Stellar diameters and temperatures – V. 11 newly characterized exoplanet host stars

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ABSTRACT

We use near-infrared interferometric data coupled with trigonometric parallax values and spectral energy distribution fitting to directly determine stellar radii, effective temperatures and luminosities for the exoplanet host stars 61 Vir, ρ CrB, GJ 176, GJ 614, GJ 649, GJ 876, HD 1461, HD 7924, HD 33564, HD 107383 and HD 210702. Three of these targets are M dwarfs. Statistical uncertainties in the stellar radii and effective temperatures range from 0.5 to 5 per cent and from 0.2 to 2 per cent, respectively. For eight of these targets, this work presents the first directly determined values of radius and temperature; for the other three, we provide updates to their properties. The stellar fundamental parameters are used to estimate stellar mass and calculate the location and extent of each system’s circumstellar habitable zone. Two of these systems have planets that spend at least parts of their respective orbits in the system habitable zone: two of GJ 876’s four planets and the planet that orbits HD 33564. We find that our value for GJ 876’s stellar radius is more than 20 per cent larger than previous estimates and frequently used values in the astronomical literature.

Key words: techniques: interferometric – stars: fundamental parameters – stars: general – stars: late-type – planetary systems – infrared: stars.

1 INTRODUCTION

In the characterization of exoplanetary systems, knowledge of particularly the stellar radius and temperature are of paramount importance as they define the radiation environment in which the planets reside, and they enable the calculation of the circumstellar habitable zone’s (HZ) location and boundaries. Furthermore, the radii and densities of any transiting exoplanets, which provide the deepest insights into planet properties, such as exoatmospheric studies or the studies of planetary interior structures, are direct functions of the radius and mass of the respective parent star. Recent advances in
sensitivity and angular resolution in long-baseline interferometry at wavelengths in the near-infrared and optical range have made it possible to circumvent assumptions of stellar radius by enabling direct measurements of stellar radius and other astrophysical properties for nearby, bright stars (e.g. Baines et al. 2008a, b, 2009, 2010; van Belle & von Braun 2009; von Braun et al. 2011a, b, 2012; Boyajian et al. 2012a, b, 2013; Huber et al. 2012, and references therein).

In this paper, we present interferometric observations (Section 2.1) that, in combination with trigonometric parallax values, produce directly determined stellar radii for 11 exoplanet host stars, along with estimates of their stellar effective temperatures based on literature photometry (Section 3). We use these empirical stellar parameters to calculate stellar masses/ages where possible, and the locations and boundaries of the system HZs (Section 3.2). We discuss the implications for all the individual systems in Section 4 and conclude in Section 5.

2 DATA

In order to be as empirical as possible in the calculation of the stellar parameters of our targets, we rely on our interferometric observations to obtain angular diameters (Section 2.1), and we fit empirical spectral templates to literature photometry to obtain bolometric flux values (Section 2.2).

2.1 Interferometric observations

Our observational methods and strategy are described in detail in section 2.1 of Boyajian et al. (2013). We repeat aspects specific to the observations of the individual targets below.

The Georgia State University Center for High Angular Resolution Astronomy (CHARA) Array (ten Brummelaar et al. 2005) was used to collect our interferometric observations of exoplanet hosts in $J$, $H$ and $K'$ bands with the CHARA Classic beam combiner in parallel with our interferometric survey of main-sequence stars (Boyajian et al. 2012b, 2013). Our requirement that any target be observed on at least two nights with at least two different baselines serves to eliminate or reduce systematic effects in the observational results (von Braun et al. 2012; Boyajian et al. 2013). We note that we were not able to adhere this strategy for HD 107383, which was only observed during one night due to weather constraints during the observing run.

An additional measure to reduce the influence of systematics is the alternating between multiple interferometric calibrators during observations to eliminate effects of atmospheric and instrumental systematics. Calibrators, whose angular sizes are estimated using size estimates from the Jean-Marie Mariotti Center Catalogue at http://www.jmmc.fr/searchcal/ (Bonneau et al. 2006, 2011; Lafrasse et al. 2010a, b), are chosen to be small sources of similar brightness as, and small angular distance to, the respective target. A log of the interferometric observations can be found in Table 1.

The uniform disc and limb-darkened angular diameters ($\theta_{\text{UD}}$ and $\theta_{\text{LD}}$, respectively; see Table 2) are found by fitting the calibrated visibility measurements (Figs 1 and 2) to the respective functions for each relation. These functions may be described as $n$th-order Bessel functions that are dependent on the angular diameter of the star, the projected distance between the two telescopes.

This includes HD 107383 whose companion’s minimum mass is around 20 Jupiter masses (Table 4) and could thus be considered a brown dwarf.

As we show in Fig. 1 and Tables 1 and 2, our angular diameter fit for GJ 614 contains literature $K'$ data obtained in 2006 and published in Baines et al. (2008a).

Calibrated visibility data are available on request.

Table 1. Log of interferometric observations.

<table>
<thead>
<tr>
<th>Star</th>
<th>UT date</th>
<th>Baseline</th>
<th>No. of Obs (filter)</th>
<th>Calibrators</th>
</tr>
</thead>
<tbody>
<tr>
<td>$61$ Vir</td>
<td>2012-04-09</td>
<td>W1/E1</td>
<td>13 (H)</td>
<td>HD 113289, HD 116928</td>
</tr>
<tr>
<td></td>
<td>2012-04-10</td>
<td>S1/E1</td>
<td>3 (H)</td>
<td>HD 113289, HD 116928</td>
</tr>
<tr>
<td>$\rho$ CriB</td>
<td>2013-05-03</td>
<td>S1/E1</td>
<td>4 (H)(2)(K')</td>
<td>HD 139389, HD 149890</td>
</tr>
<tr>
<td>GJ 176</td>
<td>2010-09-16</td>
<td>W1/E1</td>
<td>7 (H)</td>
<td>HD 29225, HD 27524</td>
</tr>
<tr>
<td></td>
<td>2010-09-17</td>
<td>W1/E1</td>
<td>2 (H)</td>
<td>HD 27534</td>
</tr>
<tr>
<td></td>
<td>2010-09-20</td>
<td>S1/E1</td>
<td>4 (H)</td>
<td>HD 29225</td>
</tr>
<tr>
<td></td>
<td>2010-11-10</td>
<td>W1/E1</td>
<td>3 (H)</td>
<td>HD 29225</td>
</tr>
<tr>
<td>GJ 614</td>
<td>2010-06-28</td>
<td>W1/E1</td>
<td>10 (H)</td>
<td>HD 144579, HD 142908</td>
</tr>
<tr>
<td></td>
<td>2010-06-29</td>
<td>W1/E1</td>
<td>6 (H)</td>
<td>HD 144579, HD 142908</td>
</tr>
<tr>
<td></td>
<td>2010-09-18</td>
<td>S1/E1</td>
<td>4 (H)</td>
<td>HD 144579</td>
</tr>
<tr>
<td>GJ 649</td>
<td>2010-06-29</td>
<td>W1/E1</td>
<td>5 (H)</td>
<td>HD 153897</td>
</tr>
<tr>
<td></td>
<td>2010-06-30</td>
<td>S1/E1</td>
<td>6 (H)</td>
<td>HD 153897, HD 150205</td>
</tr>
<tr>
<td></td>
<td>2010-07-01</td>
<td>S1/E1</td>
<td>7 (H)</td>
<td>HD 153897, HD 150205</td>
</tr>
<tr>
<td>GJ 876</td>
<td>2011-08-17</td>
<td>S1/E1</td>
<td>11 (H)</td>
<td>HD 215874, HD 217681</td>
</tr>
<tr>
<td></td>
<td>2011-08-18</td>
<td>S1/E1</td>
<td>10 (H)</td>
<td>HD 215874, HD 217681</td>
</tr>
<tr>
<td></td>
<td>2011-08-19</td>
<td>W1/E1</td>
<td>6 (H)</td>
<td>HD 215874, HD 216402</td>
</tr>
<tr>
<td></td>
<td>2011-08-20</td>
<td>W1/E1</td>
<td>6 (H)</td>
<td>HD 217861, HD 216402</td>
</tr>
<tr>
<td>HD 1461</td>
<td>2011-08-22</td>
<td>S1/E1</td>
<td>7 (H)</td>
<td>HD 966, HD 1100</td>
</tr>
<tr>
<td></td>
<td>2011-10-03</td>
<td>S1/E1</td>
<td>7 (H)</td>
<td>HD 966, HD 1100</td>
</tr>
<tr>
<td></td>
<td>2013-08-17</td>
<td>E1/W1</td>
<td>5 (H)</td>
<td>HD 966, HD 1100</td>
</tr>
<tr>
<td>HD 7924</td>
<td>2010-09-17</td>
<td>W1/E1</td>
<td>9 (H)</td>
<td>HD 9407, HD 6798</td>
</tr>
<tr>
<td></td>
<td>2011-08-21</td>
<td>W1/S1</td>
<td>6 (H)</td>
<td>HD 9407, HD 6798</td>
</tr>
<tr>
<td>HD 33564</td>
<td>2010-09-16</td>
<td>W1/E1</td>
<td>2 (H)</td>
<td>HD 29329</td>
</tr>
<tr>
<td></td>
<td>2010-09-17</td>
<td>W1/E1</td>
<td>1 (H)</td>
<td>HD 62613</td>
</tr>
<tr>
<td></td>
<td>2010-11-10</td>
<td>W1/E1</td>
<td>9 (H)</td>
<td>HD 36768, HD 46588</td>
</tr>
<tr>
<td></td>
<td>2013-08-16</td>
<td>S1/W1</td>
<td>3 (H)</td>
<td>HD 29329, HD 62613</td>
</tr>
<tr>
<td></td>
<td>2013-08-17</td>
<td>S1/W1</td>
<td>1 (H)</td>
<td>HD 29329, HD 36768, HD 46588</td>
</tr>
<tr>
<td>HD 107383</td>
<td>2013-05-06</td>
<td>E2/W2</td>
<td>3 (H)</td>
<td>HD 106661, HD 108468</td>
</tr>
<tr>
<td></td>
<td>2013-05-06</td>
<td>S2/W2</td>
<td>4 (H)</td>
<td>HD 106661, HD 104452</td>
</tr>
<tr>
<td>HD 210702</td>
<td>2013-08-18</td>
<td>S1/E1</td>
<td>5 (H)(1)(J)</td>
<td>HD 210074, HD 206043</td>
</tr>
<tr>
<td></td>
<td>2013-08-19</td>
<td>E1/W1</td>
<td>2 (H)</td>
<td>HD 210074, HD 206043</td>
</tr>
<tr>
<td></td>
<td>2013-08-22</td>
<td>E1/W1</td>
<td>7 (H)</td>
<td>HD 210074, HD 207223</td>
</tr>
<tr>
<td></td>
<td>2013-08-27</td>
<td>S1/E1</td>
<td>1 (J)</td>
<td>HD 210074, HD 207223</td>
</tr>
</tbody>
</table>

*We combine our data with the literature CHARA $K'$-band data points, observed in 2006 with the S1/E1 baseline, published in Baines et al. (2008a) – see Section 4.4 for additional details.

Note. For details on the interferometric observations, see Section 2.1.
Table 2. Stellar angular diameters.

<table>
<thead>
<tr>
<th>Star name</th>
<th>No. of Obs</th>
<th>Reduced $\chi^2$</th>
<th>$\theta_{UD} \pm \sigma$ (mas)</th>
<th>$\mu_\lambda$</th>
<th>$\theta_{LD} \pm \sigma$ (mas)</th>
<th>$\theta_{LD}$ (per cent err)</th>
</tr>
</thead>
<tbody>
<tr>
<td>61 Vir</td>
<td>16</td>
<td>0.31</td>
<td>1.037 ± 0.005</td>
<td>0.367</td>
<td>1.073 ± 0.005</td>
<td>0.4</td>
</tr>
<tr>
<td>$\rho$ CrB</td>
<td>9</td>
<td>0.46</td>
<td>0.714 ± 0.013</td>
<td>0.342</td>
<td>0.735 ± 0.014</td>
<td>1.9</td>
</tr>
<tr>
<td>GJ 176</td>
<td>14</td>
<td>0.18</td>
<td>0.441 ± 0.020</td>
<td>0.210</td>
<td>0.448 ± 0.021</td>
<td>4.6</td>
</tr>
<tr>
<td>GJ 614</td>
<td>28</td>
<td>1.16</td>
<td>0.449 ± 0.017</td>
<td>0.284</td>
<td>0.459 ± 0.017</td>
<td>3.7</td>
</tr>
<tr>
<td>GJ 649</td>
<td>18</td>
<td>1.02</td>
<td>0.472 ± 0.012</td>
<td>0.327</td>
<td>0.484 ± 0.012</td>
<td>2.5</td>
</tr>
<tr>
<td>GJ 876</td>
<td>33</td>
<td>0.32</td>
<td>0.721 ± 0.009</td>
<td>0.398</td>
<td>0.746 ± 0.009</td>
<td>1.2</td>
</tr>
<tr>
<td>HD 1461</td>
<td>16</td>
<td>0.19</td>
<td>0.483 ± 0.010</td>
<td>0.369</td>
<td>0.498 ± 0.011</td>
<td>2.1</td>
</tr>
<tr>
<td>HD 7924</td>
<td>15</td>
<td>0.19</td>
<td>0.424 ± 0.014</td>
<td>0.281</td>
<td>0.433 ± 0.014</td>
<td>3.2</td>
</tr>
<tr>
<td>HD 33564</td>
<td>16</td>
<td>1.08</td>
<td>0.629 ± 0.010</td>
<td>0.225</td>
<td>0.640 ± 0.010</td>
<td>1.6</td>
</tr>
<tr>
<td>HD 107383</td>
<td>7</td>
<td>0.16</td>
<td>1.590 ± 0.015</td>
<td>0.417</td>
<td>1.651 ± 0.016</td>
<td>1.0</td>
</tr>
<tr>
<td>HD 210702</td>
<td>16</td>
<td>1.57</td>
<td>0.845 ± 0.005</td>
<td>0.484</td>
<td>0.886 ± 0.006</td>
<td>0.6</td>
</tr>
</tbody>
</table>

*Includes CHARA classic K- data from Baines et al. (2008a). Note. $\theta_{UD}$ and $\theta_{LD}$ refer to stellar uniform-disc and limb-darkening-corrected angular diameters, respectively. $\mu_\lambda$ are the limb-darkening coefficients from Claret (2000) after an iteration based on $T_{eff}$ values. Refer to Section 2.1 for details.

Visibility is the normalized amplitude of the correlation of the light from two telescopes. It is a unitless number ranging from 0 to 1, where 0 implies no correlation, and 1 implies perfect correlation. An unresolved source would have perfect correlation of 1.0 independent of the distance between the telescopes (baseline). A resolved object will show a decrease in visibility with increasing baseline length. The shape of the visibility versus baseline is a function of the topology of the observed object (the Fourier Transform of the object’s brightness distribution in the observed wavelength band). For a uniform disc this function is a Bessel function, and for this paper, we use a simple model of a limb darkened variation of a uniform disc.

While there are methods of tracking errors through the calibration of visibility via standard statistical methods (e.g. van Belle & van Belle 2005), the principal difficulty in assessing a realistic estimate of the absolute error in visibility is the constantly changing nature of the atmosphere.

2.2 Bolometric fluxes

In this section, we report on stellar SED fits of our targets. We augment literature broad-band photometry data by using spectrophotometric data whenever available. The purpose of these SED fits is to obtain direct estimates of stellar $T_{eff}$ and $L$, as described in Section 3.1. Our procedure is analogous to the ones performed in van Belle & von Braun (2009), von Braun et al. (2011a,b, 2012) and Boyajian et al. (2012a,b, 2013).

Our SED fitting is based on a $\chi^2$-minimization of input SED templates from the Pickles (1998) to literature photometry of the star under investigation. If the literature photometry values are in magnitudes, they are converted to absolute fluxes by application of published or calculated zero-points. The filters of the literature photometry data are assumed to have a top-hat shape. That is, during the calculation of $\chi^2$, only the central filter wavelengths are correlated with the SED template’s flux value averaged over the filter transmission range in wavelength. Literature spectrophotometry data are very useful for SED fitting since multiple individual photometry data points, instead of being integrated into a single wavelength, trace out the shape of the SED in great detail. The SED template is scaled to minimize $\chi^2$ and then integrated over wavelength to obtain the bolometric flux. The code additionally produces an estimated angular diameter, which we only use as a sanity check to avoid systematic problems like the choice of wrong spectral template. Fig. 3 illustrates our procedure for the example of GJ 614.

We note that our quoted uncertainties on the bolometric flux values are statistical only. We do not (and indeed cannot) account for possible systematics such as saturation or correlated errors in the photometry, filter errors due to problems with transmission curves, or other non-random error sources. The only systematics that we can control are (1) the choice of spectral template for the SED fit, (2) the choice of which photometry data to include in the fit and (3) whether to let the interstellar reddening float during the fit or whether to set it to zero. In order to be consistent as possible, we take the following approach:

(i) All photometry data are included in the fit in principle, except when they present clear outliers in the SED. This way, we attempt to reduce systematics. In general, there are tens to hundreds of

and the wavelength of observation (Hanbury Brown et al. 1974). The temperature-dependent limb-darkening coefficients, $\mu_\lambda$, used to convert from $\theta_{UD}$ to $\theta_{LD}$, are taken from Claret (2000) after we iterate based on the effective temperature value obtained from initial spectral energy distribution (SED) fitting (see Section 2.2). Limb-darkening coefficients are dependent on assumed stellar effective temperature, surface gravity, and weakly on metallicity. When we vary the input $T_{eff}$ by 200 K and log $g$ by 0.5 dex, the resulting variations are below 0.1 per cent in $\theta_{LD}$ and below 0.05 per cent in $T_{eff}$. Varying the assumed metallicity across the range of our target sample does not influence our final values of $\theta_{LD}$ and $T_{eff}$ at all.

The values for $\theta_{UD}$ and $\theta_{LD}$ for our targets are given in Table 2. The angular diameters and their respective uncertainties are computed using MPFIT, a non-linear least-squares fitting routine in IDL (Markwardt 2009). Table 2 shows the empirical $\chi^2_{\text{reduced}}$ values of the fits shown in Figs 1 and 2 in column 3. These $\chi^2_{\text{reduced}}$ values are often calculated to be $\ll 1$ due to the difficulty of accurately defining uncertainties in the visibility measurements. Consequently, we assume a true $\chi^2_{\text{reduced}} = 1$ when calculating the uncertainties for $\theta_{UD}$ and $\theta_{LD}$, based on a rescaling of the associated uncertainties in the visibility data points. That is, the estimates of our uncertainties in $\theta_{UD}$ and $\theta_{LD}$ are based on a $\chi^2_{\text{reduced}}$ fit, not on strictly analytical calculations.
photometric measurements per target (see Table 3), and at most, we remove 1–2 data points, mostly in the $U$ band or $RI$ bands.

(ii) Interstellar extinction is set to zero for all targets, due to the small distances to the stars (see Table 4), which are adopted from van Leeuwen (2007). We cross-checked results with the ones using variable reddening, and in almost no case was there any difference. The ones for which the variable reddening produced results that are not consistent with $A_V = 0$ at the $\sim 1\sigma$ level are discussed below. For all others, letting $A_V$ vary produced $A_V = 0$.

(iii) Whenever a literature photometry datum has no quoted uncertainty associated with it, we assign it a 5 per cent random uncertainty. This is only the case for some older data sets.

(iv) The choice of spectral template is based on minimization of $\chi^2_{\text{reduced}}$ only. For about half of our targets, we linearly interpolate between the relatively coarse grid of the Pickles (1998) spectral templates to obtain a better fit and thus a more accurate value for the numerical integration to calculate the bolometric flux (indicated in Table 3). Linear interpolation never
spans more than three tenths in spectral type range (e.g. G5 to G8).

(v) Despite the fact that spectrophotometry often increases the fit’s $\chi^2_{\text{reduced}}$, we include these data whenever available (indicated in Table 3) in order to reduce the systematics in the choice of spectral template, which is determined more accurately via spectrophotometry.

Notes on individual systems with respect to SED fitting.

(i) 61 Vir. Despite the fact that we find $A_V = 0$ when letting $A_V$ float, we note that dust excess for this system was reported in Trilling et al. (2008), Bryden et al. (2009), Lawler et al. (2009) and Tanner et al. (2009). Photometry sources: Johnson et al. (1966), Golay (1972), Johnson & Mitchell (1975), Dean (1981), Haggkvist & Oja (1987), Olsen (1994), Oja (1996); spectrophotometry from Burnashev (1985).


Figure 2. Plots of calibrated interferometric visibilities and fits. See Section 2.1 for details on the interferometric observations.
Table 3. $F_{\text{BOL}}$ values from SED fitting.

<table>
<thead>
<tr>
<th>star</th>
<th>Template sp. type</th>
<th>Degrees of Freedom</th>
<th>$\chi^2_{\text{red}}$</th>
<th>$F_{\text{BOL}} \pm \sigma$ (10$^{-8}$ erg s$^{-1}$ cm$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>61 Vir</td>
<td>G6.5V</td>
<td>233$^b$</td>
<td>2.81</td>
<td>36.06 $\pm$ 0.05</td>
</tr>
<tr>
<td>$\rho$ CrB</td>
<td>G1V</td>
<td>369$^b$</td>
<td>8.74</td>
<td>18.03 $\pm$ 0.02</td>
</tr>
<tr>
<td>GI 176</td>
<td>M2.5V</td>
<td>56$^b$</td>
<td>5.95</td>
<td>1.26 $\pm$ 0.005</td>
</tr>
<tr>
<td>GI 614</td>
<td>G9IV$^v$</td>
<td>69</td>
<td>1.28</td>
<td>6.50 $\pm$ 0.02</td>
</tr>
<tr>
<td>GI 649</td>
<td>M2V</td>
<td>32</td>
<td>8.90</td>
<td>1.30 $\pm$ 0.005</td>
</tr>
<tr>
<td>GI 876</td>
<td>M3.5V$^v$</td>
<td>109</td>
<td>7.18</td>
<td>1.78 $\pm$ 0.004</td>
</tr>
<tr>
<td>HD 1461</td>
<td>G1IV</td>
<td>91</td>
<td>0.63</td>
<td>6.95 $\pm$ 0.03</td>
</tr>
<tr>
<td>HD 7924</td>
<td>K0.5V$^v$</td>
<td>16</td>
<td>2.37</td>
<td>4.14 $\pm$ 0.03</td>
</tr>
<tr>
<td>HD 33564</td>
<td>F6V</td>
<td>119$^b$</td>
<td>4.88</td>
<td>23.17 $\pm$ 0.04</td>
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<tr>
<td>HD 107383</td>
<td>K0.5III$^b$</td>
<td>68</td>
<td>1.60</td>
<td>44.49 $\pm$ 0.25</td>
</tr>
<tr>
<td>HD 210702</td>
<td>G9III$^v$</td>
<td>47</td>
<td>0.79</td>
<td>13.65 $\pm$ 0.09</td>
</tr>
</tbody>
</table>

Note. For more details, please see Section 2.2.


$^b$ Includes spectrophotometry.


(vi) GI 876. With a variable $A_V$, the SED fit for GI 876 produces slightly different results compared to the ones given in Table 3, where it was set to zero: $\chi^2_{\text{red}} = 3.81$, $F_{\text{BOL}} = (25.05 \pm 0.19) \times 10^{-8}$ erg s$^{-1}$ cm$^{-2}$ with an $A_V = 0.076 \pm 0.007$. Photometry sources: Haggkvist & Oja (1986), Golay (1972), Beichman et al. (1988), Cernis et al. (1989), Gezari et al. (1999), Kornilov et al. (1991), Cutri et al. (2003); spectrophotometry from Khartitonov, Treeshchenko & Knjazeva (1988).


(ix) HD 33564. With a variable $A_V$, the SED fit for HD 33564 produces slightly different results compared to the ones given in Table 3: $\chi^2_{\text{red}} = 3.81$, $F_{\text{BOL}} = (25.05 \pm 0.19) \times 10^{-8}$ erg s$^{-1}$ cm$^{-2}$ with an $A_V = 0.076 \pm 0.007$. Photometry sources: Haggkvist & Oja (1966), Golay (1972), Beichman et al. (1988), Cernis et al. (1989), Gezari et al. (1999), Kornilov et al. (1991), Cutri et al. (2003); spectrophotometry from Khartitonov, Treeshchenko & Knjazeva (1988).


(xi) HD 210702. With a variable $A_V$, the SED fit for HD 210702 produces marginally different results compared to the ones given in Table 3: $\chi^2_{\text{red}} = 0.77$, $F_{\text{BOL}} = (14.24 \pm 0.05) \times 10^{-8}$ erg s$^{-1}$ cm$^{-2}$ with an $A_V = 0.037 \pm 0.029$. Photometry sources: Johnson & Knuckles (1957), Johnson et al. (1966), Golay (1972), Olson...
Table 4. Stellar astrophysical parameters.

<table>
<thead>
<tr>
<th>Star</th>
<th>Radius (R_☉)</th>
<th>T_{eff} (K)</th>
<th>L (L_☉)</th>
<th>Spectral type</th>
<th>Distance (pc)</th>
<th>Metallicity [Fe/H]</th>
<th>HZ_{cons} (au)</th>
<th>HZ_{opt} (au)</th>
<th>Age (Gyr)</th>
<th>Mass (M_☉)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HD 107383</td>
<td>1.4367 ± 0.0328</td>
<td>6420 ± 50</td>
<td>3.1777 ± 0.0696</td>
<td>FT V</td>
<td>20.98 ± 0.08</td>
<td>1.73–2.88</td>
<td>1.31–3.00</td>
<td>2.2</td>
<td>1.31</td>
<td>0.0696</td>
</tr>
<tr>
<td>HD 210702</td>
<td>5.2314 ± 0.1171</td>
<td>4780 ± 18</td>
<td>12.838 ± 0.5569</td>
<td>K0 II</td>
<td>54.95 ± 0.03</td>
<td>10.8–19.4</td>
<td>8.19–20.3</td>
<td>5.9</td>
<td>5.9</td>
<td>0.0262</td>
</tr>
</tbody>
</table>

Note. For the calculations of stellar R, T_{eff} and L, please see Section 3.1. Spectral types and metallicities from Anderson & Francis (2011, 2012) unless otherwise indicated. Distances from van Leeuwen (2007). The calculations for the inner and outer boundaries of the system circumstellar HZs (conservative and optimistic) and for stellar mass and age are described in Section 3.2.

(1) = From Rojas-Ayala et al. (2012). Each value has a quoted uncertainty of 0.17 dex.

(2) = Masses calculated via equation (2), based on data in Boyajian et al. (2012b).

Beyond the range of the Y^2 isochrones.

From Maldonado, Villaver & Eiroa (2013).


3 STELLAR ASTROPHYSICAL PARAMETERS

In this section, we report on our direct measurements of stellar diameters, T_{eff}, and luminosities, based on our interferometric measurements (Section 2.1) and SED fitting to literature photometry (Section 2.2). We furthermore present calculated values wherever sensible for our targets, such as the location and extent of the respective system’s circumstellar HZ and stellar mass and age. Results are summarized in Table 4.

3.1 Direct: stellar radii, effective temperatures and luminosities

We use our measured, limb-darkening corrected, angular diameters \( \theta_{LD} \), corresponding to the angular diameter of the Rosseland, or mean, radiating surface of the star (Section 2.1, Table 2), coupled with trigonometric parallax values from van Leeuwen (2007) to determine the linear stellar diameters. Uncertainties in the physical stellar radii are typically dominated by the uncertainties in the angular diameters, not the distance.

From the SED fitting (Section 2.2, Table 3), we calculate the value of the stellar bolometric flux, \( F_{BOL} \), by numerically integration of the scaled spectral template across all wavelengths. Wherever the empirical spectral template does not contain any data, it is interpolated and extrapolated along a blackbody curve. Combination with the rewritten version of the Stefan–Boltzmann Law

\[
T_{eff}(K) = 2341 \left( F_{BOL}/\theta_{LD}^2 \right)^{1/4},
\]

where \( F_{BOL} \) is in units of \( 10^{-8} \) erg cm\(^{-2} \) s\(^{-1} \) and \( \theta_{LD} \) is in units of mas, produces the effective stellar temperatures \( T_{eff} \).

3.2 Calculated: HZs, stellar masses and ages

To first order, the HZ of a planetary system is described as the range of distances in which a planet with a surface and an atmosphere containing a modest amount of greenhouse gases would be able to host liquid water on its surface, first characterized in Kasting, Whitmire & Reynolds (1993). The HZ boundaries in this paper are calculated based on the updated formalism of Kopparapu et al. (2013a,b). Kopparapu et al. (2013a) define the boundaries based on a runaway greenhouse effect or a runaway snowball effect as a function of stellar luminosity and effective temperature, plus water absorption by the planetary atmosphere. Whichever assumption is made of how long Venus and Mars were able to retain liquid water on their respective surfaces defines the choice of HZ (conservative or optimistic). These conditions are described in more detail in Kopparapu et al. (2013a) and section 3 of Kane, Barclay & Gelino (2013). The boundaries quoted in Table 4 are the ones for both the conservative and optimistic HZ.

For stellar mass and age estimates of the early stars in our sample, we use the Yonsei–Yale (Y^2) isochrones (Yi et al. 2001; Kim et al. 2002; Demarque et al. 2004). Input data are our directly determined stellar radii and temperatures, along with the literature metallicity values from Table 4, and zero \( \alpha \)-element enhancement: \([\alpha/Fe] = 0\). We follow the arguments in section 2.4 in Boyajian et al. (2013) and conservatively estimate mass and age uncertainties of 5 per cent and 5 Gyr, respectively.

The ages of low-mass stars, however, are not sensitive to model isochrone fitting. Thus, in order to estimate the stellar masses of the KM dwarfs in our sample, we use the formalism described in section 5.4 in Boyajian et al. (2012b). We use the data from table 6 in Boyajian et al. (2012b) to derive the following equation for KM dwarfs:

\[
M = -0.0460(\pm0.0251) + 1.0930(\pm0.1481)R + 0.0064(\pm0.0472)R^2.
\]

where \( R \) and \( M \) are the stellar radius and mass in Solar units, respectively. This essentially represents the inverted form of equation 10 in Boyajian et al. (2012b) for single dwarf stars. We note that the statistical errors in the determined masses using equation (2) are dominated by the uncertainties in the coefficients and are of the order of ~30 per cent.

6 We use the online calculator at http://www3.geosc.psu.edu/~rak15/planets/
4 DISCUSSION

In this section, we briefly discuss our results on the individual systems. We compare our directly determined stellar parameters (Section 3.1 and Table 4) to quoted literature values. Statistical differences between values are calculated by adding our uncertainties and those from the literature, when available, in quadrature.

4.1 61 Vir (= HD 115617)

61 Vir hosts three planets with periods ranging from 4.2 to 124 d and minimum masses between 5.1 and 24 M_Jupiter (Vogt et al. 2010). Since the orbital distances of all three planets are less than 0.5 au from the parent star, and since the inner edge of the optimistic/conservative HZ is located at 0.69 au/0.91 au, none of the known three planets reside within the system’s HZ.

The stellar radius for 61 Vir quoted in Takeda et al. (2007, 0.98 ± 0.03 R_⊙) is statistically identical with ours. When compared to Valenti & Fischer (2005), our radius measurement is 2σ larger than their estimate (0.963 ± 0.011 R_⊙), while our temperature value is consistent with their value (5571 ± 44 K). Similarly good agreement exists with the T_eff value quoted in Ecuvilhon et al. (2006, 5577 ± 33 K).

To estimate 61 Vir’s mass and age, we use the Y^2 isochrones with our values from Table 4 as input, generated with a 0.1 Gyr step size, and a fixed metallicity of [Fe/H] = +0.01 (Table 4). Interpolation within the best-fitting isochrone gives an age of 8.6 Gyr and mass of 0.93 M_⊙.

4.2 ρ CrB (= HD 143761)

ρ CrB has a Jupiter-mass planet in a 40 d orbit (Noyes et al. 1997) whose orbital semimajor axis (0.22 au; Butler et al. 2006) is located well inside the system’s optimistic/conservative HZ’s inner boundary at 0.98 au/1.3 au.

ρ CrB’s radius was interferometrically observed in the K’ band by Baines et al. (2008a) who calculated a radius of 1.284 ± 0.082 R_⊙, well within 1σ of our value of 1.3424 R_⊙. ρ CrB’s stellar radius from Fuhrmann, Pfeiffer & Bernkopf (1998), 1.34 ± 0.05 R_⊙, also agrees within 2σ with our directly determined value. The spectroscopically determined T_eff values of Fuhrmann et al. (1998, 5821 ± 80 K) and Valenti & Fischer (2005, 5822 ± 44 K), however, come in at 1.8σ and 2.7σ above our estimate of 5665 K.

Using the same procedure as for 61 Vir (Section 4.1), we solve for best-fitting Y^2 isochrone properties for ρ CrB using a metallicity of [Fe/H] = −0.22 (Table 4). This produces estimates for stellar age of 12.9 Gyr and stellar mass of 0.91 M_⊙.

4.3 GJ 176 (= HD 285968)

The 8.8 Earth-mass planet in a 10.24 d, circular orbit around GJ 176 (Endl et al. 2008; Forveille et al. 2009) has been announced by the planet at a projected semimajor axis of less than 0.07 au from its parent star, well inside the inner boundary of the system optimistic/conservative HZ at 0.15 au/0.2 au.

Takeda (2007) estimate GJ 176’s stellar radius to be 0.46 ± 0.02 R_⊙, statistically identical to our directly determined value of 0.4525 R_⊙. The T_eff estimate in Morales, Ribas & Jordi (2008, 3520 K) is around 2σ below our value, in part due to the fact, however, that no uncertainties in T_eff are provided. Finally, our values are in agreement with T_eff = 3754 K and R = 0.40 R_⊙ from Wright et al. (2011b).

We estimate the mass of GJ 176 to be 0.45 M_⊙ via equation (2). It is possible, but extremely unreliable, to estimate isochronal ages for low-mass stars, as stated earlier in Section 3.2.

4.4 GJ 614 (= HD 145675 = 14 Her)

The planet in orbit around GJ 614 was first announced by M. Mayor in 1998 (see http://obswww.unige.ch/~udry/planet/14her.html). Butler et al. (2003) provide system characterization, including the period of around 4.7 yr with an eccentricity of 0.37 and a planet minimum mass of about 4.9 Jupiter masses. At a projected semimajor axis of 2.82 au, this planet is actually located outside the system optimistic/conservative HZ’s outer edge at 1.43 au/1.37 au, even at periastroid distance r_peri = (1 − e)a = 1.78 au.

Baines et al. (2008a) interferometrically determine GJ 614’s physical radius to be 0.708 ± 0.085 R_⊙, which is around 1.9σ below our value. Those data are taken in the K’ band, where GJ 614’s small angular diameter is only marginally resolved. Since the Baines et al. (2008a) K’ band data alone thus do not constrain the angular diameter fit as well as our H data, we combine the Baines et al. (2008a) visibilities along with our own to determine GJ 614’s angular diameter in Table 2 and linear radius in Table 4. Our H-band data enable increased resolution due to the shorter wavelengths (see Fig. 1, where the Baines et al. 2008a calibrated visibilities are represented by the cluster of data points with relatively large scatter at shorter effective baseline values).

There are a large number (many tens on Vizier) of T_eff values in the literature with estimates between 4965 and 5735 K, which average 509 K falls in the middle of a fairly large range in the literature values: Lafrasse et al. (2008a), 4185 ± 161 K, and Morales et al. (2008a), 5600 K (Heiter & Luck 2003). Our value of 5518 ± 102 K falls towards the upper tier of the literature range.

Using equation (2), we derive the stellar mass for GJ 614 to be 0.91 M_⊙.

4.5 GJ 649

GJ 649 hosts a 0.33 M_Jupiter planet in an eccentric 598.3 d orbit with a semimajor axis of 1.135 au (Johnson et al. 2010). We calculate GJ 649’s circumstellar optimistic/conservative HZ’s outer boundary to be 0.44 au/0.42 au, putting the planet well beyond the outer edge of the system HZ, even at periastron.

Our estimate for GJ 649’s radius is 0.5387 ± 0.0157 R_⊙, which falls in the middle between the values in Takeda (2007, 0.49 ± 0.03 R_⊙), Zakhozajh (1979, 0.49 R_⊙) and Houdene (2010, 0.616 ± 0.026 R_⊙).

Our effective temperature for GJ 649 is 3590 ± 45 K, which also falls in the middle of a fairly large range in the literature values: Lafasse et al. (2010a, 3370 K), Ammons et al. (2006, 4185 ± 334 K), Soubian et al. (2010, 3717, 3782 ± 58 K), Morales et al. (2008, 3670 K) and Houdene (2010, 3503 ± 50 K).

Applying equation (2) to our radius value produces a mass value for GJ 649 of 0.54 M_⊙.

4.6 GJ 876

The late-type, multiplanet host GJ 876 has been studied extensively. There are four planets in orbit around the star (Correia et al. 2010; Rivera et al. 2010b) at an inclination angle with respect to the plane of the system of 59.5. The planet masses range from 6.83 M_Jupiter to 2.28 M_Jupiter in orbital distances that range from 0.02 to 3.3 au with periods between 1.94 and 124.26 d (Rivera et al. 2010b). Based on
Finally, we derive a mass for GJ 876 of $M = 0.37 M_\odot$ using equation (2).

4.7 HD 1461

HD 1461 hosts two super-Earths in close proximity to both the star and each other: a 7.6 Earth-mass planet in a 5.8 d orbit (Rivera et al. 2010a) and a 5.9 Earth-mass planet in a 13.5 d orbit (Mayor et al. 2011, both are minimum masses). Their semimajor axes are 0.063 and 0.112 au, respectively, all well inside HD 1461’s HZ, whose inner optimistic/conservative boundary is at 0.83 au/1.10 au.

Our radius estimate of 1.1987 ± 0.0275 R_⊙ is larger at the $>2\sigma$ level than both radius estimates in the literature: 1.11 ± 0.04 R_⊙ (Takeda 2007) and 1.0950 ± 0.0260 R_⊙ (Valenti & Fischer 2005).

We measure an effective temperature for HD 1461 to be 5486 ± 52 K, which falls below all of the many temperature estimates in the literature for HD 1461 – a sensible consequence given that our radius is larger than literature estimates. The Soubran et al. (2010) catalogue alone has 13 different $T_{eff}$ values, ranging from 5683 to 5929 K. Additionally, we find temperature estimates of 5688 K (Holmberg, Nordström & Andersen 2009), 5666 ± 42 K (Prugniel, Vauglin & Koleva 2011) and 5588 ± 64 K (Koleva & Vazdekis 2012).

We use the $Y^2$ isochrones following the method described in Section 3.2 to derive and age and mass of HD 1461. We obtain a mass of 0.94 M_⊙ and an age of 13.8 Gyr.

4.8 HD 7924

The super-Earth ($M \sin i = 9.26 M_\oplus$) orbits HD 7924 at a period of 5.4 d and at a semimajor axis of 0.057 au (Howard et al. 2009). The inner boundary of the optimistic/conservative HD 7924 system is at 0.47 au/0.62 au from the star, well beyond the planetary orbit.

The radius of HD 7924 has been estimated to be $R = 0.78 ± 0.02 R_\odot$ (Takeda 2007), which is identical to our direct measurement of $R = 0.7821 R_\odot$. The radius estimate of $R = 0.754 R_\odot$ (Valenti & Fischer 2005) is slightly below but consistent with our value.

Our effective temperature measurement $T_{eff} = 5075$ K falls into the middle of a large effective temperature range present in the literature for HD 7924: 4550 K (Lafrasse et al. 2010a), 4750 K (Wright et al. 2003), 5111_13 K (Ammons et al. 2006), 5121–5177 K (six entries; Soubran et al. 2010), 5177 K (Petigura & Marcy 2011), 5177 K (Valenti & Fischer 2005), 5153 ± 5.8 K (Kovtyukh, Soubran & Belik 2004) and 5165 K (Mishenina et al. 2008, 2012).

Our derived mass from equation (2) is $M = 0.81 M_\odot$.

4.9 HD 33564

HD 33564 hosts a $M \sin i = 9.1 M_\text{Jup}$ planet in an eccentric 388 d orbit (Galland et al. 2005) and an orbital semimajor axis of 1.1 au. Since the orbital eccentricity is 0.34, its apastron distance is $r_{ap} = (1 + e)a = 1.47$ au. While the conservative HZ is located beyond HD 33564b’s orbit, the planet spends around 43 per cent of its orbital duration inside the optimistic HZ, whose inner edge is at 1.31 au.

HD 33564’s system architecture is shown in Fig. 5.

We measure HD 33564’s radius to be $1.4712 ± 0.0219 R_\odot$. This is consistent with the previous estimate based on SED fitting by van Belle & von Braun (2009) of $R = 1.45 ± 0.03 R_\odot$.

Our value for the effective temperature of HD 33564 is 6346 ± 44 K, which is largely consistent with the considerable number of estimates available in the literature: $T_{eff} = 6440$ K (Wright...
4.11 HD 210702
HD 210702 hosts a $M \sin i = 1.9 M_{\mathrm{Jup}}$ planet in a low-eccentricity, 355 d orbit (Johnson et al. 2007). With a semimajor axis of 1.2 au, the planet does not enter the system conservative or optimistic HZs, even at apastron.

We measure HD 210702’s radius and $T_{\text{eff}}$ to be 5.2314 ± 0.1171 $R_{\odot}$ and 4780 ± 18 K, respectively, which is consistent with the interferometric (CHARA $K'$-band) values published in Baines et al. (2009, 5.17 ± 0.15 $R_{\odot}$ and 4859 ± 62 K), as well as the radius estimated in the XO-Rad catalogue of van Belle & von Braun (2009, 5.20 ± 0.31 $R_{\odot}$). Allende Prieto & Lambert (1999) quote 5.13 $R_{\odot}$ and 4897 K (with error estimates for all stars in their catalogue of 6 per cent in radius and 2 per cent in $T_{\text{eff}}$) – also consistent with our direct values. Other estimates from Johnson et al. (2007, 4.45 ± 0.07 $R_{\odot}$ and 5510 ± 44 K) and Maldonado et al. (2013, 4.7 $R_{\odot}$ and 4993 K) are lower in radius and higher in effective temperature than our directly determined values.

Application of the $Y^2$ isochrones with input values from Table 4 for HD 210702 returns a stellar age of 5 Gyr and a mass of 1.29 $M_{\odot}$.

5 SUMMARY AND CONCLUSION
A very large fraction of the information on extrasolar planets that has been gathered over the course of the last 20 yr is purely due to the study of the effects that the planets have on their respective parent stars. That is, the star’s light is used to characterize the planetary system. In addition, the parent star dominates any exoplanet system as the principal energy source and mass repository. Finally, physical parameters of planets are almost always direct functions of their stellar counterparts. These aspects assign a substantial importance to studying the stars themselves: one at best only understands the planet as well as one understands the parent star.

In this paper, we characterize 11 exoplanet host star systems with a wide range in radius and effective temperature, based on a 3.5 yr long observing survey with CHARA’s Classic beam combiner. For the systems with previously published direct diameters ($\rho$ CrB, GJ 614 and HD 210702), we provide updates based on increased data quantity and improved performance by the array. For the rest of the systems, only indirectly determined values for radius and effective temperature are present in the literature (if any exist at all). Our thus determined stellar astrophysical parameters make it possible to place our sample of exoplanet host stars on to an empirical Hertzsprung–Russell (H–R) diagram. In Fig. 6, we show our targets along with interferometrically determined parameters of previously published exoplanet hosts and other main-sequence stars with diameter uncertainties of less than 5 per cent.

Due to the relatively low number of stars per spectral type in our sample, and due to the large variance among radius and temperature values quoted in the literature, it is impossible to quantify trends in terms of how indirectly determined values compare with direct counterparts as a function of spectral type, such as the ones documented in Boyajian et al. (2013). For the latest spectral type in our sample, and arguably the most interesting system in terms of exoplanet science, GJ 876, our directly determined stellar radius is significantly (>20 per cent) larger than commonly used literature equivalents (Section 4.6).

We use our directly determined stellar properties to calculate stellar mass and age wherever possible, though the associated uncertainties are large for the KM dwarfs (Section 3.2). Calculations of system HZ locations and boundaries, based on stellar luminosity and effective temperature, show that (1) GJ 876 hosts two planets...
Figure 6. Empirical H–R diagram for all main-sequence stars with interferometrically determined radii whose random uncertainties are smaller than 5 per cent, as published and compiled in Boyajian et al. (2012b, 2013). The diameter of each data point is representative of the respective stellar radius. Error bars in effective temperature and luminosity are smaller than the size of the data points. Previously published exoplanet host stars are identified in blue (Kervella et al. 2003; Baines et al. 2008a, 2009; van Belle & von Braun 2009; von Braun et al. 2011a,b, 2012; Boyajian et al. 2013; Henry et al. 2013). The exoplanet host stars that are presented in this work are shown in red. Stars that do not host any published exoplanets are shown in grey.

who spend all or large parts of their orbital duration in the system HZ, whereas (2) the planet orbiting HD 33564 spends a small part of its period in the stellar HZ as its elliptical orbit causes it to periodically dip into it around apastron passage.

CHARA’s continuously improving performance in both sensitivity and spatial resolution increasingly enables the direct measurements of stellar radii and effective temperatures further and further into the low-mass regime to provide comparison to stellar parameters derived by indirect methods, and indeed calibration of these methods themselves.

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