The primordial and evolutionary abundance variations in globular-cluster stars: a problem with two unknowns

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ABSTRACT
We demonstrate that among the potential sources of the primordial abundance variations of the proton-capture elements in globular-cluster stars proposed so far, such as the hot-bottom burning in massive asymptotic giant branch stars and H burning in the convective cores of supermassive and fast-rotating massive main-sequence (MS) stars, only the supermassive MS stars with $M > 10^4 \, M_\odot$ can explain all the observed abundance correlations without any fine-tuning of model parameters. We use our assumed chemical composition for the pristine gas in M13 (NGC 6205) and its mixtures with 50 and 90 per cent of the material partially processed in H burning in the $6 \times 10^4 \, M_\odot$ MS model star as the initial compositions for the normal, intermediate, and extreme populations of low-mass stars in this globular cluster, as suggested by its O–Na anticorrelation. We evolve these stars from the zero-age MS to the red giant branch (RGB) tip with the thermohaline and parametric prescriptions for the RGB extra mixing. We find that the $^3$He-driven thermohaline convection cannot explain the evolutionary decline of [C/Fe] in M13 RGB stars, which, on the other hand, is well reproduced with the universal values for the mixing depth and rate calibrated using the observed decrease of [C/Fe] with $M_V$ in the globular cluster NGC5466 that does not have the primordial abundance variations.

Key words: stars: abundances – stars: evolution – stars: interiors.

1 INTRODUCTION
There is a long-standing problem in stellar astrophysics – understanding the origin of the abundance anomalies of the proton-capture elements, such as C, N, O, Na, Mg, Al, and their isotopes in globular-cluster (GC) stars (e.g. Kraft 1979, 1994; Gratton, Carretta & Bragaglia 2012). The fact that the anomalous abundance variations display clear anticorrelations between C and N, O and Na, Na and F, O and Al, as well as Al and Mg is unanimously interpreted as a strong evidence for all of them to have been produced in hydrogen burning at a sufficiently high temperature, so that reactions of the NeNa and MgAl cycles were able to compete with the CNO cycle (Denissenkov et al. 1998; Prantzos, Charbonnel & Iliadis 2007). Given that some of these anticorrelations are found in low-mass main-sequence (MS) stars in the present-day GCs (Briley et al. 1996, 2004; Cannon et al. 1998; Gratton et al. 2001; Briley, Cohen & Stetson 2002), the required high-temperature H burning must have occurred in their more massive siblings in the past. Because anticorrelations of the same magnitude are observed in a same GC both in MS and red giant stars (Gratton et al. 2001; Dobrovol’skas et al. 2014), the latter possessing deep convective envelopes, quite significant fractions of material lost by the massive stars that had produced those anticorrelations must have been mixed with the pristine gas in the young GC before the low-mass stars formed out of that mixture.

Three types of H burning in stars have been proposed as possible sources of the primordial abundance variations of p-capture elements in GCs: hot-bottom burning (HBB) in massive asymptotic giant branch (AGB) stars (D’Antona, Gratton & Chieffi 1983), H burning in convective cores of rapidly rotating massive MS stars (Decressin et al. 2007) and, more recently, core H burning in supermassive MS stars with masses $M > 10^4 \, M_\odot$ (Denissenkov & Hartwick 2014). Fast rotation with a nearly break-up velocity in the second case plays a twofold role: first, it drives rotation-induced mixing in the radiative envelope, thus bringing H-burning products from the convective core to the surface and, secondly, it

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leads to equatorial mass-loss with a relatively low velocity caused by the centrifugal force. The second property, like the assumed low-velocity mass-loss by the AGB stars, is required to explain the retention of the mass lost by the massive stars in the shallow potential well of the young GC. It is also assumed that the massive AGB and MS stars had migrated to the GC centre, as a result of dynamical friction, before they deposited the products of H burning to the GC interstellar medium. The last assumption is usually used to interpret the larger fraction of low-mass stars with the stronger abundance anomalies in the cores of some GCs (e.g. Milone et al. 2012).

A solution of the problem of the primordial abundance variations in GCs should be divided into two steps. The first step is to find the right massive star candidate, such that when we dilute its H-burning yields with the GC pristine gas we get individual abundances and correlations between them consistent with all the relevant observational data for GCs. The second step is to understand how the required massive stellar objects formed and functioned, how their lost mass was retained and mixed with the pristine gas in GCs, and how significant fractions of low-mass stars made out of those gas mixtures survived and got distributed among their unpolluted counterparts in GCs by the present day. In the framework of this two-step solution, the early disc accretion model of Bastian et al. (2013a) and the model of massive binaries of de Mink et al. (2009) are not considered by us as additional possibilities to explain the origin of the p-capture element abundance anomalies in GCs, because either of these models still uses massive MS stars as the H-burning source of those anomalies. Other details of these models are relevant only to the second step. However, we think that it is not worth discussing any solution details pertaining to the second step until the first step is completed, especially, given that there are a lot of observational data on abundances of p-capture elements and their isotopes in GC stars to constrain the solution on the first step, whereas there are no direct observational data on the formation of the first generation stars in GCs.

In this paper, we conclude that the massive AGB and MS stars are not the best candidates for the origin of the primordial abundance variations in GCs, because they fail to reproduce the correlations between the abundances of Al and Mg isotopes, that have recently been reported by Da Costa, Norris & Yong (2013) and now include stars from five GCs. We demonstrate that this failure is a consequence of temperatures of H burning in these objects that are either too high (in the case of AGB stars) or too low (in the case of massive MS stars). On the other hand, the hypothetical supermassive MS stars with $M > 10^7 M_\odot$ have the right temperature to nicely reproduce not only the Mg–Al anticorrelation, but also all the other observed abundance anomalies of the p-capture elements in GCs, including enhanced He abundances (Denissenkov & Hartwick 2014).

The primordial variations of the C and N abundances and $^{12}$C/$^{13}$C isotopic ratio in GCs are obscured by their evolutionary changes that occur in low-mass stars, both in GCs and in the field, on the upper red giant branch (RGB) above the bump luminosity. These changes are caused by some extra mixing that operates in radiative zones of RGB stars between the H-burning shell (HBS) and the bottom of the convective envelope (BCE). It results in the decreasing surface C abundance and $^{12}$C/$^{13}$C ratio and increasing N abundance when the star climbs the upper RGB and its luminosity increases. At the bump luminosity, the HBS, advancing in mass, erases a mean molecular weight ($\mu$) discontinuity left behind by the BCE at the end of the first dredge-up. This discontinuity probably prevents extra mixing from reaching the HBS on the lower RGB. Above the bump luminosity, the $\mu$-profile in the radiative zone is uniform everywhere, except the vicinity of the HBS, where the reaction $^3$He($^3$He,2p)$^4$He produces its local depression of the order of $\Delta \mu \sim 10^{-4}$ (Eggleton, Dearborn & Lattanzio 2006). This $\mu$-depression should drive thermohaline mixing that was proposed for the role of RGB extra mixing by Charbonnel & Zahn (2007) who actually assumed that its associated fluid parcels (‘salt fingers’) had a ratio of their vertical length to horizontal diameter $a = l/d \sim 6.2$. However, numerical simulations of thermohaline convection by Denissenkov (2010) have shown that the aspect ratio of salt fingers in RGB stars is rather $a \lesssim 1$. Given that the diffusion coefficient for thermohaline convection $D_{th}$ is proportional to $a^2$, it turns out to be too inefficient for the RGB extra mixing.

In this work, we have chosen M13 (NGC 6205) as an exemplary instance of GCs with abundance anomalies of p-capture elements, because it is one of a few GCs that show the most extreme primordial abundance anomalies. We assume that its low-mass stars had been formed out of mixtures of the pristine gas and a varying fraction of material lost by a supermassive MS star with $M > 10^4 M_\odot$, as described by Denissenkov & Hartwick (2014). Following Johnson & Piłachowski (2012), we use the O–Na anticorrelation for M13 stars to subdivide them into three populations according to the strength of their primordial abundance anomalies: a normal (or primordial) population is made of the pristine gas, while intermediate and extreme populations contain, respectively, 50 and 90 per cent of material from the supermassive star mixed with the pristine gas. Then, we allow the low-mass stars belonging to the different populations to evolve from the zero-age MS to the RGB tip. The RGB extra mixing is modelled either using the thermohaline diffusion coefficient (equation 25) from Denissenkov (2010) with the salt-finger aspect ratios $a \gtrsim 7$ that provide the most efficient mixing or using the observationally constrained parametric prescription from Denissenkov & Pinnoneault (2008) and Denissenkov (2012) that employs the same mixing depth as in the thermohaline case, i.e. $\log_{10}(r_{mix}/R) = -1.35$, and diffusion coefficient $D_{mix} = \alpha K$, where $K$ is the thermal diffusivity and $\alpha = 0.01–0.1$ is the free parameter. This simple model that focuses on the nucleosynthesis part (the first step) of the solution takes into account both the primordial and evolutionary abundance variations of the p-capture elements in GC stars. We compare its predictions with the relevant observational data not only for M13 but also for other GCs.

2 COMPUTATIONAL METHOD

The MS evolution of supermassive stars with $M > 10^4 M_\odot$ is calculated using the revision 5329 of the stellar evolution code of MESA (Paxton et al. 2011, 2013), as described by Denissenkov & Hartwick (2014). In particular, because the MS stars with $M > 10^4 M_\odot$ are objects with super-Eddington luminosities, in which the radiation pressure dominates over the gas pressure, we use the MET++ prescription for convection that is recommended in MESA for such cases (see section 7.2 in Paxton et al. 2013). Other input physics data and assumptions in our calculations of supermassive MS stars are the same that we use to calculate low-mass star models.

The evolution of low-mass stars is computed using the older MESA revision 3251. The RGB extra mixing is modelled with the thermohaline diffusion coefficient $D_{th}$ and parametric prescription $D_{mix} = \alpha K$. Model-smoothing parameters were adjusted by Denissenkov (2010) for the MESA revision 3251 to reproduce

http://mesa.sourceforge.net

\footnote{1 http://mesa.sourceforge.net}
Figure 1. Panel A: the O–Na anticorrelation for the M13 RGB stars (the blue, green, and red circles) from Johnson & Pilachowski (2012) is compared with the dilution curves (the magenta star symbols, squares, and diamonds connected by the solid black curves) obtained by mixing the abundances from the M13 pristine gas (the lower-right ends of the curves) with a varying fraction (from 0 to 100 per cent) of material from the supermassive MS stars with the masses $5 \times 10^4 M_{\odot}$ (the star symbols), $6 \times 10^4 M_{\odot}$ (the squares), and $7 \times 10^4 M_{\odot}$ (the diamonds). The black asterisks connected by the dashed line are the theoretical data for the massive AGB stars with the indicated initial masses from Ventura & D’Antona (2009). Panels B, C, and D: the Al abundances and Mg isotopic ratios for the 33 RGB stars from 5 GCs collected by Da Costa et al. (2013, the large single symbols, as identified in panel D) are compared with the theoretical predictions from the supermassive MS and massive AGB stars, as explained for panel A.

The results of his COMSOL high-resolution test simulations of thermohaline mixing in low-mass RGB stars. Stellar evolution codes from both revisions are run with the same nuclear network that includes 31 isotopes from $^1$H to $^{28}$Si coupled by 60 reactions of the pp chains, CNO, NeNa, and MgAl cycles. For solar composition, we use the elemental abundances of Grevesse & Sauval (1998) with the isotopic abundance ratios from Lodders (2003). The chemical composition of the pristine gas in M13 is obtained from the solar composition using $[\text{Fe/H}] = -1.53$ for the metallicity of M13 as a scaling factor and $[\alpha/\text{Fe}] = +0.4$ for the abundances of $\alpha$-elements ($^{16}$O, $^{20}$Ne, $^{24}$Mg etc.). We also assume $[\text{Na/Fe}] = -0.4$ and the solar Mg isotopic ratios in the initial composition of the pristine gas because these values are suggested by the observational data.

\[ [\text{A/B}] = \log_{10}(N(\text{A})/N(\text{B})) - \log_{10}(N(\odot)(\text{A})/N(\odot)(\text{B})), \]

where $N(\text{A})$ and $N(\text{B})$ are number densities of the nuclides A and B.

(e.g. Fig. 1). This initial chemical composition is slightly different from that used by Denissenkov & Hartwick (2014), therefore we have re-calculated our supermassive MS models. With the MESA kap pre-processor, we have generated opacity tables appropriate for our initial composition, i.e. the ones based on the Grevesse & Sauval (1998) solar abundances with $[\alpha/\text{Fe}] = +0.4$, and employed them for all of our mixtures. The pre-processor uses the corresponding OPAL opacity tables (Iglesias & Rogers 1993, 1996) and low-temperature molecular opacities of Ferguson et al. (2005) as input data. For the convective mixing length, we have chosen the MESA solar calibrated parameter $\alpha_{\text{MLT}} = 1.92$ in the Henyey, Vardya & Bodenheimer (1965) MLT prescription. Atmospheric boundary conditions are calculated in the approximation of Krishna Swamy (1966).

For M13 isochrone calculations, we have used the VICTORIA stellar evolution code (VandenBerg et al. 2012), because it treats both atomic diffusion and its counteracting turbulent mixing, whereas MESA code does not include the latter, which leads to an excessive...
depletion of the surface He abundance on the MS.\textsuperscript{3} Therefore, in our \textsc{mesa} calculations of the evolution of low-mass stars we neglect atomic diffusion. This increases the effective temperature at the MS turn-off, but does not lead to important differences, except for Li, in the evolution of surface composition on the RGB as compared to the \textsc{victoria} models, because the first dredge-up erases most of the surface abundance changes produced by atomic diffusion on the MS.

\section{Three Populations of Low-Mass Stars in M13}

Following Denissenkov & Hartwick (2014), we calculate the evolution of supermassive MS stars with $M > 10^8 \, \text{M}_\odot$ only until the He mass fraction at the surface and, because these stars are fully convective, also at the centre has reached the value $Y = 0.40$, which is close to the maximum He abundances reported in the present-day GC stars (Pasquini et al. 2011; King et al. 2012). The corresponding ages of the supermassive stars are less than $10^8$ years. Their initial chemical composition is assumed to be that of the M13 pristine gas, which is equivalent to the composition of the M13 normal population in Table 1.

\textsuperscript{3} When very close to the same physics is assumed, the \textsc{victoria} and \textsc{mesa} codes predict nearly identical evolutionary tracks and lifetimes for stars of a given mass and chemical composition (see VandenBerg et al. 2012).

We explicitly require that the evolution of supermassive stars is terminated early on the MS, because the observed abundance patterns in GCs are characteristic of incomplete H burning (Denissenkov et al. 1998; Prantzos et al. 2007). As proposed by Denissenkov & Hartwick (2014), the most likely cause of such an early death of supermassive stars could be their fragmentation by a diffusive mode of the Jeans instability (Thompson 2008), which does not depend on a spatial scale and grows on the right time-scale of the order of a few $10^7$ years.

The filled blue, green, and red circles in Fig. 1(A) represent, respectively, the normal (or primordial), intermediate, and extreme populations of low-mass RGB stars in M13, according to the selection criteria used by Johnson & Pilachowski (2012). Together, they form the O–Na anticorrelation which, like the other correlations between the p-capture elements and their isotopes, is usually interpreted as a result of mixing of the pristine gas with different fractions of material lost by massive stars that had taken place in the young GC, before those low-mass stars formed. In this interpretation, one end of a correlation, e.g. the lower-right end of the O–Na anticorrelation, gives abundances in the pristine composition ($\text{[O/Fe]} \approx +0.4$ and $\text{[Na/Fe]} \approx -0.4$ for M13), while the other end points towards abundances characteristic of the polluting star ($\text{[O/Fe]} \lesssim -1$ and $\text{[Na/Fe]} \approx +0.4$ for M13).

The filled magenta star symbol, square, and diamond at the left ends of solid black curves in Fig. 1(A) give the O and Na abundances in the MS stars with the masses $5 \times 10^4$, $6 \times 10^4$, and $7 \times 10^4 \, \text{M}_\odot$ at $Y = 0.40$. The same symbols at other locations along the solid

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Table 1. Initial mass fractions of isotopes used in our calculations.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Norm. pop.$^a$</th>
<th>Int. pop.$^b$</th>
<th>Extr. pop.$^c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1\text{H}$</td>
<td>0.748 815</td>
<td>0.674 238</td>
<td>0.614 576</td>
</tr>
<tr>
<td>$^3\text{He}$</td>
<td>$6.554 \times 10^{-5}$</td>
<td>$3.277 \times 10^{-5}$</td>
<td>$6.554 \times 10^{-6}$</td>
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<tr>
<td>$^4\text{He}$</td>
<td>0.250 000</td>
<td>0.324 641</td>
<td>0.384 353</td>
</tr>
<tr>
<td>$^6\text{Li}$</td>
<td>1.045 86 $\times 10^{-9}$</td>
<td>5.232 63 $\times 10^{-10}$</td>
<td>1.051 87 $\times 10^{-10}$</td>
</tr>
<tr>
<td>$^9\text{O}$</td>
<td>9.501 77 $\times 10^{-6}$</td>
<td>5.779 48 $\times 10^{-5}$</td>
<td>2.801 65 $\times 10^{-5}$</td>
</tr>
<tr>
<td>$^{12}\text{C}$</td>
<td>1.153 10 $\times 10^{-9}$</td>
<td>3.509 72 $\times 10^{-6}$</td>
<td>5.395 01 $\times 10^{-6}$</td>
</tr>
<tr>
<td>$^{25}\text{Mg}$</td>
<td>2.805 40 $\times 10^{-5}$</td>
<td>3.529 88 $\times 10^{-4}$</td>
<td>6.129 36 $\times 10^{-4}$</td>
</tr>
<tr>
<td>$^{28}\text{Si}$</td>
<td>1.130 97 $\times 10^{-7}$</td>
<td>6.435 48 $\times 10^{-8}$</td>
<td>2.743 31 $\times 10^{-8}$</td>
</tr>
<tr>
<td>$^{16}\text{O}$</td>
<td>6.554 65 $\times 10^{-4}$</td>
<td>3.312 00 $\times 10^{-4}$</td>
<td>7.178 76 $\times 10^{-5}$</td>
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<td>$^{18}\text{O}$</td>
<td>1.033 84 $\times 10^{-7}$</td>
<td>5.946 62 $\times 10^{-8}$</td>
<td>2.433 19 $\times 10^{-8}$</td>
</tr>
<tr>
<td>$^{19}\text{F}$</td>
<td>5.886 41 $\times 10^{-7}$</td>
<td>2.943 23 $\times 10^{-7}$</td>
<td>5.886 87 $\times 10^{-8}$</td>
</tr>
<tr>
<td>$^{20}\text{Ne}$</td>
<td>1.357 40 $\times 10^{-4}$</td>
<td>1.313 17 $\times 10^{-4}$</td>
<td>1.277 79 $\times 10^{-4}$</td>
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<tr>
<td>$^{21}\text{Ne}$</td>
<td>1.360 18 $\times 10^{-5}$</td>
<td>6.964 19 $\times 10^{-8}$</td>
<td>1.654 12 $\times 10^{-8}$</td>
</tr>
<tr>
<td>$^{22}\text{Ne}$</td>
<td>4.370 84 $\times 10^{-6}$</td>
<td>2.213 63 $\times 10^{-6}$</td>
<td>4.878 57 $\times 10^{-7}$</td>
</tr>
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<td>$^{23}\text{Na}$</td>
<td>1.161 95 $\times 10^{-6}$</td>
<td>4.760 80 $\times 10^{-6}$</td>
<td>7.639 88 $\times 10^{-6}$</td>
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<tr>
<td>$^{24}\text{Mg}$</td>
<td>4.377 80 $\times 10^{-5}$</td>
<td>2.349 66 $\times 10^{-5}$</td>
<td>7.271 60 $\times 10^{-6}$</td>
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<td>$^{25}\text{Mg}$</td>
<td>5.774 73 $\times 10^{-6}$</td>
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<td>$^{26}\text{Mg}$</td>
<td>6.607 31 $\times 10^{-6}$</td>
<td>1.364 72 $\times 10^{-5}$</td>
<td>1.927 91 $\times 10^{-5}$</td>
</tr>
<tr>
<td>$^{27}\text{Al}$</td>
<td>2.017 55 $\times 10^{-6}$</td>
<td>2.256 30 $\times 10^{-5}$</td>
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<td>5.694 95 $\times 10^{-5}$</td>
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<td>6.006 38 $\times 10^{-5}$</td>
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<tr>
<td>$^{56}\text{Fe}$</td>
<td>3.928 93 $\times 10^{-5}$</td>
<td>3.928 93 $\times 10^{-5}$</td>
<td>3.928 93 $\times 10^{-5}$</td>
</tr>
<tr>
<td>$^{56}\text{Ni}$</td>
<td>4.334 57 $\times 10^{-5}$</td>
<td>4.334 57 $\times 10^{-5}$</td>
<td>4.334 57 $\times 10^{-5}$</td>
</tr>
</tbody>
</table>

\textit{Notes.} $^a$ The heavy-element mass fraction for this and the other two mixtures is $Z \approx 0.0011$. $^b$ For the intermediate population, we assume a mixture of 50 per cent of the abundances from the normal population with 50 per cent of the abundances from our $6 \times 10^7 \, \text{M}_\odot$ MS star model when its He abundance has increased to $Y = 0.40$. $^c$ For the extreme population, we assume a mixture of 10 per cent of the abundances from the normal population with 90 per cent of the abundances from our $6 \times 10^7 \, \text{M}_\odot$ MS star model when its He abundance has increased to $Y = 0.40$. 

\[ (\text{C} + \text{N} + \text{O})/\text{H} \]
black curves show the results of these final abundances having been mixed with 10, 20, . . . , 90, and 100 percent of the O and Na abundances from the pristine gas. As the abundance mixtures represent the normal, intermediate, and extreme populations of stars in M13, we choose those with 0, 50, and 90 percent of material from the MS star with \( M = 6 \times 10^4 M_\odot \) (the first, sixth, and tenth magenta squares on the middle solid black curve counting from its lower-right end). Their corresponding initial isotopic mass fractions are given in Table 1. We make such a discrete choice of the initial chemical composition only for simplicity, while keeping in mind that, actually, there are no sharp boundaries between the three populations in Fig. 1(A).

The different single symbols in Figs 1(B)–(D) form the discernable dependences of the Mg isotopic ratios on the Al abundance. They represent observational data for 33 RGB stars from 5 GCs, including M13, that have recently been collected by Da Costa et al. (2013). The filled magenta squares and diamonds connected by the solid black curves show the Al and Mg isotopic abundances for the same supermassive star models and mixtures as in Fig. 1(A).

The four panels in Fig. 1 illustrate the fact, already discussed by Denissenkov & Hartwick (2014), that the supermassive stars with \( M > 10^5 M_\odot \) reproduce all the primordial abundance variations of \( p \)-capture elements in GCs surprisingly well. The two filled black triangles in Fig. 1 are M13 stars that belong to the extreme population. From their locations in the four panels, we conclude that all the six abundances in these stars are consistent with the supermassive star hypothesis.

Denissenkov & Hartwick (2014) have noted that the success of the supermassive star models in the reproducing of the primordial abundance variations of the \( p \)-capture elements in GCs, including the Mg–Al anticorrelation, is not surprising, because these models have central temperatures in the right range for this, \( 74 \times 10^4 K \leq T_c \leq 78 \times 10^4 K \), as was first shown by Denissenkov et al. (1998) in their ‘black box’ solution and, later, independently confirmed by Prantzos et al. (2007). However, in both of the cited papers the H burning was considered to take place at a constant temperature, and, as a result, the required final abundances were reached when less than 5 percent of H was consumed, which would not be sufficient to explain the He enhancements of up to \( Y \approx 0.4 \) measured in some GCs. In the fully convective supermassive MS stars, as much as 20 percent of H can be transformed into He, thus changing \( Y \) from its initial value 0.25 to 0.4, by the moment when the \( p \)-capture elements and their isotopes still have the required abundances. The filled magenta squares in Fig. 2 show the He and O abundances in the mixtures of the M13 pristine gas with different fractions of the material from the supermassive MS stars. According to this figure, the normal, intermediate, and extreme populations of stars in M13 should have \( Y = 0.25, 0.32, \) and 0.38, respectively (see Table 1). These values agree with the He abundances in the blue horizontal branch stars in the GC NGC2808 measured by Marino et al. (2014).

4 SUPERMASSIVE MS STARS WITH \( M > 10^5 M_\odot \) VERSUS MASSIVE MS AND AGB STARS

4.1 The Al abundance and Mg isotopic ratios

Hydrogen burning in the convective cores of MS stars with \( M \leq 10^5 M_\odot \) that occurs at \( T_c \leq 60 \times 10^6 K \) as long as \( Y < 0.40 \) can result only in a marginal depletion of the \( ^{24} \text{Mg} \) abundance. Therefore, neither the fast-rotating massive MS stars with \( 20 M_\odot \leq M \leq 120 M_\odot \) (Decressin et al. 2007), nor the stars with

![Figure 2](https://academic.oup.com/mnras/article-abstract/448/4/3314/963606/10.1093/mnras/stz2028)

**Figure 2.** The same theoretical plot as in Fig. 1(A), but for the He mass fraction \( Y \). The dilution curves for the different supermassive MS star models are overlaying one another.

\( M = 20 M_\odot \) in close binaries (de Mink et al. 2009), nor the very massive MS stars with \( M \sim 10^3 M_\odot \) (Sills & Glebbeek 2010), all of which have been proposed as the potential sources of the primordial abundance variations of the \( p \)-capture elements in GCs, can actually reproduce the observed patterns between the abundances of Al and Mg isotopes in Figs 1(B)–(D).

The four asterisks connected by the dashed line in Figs 1(B)–(D) represent the theoretical data for the AGB stars with the initial masses 5.0, 5.5, 6.0, and 6.3 \( M_\odot \) and heavy-element mass fraction \( Z = 10^{-3} \), which is close to that of M13 stars, from table 2 of Ventura & D’Antona (2009). They are located far away from the observed dependences which, on the other hand, are very well matched by the H-burning yields from the supermassive stars. Unlike the massive MS stars, the problem with the massive AGB stars is that the HBB of H in their convective envelopes occurs at too high temperatures, \( T_{\text{HBB}} \gtrsim 10^{8} K \). In Fig. 3, we have plotted the \( (p, \gamma) \) reaction rates

![Figure 3](https://academic.oup.com/mnras/article-abstract/448/4/3314/963606/10.1093/mnras/stz2028)

**Figure 3.** The \( (p, \gamma) \) reaction rates (cm³ s⁻¹ mol⁻¹) for \( ^{24} \text{Mg} \) (the solid blue, black, and red curves, the last two overlaying one another) and \( ^{25} \text{Mg} \) (the dashed curves of the same colours) from the different compilations indicated in the legend box (including Iliadis et al. 2001 and NACRE data from Angulo et al. 1999), as well as for \( ^{16} \text{O} \) (the green curve) from Iliadis et al. (2010). These data have been found using a web interface to the JINA REACLIB default library (Cyburt et al. 2010). The black circles are the most recent data for the reaction \( ^{24} \text{Mg}(p, \gamma)^{25} \text{Al} \) from Straniero et al. (2013).
as functions of $T_0 = T/10^4$ K for $^{24}$Mg (the solid blue, black, and red curves), $^{25}$Mg (the dashed curves), and $^{16}$O (the green curve) taken from the most recent experimental data compilations that we found using a web interface\footnote{https://groups.nscl.msu.edu/jina/reaclib/db/} to the JINA REACLIB default library (Cyburt et al. 2010). This figure shows that at $T_0 \lesssim 0.06$ the reaction $^{24}$Mg(p,$\gamma$) is more than three orders of magnitude slower than $^{16}$O(p,$\gamma$). This explains why H burning in massive MS stars is not accompanied by the required depletion of $^{24}$Mg. On the other hand, during the HBB in the massive AGB stars at $T_0 \gtrsim 0.1$ the rate of the reaction $^{25}$Mg(p,$\gamma$) exceeds that of $^{16}$O(p,$\gamma$). This should lead to a faster destruction of the most abundant magnesium isotope $^{24}$Mg than $^{16}$O, which could be a problem, because [O/Fe] usually exhibits much lower values than [Mg/Fe] in GCs (Denissenkov & Herwig 2003), unless the $^{24}$Mg destruction would lead to a commensurate accumulation of $^{25}$Mg. This is exactly what happens in the massive AGB stars, because the reaction $^{25}$Mg(p,$\gamma$) at $T_0 \gtrsim 0.1$ is slower than both the p-captures by $^{24}$Mg and $^{16}$O (Fig. 3). Only at $T_0 \approx 0.075$, which is close to the central temperatures in supermassive MS stars with $M > 10^4$ $M_\odot$, do we find the right relative rates of the above three reactions, which guarantees that when $^{16}$O is destroyed, a smaller amount of $^{24}$Mg can also be burned, while the freshly produced $^{25}$Mg will be rapidly converted into $^{26}$Mg because its p-capture rate is higher than that of $^{24}$Mg(p,$\gamma$). This explains why both the massive MS and AGB stars fail to reproduce the observed (anti)correlations between the abundances of Al and Mg isotopes, while the H burning in the supermassive MS stars with $M > 10^4$ $M_\odot$ does the work.

The black circles in Fig. 3 present the new rate for the reaction $^{25}$Mg(p,$\gamma$)$^{26}$Al from Straniero et al. (2013), which is approximately two times as large as the older rates in the range of $T_0$ characteristic of the HBB in the massive AGB stars. Ventura, Carini & D’Antona (2011) estimated that with such the increase of this reaction rate they could obtain the [Mg/Fe] depletion and [Al/Fe] enhancement in a better agreement with observations. However, their Mg isotopic ratios in this case, $^{25}$Mg/Mg = 90 per cent and $^{26}$Mg/Mg = 5.4 per cent, as well as the ratios $^{25}$Mg/Mg = 76 per cent and $^{26}$Mg/Mg = 5.6 per cent from the super-AGB star with the initial mass 8 $M_\odot$ are still far away from the observed ones.

4.2 The O–Na anticorrelation

The minimum value of [O/Fe], that is still accompanied by a relatively high value of [Na/Fe] to fit the O–Na anticorrelation, obtained in the massive AGB models with the HBB is close to $-0.5$ (see the asterisks connected by the dashed line in Figs 1(A) and 2). This presents another problem for the massive AGB star population hypothesis because some stars in M13 (the red circles to the left of [O/Fe] = $-0.5$), as well as stars in a few other GCs, possess much lower O abundances. To solve this problem, D’Antona & Ventura (2007) have proposed that the low-mass stars from the extreme population of GCs experience deeper extra mixing on the RGB than their counterparts from the normal population because the higher initial He abundance in the former (Fig. 2) should reduce the $\mu$-discontinuity that prevents the RGB extra mixing from penetrating deep into the HBS. However, what really matters when one considers extra mixing in the radiative zone of an RGB star is its ability to overcome the restoring Archimedes force that is proportional to $\Delta \mu/\mu_{\text{BCE}}$, where $\Delta \mu = \mu(r) - \mu_{\text{BCE}}$, provided that $D_{\text{mix}} \ll K$ which is true for the RGB extra mixing (see the next section). In

**Figure 4.** The relative changes of the mean molecular weight and oxygen abundance in the radiative zones of the normal (the blue curve) and extreme, i.e. He-rich, (the red curve) population RGB models with the masses 0.8 and 0.65 $M_\odot$, respectively, immediately above the bump luminosity. The $\mu$ ratio drops vertically in the vicinity of the HBS, where the reaction $^3$He($^4$He,$\gamma$)$^7$He decreases $\mu$ locally, but the O abundance has not changed yet. Deeper in the HBS (to the right), the CNO cycle increases both $\mu$ and the relative deviation of the O abundance from its value at the BCE. Fig. 4, we compare the ratios $\Delta \mu/\mu_{\text{BCE}}$ plotted as functions of a relative deviation of the local O mass fraction from its value at the BCE (we use the positive difference $\Delta X(O) = X_{\text{BCE}}(O) - X(O)$) in our 0.8 and 0.65 $M_\odot$ RGB models with the normal and extreme initial compositions from Table 1 immediately above the bump luminosity. Both quantities remain zero until we reach the vicinity of the HBS, where the $\mu$-profile has the depression caused by the reaction $^3$He($^4$He,$\gamma$)$^7$He. There, the $\mu$ ratio drops vertically because there are no changes of the O abundance yet. When we move further to the right into the HBS, the H burning in the CNO cycle increases both $\Delta \mu$ and $\Delta X(O)$. Fig. 4 shows that the $\Delta \mu/\mu_{\text{BCE}}$ ratio increases faster in the extreme population RGB model, which means that its chemical structure does not facilitate the penetration of extra mixing deeper into the HBS and dredge up more material with a deficit in O, as compared to the normal population RGB model. Moreover, in order to attain the same level of the surface O depletion, if it is required by observations, the extreme population RGB star must have more powerful extra mixing, e.g. if the RGB extra mixing is driven by rotation, then the extreme population stars in GCs must rotate faster than their normal population counterparts by some reason, which is difficult to understand.

In the hypothesis that proposes the supermassive MS stars with $M > 10^4$ $M_\odot$ as the source of the primordial abundance variations in GCs, it is sufficient to assume that some low-mass stars in GCs were formed out of more than 90 percent of the material lost by these supermassive stars (Fig. 1A). We also note that the total CNO abundances in the M13 RGB stars measured by Cohen & Méféndez (2005), namely the $[$C+N+O$/H]$ ratios between $-1.4$ and $-1.1$ with the average value $-1.23$, are very close to those in our Table 1.

Like massive MS stars, the supermassive stars destroy Li very quickly. That could be a problem for a hypothesis invoking such stars as the source of the primordial abundance variations in GCs if low-mass stars with low O and high Li abundances were found in a GC. The most likely pollution source in that case would be massive AGB stars, because they produce Li via the convective $^7$Be-mechanism (Cameron & Fowler 1971), while destroying O in
the HB stars (e.g. Ventura & D'Antona 2010). However, there are no stars found in GCs yet that are O-poor and Li-rich at the same time. On the contrary, there are spectroscopic observations of MS turn-off stars in 47 Tuc (Dobrovolskis et al. 2014) and NGC 6752 (Shen et al. 2010), the latter having [Fe/H] close to that of M13, that reveal O−Li correlations, as if material with depleted abundances of both O and Li had been mixed with the pristine gas in these GCs. Although Shen et al. (2010) claim that the characteristic of their obtained O−Li correlation for NGC 6752 MS turn-off stars requires that the polluting gas had been enriched in Li, we notice that a large number of their measured Li abundances have values that are nearly 0.2−0.3 dex higher than those of both the Spite plateau Li abundance (Spite & Spite 1982) and Li abundances in MS turn-off stars from the same GC reported by Gruyters, Nordlander & Korn (2014). After their reduction by 0.2−0.3 dex, the (logarithmic) Li abundances from Shen et al. (2010) will probably show a one-to-one correlation with [O/Fe], as predicted by mixing of pristine gas with material from massive and supermassive MS stars.

5 THE EVOLUTIONARY ABUNDANCE VARIATIONS IN M13 RGB STARS

We have used the VICTORIA stellar evolution code to generate three 11.5 Gyr isochrones for the combinations of the initial He mass fraction, metallicity and α-element enhancement that are close to those assumed for the extreme, intermediate, and normal populations of low-mass stars in M13 (Table 1). These isochrones are transformed to the Hubble Space Telescope (HST) Advanced Camera for Surveys (ACS) photometric system (using the colour−$T_{\text{eff}}$ relations given by Casagrande & Vandenberg (2014)) and compared with the HST colour−magnitude diagram (CMD) data for M13 in Fig. 5, for which we have selected the 100 stars in each 0.10 mag bin that have the smallest error bars on the observed magnitudes and colours. The RGB segments of the isochrones appear to be redder than their corresponding observed CMD. A number of possible sources of this discrepancy are discussed in section 6.1.2 of the paper of Vandenberg et al. (2013). It is also possible that a majority of M13 stars have initial chemical composition that is actually closer to the intermediate one, with $Y \approx 0.33$, as suggested by the O−Na anticorrelation in Fig. 1(A). In this case, the fiducial theoretical isochrone for M13 should be the green one.

The individual evolutionary tracks of stars with the masses 0.65, 0.7, and 0.8 $M_\odot$ calculated for the first, second, and third initial compositions, respectively, are found to have RGBs coinciding with the RGBs of their corresponding isochrones (Fig. 6). Therefore, we have chosen these masses as the initial ones for our study of the evolutionary abundance variations, caused by the RGB extra mixing, in the M13 low-mass stars belonging to the extreme, intermediate, and normal populations, using for them the initial abundances from Table 1. In this study, we employ the revision 3251 of MESA instead of the VICTORIA code because the latter cannot model extra mixing on the upper RGB, although the VICTORIA code produces better isochrones than MESA because it accounts for the atomic diffusion and its counteracting turbulent mixing on the MS, which has not yet been implemented in the MESA code.

The red, green, and blue curves in Fig. 7(A) show the evolution of the surface C abundance in the models representing the three populations of low-mass stars in M13, in which the RGB extra mixing has been modelled using the thermohaline diffusion coefficient (equation 25) from Denissenkov (2010). We have used the salt-finger aspect ratio $a = 7$ that gives the maximum possible depletion of [C/Fe] in these models. The open blue diamonds in Fig. 7(A) are M13 MS turn-off and subgiant stars for which the [C/Fe] values were determined by Briley et al. (2002). They demonstrate a pattern characteristic of the equilibrium CNO cycle – the floor at [C/Fe] $\approx −0.8$ (Denissenkov et al. 1998), which supports the idea that the primordial abundance variations in GC stars were produced in H burning at a sufficiently high temperature for the CNO cycle to reach equilibrium. The rest of the symbols in Fig. 7(A), except the red star symbols that represent low-mass AGB stars, present the [C/Fe] data for RGB stars with $M_\gamma < +0.8$ compiled by Smith & Briley (2006) from the literature. We have used $(m − M)_\gamma = 14.42$ as the distance modulus for M13 and applied the correction $\Delta[C/Fe] = +0.4$ to all of the RGB [C/Fe] values, as
Figure 7. Panel A: the evolution of the surface C abundance in the stars with the masses $0.8\,M_\odot$ (blue), $0.7\,M_\odot$ (green), and $0.65\,M_\odot$ (red) and initial chemical compositions from the second, third, and fourth columns of Table 1, respectively, calculated from the MS to the RGB tip. The RGB extra mixing is modelled using the thermohaline diffusion coefficient (equation 25) from Denissenkov (2010) with the salt-finger aspect ratio $a = 7$ that produces the maximum possible decrease of [C/Fe]. The results of these calculations are compared with the observational data for M13 MS turn-off and subgiant stars (the open blue diamonds) and RGB stars (the other symbols, except the red star symbols which represent AGB stars) from Briley et al. (2002). Panel B: the same as in panel A, but for the stars of the single stellar population GC NGC5466 observed by Shetrone et al. (2010) and $a = 10$. Panel D: the same as in panel B, but here we have also used the parametric prescription for the RGB extra mixing with the mixing depth $\log_{10}(r_{\text{mix}}/R_\odot) = -1.35$ (same as for the thermohaline mixing) and diffusion coefficient $D_{\text{mix}} = \alpha K$, where $K$ is the radiative diffusivity and $\alpha = 0.02$ (magenta line) and 0.03 (black line). Panel C: the same as in panel D, but for the M 13 stars with the dashed and solid curves representing the cases of $\alpha = 0.02$ and 0.03.

We see that the thermohaline convection driven by the $^3$He burning produces shallow evolutionary declines of [C/Fe] incompatible with the observational data for the M13 RGB stars. From Fig. 7(B), the same conclusion can be made for the NGC 5466 stars studied by Shetrone et al. (2010), for which the value of $a = 10$ gives a maximum effect for the evolutionary depletion of [C/Fe] on the upper RGB (our defined maximal-mixing salt-finger aspect ratio slightly depends on the metallicity). This GC is unique because it does not appear to have any primordial abundance variations (Shetrone et al. 2010). This conclusion has recently been supported by the new HST photometric data extended to include UV passbands, according to which NGC 5466 does not appear to have multiple CMDs (Piotto et al. 2015). Therefore, it can be used to calibrate the depth and rate of the RGB extra mixing. To eliminate the mixing depth as a free parameter, we assume that it is equal to the almost universal depth that we usually find for the $^3$He-driven thermohaline convection in upper RGB models of different metallicities, i.e. $\log_{10}(r_{\text{mix}}/R_\odot) = -1.35$. This depth guarantees that only the products of H burning in the CN branch of the CNO cycle are dredged up from the HBS to BCE, as indicated by observations. In the absence of a good candidate for the mechanism of the RGB extra mixing, we assume that it can be modelled as a diffusion process with a diffusion coefficient $D_{\text{mix}}$ proportional to the radiative diffusivity

$$K = \frac{4\alpha c T^3}{3\kappa C_p \rho^2},$$

where $\alpha$ is the radiation constant, $c$ the speed of light, $\kappa$ is the Rosseland mean opacity, $C_p$ is the specific heat at constant pressure.
and $\rho$ is the density. This assumption makes sense as long as the RGB extra mixing operates on a thermal time-scale, when the radiative heat diffusion facilitates it by reducing temperature contrasts between rising and sinking fluid parcels. In Fig. 7(D), the magenta and black curves are obtained with $D_{\text{mix}} = \alpha K$ for $\alpha = 0.02$ and 0.03, respectively. When we employ the same parameters of the RGB extra mixing in models of the M13 low-mass stars, we get very good agreement with the observational data (Fig. 7 C), in spite of the fact that the two GCs have different metallicities, $[\text{Fe/H}] = -1.53$ for M13 and $[\text{Fe/H}] = -2.2$ for NGC 5466.

## 6 CMDs for the Three Populations of Low-Mass Stars in M13

The colour difference between our theoretical isochrones for the normal and extreme populations of low-mass stars in M13 in Fig. 5 is approximately as large as the width of its CMD observed with the *HST* ACS. Therefore, our assumption that M13 has the populations of low-mass stars with $Y$ varying between 0.25 and 0.38 cannot be rejected on the basis of these photometric data.\(^5\) This conclusion appears even more true when we take into consideration that both the extreme and normal populations are likely to be poorly presented in M13, as compared to its intermediate population. This possibility is supported by the fact that the O–Na anticorrelation for M13 in Fig. 1(A) includes 63 per cent of the intermediate-population RGB stars, while the normal and extreme populations contribute only 15 and 22 per cent (Johnson & Pilachowski 2012). Also, the M13 RGB stars with the most extreme abundance anomalies are predominantly located near the RGB tip, where the three isochrones almost converge.

We remind the reader that our subdivision of the M13 low-mass stars into the three distinct populations is an approximation that has been made using the rather arbitrary selection criteria. In fact, we assume that variations of abundances of the p-capture elements in the material out of which low-mass stars formed in GCs were smooth, reflecting dilution of the GC pristine gas by gas partially processed in supermassive MS stars, with a random but continuously varying fraction of the latter. This assumption seems to contradict to multiple CMDs found in a constantly increasing number of GCs (e.g. Monelli et al. 2013; Piotto et al. 2015). Except a small number of GCs with intrinsic and almost discrete variations of $[\text{Fe/H}]$ (e.g. $\omega$ Cen and M22) or $[\text{C+N+O}/\text{Fe}]$ (NGC2808), which can only be explained by several star formation episodes, multiple CMDs in most other GCs with nearly constant $[\text{Fe/H}]$ and $[\text{C+N+O}/\text{Fe}]$ values in each of them can probably be explained by the fact that their narrow-band photometric observations filter out some special spectral features characteristic of the p-capture element abundance anomalies. At least, this seems to be true when one uses the new photometric index $c_{U,B,I} = (U - B) - (B - I)$ that has recently been proposed by Monelli et al. (2013) to help reveal multiple stellar populations in GCs more easily. Indeed, as Sbordone et al. (2011) have shown, the enhanced abundance of N in the population of stars polluted by the products of H burning (the intermediate and primordial populations in our case) leads to a significant increase of the $U$ magnitude because of the higher concentrations of the CN and NH molecules in these stars that absorb more light in the $U$ band. From our point of view, this is like looking at a rainbow, that has a smooth transition of colour from red to violet, through an anaglyph glasses and, as a result, seeing discrete red and cyan stripes. Our hypothesis is indirectly supported by Bastian et al. (2013b) who have not found evidence for ongoing star formation within any of 130 Galactic and extragalactic young massive clusters surveyed by them.

From Fig. 7(C), it is seen that the evolutionary tracks of the low-mass stars belonging to the different populations in M13 have slightly different bump luminosities, at which the RGB extra mixing starts to operate, increasing with $Y$. A differential luminosity function constructed for the normal population (the blue curve in Fig. 8) has a bump at $V \approx 14.7$ which is very close to its observed location in M13, $V \approx 14.75$, as reported by Sandquist et al. (2010), or $V \approx 14.77$, according to the most recent *HST* GC data analysis by Nataf et al. (2013). However, if the M13 low-mass stars represent a mixture of the three populations, then location and width of the bump depend on its relative strength and a number of stars in the different populations. It turns out that the former decreases when $Y$ increases. The red, green, and blue curves in Fig. 8 show the differential luminosity functions for the M13 three populations constructed assuming that they include 22, 63, and 15 per cent of all the M13 stars, like in its O–Na anticorrelation (Johnson & Pilachowski 2012), as well as their composite luminosity function (the black histogram).

![Figure 8. The theoretical differential luminosity functions for the evolutionary tracks from Fig. 7(A) (the blue, green, and red histograms) constructed assuming that the three populations contribute 15, 63, and 22 per cent to the total population of stars in M 13, like in its O–Na anticorrelation (Johnson & Pilachowski 2012), as well as their composite luminosity function (the black histogram).](https://academic.oup.com/mnras/article-abstract/448/4/3314/963606/figure8)

\(^5\)The implications of high helium abundances and/or a wide range in $Y$ for the horizontal branch of M13 will be considered in a separate paper (Denissenkov et al., in preparation). Some studies (e.g. Catelan et al. 2009) have argued in support of a normal helium content for this cluster, while others have estimated $Y_{\text{max}} \approx 0.31$ for M13 HB stars (e.g. Dalessandro et al. 2013).

## 7 Conclusion

In this paper, we have elaborated on the hypothesis, recently proposed by Denissenkov & Hartwick (2014), that the primordial abundance anomalies of the p-capture elements and their isotopes in GC stars had been produced during a short time ($\sim 10^7$ years) of H burning in the fully convective supermassive MS stars with $M > 10^4 M_\odot$. Because such stars are supported against the force of gravity almost
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REFERENCES

Paxton B., Bildsten L., Dotter A., Herwig F., Lesaffre P., Timmes F., 2011, 
ApJS, 192, 3
Paxton B. et al., 2013, ApJS, 208, 4
Piotto G. et al., 2015, AJ, 149, 91
VandenBerg D. A., Bergbusch P. A., Dotter A., Ferguson J. W., Michaud G., 
775, 134

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