

Relativistic effects and dark matter in the Solar system from observations of planets and spacecraft

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ABSTRACT

The high precision of the latest version of the planetary ephemeris Ephemerides of the Planets and the Moon (EPM2011) enables one to explore more accurately a variety of small effects in the Solar system. The processing of about 678 thousand of position observations of planets and spacecraft for 1913–2011 with the predominance of modern radar measurements resulted in improving the PPN parameters, dynamic oblateness of the Sun, secular variation of the heliocentric gravitational constant GM_{\odot} , and the stronger limits on variation of the gravitational constant G . This processing made it possible to estimate the potential additional gravitational influence of dark matter on the motion of the Solar system bodies. The density of dark matter ρ_{dm} , if any, turned out to be substantially below the accuracy achieved by the present determination of such parameters. At the distance of the orbit of Saturn the density ρ_{dm} is estimated to be under $1.1 \times 10^{-20} \text{ g cm}^{-3}$, and the mass of dark matter in the area inside the orbit of Saturn is less than $7.9 \times 10^{-11} M_{\odot}$ even taking into account its possible tendency to concentrate in the centre.

Key words: relativistic processes – astrometry – ephemerides – Sun: fundamental parameters – dark matter.

1 INTRODUCTION

The possibility to test and refine various relativistic and cosmological effects from the analysis of the motion of the Solar system bodies is due to the present metre accuracy radio techniques (Standish 2008) and millimeter accuracy laser techniques (Murphy et al. 2008) for the distance measurements. Just these techniques have provided an observational foundation of the contemporary high-precision theories of planetary motions.

The numerical theories of planetary motions have been improved and developed by several groups in different countries and their accuracy is constantly growing. The progress is related with the increase of the number of high-precision radio observations and the inclusion of a number of small effects (perturbations from a set of asteroids, the solar oblateness perturbations, etc.) in constructing a dynamic model of the Solar system. The radio technical observations, having much higher accuracy as compared with the optical ones, are commonly used now in astrometric practice. These high-precision measurements covering more than 50 year time interval allow us to find the orbital elements, masses and other parameters determining the motion of the bodies. Moreover, they also give a possibility to check some relativistic parameters to estimate the secular change of the heliocentric gravitation constant and to examine

the presence of dark matter in the Solar system. The last point is of particular importance for the contemporary cosmological theories. A more accurate and extensive set of observations permits us not only to determine the relativistic perihelion precession of planets, but also to estimate the oblateness of the Sun with the corresponding contribution into the drift of the perihelia. Moreover, these observations provide a means for finding the secular variation of the heliocentric gravitational constant GM_{\odot} and the constraint on the secular variation of the gravitational constant G (Pitjeva & Pitjev 2012). In addition, these precise observations enable us to consider the assumption of the presence of dark matter in the Solar system and to estimate the upper limits of its mass and density.

The present research has been performed on the basis of the current version of the numerical Ephemerides of the Planets and the Moon (EPM2011) of the Institute of Applied Astronomy of Russian Academy of Sciences (IAA RAS).

2 THE PLANETARY EPHEMERIS EPM2011

Numerical EPM had started in the 1970s. Each subsequent version is characterized by additional new observations, refined values of the orbital elements and masses of the bodies, an improved dynamical model of the celestial bodies motion, as well as a more advanced reduction of observational data.

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Table 1. VLBI observations of near-planet spacecraft at the ICRF background quasars.

Planet	Spacecraft	Interval of observations	Number of observations
Venus	Magellan	1990–1994	$18(\alpha + \delta)$
	VEX	2007–2010	$29(\alpha + \delta)$
Mars	Phobos	1989	$2(\alpha + \delta)$
	MGS	2001–2003	$15(\alpha + \delta)$
	Odyssey	2002–2010	$86(\alpha + \delta)$
	MRO	2006–2010	$41(\alpha + \delta)$
Saturn	Cassini	2004–2009	$22(\alpha, \delta)$

All presently used main planetary ephemerides DE (Standish 1998), EPM (Pitjeva 2005a) and INPOP (Fienga et al. 2008) are based on General Relativity involving the relativistic equations of celestial bodies motion and light propagation as well as the relativistic time-scales. In addition, these ephemerides involve estimating from observations some parameters (β , γ , G) to check their compatibility with General Relativity.

The current EPM2011 ephemerides were constructed using approximately 680 thousand data (1913–2011) of different types. The equations of the bodies motion were taken within the parametrized post-Newtonian N -body metric in the barycentric coordinate system – BCRS (Brumberg 1991), the same as that of DE. Integration in Barycentric Dynamical Time (TDB) time-scale (see the IAU2006 resolution B3) was performed using Everhart’s method over the 400 year interval (1800–2200) with the lunar and planetary integrator of the ERA-7 software package (Krasinsky & Vasilyev 1997). The EPM ephemerides including also the time differences TT–TDB, and seven additional objects, namely, Ceres, Pallas, Vesta, Eris, Haumea, Makemake and Sedna are available via FTP by means of <ftp://quasar.ipa.nw.ru/incoming/EPM/>.

Since the basic observational data for producing the next version of the planetary ephemerides EPM2011 were mainly related to the spacecraft, the control of the orientation of the EPM2011 ephemerides with respect to the ICRF frame has required a particular attention. For this purpose, we have used the very long baseline interferometry (VLBI) observations of spacecraft near planets at the background of quasars whose coordinates are given in the ICRF frame (Table 1), where (α, δ) are two-dimensional measurements, $(\alpha + \delta)$ being one-dimensional measurements of the α and δ combination (the position of the planet is observed to be displaced from the base ephemeris (DE405) by a correction measured counter-clockwise along a line at an angle to the right ascension axis, see Folkner 1992).

The accuracy of such observations increased to tenths of mas (1 mas = 0.001 arcsec) for Mars and Saturn in 2001–2010 (Jones et al. 2011) enabling us to improve the orientation of EPM ephemerides (Table 2) in the same way, as it was done by Standish (1998). The angles of rotation of the Earth–Moon barycentre vector about the x -, y -, z -axes of the BCRS system were obtained from VLBI observations described above.

Table 2. The angles of rotation of the EPM2011 ephemerides to ICRF (1 mas = 0.001 arcsec).

Interval of observations	Number of observations	ε_x (mas)	ε_y (mas)	ε_z (mas)
1989–2010	213	-0.000 ± 0.042	-0.025 ± 0.048	0.004 ± 0.028

More than 270 parameters are estimated in the planetary part of EPM2011 ephemerides as follows:

- (i) the orbital elements of planets and satellites of the outer planets,
- (ii) the value of the astronomical unit (au) or GM_\odot ,
- (iii) the angles of orientation of the EPM ephemerides with respect to the ICRF system,
- (iv) parameters of the Mars rotation and the coordinates of the three Mars landers,
- (v) masses of 21 asteroids, the average density of the taxonomic class of asteroids (C, S, M),
- (vi) the mass and radius of the asteroid ring and the mass of the trans-neptunian object (TNO) ring,
- (vii) the mass ratio of the Earth and Moon,
- (viii) the quadrupole moment of the Sun and the solar corona parameters for different conjunctions of the planets with the Sun,
- (ix) the coefficients for the Mercury topography and the corrections to the level surfaces of Venus and Mars,
- (x) coefficients for the additional phase effect of the outer planets.

In the lunar part of EPM ephemerides about 70 parameters are estimated from LLR data (see for example, Krasinsky, Prokhorenko & Yagudina 2011). All estimated parameters in both parts are consistent within the frame of the combined theory of motion of the planets and the Moon given by the EPM ephemerides.

The initial parameters of EPM2011 represented the constants adopted by the IAU GA 27 (Luzum et al. 2011) as the current best values for ephemeris astronomy. Among them five constants were resulted from the ephemeris improvement of DE and EPM ephemerides (Pitjeva & Standish 2009). At present, these five parameters adjusted from processing all observations for EPM2011 are as follows: the masses of the largest asteroids, i.e. $M_{\text{Ceres}}/M_\odot = 4.722(8) \times 10^{-10}$, $M_{\text{Pallas}}/M_\odot = 1.047(9) \times 10^{-10}$, $M_{\text{Vesta}}/M_\odot = 1.297(5) \times 10^{-10}$; ratio of the masses of the Earth and Moon $M_{\text{Earth}}/M_{\text{Moon}} = 81.300\,567\,63 \pm 0.000\,000\,05$; the value of the au in metres $\text{au} = (149\,597\,870\,695.88 \pm 0.14)$ or the heliocentric gravitation constant $GM_\odot = (132\,712\,440\,031 \pm 1) \text{km}^3 \text{s}^{-2}$.

Presently, in accordance with the IAU 2012 resolution B2 the au is re-defined by fixing its value. Up to now, both values of au and the heliocentric gravitation constant (GM_\odot) were connected. It was possible to determine the au value and to calculate the value of GM_\odot from it, or vice versa, to determine GM_\odot and to calculate the value of au from it. Here, the values of au and GM_\odot are given as in the paper Pitjeva & Standish (2009) published before the IAU 2012 resolution B2. At present, only the value of GM_\odot is estimated from observations.

A serious problem in developing modern planetary ephemerides arises due to the necessity to take into account the perturbations caused by asteroids. The factors affecting the planetary motions and needed to be included in developing high-precision ephemerides are of particular consideration in this paper. The hazard near-Earth asteroids are relatively small ($D < 5$ km) and their perturbations do not affect practically the Earth motion. That is why they are not examined in this paper. The main asteroid belt substantially

affecting the motion of Mars and other planets is modelled in the EPM ephemerides by using the motion of 301 large asteroids and a homogeneous material ring representing the influence of all other numerous small asteroids (Krasinsky et al. 2002; Pitjeva 2010a). The parameters characterizing the ring of small asteroids (its mass and radius) were determined from the analysis of observations resulting in the values:

$$M_{\text{ring}} = (1.06 \pm 1.12) \times 10^{-10} M_{\odot} (3\sigma),$$

$$R_{\text{ring}} = (3.57 \pm 0.26) (3\sigma) \text{ au.}$$

The total mass of the asteroid main belt represented by the sum of the mass of 301 largest asteroids and the homogeneous material ring (involving the main uncertainty) is

$$M_{\text{belt}} = (12.29 \pm 1.13) \times 10^{-10} M_{\odot} (3\sigma), \text{ that is } \approx 3M_{\text{Ceres}}. \quad (1)$$

This value of M_{belt} is close to $M_{\text{belt}} = (13.3 \pm 0.2) \times 10^{-10} M_{\odot} (\sigma)$, obtained from the Mars ranging data in the paper by Kuchynka & Folkner (2013) by means of another method in estimating the masses of 3714 individual asteroids.

Hundreds of TNO discovered in recent years also affect the motion of the planets, especially outer ones. A dynamic model of EPM ephemerides includes Eris (the planet-dwarf found in 2003 and surpassing Pluto by its mass) and the 20 largest TNO into the process of the simultaneous integration. Perturbations from other TNO are modelled by the perturbation from a homogeneous ring located in the ecliptic plane with the radius of 43 au and the mass estimated in (Pitjeva 2010a). The mass of the TNO ring found from the analysis of observations amounts to

$$M_{\text{TNO ring}} = (501 \pm 249) \times 10^{-10} M_{\odot} (3\sigma).$$

The total mass of all TNO including the mass of Pluto, 21 largest TNO and the TNO ring comes to

$$M_{\text{TNO}} = (790 \pm 250) \times 10^{-10} M_{\odot} (3\sigma), \text{ that is } \approx 164M_{\text{Ceres}} \text{ or } \approx 2M_{\text{Moon}}. \quad (2)$$

In addition to the mutual perturbations of the major planets and the Moon, the EPM2011 dynamic model includes

- (i) the perturbations of the 301 most massive asteroids,
- (ii) the perturbations from the remaining minor planets of the main asteroid belt modelled by a homogeneous ring,
- (iii) the perturbations from the 21 largest TNO,
- (iv) the perturbations from the remaining TNO modelled by a uniform ring at the average distance of 43 au,
- (v) the relativistic perturbations,
- (vi) the perturbation due to the oblateness of the Sun estimated in EPM2011 fitting as ($J_2 = 2 \times 10^{-7}$).

3 OBSERVATION DATA AND THEIR REDUCTIONS

The total amount of the high-precision observations used for fitting EPM2011 has been increased due to the recent data. They include 677 670 positional measurements of different types for 1913–2011 from classic meridian measurements to modern spacecraft tracking data (Table 3).

Radar measurements (the detailed description of them is given in Pitjeva 2005a, 2013) have a high accuracy. At present, the relative accuracy $\sim 10^{-12}$ for the spacecraft trajectory measurements became usual, exceeding the accuracy of classical optical measurements by five orders of magnitude. However, in general only

Table 3. The observational material.

Planet	Radio observations		Optical observations	
	Time interval	Number	Time interval	Number
Mercury	1964–2009	948	–	–
Venus	1961–2010	40 061	–	–
Mars	1965–2010	578 918	–	–
Jupiter+4 sat.	1973–1997	51	1914–2011	13 364
Saturn+9 sat.	1979–2009	126	1913–2011	15 956
Uran+4 sat.	1986	3	1914–2011	11 846
Neptun+1 sat.	1989	3	1913–2011	11 634
Pluton	–	–	1914–2011	5660
Total	–	620 110	–	57 560

Table 4. Additional perihelion precessions from observations of planets and 19 spacecraft.

Planets	$\dot{\pi}$ (mas yr ⁻¹)	$ \sigma_{\dot{\pi}}/\dot{\pi} $
Mercury	-0.020 ± 0.030	1.5
Venus	0.026 ± 0.016	0.62
Earth	0.0019 ± 0.0019	1.0
Mars	-0.00020 ± 0.00037	1.9
Jupiter	0.587 ± 0.283	0.48
Saturn	-0.0032 ± 0.0047	1.5

Mercury, Venus and Mars are provided with radio observations. Initially, the surfaces of these planets were radio located from 1961 to 1995. Later on many spacecraft passed by, orbited or landed to these planets. A large portion of the spacecraft data were used to get the astrometric positions. There are much less radio observations for Jupiter and Saturn, and only one set of the three-dimensional normal points (α , δ , R) obtained from the Voyager-2 spacecraft are available for Uranus and Neptune. Therefore, the optical observations are still of great importance for the outer planets. Thereby, the varied data of 19 spacecraft were used for constructing the EPM2011 ephemerides and estimating the relevant parameters, in particular, the additional perihelion precessions of the planets (see Table 4).

The recent data from the spacecraft have been added to the previous ones for the latest version of the EPM ephemerides. It involves data related to Odyssey, Mars Reconnaissance Orbiter (MRO; Konopliv et al. 2011), Mars Express (MEX), Venus Express (VEX) and, more specifically, VLBI observations of Odyssey and MRO, three-dimensional normal points of Cassini and Messenger observations, along with the CCD observations of the outer planets and their satellites obtained at Flagstaff and Table Mountain observatories. The most part of observations were taken from the data base of JPL/Caltech created by Dr Standish and continued by Dr Folkner. MEX and VEX data provided by ESA became available thanks to Dr. Fienga's kindness (private communications of T. Morlay to A. Fienga).

The detailed description of methods for all reductions of planetary observations (both optical and radar ones) was given by Standish (1990). This is a basic paper in the field of planetary observations discussion. In the EPM ephemerides some reductions changed slightly are described in Pitjeva (2005a, 2013). All necessary reductions listed therein were introduced into actual observation data as follows:

Reductions of the radar observations:

- (i) the reduction of moments of observations to a uniform time-scale;

(ii) the relativistic corrections – the time-delay of propagation of radio signals in the gravitational field of the Sun, Jupiter and Saturn (Shapiro effect) and the reduction of TDB time (ephemeris argument) to the observer's proper time;

(iii) the delay of radio signals in the troposphere of the Earth;

(iv) the delay of radio signals in the plasma of the solar corona;

(v) the correction for the topography of the planetary surfaces (Mercury, Venus, Mars).

Reductions of the optical observations:

(i) the transformation of observations to the ICRF frame catalogue differences \Rightarrow FK4 \Rightarrow FK5 \Rightarrow ICRF;

(ii) the relativistic correction for the light bending of the Sun;

(iii) the correction for the additional phase effect.

4 RESULTS

4.1 Estimates of relativistic effects

Some small parameters are determined (in addition to the orbital elements of the planets) while constructing the EPM ephemerides using new observations and the method similar to (Pitjeva 2005a,b). In most cases, the parameters can be found from the analysis of the secular changes of the orbital elements. Therefore, the uncertainties of their determination decrease with increasing the time interval of observations.

The simplified relativistic equations of the planetary motion were derived more than 30 years ago in different coordinate systems of the Schwarzschild metric supplemented with coordinate parameter α to specify standard, harmonic, isotropic or any other coordinates. These equations were described in Brumberg (1972, 1991). For example, the integration exposed in Oesterwinter & Cohen (1972) was made in the standard coordinates ($\alpha = 1$). However, planetary coordinates turned out to be essentially different for the standard and harmonic systems. It was shown in Brumberg (1979) that the ephemeris construction and processing of observations should be done in the same coordinate system resulting to the relativistic effects not dependent on the coordinate system (effacing of parameter α). Later on, the resolutions of IAU (1991, 2000) recommended to use the harmonic coordinates for BCRS. In accordance with the IAU 2000 resolution B1.3, all modern planetary ephemerides are constructed in the harmonic coordinates for BCRS – the barycentric (for the Solar system) coordinate system.

The parameters of the PPN formalism β, γ used to describe the metric theories of gravity must be equal to 1 in General Relativity. The values of parameters β, γ were obtained simultaneously by using the EPM2011 ephemerides and the updated data base of high-precision observations (Table 3) from the relativistic periodic and secular variations of the orbital elements, as well as the Shapiro effect. Certainly, the periodic variations of the orbital elements are smaller than the secular ones but they are of importance to compute the planetary motion. We derived expressions for the partial derivatives of the orbital elements with respect to β and γ using the analytical formulas for the relativistic perturbations of the elements, including the secular and principal periodic terms given in Brumberg (1972). This technique enabled us to get actually the values for β and γ . Moreover, in the 80s of the last century, we tested relativistic effects by processing the observations available at that time. It turned out that the relativistic ephemeris for any observed planet provided a considerably better fit of observations (by 10 per cent) than the Newtonian theory even if the latter incor-

porated the observed perihelion secular motion (Krasinsky et al. 1986).

The obtained values of β and γ read

$$\beta - 1 = -0.00002 \pm 0.00003,$$

$$\gamma - 1 = +0.00004 \pm 0.00006 (3\sigma). \quad (3)$$

The uncertainties in (3) significantly decreased as compared with the results for the EPM2004 (Pitjeva 2005b) and EPM2008 (Pitjeva 2010b)

$$|\beta - 1| < 0.0002, \quad |\gamma - 1| < 0.0002.$$

In Fienga et al. (2011) based on the INPOP10a planetary ephemeris, these parameters were determined separately fixing one of these two values, either $\beta = 1$ or $\gamma = 1$. Yet the ephemeris fitting results in

$$\beta - 1 = -0.000062 \pm 0.000081,$$

$$\gamma - 1 = +0.000045 \pm 0.000075.$$

For comparison, we also quote the new γ value obtained by using Very Long Baseline Array measurements of radio sources by Fomalont et al. (2009), i.e. $\gamma = 0.9998 \pm 0.0003$.

All the obtained values of β, γ are in the close vicinity of 1 within the limits of their uncertainties. As the uncertainties of these parameters decrease, the range of possible values of the PPN parameters narrows, imposing increasingly stringent constraints on the gravitation theories alternative to General Relativity.

4.2 Estimations of the solar dynamic oblateness

The solar oblateness produces the secular trends in all elements of the planets with the exception of their semimajor axes and eccentricities (see, for example, Brumberg 1972). Therefore, the dynamic solar oblateness can be determined together with other parameters from observations in constructing a theory of planetary motion. The quadrupole moment of the Sun characterizing the solar oblateness was found in EPM2011 to be

$$J_2 = (2.0 \pm 0.2) \times 10^{-7} (3\sigma), \quad (4)$$

that is close to the previous result (Pitjeva 2005a,b) for EPM2004 $J_2 = (1.9 \pm 0.3) \times 10^{-7}$ and the result of INPOP10a (Fienga et al. 2011) $J_2 = (2.40 \pm 0.25) \times 10^{-7} (1\sigma)$.

4.3 Estimations of the secular changes of GM_\odot and G

The value of the secular change of the heliocentric gravitational constant GM_\odot has been updated for the expanded data base and the improved dynamical model of planetary motions (EPM2011). The determination of secular variation GM_\odot was carried out by the method exposed in detail in (Pitjeva & Pitjev 2012) dealing with the EPM2010 planetary ephemerides.

The GM_\odot change was determined by the weighted method of the least squares with all the basic parameters of the EPM2011 ephemerides. In determining GM_\odot , it was taken into account that the acceleration between the Sun and any planet varies with time when GM_\odot is changing, but the acceleration between any two planets remains unchanged. This is different from the situation when one looks for the G change involving the corresponding change of the accelerations of all bodies. It should be noted that when we determine \dot{G} using the planetary motions (Pitjeva & Pitjev 2012), it is the Sun that contributes most of all. Indeed, the equations of

planetary motion include the products of the masses of bodies and the gravitational constant, the main term exceeding other terms by several orders of magnitude is that for the Sun (GM_{\odot}). Therefore, as the GM_{\odot} term dominates, it is impossible to separate the change of G from the change of GM_{\odot} considering only the motion of the planets (Pitjeva & Pitjev 2012). However, if the change of the solar mass (M_{\odot}) may be estimated from the independent astrophysical data, then based on the change of the GM_{\odot} and the limits of the M_{\odot} change, the limits of the gravitation constant (G) change can be obtained taking into account the following relation

$$\dot{G}M_{\odot}/GM_{\odot} = \dot{G}/G + \dot{M}_{\odot}/M_{\odot} \quad (5)$$

(see details in Pitjeva & Pitjev 2012).

Contrary to \dot{G} , it is the change of GM_{\odot} that can be determined more accurately and reliably using the planetary motions. To control the stability of the solution for $\dot{G}M_{\odot}$ and to obtain the more reliable error, we considered various fitting versions with different numbers of the parameters (the number of the adjusted masses of asteroids, perihelion precessions, etc.). The time-decrease of GM_{\odot} was found to be

$$\dot{G}M_{\odot}/GM_{\odot} = (-6.3 \pm 4.3) \times 10^{-14} \text{ yr}^{-1} (2\sigma). \quad (6)$$

This decrease is caused mostly by the loss of the solar mass M_{\odot} through radiation and the solar wind. The estimate of the uncertainty for this value is more reliable and larger than in Pitjeva & Pitjev (2012). Analysis of versions with different numbers of the fitting parameters demonstrates that the value of $\dot{G}M_{\odot}$ and its uncertainty are the most sensitive to the parameters related to the main asteroid belt, i.e. the amount of the adjusted masses of the selected large asteroids and the estimated characteristics of the ring representing the effect of the small asteroids. The value obtained by Folkner (Konopliv et al. 2011) for DE423 ephemerides from the Mars ranging data is

$$\dot{G}M_{\odot}/GM_{\odot} = (1 \pm 16) \times 10^{-14} \text{ yr}^{-1}.$$

The uncertainty of this value is larger (and may be more reliable) due to taking into account the uncertainties of many other asteroid masses that are not estimated.

Estimation of \dot{M}_{\odot} has been made by means of the astrophysical data using the values for the average solar radiation and solar wind, and amount of comet and asteroid matter falling on the Sun. The obtained limits of the possible change of M_{\odot} can be bounded by the inequality (Pitjeva & Pitjev 2012)

$$-9.8 \times 10^{-14} < \dot{M}_{\odot}/M_{\odot} < -3.6 \times 10^{-14} \text{ yr}^{-1}. \quad (7)$$

This interval may be narrowed due to the more accurate estimation of matter falling on the Sun. From (6) and taking into account (5) and (7), the \dot{G}/G value is found to be within the interval (with the 95 per cent probability)

$$-7.0 \times 10^{-14} < \dot{G}/G < +7.8 \times 10^{-14} \text{ yr}^{-1}. \quad (8)$$

The interval (8) imposes the more rigid limits on the possible change of G than the results of the determination \dot{G} obtained from processing lunar laser observations by Turyshev & Williams (2007)

$$\dot{G}/G = (6 \pm 7) \times 10^{-13} \text{ yr}^{-1}$$

and Hofmann, Muller & Biskupek (2010)

$$\dot{G}/G = (-7 \pm 38) \times 10^{-14} \text{ yr}^{-1}.$$

4.4 Estimations of dark matter in the Solar system

It is proposed in the modern cosmological theories that the bulk of the average density of the Universe falls on dark energy (about 73 per cent) and the dark matter 23 per cent, whereas the baryon matter contains about 4 per cent (Kowalski et al. 2008). The nature of dark matter is non-baryon and its properties are hypothetical (Bertone, Hooper & Silk 2005; Peter 2012).

Despite the possible absence or the very weak interaction of dark matter with ordinary matter, it must possess the capacity of gravity, and its presence in the Solar system can be manifested through its gravitational influence on the body motion. Attempts to detect the possible influence of dark matter on the motion of objects in the Solar system have already been made (Anderson et al. 1989, 1995; Nordtvedt, Mueller & Soffel 1995; Khriplovich & Pitjeva 2006; Sereno & Jetzer 2006; Khriplovich 2007; Frere, Ling & Vertongen 2008).

The additional gravitational influence may depend on the density of dark matter, its distribution in space, etc. We assume, as it is usually done (Anderson et al. 1989, 1995; Gron & Soleng 1996; Khriplovich & Pitjeva 2006; Frere et al. 2008), that dark matter is distributed in the Solar system spherically symmetric relative to the Sun. Then, we may suppose that any planet at distance r from the Sun can undergo an additional acceleration from invisible matter along with the accelerations from the Sun, planets, asteroids, TNO

$$\ddot{\mathbf{r}}_{\text{dm}} = -\frac{GM(r)_{\text{dm}}}{r^3} \mathbf{r},$$

where $M(r)_{\text{dm}}$ is the mass of the additional matter in a sphere of radius r around the Sun.

Testing the presence of the additional gravitational environment can be carried out either by finding the additional acceleration, as was made, for example, in Nordtvedt et al. (1995) and Anderson et al. (1989) or the additional perihelion drift (for example, Gron & Soleng 1996).

The first method determines actually if there is any extra mass inside the spherically symmetric volume, in addition to the masses of the Sun, planets and asteroids already taken into account. Any detected correction to the central attracting mass (or to the heliocentric gravitational constant GM_{\odot}) from the observational data separately for each planet would result in its increased value in accordance with the additional mass within the sphere with the mean radius of the planetary orbit.

The second way is related with an unclosed trajectory of motion in the presence of the additional gravitational medium and the drift of the positions of the pericentres and apocentres from revolution to revolution in contrast to the purely Keplerian case of the two-body problem. Denoting the integrals of energy and area by E , J and the spherically symmetric potential by $U(r)$, the equations of motion of a unit mass along the radius r and along the azimuthal coordinate θ can be written, respectively, (Landau & Lifshitz 1969)

$$\dot{r} = (2[E + U(r)] - J^2/r^2)^{1/2}, \quad (9)$$

$$\frac{d\theta}{dr} = \frac{J/r^2}{(2[E + U(r)] - J^2/r^2)^{1/2}}. \quad (10)$$

In the Keplerian two-body problem, the oscillation periods along the radius r (from the perihelion to the apocentre and back) and along azimuth θ around the centre coincide, and the positions of the pericentre and apocentre are not displaced from revolution to revolution. The additional gravitating medium leads to a shorter radial period and a negative drift of the position of the pericentre and apocentre (in a direction opposite to the planetary motion). The

perihelion precession for the uniformly distributed matter ($\rho_{\text{dm}} = \text{const}$) depends on the orbital semimajor axis a and eccentricity e of the planetary orbit (Khriplovich & Pitjeva 2006)

$$\Delta\theta_0 = -4\pi^2 \rho_{\text{dm}} a^3 (1 - e^2)^{1/2} / M_{\odot}, \quad (11)$$

where $\Delta\theta_0$ is the perihelion drift for one complete radial oscillation.

Estimations of the density and mass of dark matter are produced often under the assumption that it changes very slowly or is constant within the Solar system, i.e. under the assumption of the uniform distribution of dark matter. A number of papers (Lundberg & Edsjo 2004; Peter 2009; Iorio 2010) assume the concentration of dark matter to the centre and even its capture and dropping on the Sun. The latter assumption should be made with caution. In the Section 4.3 (as well as in Pitjeva & Pitjev 2012), it was found that the heliocentric gravitational constant GM_{\odot} decreases, so there is a stringent limitation on the amount of possible dark matter dropping on the Sun. The constraint on the possible presence of dark matter inside the Sun (no more than 2–5 per cent of the solar mass) was also obtained in Kardashev, Tutukov & Fedorova (2005), where the physical characteristics of the Sun have been carefully analysed.

Both approaches have been applied in this work. The more sophisticated consideration is given in Pitjev & Pitjeva (2013).

The corrections to the additional perihelion precession and to the central mass were obtained by fitting the EPM2011 ephemerides to about 780 thousand of observations of the planets and spacecraft (Table 3). The fitting was done by the weighted method of the least squares. The various test solutions differing from one another by the sets of the adjusted parameters were considered for obtaining the reliable values of these parameters and their uncertainties (σ_i) in the same manner as for getting the GM_{\odot} estimation.

The resulting values are exceeded by their uncertainties (σ) indicating that the dark matter density ρ_{dm} , if any, is very small being lower than the accuracy of these parameters achieved by the modern determination. The obtained opposite signs for the values $\Delta\pi$ and ΔM_0 for the various planets also show the smallness of such effects.

The relative uncertainties in the corrections to the central mass from the observations separately for each planet were significantly greater than that for the additional perihelion precessions exceeding the corrections to the central mass themselves in several times or even by several orders of magnitude. It should be remembered that the integral estimation of the dark matter mass falling into a spherically symmetric (relative to the Sun) volume depends on the accuracy of knowledge of all body masses into this volume. Basically, it is the inaccurate knowledge of the masses of asteroids.

More accurate results were obtained for estimates of the perihelion precessions (Table 4) allowing us to estimate the local density of dark matter at the mean orbital distance of a planet. Here, the uncertainties of determination of the corrections are comparable with the values themselves. Therefore, the estimates from Table 4 were actually used.

The investigation of the additional perihelion precession of the planets was carried out taking into account all other known effects affecting the perihelion drift. Indeed, if there is an additional gravitating medium, then a negative drift of the perihelion and aphelion occurs from revolution to revolution in accordance with the formula (11). Since the growth of the perihelion drift is accumulated, this criterion can be sensitive enough for verifying the presence of additional matter.

All the uncertainties of Table 4 are comparable or larger than the absolute values obtained for perihelion precessions. These uncertainties σ_{π} may be treated as the upper limits for the possible

Table 5. Estimates of the density ρ_{dm} obtained from $\sigma_{\Delta\pi}$ for the perihelion precessions.

Planets	σ_{π} (arcsec yr ⁻¹)	ρ_{dm} (g cm ⁻³)
Mercury	0.000 030	$<9.3 \times 10^{-18}$
Venus	0.000 016	$<1.9 \times 10^{-18}$
Earth	0.000 0019	$<1.4 \times 10^{-19}$
Mars	0.000 000 37	$<1.4 \times 10^{-20}$
Jupiter	0.000 283	$<1.7 \times 10^{-18}$
Saturn	0.000 0047	$<1.1 \times 10^{-20}$

additional drifts of the secular motion of the perihelia, and can give the upper limit for the density of the distributed matter by using (11). The resulting estimates ρ_{dm} are shown in Table 5.

The data based on the estimates for the Earth, Mars and Saturn yield the most stringent constraints on the density ρ_{dm} . The high-precision series of observations of Saturn appeared when the Cassini spacecraft arrived to it in 2004. There is the large and long set of observations of Mars associated with many spacecraft on its surface and around it. The Earth orbit improvement is based on all observations as the observations are made from the Earth. Assuming the homogeneous distribution ρ_{dm} in the Solar system the most stringent constraint $\rho_{\text{dm}} < 1.1 \times 10^{-20}$ g cm⁻³ is obtained from the data for Saturn. Then the mass M_{dm} within the spherical volume with the size of Saturn's orbit is

$$M_{\text{dm}} < 7.1 \times 10^{-11} M_{\odot}. \quad (12)$$

This value is about two times smaller than the uncertainty of the obtained total mass of the main asteroid belt (1).

Another version can be considered when a continuous medium has some concentration to the centre of the Solar system. Investigations under the assumption of density concentration to the centre have already been carried out, for example, by Frere et al. (2008). We have taken the model for ρ_{dm} with the exponential dependence on the distance r

$$\rho_{\text{dm}} = \rho_0 \times e^{-cr}, \quad (13)$$

where ρ_0 is the central density and c is a positive parameter characterizing an exponential decrease of the density to the periphery. The value of $c = 0$ corresponds to a uniform density. Function (13) is everywhere finite and has no singularities at the centre and on the periphery. The mass inside a sphere of radius r for distribution (13) is

$$M_{\text{dm}} = 4\pi\rho_0 \times \frac{2 - e^{-cr}(c^2r^2 + 2cr + 2)}{c^3}. \quad (14)$$

In spite of the presence of c^3 in the denominator this expression does not have singularities for $c \rightarrow 0$. The formula (14) transforms therewith into the expression for the mass of a homogeneous sphere.

The values in Table 5 may be considered as the limits of the density ρ_{dm} at various distances. In a relatively narrow interval of the radial distances caused by the eccentricity of the planetary orbit, the density of dark matter can be considered to be approximately constant. The potential existence of the dark matter M_{dm} distributed between the Sun and the orbit of a planet gives very small contribution (the tenths or elevenths fraction of the magnitude) to the total attractive central mass determined by the solar mass. Therefore, one can use the formula (11) and obtain the local restrictive estimations for ρ_{dm} in the neighbourhood of the planet orbit (Table 5).

With the assumption of the concentration to the centre, the estimate of the mass of dark matter within the orbit of Saturn was determined from the evaluation of the masses within the two intervals, i.e. from Saturn to Mars and from Mars to the Sun. For this purpose, the most reliable data of Table 5 for Saturn ($\rho_{\text{dm}} < 1.1 \times 10^{-20} \text{ g cm}^{-3}$), Mars ($\rho_{\text{dm}} < 1.4 \times 10^{-20} \text{ g cm}^{-3}$) and Earth ($\rho_{\text{dm}} < 1.4 \times 10^{-19} \text{ g cm}^{-3}$) were used. Based on the data for Saturn and Mars a very flat trend of the density curve (13) between Mars and Saturn was obtained with $\rho_0 = 1.47 \times 10^{-20} \text{ g cm}^{-3}$ and $c = 0.0299 \text{ au}^{-1}$. From these parameters, the mass in the space between the orbits of Mars and Saturn is $M_{\text{dm}} < 7.33 \times 10^{-11} M_{\odot}$. The obtained trend of the density curve (13) in the interval between Mars and the Sun gives a steep climb to the Sun according to the data for Earth and Mars with the parameters $\rho_0 = 1.17 \times 10^{-17} \text{ g cm}^{-3}$ and $c = 4.42 \text{ au}^{-1}$. For these parameters, the mass (14) between the Sun and the orbit of Mars is $M_{\text{dm}} < 0.55 \times 10^{-11} M_{\odot}$.

Summing masses for both intervals, the upper limit for the total mass of dark matter was estimated as $M_{\text{dm}} < 7.88 \times 10^{-11} M_{\odot}$ between the Sun and the orbit of Saturn, taking into account its possible tendency to concentrate in the centre. This value is less than the uncertainty $\pm 1.13 \times 10^{-10} M_{\odot}$ (3σ) of the total mass of the asteroid belt. The value M_{dm} does not change perceptibly compared to the hypothesis of a uniform density (12), although the trend of the density curve in the second case provides the significant (by three orders of magnitude) increase to the centre.

5 CONCLUSION

The estimations of the gravitational PPN parameters, the solar oblateness, the secular change of the heliocentric gravitation constant GM_{\odot} and the gravitation constant G , as well as the possible gravitational influence of dark matter on the motion of the planets in the Solar system have been made on the basis of the EPM2011 planetary ephemerides of IAA RAS using about 678 000 positional observations of planets and spacecraft, mostly ranging ones.

The PPN parameters turned out to be $\beta - 1 = -0.00002 \pm 0.00003$, $\gamma - 1 = +0.00004 \pm 0.00006$ (σ). Our estimation for the change of the heliocentric gravitational constant is $\dot{GM}_{\odot}/GM_{\odot} = (-6.3 \pm 4.3) \times 10^{-14} \text{ yr}^{-1}$ (2σ). It was found also that the limits for the time variation of the gravitational constant G are $-7.0 \times 10^{-14} < \dot{G}/G < +7.8 \times 10^{-14}$ (2σ) yr^{-1} .

The mass and the level of dark matter density in the Solar system, if any, was obtained to be substantially lower than the modern uncertainties of these parameters. The density of dark matter was found to be lower than $\rho_{\text{dm}} < 1.1 \times 10^{-20} \text{ g cm}^{-3}$ at the distance of the Saturn orbit, and the mass of dark matter in the area inside the orbit of Saturn is less than $7.9 \times 10^{-11} M_{\odot}$, even taking into account its possible tendency to concentrate in the centre.

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