Rapid approach to the quantitative determination of nocturnal ground irradiance in populated territories: a clear-sky case

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ABSTRACT

A zero-order approach to the solving of the radiative transfer equation and a method for obtaining the horizontal diffuse irradiance at night-time are both developed and intended for wide use in numerical predictions of nocturnal ground irradiance in populated territories. Downward diffuse radiative fluxes are computed with a two-stream approximation, and the data products obtained are useful for scientists who require rapid estimations of illumination levels during the night. The rapid technique presented here is especially important when the entire set of calculations is to be repeated for different lighting technologies and/or radiant intensity distributions with the aim of identifying high-level illuminance/irradiance, the spectral composition of scattered light or other optical properties of diffuse light at the ground level. The model allows for the computation of diffuse horizontal irradiance due to light emissions from ground-based sources with arbitrary spectral compositions. The optical response of a night sky is investigated using the ratio of downward to upward irradiance, $R_{\perp,\lambda}(0)$. We show that $R_{\perp,\lambda}(0)$ generally peaks at short wavelengths, thus suggesting that, e.g., the blue light of an LED lamp would make the sky even more bluish. However, this effect can be largely suppressed or even removed with the spectral sensitivity function of the average human eye superimposed on to the lamp spectrum. Basically, blue light scattering dominates at short optical distances, while red light is transmitted for longer distances and illuminates distant places. Computations are performed for unshielded as well as fully shielded lights, while the spectral function $R_{\perp,\lambda}(0)$ is tabulated to make possible the modelling of various artificial lights, including those not presented here.

Key words: light pollution.

1 INTRODUCTION

The light-emitting areas are constantly increasing over the world, thus dark night skies are becoming rare in many places. A direct consequence of this phenomenon is a constant increase of the over-illumination of the ambient environment. This problem currently can no longer be overlooked and has become one of the important environmental risks even in the public’s eye and requires a thorough treatment (see e.g. the IDA initiative, Alvarez del Castillo, Crawford & Davis 2003). The other priority research area this initiative addresses is linking excessive lighting and the environment in public awareness and the need for prioritizing environmental concerns. Regional and global over-illumination and changes in the skyglow are currently two of the most pressing environmental concerns discussed by the LoNNe consortium and they are also the subject of international field campaigns (e.g. Bará et al. 2015).

The light emitted from heterogeneously distributed artificial sources spreads over a city and its surroundings, and thus disrupts fragile ecosystems in many respects (Rich & Longcore 2006). The diffuse light is almost ubiquitous in highly industrialized territories, including urban, suburban and rural regions. An exact numerical prediction of illuminance levels in such areas generally appears difficult or rather impossible because of the huge number of isolated ground-based light sources (individual lamps, and small radiating areas like villages, towns and cities) that all contribute to the sky radiance. However, a large population of radiating sources can be approximated by a quasi-homogeneously light-emitting surface. Then the vector radiative transfer equation (vRTE) can be reduced to the scalar (1D) RTE, which is easy to solve even for a vertically inhomogeneous atmosphere. Unfortunately, the numerical computations may still appear time-consuming because of the...
low-convergent recurrent solution schema for an optically thick atmosphere. Therefore, a further simplification of radiative transport is to be applied when the eigenvalues are far from being low.

Here we proceed with the two-stream approximation, which is the first choice approach to solving the RTE. This method provides only the upward and downward radiative fluxes at a set of discrete altitudes and benefits from its simple concept and inexpensive numerical runs (Rybicki & Lightman 2004). In particular, the two-stream approximation has been applied when other single-scattering models fail in reproducing the observed phenomena. It has also been employed in global models to estimate the radiative budget in a sun-illuminated atmosphere or in modelling stellar atmospheres (Gudiksen et al. 2011). Even if two-stream radiation schemes are subject to persistent biases, the error margins are within acceptable limits (e.g. the bias in radiative fluxes is of the order of 10–20 per cent, Randles et al. 2013).

Kocifaj (2016) has adopted an inverse coordinate system to solve the RTE in the two-stream approximation. The solution principles for daylight and nightlight atmospheres are the same, thus the techniques originally developed for daylighting can be successfully reused in modelling irradiances in the night-time regime. This allows for rapid numerical simulations, as the conventional two-stream codes (e.g. those based on the Eddington approximation) are optimized well to solve wide spectrum of radiative transfer problems (Lu, Zhang & Li 2009; Shi & Nakajima 2015). Unlike the homogenized emission function used in our previous study (Kocifaj 2012), we now use an extended solution for the anisotropic radiative intensity distribution and a set of typical spectra. Downward diffuse radiative fluxes are advantageously computed relative to the flux emitted upwards from the ground-based sources, thus the ratios determined in this paper can be easily used to model the downwelling diffuse radiation as a linear product of the above ratio and the flux density of upwardly emitted light. Any computational result obtained this way for specific lamp spectra and atmospheric stratification can be (linearly) rescaled using the ratios found here. It is not the intention in this paper to model direct artificial light that can only be observed in places near a light source. Instead, the present study is directed toward diffuse light that spreads at large distances and is observed as a background veiling luminance. Such light noise can contribute to the illumination of otherwise dark places and make astronomical observations generally difficult.

2 RTE FOR A VERTICALLY STRATIFIED INHOMOGENEOUS ATMOSPHERE: BASIC RELATIONS

The spectral radiance \( I_0 (z, A) \) (W m\(^{-2}\) nm\(^{-1}\) sr\(^{-1}\)) of a light beam propagating from the ground upwards decays exponentially as a function of the optical depth \( \tau_z \) (see the first terms in the right-hand sides in the equation below). However, the secondary (scattered) light beams contribute to the total radiance as evident from the second and third terms in the RHS of equation (1):

\[
\cos \frac{d}{d\tau_z} \left( I_0 (z, A) + \frac{\tau_z}{4\pi} \int_{-\pi/2}^{\pi/2} p_z (z, A, z') \right) = -I_0 (z, A) + \frac{\tau_z}{4\pi} \int_{-\pi/2}^{\pi/2} p_z (z, A, z') dA' dz' + \frac{\tau_z}{4\pi} \Phi_{p,0} p_z(z, A, z_0, A_0) \exp \left( \frac{-\tau_z}{\cos z_0} \right),
\]

where \( z, A \) and \( z_0, A_0 \) are the observational and light-source zenith and azimuth angles when seeing from the top of the atmosphere to the ground. Negative values of \( z \) are for downward intensities. The optical depth is zero at the ground and increases as a function of altitude. The last term in equation (1) represents the contribution of a light source with the spectral flux density \( \Phi_{p,0} (W m^{-2} nm^{-1}) \). Here, \( \tau_z(z, A, z')/4\pi \) is the wavelength of the incident radiation. The scattering phase function \( p_z(z, A, z') \) characterizes the probability that a beam propagating along direction \( (z', A') \) is scattered toward direction \( (z, A) \).

An atmospheric environment belongs to a class of random scattering media where \( \tau_z \) is a sum of the optical depths of all individual components, such as air molecules (\( \tau_{R,\lambda} \)), aerosols (\( \tau_{S,\lambda} \)), water vapour (\( \tau_{W,\lambda} \)), ozone (\( \tau_{O,\lambda} \)), etc. Then \( \tau_z \) is determined as a weighted contribution of all constituents:

\[
\tau_z(z, A, z') = \frac{\int \tau_{R,\lambda} p_{R,\lambda} A_{R,\lambda} + \tau_{S,\lambda} p_{S,\lambda} A_{S,\lambda} + \cdots}{\int \tau_{R,\lambda} A_{R,\lambda} + \tau_{S,\lambda} A_{S,\lambda} + \cdots},
\]

because the single scattering albedo of all atmospheric gases approaches unity. In the Eddington approximation, the radiance is expanded in Legendre polynomials, and only two terms are adopted to replace the integral in the RHS of equation (1). The radiance is then formulated as a linear combination of two azimuthally averaged functions (Thomas & Stamnes 2006):

\[
I_0 (\tau_z, z) = I_0^0 (\tau_z) + (\cos z) I_0^1 (\tau_z).
\]

It is also convenient to expand the scattering phase function in Legendre polynomials:

\[
p_z(z, A, z') = \sum_{m=0}^\infty \sum_{\ell=0}^\infty \chi^{m,\ell} P_m^\ell (\cos z) P_m^\ell (\cos z')
\times \cos [m (A - A')],
\]

where \( P_m^\ell \) is an associated Legendre polynomial of order \( \ell \) and degree \( m \), and \( \chi^{m,\ell} \) is an expansion coefficient that for an azimuthally averaged Henyey–Greenstein phase function reduces to (2\( \ell + 1 \)\( g_o^2 \), thus resulting in

\[
p_z(z, A, z') = \sum_{\ell=0}^\infty \left( 2\ell + 1 \right) g^2 P_\ell^0 (\cos z) P_\ell^0 (\cos z').
\]

with \( g_o \) being the dimensionless asymmetry parameter. The form of equation (3) guarantees that all terms vanish when equation (1) is integrated over all azimuth and zenith angles. This is why the phase function can be approximated as

\[
p_z(z, z') = 1 + 3 g_o \cos (z) \cos (z'),
\]

and the solution functions for both \( I_0^0 \) and \( I_0^1 \) have the form

\[
I_0 (0,0) = C(0,1) e^{k_i} + D(0,1) e^{-k_i} + \Phi(0,1) e^{1/k_o},
\]

where the constants \( C(0,1) \) and \( D(0,1) \) can be derived by applying boundary conditions to the solution function, and \( k = \sqrt{M (1 - \sigma_z)} \). The parameter \( \Phi(0,1) \) depends on the input parameters, such as the flux density of radiation emerging from the ground, the single scattering albedo of the atmosphere, etc. (for more details see the paper by Jiménez-Aquino & Varela 2005).

A ground-based light source with unidirectional emission can be simulated as

\[
I_0 (\tau_z = 0, -z, A) = \frac{\Phi_{p,0}}{4\pi} \delta (z - z_0) \delta (A - A_0),
\]

where \( \delta \) is the Dirac delta function, so \( I_0 = \Phi_{p,0}/(4\pi) \) if \( z = z_0 \), while \( I_0 = 0 \) in all other cases. The optical depth \( \tau_z \) is measured
from the ground upwards. The sum of the direct and ground-reflected components of the upwardly directed emissions is interpreted here as the initial radiation, while diffuse light occurs only due to the interaction of the initial radiation with the atmospheric environment. Therefore, no diffuse light is emitted from the ground:

\[ I_x (t_x = 0, -z) = 0 \]  

(9)

and also no diffuse light emerges from the top of the atmosphere, i.e.

\[ I_x (t_x = T_x, z) = 0. \]  

(10)

Here \( T_x \) is the optical depth of the entire atmosphere. Applying an inverse coordinate system to the solved light-pollution problem, we can immediately find that diffuse irradiance escaping from the top of the Earth’s atmosphere is

\[
\Phi_{\text{up}, \lambda} (T_x, z_0) = 2 \pi \int_0^1 [I_x^0 (T_x) + \mu I_x^1 (T_x)] \mu \, d\mu
\]

\[
= \pi \left[ I_x^0 (T_x) + \frac{2}{3} I_x^1 (T_x) \right],
\]

(11)

where \( \mu = \cos z \). The total irradiance at the top of atmosphere directed upwards is then

\[
\Phi_{\text{up}, \lambda} (T_x, z_0) = \pi \left[ I_x^0 (T_x) + \frac{2}{3} I_x^1 (T_x) + (\cos z_0) \Phi_{\lambda,0} (z_0) \right]
\]

\[
\times \exp \left\{ - \frac{T_x}{\cos z_0} \right\},
\]

(12)

while the downward diffuse irradiance at the ground is

\[
\Phi_{\text{down}, \lambda} (0, z_0) = 2 \pi \int_0^{-1} [I_x^0 (0) + \mu I_x^1 (0)] \mu \, d\mu
\]

\[
= \pi \left[ I_x^0 (0) - \frac{2}{3} I_x^1 (0) \right].
\]

(13)

Consider now that the radiative flux on a surface oriented normally to the emitted beams is \( \Phi_{\lambda,0} (z_0) \). Then, the horizontal diffuse irradiance at the ground can be obtained analogously as an integral over the upper hemisphere:

\[
\Phi_{\text{diffuse}, \lambda} (0, z_0) = \frac{n/2}{0} \Phi_{\text{down}, \lambda} (0, z_0) \sin (z_0) \, d\zeta_0,
\]

(14)

while the total uplight (introduced earlier as the initial radiation) is

\[
\Phi_{\text{initial}, \lambda} = \int_0^{\pi/2} \Phi_{\lambda,0} (z_0) \cos (z_0) \sin (z_0) \, d\zeta_0,
\]

(15)

where the coefficient of proportionality \( 2 \pi \) disappeared because the normalized radiation is computed as \( \Phi / (2\pi) \) in both the cases above. The use of radiant intensity dates back to the point-source approximation and is traditionally used by illumination engineers to model optical emissions from light sources, such as bulbs. However, equation (14) is used to compute the vertical component of the downwelling diffuse radiation that is normally the subject of RTE calculations. Therefore, analogously to equation (14), we introduced equation (15), which characterizes the upward horizontal irradiance due to emissions from ground-based light sources.

It is convenient to determine \( \Phi_{\text{diffuse}, \lambda} (0) \) relative to the initial emissions \( \Phi_{\text{initial}, \lambda} \), as this allows for immediate estimation of the spectral diffuse irradiance as a function of the total spectral power radiated upwards from a ground-based light source. This ratio is advantageous in modelling when the diffuse radiation is to be estimated based on the known inventory of light sources or using other a priori information, e.g. information about the average output in lumens per head of population. For instance, Narisada & Schreuder (2004) assumed 1500 lumens per inhabitant for a few highly illuminated sports fields in the Netherlands. If two light sources (1) and (2) differ in only the emitted powers \( \Phi_{\text{initial}, \lambda} \) and \( \Phi_{\text{initial}, \lambda} \), then the diffuse irradiance at the ground would be \( \Phi_{\text{diffuse}, \lambda} (0) = f_\lambda \cdot \Phi_{\text{diffuse}, \lambda} (0) \), where the linear coefficient of proportionality is \( f_\lambda = \Phi_{\text{initial}, \lambda} / \Phi_{\text{initial}, \lambda} \).

3 FUNDAMENTALS OF THE SOLUTION CONCEPT, INPUT DATA SETS AND OUTPUTS

By configuring the model as

\[
R_{\perp, \lambda} (0) = \frac{\Phi_{\text{diffuse}, \perp, \lambda} (0)}{\Phi_{\text{initial}, \perp, \lambda}} = \frac{\pi/2}{0} \int_0^{\pi/2} \Phi_{\text{down}, \lambda} (0, z_0) \sin (z_0) \, d\zeta_0
\]

\[
= \frac{\pi/2}{0} \int_0^{\pi/2} \Phi_{\lambda,0} (z_0) \cos (z_0) \sin (z_0) \, d\zeta_0,
\]

(16)

the total horizontal broad-band irradiance recorded by a measuring device can be theoretically predicted for any lamp spectra \( \Phi_{\lambda,0} \) as:

\[
\Phi_{\text{diffuse}, \perp, \lambda} (0) = \int_0^{\lambda_1} R_{\perp, \lambda} (0) \Phi_{\text{initial}, \perp, \lambda} S, \, d\lambda,
\]

(17)

where \( R_{\perp, \lambda} (0) \) is the ratio provided in this paper for all of the colours in the visible (i.e. for wavelengths \( \lambda \) ranging from 400 to 800 nm), \( \Phi_{\text{initial}, \perp, \lambda} \) is the total spectral emission at a corresponding wavelength (see equation 15), and \( S, \lambda \) is the spectral sensitivity of the (photo)sensor used. Two light sources having the same radiant intensity distribution can be characterized by \( f_\lambda = \Phi_{\text{initial}, \perp, \lambda} / \Phi_{\text{initial}, \perp, \lambda} \), which is more or less consistent with the ratio of the total spectral powers found in technical specifications of light sources.

It is, therefore, useful to focus exclusively on the spectral behaviour of \( R_{\perp, \lambda} (0) \), which is independent of lamp spectra because the wavelength-dependent part of \( \Phi_{\lambda,0} \) occurs in the numerator as well as in the denominator of equation (16), so it disappears after computing the ratio.

Eddington’s approximation is one of the most well-known two-stream solvers for radiative transfer problems, where a plane-parallel atmosphere is divided into a finite number of isolated layers. Each layer is characterized by the optical properties of all gaseous and solid-phase constituents. Specifically, the extinction by gases is derived from the Rayleigh scattering by air molecules and absorption by ozone, water vapour, NO\(_x\) and other atmospheric gases. The optical signatures of solid-phase materials (either water soluble or insoluble) can be interpreted in terms of scattering and absorption properties using exact electromagnetic scattering theories (Mie theory, T-matrix otherwise known as the extended boundary condition method or the null-field method, and the discrete dipole approximation, which is a finite element method in which the continuum target is replaced by an array of point dipoles; Kahnert 2003). The input parameters to Eddington’s approximation involve vertical stratification of atmospheric constituents, the size distribution of aerosol particles, their single scattering albedo, an asymmetry parameter, as well as the complex refractive index of the aerosols. We emphasize that the latter three parameters depend on the
wavelength, like the other optical characteristics, such as ground-based light emissions, the Rayleigh-scattering cross section, the volume-scattering coefficient, and ozone and water vapour attenuation coefficients.

One of the major strengths of the Eddington approximation is that the solution remains simple even if a cloudy layer or hazy conditions are considered. A cloud deck can be incorporated by modifying the aerosol microphysics in a layer. For instance, the aerosol particles suspended in an atmospheric boundary layer usually have a size distribution $s(r) \propto r^2 \exp[-15\sqrt{r}]$ with a modal radius $r_M$ of about 0.07 $\mu$m, while water droplets in a cloud are much larger in size with $r_M \approx 4.0 \mu$m and size distribution $s(r) \propto r^6 \exp[-1.5\times r]$. In both cases, the particle radius $r$ is in micrometres. Clouds droplets composed of liquid water and aerosol particles also differ in their dielectric function, specifically the absorption coefficient, which is several orders of magnitude smaller for water droplets (Kokhanovsky 2004) than for organic, urban or background aerosols (Guyon et al. 2003; Raut & Chazette 2008).

The core solution and data model are well separated from each other, thus no additional implementation is necessary when altering the model configuration. This makes the numerical tool easily reusable and well adaptable to various situations that can occur in nature. The outputs involve either spectral or broad-band radiative fluxes determined by integrating the corresponding spectral quantities over a dedicated spectral band. The radiative fluxes are computed on a regular grid $[z_0, \lambda] \times [r, \lambda]$ with predetermined, not necessarily uniform, spacing. In such a case, $N \times M$ combinations are computed in a single loop. For instance, the computation time of one scenario with $20 \times 20$ cases takes about one minute on a personal computer based on an Intel Core i7. The ratio of downward to upward irradiances generally peaks at short wavelengths, thus implying an overall bluish effect. $R_{\perp,\lambda}(0, z_0)$ approaches its maximum at zenith angles $z_0 = 60–70^\circ$ or at $z_0 > 70^\circ$. Undoubtedly the downward diffuse irradiance in the blue part of the spectrum is due to efficient scattering by both air molecules and aerosols, while a decrease of the $R$ factor in the red is due to a lower scattering efficiency as well as enhanced absorption by ozone and water vapour. This implies that a clear sky preferably becomes bluer if observations are made under low-turbidity conditions at short distances from light sources (see e.g. Fig 1a). Of course, it is expected that all wavelengths contribute equivalently to the total emission, i.e. there are no dominant emission peaks in the red part of the spectrum. The situation changes if $\cos(z_0) < 0.1$ (see the spectral behaviour of the $R$ factor in Fig. 1a) or when the turbidity increases (see Fig. 1b). In the latter case, the limit value of $\cos(z_0)$ shifts to $\approx 0.2$. However, a transition from a bluish effect to signal reddening may appear when diffuse light appears due to illumination from distant light sources. This is because blue rays undergo a rapid intensity decay when they traverse through the atmosphere.

4 NUMERICAL RESULTS

By defining the spectral directional ratio $R_{\perp,\lambda}(0, z_0)$:

$$R_{\perp,\lambda}(0, z_0) = \frac{\Phi_{\text{down},\lambda}(0, z_0)}{\Phi_{\text{up},\lambda}(z_0) \cos(z_0)},$$

we can compute the integral ratio $R_{\perp,\lambda}(0)$ (equation 16) for an arbitrary emission function independent of its normalization. Let us now formulate $\Phi_{\text{up},\lambda}(z_0)$ as a product of two functions $H_{\text{up},\lambda}$ and $\nu(z_0)$, where the first depends exclusively on the wavelength, while the second is a function of zenith angle. Here, $\nu(z_0)$ is an (uncalibrated) directional emission function that is inversely proportional to $\nu(z_0)$. However, due to the linearity of the radiative transfer equation, $\Phi_{\text{down},\lambda}(0, z_0)$ increases proportionally to the number of photons emitted towards the zenith angle $z_0$. This is why $R_{\perp,\lambda}(0, z_0)$ is not affected by how the directional emission function is normalized. Computational results for the US Standard Atmosphere (Anderson et al. 1986) are shown in Fig. 1. Small ripples are due to both Mie scattering and interpolation applied to the data originally computed on a regular grid. See the guidelines for Surfer Gridding Methods for more details (Golden Software 2010).

Ozone and water vapour absorption, Rayleigh scattering as well as the aerosol optics were all combined to take into account the standardized vertical structure of the atmosphere. The model of slightly absorbing particles used in our numerical experiment is very representative for a humid atmosphere, water-like and maritime aerosols, sea salt, ammonium sulphate and also illite. Therefore, the ratios can be used to model maritime, continental, stratospheric and desert environments except for regions with heavily polluted air where contamination originates from industrial production or combustion and, thus, contains strongly absorbing (e.g. carbonaceous) particles. The reference aerosol optical depth (AOD) at the nominal wavelength of $\lambda_{\text{nom}} = 0.5 \mu$m is taken to be either 0.1 (see Fig. 1a) or 0.5 (Fig. 1b). The latter imitates a turbid atmosphere, while AOD = 0.1 is applicable to slightly polluted regions. The aerosol optical depth is inversely proportional to the wavelength $\lambda$ (i.e. the Ångström exponent $\approx 1$) to be consistent with a set of experiments made in different areas (e.g. Pesava, Horvath & Kasahara 2001).

![Figure 1. Theoretical values of $R_{\perp,\lambda}(0, z_0)$ computed for the US Standard Atmosphere considering slightly absorbing (maritime-like) aerosols with a modal radius of 0.07 $\mu$m. (a) Simulates low-turbidity conditions with AOD = 0.1, while (b) is for a turbid atmosphere with AOD = 0.5 (both computed at a nominal wavelength of 0.5 $\mu$m).](https://i.imgur.com/ExampleImage.jpg)
Earth’s atmosphere at inclined trajectories. A large portion of blue light is scattered at short distances from a radiating source, while red light transmits over longer distances and illuminates distant places. Note that reddening of the sky can also appear at short distances from a light source if, however, the observations are made under cloudy or overcast conditions (Kyya et al. 2012). Such a reddening is related to the artificial light beams that are diffusely reflected by a cloud deck and directed downwards. The downward irradiance is then due to the superposition of diffuse and reflected light, which appears dominant thanks to the short optical beam paths.

4.1 Light sources with direct uplight

The worldwide trend to constrain or even reduce direct uplight helps to minimize light pollution levels, but unfortunately poorly shielded street lights have been a persistent problem in our society for years. Advertisement boards are disturbing factors to those who are dealing with the environmental impact of artificial light pollution. Poorly aimed fixtures cast light at low elevation angles, thus causing indirect (diffuse) illumination of otherwise dark places.

Following the concept of $\Phi_{\lambda}(z_0) = H_{\lambda,0} v(z_0)/\cos(z_0)$ introduced above, the integral ratio (equation 16) is

$$R_{\perp,\lambda}(0) = \frac{\int_0^{\pi/2} R_{\perp,\lambda}(0, z_0) v(z_0) \sin(z_0) \, dz_0}{\int_0^{\pi/2} v(z_0) \sin(z_0) \, dz_0},$$

(19)

where $v(z_0)$ can be expressed in arbitrary units, e.g. one can use a zenith normalized $v(z_0)$. Although the radiant flux emitted per unit solid angle can be drawn as an intensity solid for many artificial light sources, the effect of isolated light in a heterogeneous environment has still not been satisfactorily explored. The problem of bulk emissions is quite complex because of the variety of possible configurations. Basically, no general approach to this problem exists even today. This is why oversimplified models are still in use.

For instance, Garstang’s function (Garstang 1989)

$$v_G(z_0) \propto 2 G (1 - F) \cos(z_0) + 0.554 F z_0^2,$$

(20)

may be useful as a zero-order approximation to imitate light sources with and/or without direct uplight. Here $G$ is the fraction of the light that is isotropically reflected from the ground and $F$ is the fraction radiated directly into the upward hemisphere (Garstang 1986). Garstang developed his model using observations in Boulder, Colorado, and found $G = 0.15$ and $F = 0.15$, most likely due to light emissions from Denver (Kerola 2006). Although a few experiments indicate that realistic emission functions may exhibit more complex behaviour compared to Garstang’s model, an appropriate analytical formula for $v_G(z_0)$ is still missing.

It is our intention to demonstrate differences only between the optical effects of poorly shielded and well-shielded light fixtures, thus $v_G(z_0)$ appears convenient for such a comparison. We have computed $R_{\perp,\lambda}(0)$ for a horizontally stratified atmosphere with an atmospheric data model consistent with that of the US Standard Atmosphere. The computational results are plotted as a function of wavelength in Fig. 2.

Fig. 2 reveals the spectral behaviour of $R_{\perp,\lambda}(0)$ under low-turbidity conditions for an aerosol optical depth inversely proportional to the wavelength assuming AOD = 0.1 at the reference wavelength of 500 nm. Four combinations of uplight fraction ($F$) and ground-reflectance ($G$) are taken into consideration to cover a few distinct situations: (1) $F = 0.15$ and $G = 0.15$ date from Garstang’s original model, (2) $F = 0.10$ and $G = 0.07$ represent the most common situation (see e.g. Luginbuhl et al. 2009 for uplight and Aubé, Roby & Kocifaj 2013 for the typical ground reflectance for a city in summer), (3) $F = 0.20$ and $G = 0.07$ are for cities with inadequate shielding of direct upward emissions, such as Los Mochis in Mexico (Kocifaj, Solano-Lamphar & Kundracik 2015), while (4) $F = 0.00$ and any value of $G$ will be analysed in Section 3 as a case of well-shielded light. Basically, the factor $R_{\perp,\lambda}(0)$ decreases as the wavelength approaches the red edge of the visible spectrum, while the functional dependence is not known explicitly. However, it is very similar to $C \lambda^{-2.5}$ with $C$ being $\approx 0.025$. This means that the blue light of a LED lamp makes the sky even more bluish. However, this effect can be largely suppressed or even removed with the spectral sensitivity function of the average human eye superimposed on the lamp spectrum. Then the human visual perception of the night sky brightness may change thanks to the shift of the peak position to wavelengths a bit longer than 450 nm. To model visual effects, one needs to introduce the function $S_V$ into equation (17). In most cases (for luminance levels below $10^{-5}$ cd m$^{-2}$), $S_V$ can be replaced by a scotopic luminosity function $V'$, so in analogy with the definition of $\Phi_{\lambda,0}(z_0) = H_{\lambda,0} v(z_0)/\cos(z_0)$ for a ground-based light source, we may express the diffuse irradiance at the ground as

$$\Phi_{\perp,\lambda,\text{down}}(0) = \left( \int_0^{\pi/2} v(z_0) \sin(z_0) \, dz_0 \right) \times \left( \int_0^{2\pi} R_{\perp,\lambda}(0) H_{\lambda,0} V' \, d\lambda \right),$$

(21)

where $H_{\lambda,0}$ is the emitted radiant energy per ground unit area (W m$^{-2}$). This accounts for direct emissions and also reflected
radiation, both of which are directed to the upward hemisphere. The effect of the atmosphere on the spectral composition of ground-reaching diffuse radiation can be interpreted in terms of two functions introduced in Fig. 3, where the lamp spectra $H_{\lambda,0}$ are compared against the product of $R_{\perp,\lambda}(0)$ and $V'_\lambda$. The total diffuse irradiance at a horizontal surface is an integral of $R_{\perp,\lambda}(0) H_{\lambda,0} V'_\lambda$ over a dedicated spectral band, as evident from equation (21).

It can be seen from Fig. 3(a) that the most important narrow peak in an LED lamp spectrum will be partially damped after the interaction of ground-based emissions with an atmospheric environment. An even more significant suppression of blue light would be manifest for cool white street lights with intensified yellow emissions compared to blue wavelengths. On the other hand, green light emitted from a high-pressure sodium lamp will be amplified when seeing the scattered light of a night sky. Red light will mainly be diminished.

Undoubtedly $R_{\perp,\lambda}(0)$ is a useful tool for the rapid determination of nocturnal ground irradiance in populated territories. Therefore, some of the data functions are also presented in tabulated form (see Tables 1 and 2).

It has been shown that an increase of the atmospheric turbidity results in a slight decrease of $R_{\perp,\lambda}(0)$ at short wavelengths, but the red light is almost unaffected (Fig. 4). This is similar to what we know as an effect of distant sources. A light beam transported over a long distance has multiple interactions with air molecules and aerosols, implying a rapid intensity decay. The optical cross section for blue light exceeds that for red light by a factor of 10 or even more (e.g. $\approx 16$ in the Rayleigh regime), thus causing a loss of blue light. High turbidity means high aerosol concentrations, which in turn increases the probability of an interaction between airborne aerosols and the beams of electromagnetic radiation. The effect we have analysed in Fig. 4 is similar to that discussed above, but the amplitude of the effect is very low due to significantly shortened optical paths. A drop at a wavelength of 730 nm is due to absorption by water vapour (see Fig. 4 and also Fig. 2).

### Table 1. Theoretical values of $R_{\perp,\lambda}(0)$ computed under low- and high-turbidity conditions for non-absorbing particles using data presented in Fig. 1. The uplight fraction is $F = 0.15$, while the fraction of reflected light is $G = 0.15$.

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>Low-turbidity conditions $R_{\perp,\lambda}(0)$</th>
<th>High-turbidity conditions $R_{\perp,\lambda}(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>350</td>
<td>0.300</td>
<td>0.255</td>
</tr>
<tr>
<td>370</td>
<td>0.271</td>
<td>0.226</td>
</tr>
<tr>
<td>390</td>
<td>0.245</td>
<td>0.202</td>
</tr>
<tr>
<td>410</td>
<td>0.222</td>
<td>0.181</td>
</tr>
<tr>
<td>430</td>
<td>0.200</td>
<td>0.163</td>
</tr>
<tr>
<td>450</td>
<td>0.182</td>
<td>0.148</td>
</tr>
<tr>
<td>470</td>
<td>0.165</td>
<td>0.135</td>
</tr>
<tr>
<td>490</td>
<td>0.152</td>
<td>0.125</td>
</tr>
<tr>
<td>510</td>
<td>0.136</td>
<td>0.114</td>
</tr>
<tr>
<td>530</td>
<td>0.126</td>
<td>0.106</td>
</tr>
<tr>
<td>550</td>
<td>0.116</td>
<td>0.099</td>
</tr>
<tr>
<td>570</td>
<td>0.101</td>
<td>0.089</td>
</tr>
<tr>
<td>590</td>
<td>0.096</td>
<td>0.069</td>
</tr>
<tr>
<td>610</td>
<td>0.093</td>
<td>0.084</td>
</tr>
<tr>
<td>630</td>
<td>0.086</td>
<td>0.080</td>
</tr>
<tr>
<td>650</td>
<td>0.073</td>
<td>0.073</td>
</tr>
<tr>
<td>670</td>
<td>0.081</td>
<td>0.078</td>
</tr>
<tr>
<td>690</td>
<td>0.076</td>
<td>0.076</td>
</tr>
<tr>
<td>710</td>
<td>0.054</td>
<td>0.061</td>
</tr>
<tr>
<td>730</td>
<td>0.025</td>
<td>0.036</td>
</tr>
<tr>
<td>750</td>
<td>0.068</td>
<td>0.072</td>
</tr>
</tbody>
</table>

### 4.2 A case of well-shielded light

Although a few of the present lighting technologies allow for efficient elimination of all light emitted upwards, cities or towns with no direct uplight ($F = 0$) are hypothetical rather than realistic. This is because the light beams originate from many sources, including buildings, cars, advertisement boards and also outdated streetlight installations. Indeed, fully shielded luminaires producing no direct
As Table 1, but for $F = 0.15$ and $G = 0.07$.

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>Low-turbidity conditions ($\text{AOD} = 0.1$)</th>
<th>High-turbidity conditions ($\text{AOD} = 0.5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>350</td>
<td>0.306</td>
<td>0.249</td>
</tr>
<tr>
<td>370</td>
<td>0.280</td>
<td>0.223</td>
</tr>
<tr>
<td>390</td>
<td>0.256</td>
<td>0.200</td>
</tr>
<tr>
<td>410</td>
<td>0.235</td>
<td>0.181</td>
</tr>
<tr>
<td>430</td>
<td>0.214</td>
<td>0.163</td>
</tr>
<tr>
<td>450</td>
<td>0.197</td>
<td>0.149</td>
</tr>
<tr>
<td>470</td>
<td>0.180</td>
<td>0.137</td>
</tr>
<tr>
<td>490</td>
<td>0.167</td>
<td>0.127</td>
</tr>
<tr>
<td>510</td>
<td>0.151</td>
<td>0.117</td>
</tr>
<tr>
<td>530</td>
<td>0.141</td>
<td>0.110</td>
</tr>
<tr>
<td>550</td>
<td>0.130</td>
<td>0.103</td>
</tr>
<tr>
<td>570</td>
<td>0.113</td>
<td>0.093</td>
</tr>
<tr>
<td>590</td>
<td>0.076</td>
<td>0.072</td>
</tr>
<tr>
<td>610</td>
<td>0.107</td>
<td>0.089</td>
</tr>
<tr>
<td>630</td>
<td>0.099</td>
<td>0.085</td>
</tr>
<tr>
<td>650</td>
<td>0.083</td>
<td>0.077</td>
</tr>
<tr>
<td>670</td>
<td>0.094</td>
<td>0.083</td>
</tr>
<tr>
<td>690</td>
<td>0.088</td>
<td>0.081</td>
</tr>
<tr>
<td>710</td>
<td>0.061</td>
<td>0.065</td>
</tr>
<tr>
<td>730</td>
<td>0.027</td>
<td>0.037</td>
</tr>
<tr>
<td>750</td>
<td>0.080</td>
<td>0.077</td>
</tr>
</tbody>
</table>

**Figure 4.** Theoretical values of $R_{\perp,\lambda}(0)$ and $R_{\parallel}(0)$ computed under low (solid lines) and high (dashed lines) turbidity conditions. The fractions of direct upright ($F$) and reflected light ($G$) are given in the figure legend.

Table 3. As Table 1, but for well-shielded light with $F = 0.0$ and an arbitrary value of $G$.

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>Low-turbidity conditions ($\text{AOD} = 0.1$)</th>
<th>High-turbidity conditions ($\text{AOD} = 0.5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>350</td>
<td>0.284</td>
<td>0.270</td>
</tr>
<tr>
<td>370</td>
<td>0.246</td>
<td>0.236</td>
</tr>
<tr>
<td>390</td>
<td>0.214</td>
<td>0.207</td>
</tr>
<tr>
<td>410</td>
<td>0.186</td>
<td>0.182</td>
</tr>
<tr>
<td>430</td>
<td>0.161</td>
<td>0.161</td>
</tr>
<tr>
<td>450</td>
<td>0.141</td>
<td>0.144</td>
</tr>
<tr>
<td>470</td>
<td>0.124</td>
<td>0.129</td>
</tr>
<tr>
<td>490</td>
<td>0.109</td>
<td>0.117</td>
</tr>
<tr>
<td>510</td>
<td>0.095</td>
<td>0.104</td>
</tr>
<tr>
<td>530</td>
<td>0.085</td>
<td>0.096</td>
</tr>
<tr>
<td>550</td>
<td>0.076</td>
<td>0.088</td>
</tr>
<tr>
<td>570</td>
<td>0.065</td>
<td>0.078</td>
</tr>
<tr>
<td>590</td>
<td>0.049</td>
<td>0.062</td>
</tr>
<tr>
<td>610</td>
<td>0.055</td>
<td>0.071</td>
</tr>
<tr>
<td>630</td>
<td>0.050</td>
<td>0.067</td>
</tr>
<tr>
<td>650</td>
<td>0.043</td>
<td>0.061</td>
</tr>
<tr>
<td>670</td>
<td>0.044</td>
<td>0.063</td>
</tr>
<tr>
<td>690</td>
<td>0.040</td>
<td>0.060</td>
</tr>
<tr>
<td>710</td>
<td>0.032</td>
<td>0.050</td>
</tr>
<tr>
<td>730</td>
<td>0.019</td>
<td>0.031</td>
</tr>
<tr>
<td>750</td>
<td>0.033</td>
<td>0.055</td>
</tr>
</tbody>
</table>

Conclusively, to what we have printed in Tables 1 and 2, we provide a new data set for well-shielded light in Table 3.

**5 CONCLUSIONS**

A number of devices are employed in environmental monitoring, but only a few are fabricated for night-time use. There already exists a vast database of Sky Quality Meter (SQM) data that closely relates to the zenith radiance. However, such data suffer from low information content because directional radiance cannot characterize the total amount of light that reaches the ground surface. The diffuse downwelling radiation is of high interest due to its direct consequences in astronomy, as the diffuse light (a veiling luminance) can preclude not only professional but also amateur observations at many places and even during clear moonless nights.

Illuminance measurements are difficult to perform in the field, but these data play a key role in the characterization of nocturnal environments. A linking of total illumination and other data products (e.g. SQM data) was, therefore, one of the legitimate motivations for a few targeted studies (e.g. Kociñ, Posch & Solano-Lamphar 2015a). In spite of an apparent correlation between the aforementioned quantities, the above methods suffer from an uncertainty that is difficult to eliminate. This is why there is a great demand for methods that allow for practical estimation and predictions of ground-reaching diffuse irradiance for a particular territory.

In this paper, a new rapid computational technique is developed to simulate spectral and broad-band downward radiative fluxes based on information about emissions from artificial light sources. Unlike the information on per capita light output in lumens normally required as an input to the conventional solution methods (Garstang 1986; Kerola 2006), we proceed with a statistical approach that is more appropriate for characterizing a number of light sources dispersed over a large territory and it represents a simplistic input to the RTE solution scheme. The modelling of ground-reaching diffuse radiation in densely populated territories is generally difficult,
partly due to theoretical constraints when many light sources scattered everywhere around are to be analysed and partly because of CPU requirements in mass numerical computations. This is why an approximate approach is needed. Among many others, the solution method and the resulting tables introduced in our work may be a very convenient approach to rapid modelling. In particular, the tables can help modellers to minimize the computational effort otherwise spent in obtaining the same input data we provide in our paper. The tabulated data are independent of wavelength, thus can be used repeatedly for different spectra of ground-based light sources.

Ground-based light emissions are expressed as average watts per square metre (or W m\(^{-2}\) nm\(^{-1}\) for spectral radiative fluxes). The output diffuse irradiances at a horizontal surface are correspondingly obtained as average watts per square metre. The data products provided represent an ideal platform for qualification and quantification of different lighting technologies in terms of their impacts on a night-time environment. The data products involve the spectral ratios of \(R_{\perp,\lambda}(0)\), which represent a basis for determining the diffuse irradiance at the ground. The computations can be easily made for different spectra \(H_{\lambda,0}\) using, e.g., equation (21), where the radiant intensity distribution \(v(z_0)\) is expected to be independent of \(H_{\lambda,0}\). However, the scotopic luminosity function \(V'_\lambda\) should be replaced by the spectral sensitivity \(S_\lambda\) of a detector if modelling an objective device.

We expect lighting engineers, astronomers and environmental scientists will enrich the tables of \(R_{\perp,\lambda}(0)\) with new data products obtained for other aerosol microphysics, turbidities or stratifications of atmospheric constituents.

ACKNOWLEDGEMENTS

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