pm 0.4$ pc and an age of 224 ± 25 Myr. The cluster is found to be heavily reddened with $E(B-V)$ = 0.82 ± 0.02. The mean proper motion was estimated to be $\mu_\alpha = -3.29 \pm 0.35$ mas yr$^{-1}$, $\mu_\delta = -1.24 \pm 0.28$ mas yr$^{-1}$. Our analysis suggests that the cluster is slightly younger and more reddened than previously thought. It is important to note that because we limit determinations to the most probable cluster members, the errors in the estimates of various cluster parameters have been considerably reduced.

The reddening law in the direction of the cluster was found to be normal at longer wavelengths but anomalous at shorter wavelengths. In general, we found a slightly higher total-to-selective extinction $R_V = 3.3$ towards NGC 559. The larger value of $R_V$ could be caused by a bigger than average grain size. Polarimetric data would be useful to ascertain the size and behaviour of the dust grains. From the combined optical and near-infrared data, we obtained colour excesses of $E(V-K) = 2.14 \pm 0.02$, $E(J-K) = 0.37 \pm 0.01$ and $E(B-V) = 0.76 \pm 0.04$, in the direction of NGC 559.

The MF for MS stars in the cluster is not uniform over the entire region and found in the range $1.64 \geq \Gamma \geq -2.14$ for the mass range $1.0 \leq M/M_\odot < 3.7$. The MF slope of the core region is in agreement with the Salpeter value, but it is found to be steeper in the corona and in the cluster as a whole. This suggests mass segregation in MS stars due to the dynamical evolution of the cluster. A deficiency of low-mass stars as well as very massive stars was found in the core region of the cluster. The age of the cluster was found to be much higher than the relaxation time of 19.2 Myr, which implies that the cluster is dynamically relaxed. An improvement in the cluster

### Table 11. Parameters used to estimate $T_E$ for the cluster NGC 559.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probable members ($N$)</td>
<td>202</td>
</tr>
<tr>
<td>Total cluster mass</td>
<td>344 M$_\odot$</td>
</tr>
<tr>
<td>Cluster half-radius ($R_h$)</td>
<td>2.3 pc</td>
</tr>
<tr>
<td>Mean stellar mass ($\bar{m}$)</td>
<td>1.7 M$_\odot$</td>
</tr>
<tr>
<td>Dynamical relaxation time ($T_E$)</td>
<td>19.2 Myr</td>
</tr>
<tr>
<td>Age of the cluster</td>
<td>224 Myr</td>
</tr>
</tbody>
</table>
parameters and knowledge of dynamical evolution should allow a better understanding of star formation in NGC 559.

In a forthcoming paper, we will report on stellar variability in NGC 559 from 35 nights taken over three years from 2010 to 2012.

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