Closure to "Computational Analysis for Mixed Convective Flows of Viscous Fluids With Nanoparticles" (Farooq, U., Lu, D. C., Ahmed, S., and Ramzan, M., 2019, ASME J. Therm. Sci. Eng. Appl., 11(2), p. 021013)

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The authors regret in the published paper referenced above and agree with the discussion by Pantokratoras (2019, "Discussion: "Computational Analysis for Mixed Convective Flows of Viscous Fluids With Nanoparticles" (Farooq, U., Lu, D. C., Ahmed, S., and Ramzan, M., 2019, ASME J. Therm. Sci. Eng. Appl., 11(2), p. 021013)," ASME J. Therm. Sci. Eng. Appl., 11(5), p. 055503). In this Closure, the non-similar mathematical model is developed to describe the mixed convective nanofluid flow over vertical sheet which is stretching at an exponential rate. In the published article referenced above, similarity transformations are utilized to convert the governing nonlinear partial differential equations (PDEs) into ordinary differential equations (ODEs). The important physical numbers such as magnetic field (M^2), Brownian motion parameter (N_b), thermophoresis (N_t), Eckert number (Ec), ratio of mass transfer Grashof to heat transfer Grashof (N), buoyancy parameter (λ), and Reynolds number (Re) appearing in the dimensionless ODEs are still functions of coordinate "x"; therefore, the problem is non-similar. In this corrigendum, the non-similar model is developed by using ξ(x) as non-similarity variable and η(x, y) as pseudo-similarity variable. The dimensionless non-similar model is numerically simulated by employing local non-similarity via bvp4c. The graphical results show no change in behavior. The important thermal and mass transport quantities such as Nusselt number and Sherwood number have been computed for the non-similar model, and results are compared with the published article.

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Keywords: non-similar modeling, mixed convection, exponentially stretching surface, bvp4c, heat and mass transfer, magnetohydrodynamic (MHD)

1 Introduction

In the literature, most of the boundary layer problems have been tackled using similarity transformations which change the governing partial differential equations (PDEs) to ordinary differential equations (ODEs) [1,2]. Numerical and analytical methods are generally used to evaluate these ODEs. Analytical methods are mostly used to solve similar flows due to calculation and ideal simplicity. Sometimes it may happen that after applying similarity variables, the physical parameters in the governing equations are still functions of coordinate "x" (see Refs. [3,4]), and these problems are known as non-similar. Despite of this fact that non-similar flows are extensive in nature and they have vast applications in real life, there are still lesser publications in this direction compared with similar flows. Many numerical techniques have been developed which can tackle non-similar flows. The most prominent technique is known as local non-similarity established by Sparrow et al. [5]. In this corrigendum, local non-similarity via bvp4c is employed. The main drawback of the local similarity approach is that it neglects the non-similar part. To overcome this drawback, Sparrow and Yu [6] presented a new technique termed as "method of local non-similarity" to obtain results for non-similar flows. Later non-similar flows were studied by numerous authors (see for instance, Refs. [5,7–11]). In this corrigendum, non-similar transformations have been proposed for mixed convective boundary layer flow over vertical surface which is stretching at an exponential rate. The local non-similarity technique via bvp4c is employed to obtain non-similar solutions.

2 Mathematical Formulation

Consider two-dimensional, laminar, incompressible magnetized nanofluid flow which is produced by the exponential expansion of the sheet with velocity "U_w". The buoyancy in terms of temperature and concentration is incorporated in the momentum equation. The impacts of magnetic and viscous dissipations are incorporated in the energy equation. The physical interpretation of the model is described as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\left(\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma B_o^2}{\nu_f}u+g\beta_c(C-C_\infty)+g\beta_T(T-T_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial y^2}\right)+\frac{\mu}{\rho_f C_p}\left(\frac{\partial u}{\partial y}\right)^2+\tau D_B\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)+\frac{\tau D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2+\frac{\sigma B_o^2}{C_p\rho_f}u^2 \tag{3}$$

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$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial y^2}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

In preceding expressions, u and v represent velocities in x- and y-directions, and α, ν, T, σ, C_p, T_∞, D_B, C, D_T, and ρ_f represent thermal diffusivity, kinematic viscosity, temperature, electrical conductivity, specific heat, freestream temperature, Brownian diffusion coefficient, nanoparticle volume fraction, thermophoretic diffusion coefficient, and fluid density, respectively.

The relevant conditions take the following forms:

$$\text{At } y=0\,,\quad u=U_w(x)=U_0\exp\left(\frac{x}{l}\right),\quad v=0,\quad C=C_w(x),$$

$$-k\frac{\partial T}{\partial y}=h_f\,(T_w-T) \tag{5}$$

$$\text{As } y\to\infty,\quad v\to 0,\quad u\to 0,\quad T\to T_\infty,\quad C\to C_\infty \tag{6}$$

where

$$T_w=T_\infty+T_0\exp\left(\frac{2x}{l}\right),\quad C_w=C_\infty+C_0\exp\left(\frac{2x}{l}\right)$$

In earlier mentioned conditions, C_w and T_w are the concentration and temperature on the wall and h_f is the convective heat transfer coefficient. Introducing two dimensionless numbers ξ and η the non-similarity variable and pseudo-similarity variable, respectively, non-similarity transformations are then proposed as

$$\begin{cases}\xi=\exp\left(\frac{x}{l}\right),\quad \eta=\sqrt{\frac{U_0}{2\nu l}}\,y\,\exp\left(\frac{x}{2l}\right)\\ u=U_0\frac{\partial f}{\partial\eta}(\xi,\eta)\,\exp\left(\frac{x}{l}\right)\\ v=-\sqrt{\frac{\nu U_0}{2l}}\exp\left(\frac{x}{2l}\right)\left(f(\xi,\eta)+\eta\frac{\partial f}{\partial\eta}+2\xi\frac{\partial f}{\partial\xi}\right)\\ T=\theta(\xi,\eta)(T_w-T_\infty)+T_\infty\\ C=\phi(\xi,\eta)(C_w-C_\infty)+C_\infty\end{cases} \tag{7}$$

Using Eq. (7) in Eqs. (1)–(6), they are transformed as follows:

$$\frac{\partial^3 f}{\partial\eta^3}+f\frac{\partial^2 f}{\partial\eta^2}-2\left(\frac{\partial f}{\partial\eta}\right)^2-2M^2\xi^{-1}\frac{\partial f}{\partial\eta}+2\lambda(\theta+N\phi)=2\xi\left(\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\xi\partial\eta}-\frac{\partial^2 f}{\partial\eta^2}\frac{\partial f}{\partial\xi}\right) \tag{8}$$

$$\frac{\partial^2\theta}{\partial\eta^2}+\mathrm{Pr}\left(f\frac{\partial\theta}{\partial\eta}-4\theta\frac{\partial f}{\partial\eta}+2M^2\mathrm{Ec}\xi^{-1}\left(\frac{\partial f}{\partial\eta}\right)^2+\xi^2\left(N_b\frac{\partial\phi}{\partial\eta}\frac{\partial\theta}{\partial\eta}+N_t\left(\frac{\partial\theta}{\partial\eta}\right)^2\right)+\mathrm{Ec}\left(\frac{\partial^2 f}{\partial\eta^2}\right)^2\right)=2\mathrm{Pr}\xi\left(\frac{\partial f}{\partial\eta}\frac{\partial\theta}{\partial\xi}-\frac{\partial\theta}{\partial\eta}\frac{\partial f}{\partial\xi}\right) \tag{9}$$

$$\frac{\partial^2\phi}{\partial\eta^2}+\frac{N_t}{N_b}\frac{\partial^2\theta}{\partial\eta^2}-\mathrm{Sc}\left(4\phi\frac{\partial f}{\partial\eta}-f\frac{\partial\phi}{\partial\eta}\right)=2\mathrm{Sc}\xi\left(\frac{\partial f}{\partial\eta}\frac{\partial\phi}{\partial\xi}-\frac{\partial\phi}{\partial\eta}\frac{\partial f}{\partial\xi}\right) \tag{10}$$

The boundary conditions are as follows:

$$\begin{aligned}&\frac{\partial f}{\partial\eta}(\xi,0)=1,\quad f(\xi,0)+2\xi\frac{\partial f}{\partial\xi}(\xi,0)=0,\quad \phi(\xi,\infty)=0,\quad \phi(\xi,0)=1\\ &\frac{\partial f}{\partial\eta}(\xi,\infty)=0,\quad \theta(\xi,\infty)=0,\quad \frac{\partial\theta}{\partial\eta}(\xi,0)=-\gamma\xi^{-1/2}(1-\theta(\xi,0))\end{aligned} \tag{11}$$

The parameters Re, λ, N_b, Pr, N_t, M, Le, N, and Ec represent Reynolds number, buoyancy parameter, Brownian motion, Prandtl number, thermophoresis, magnetic field, Lewis number, ratio between mass transfer Grashof number and heat transfer Grashof number, and Eckert number, respectively. The dimensionless parameters are represented as

$$\begin{aligned}&\lambda=\frac{\mathrm{Gr}}{\mathrm{Re}^2},\quad \mathrm{Pr}=\frac{\nu}{\alpha},\quad N_b=\frac{\tau D_B C_0}{\nu},\quad N_t=\frac{\tau T_o D_T}{\nu T_\infty},\quad M^2=\frac{\sigma B_o^2 l}{\rho_f U_o},\quad \mathrm{Ec}=\frac{U_o^2}{C_p T_o},\quad \mathrm{LePr}=\mathrm{Sc}\\ &\mathrm{Sc}=\frac{\nu}{D_B},\quad \mathrm{Re}=\frac{lU_o}{\nu},\quad N=\frac{\mathrm{Gr}^*}{\mathrm{Gr}},\quad \gamma=\frac{h_f}{k}\sqrt{\frac{2\nu l}{U_o}},\quad \mathrm{Gr}=\frac{g\beta_T T_0 l^3}{\nu^2},\quad \mathrm{Gr}^*=\frac{g\beta_c C_0 l^3}{\nu^2}\end{aligned} \tag{12}$$

The local Nusselt number, skin friction, and local Sherwood number coefficients are

$$\mathrm{Nu}=\frac{xq_I}{k(T_1-T_\infty)},\quad C_f=\frac{2\tau_I}{\rho U_I^2},\quad \mathrm{Sh}=\frac{xj_I}{D_B(C_I-C_\infty)} \tag{13}$$

where j_I is the mass flux, τ_I is the wall skin friction, and q_I is heat flux from surface, defined below:

$$\tau_I=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0},\quad j_I=-D_B\left(\frac{\partial C}{\partial y}\right)_{y=0},\quad q_I=-k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{14}$$

$$\begin{aligned}&(2\mathrm{Re})^{(1/2)}C_f=\sqrt{2}f''(\xi,0),\\ &\mathrm{Re}^{-(1/2)}\mathrm{Sh}=-\sqrt{\frac{1}{2}}\ln(\xi)\phi'(\xi,0),\\ &\mathrm{Re}^{-(1/2)}\mathrm{Nu}=-\sqrt{\frac{1}{2}}\theta'(\xi,0)\ln(\xi)\end{aligned} \tag{15}$$

3 Non-Similarity Method

To interpret several non-similarity boundary layers, many scientists utilize this method (see for instance, Refs. [12–14]). The principle of local similarity is usually considered for outcomes of non-similarity boundary layer. With the help of the local non-similar technique, we have approximated non-similar PDEs through ODEs. In this method, the right-edge of Eqs. (8)–(10) are supposed to be sufficiently small; therefore, it can be estimated to

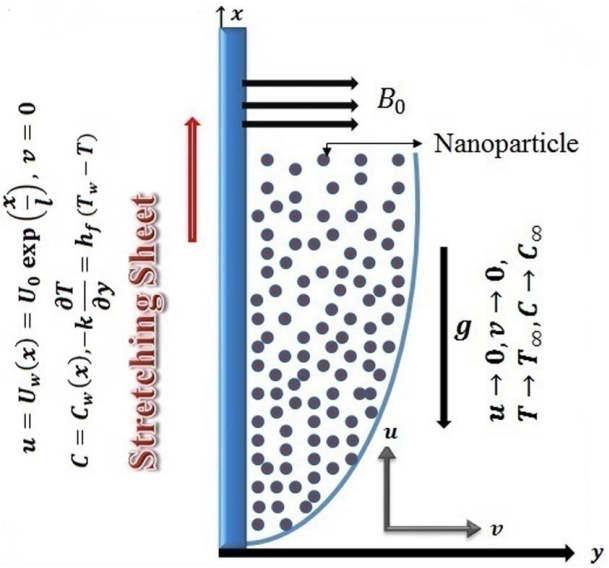

Fig. 1 Physical configuration

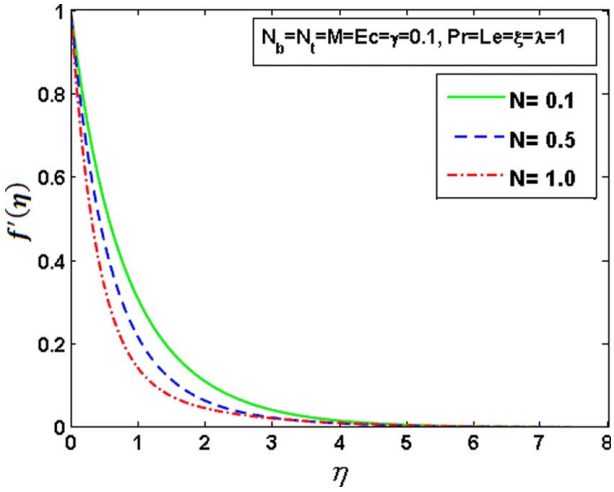

Fig. 2 Graph of $f'(\eta)$ for distinct values of N

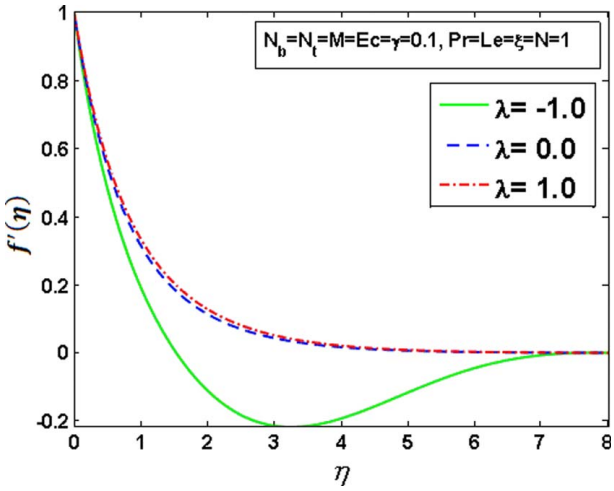

Fig. 3 Graph of $f'(\eta)$ for distinct values of λ

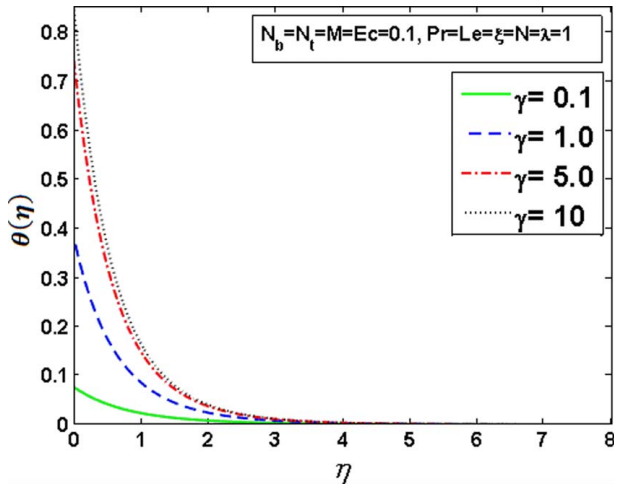

Fig. 4 Graph of $\theta(\eta)$ for distinct values of γ

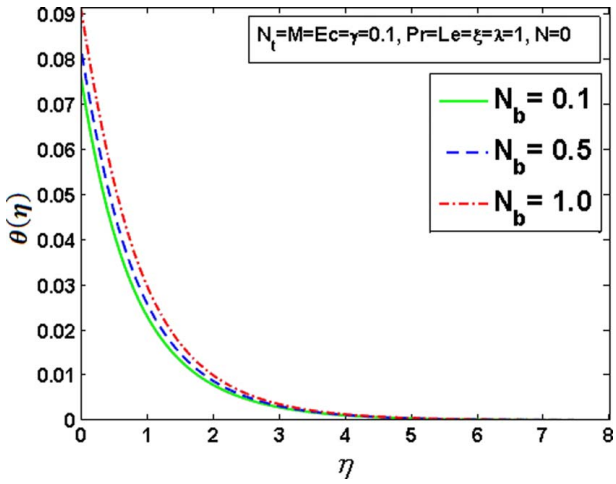

Fig. 5 Graph of $\theta(\eta)$ for distinct values of N_b

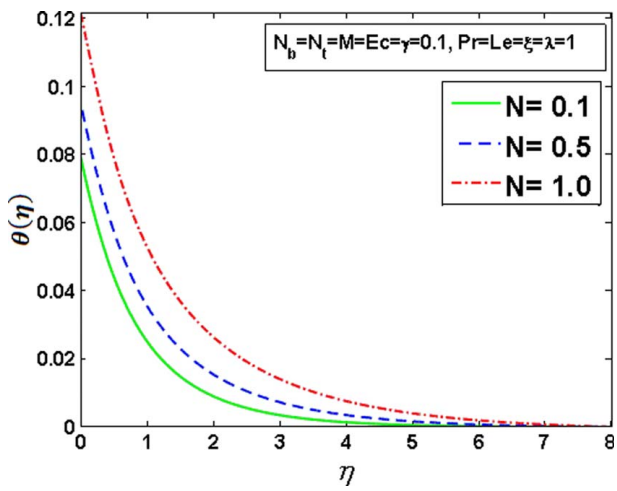

Fig. 6 Graph of $\theta(\eta)$ for distinct values of N

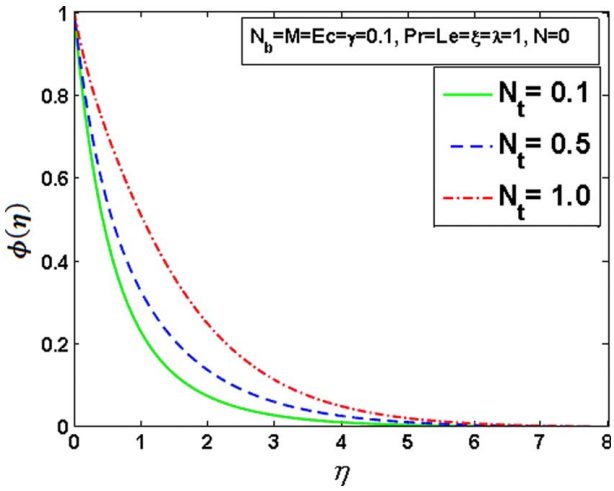

Fig. 7 Graph of $\phi(\eta)$ for distinct values of N_t

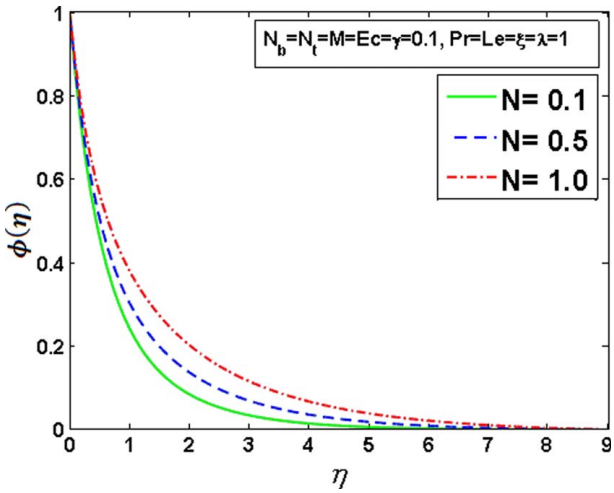

Fig. 8 Graph of $\phi(\eta)$ for distinct values of N

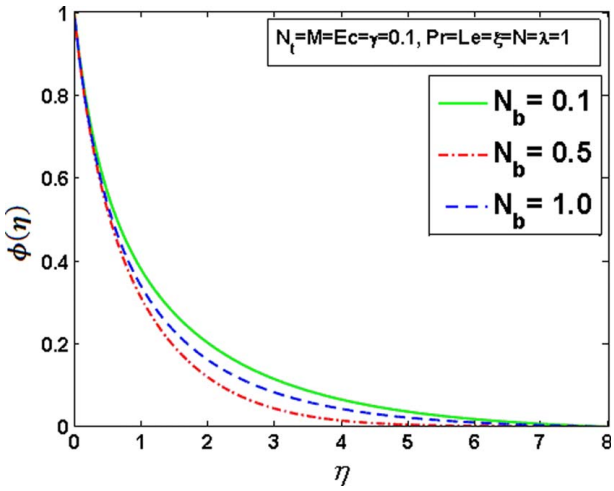

Fig. 9 Graph of $\phi(\eta)$ for distinct values of N_b

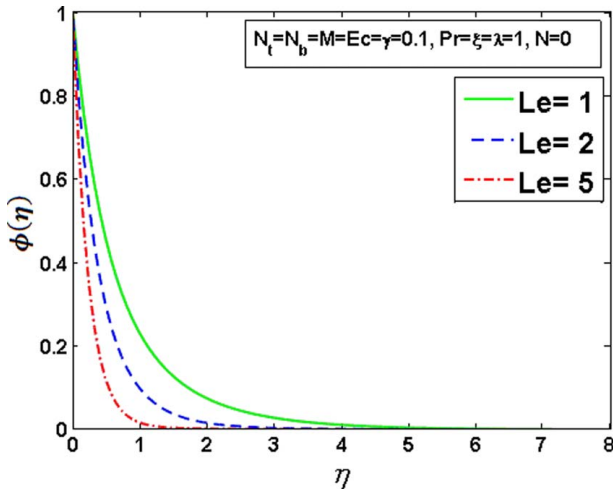

Fig. 10 Graph of $\phi(\eta)$ for distinct values of Le

zero, and hence leading PDEs become ODEs. The final resulting non-dimensional set of coupled nonlinear PDEs is then resolved with the utilization of bvp4c function in MATLAB. A technique for finding locally non-similar boundary layer is discussed and demonstrated by Farooq et al. [15].

4 Results and discussion

Local non-similarity via bvp4c is utilized for the influences of numerous parameters on temperature profile θ, velocity profile f', and concentration profile ϕ. Figures 1–10 are generated for distinct values of γ, N, N_b, N_t, λ, and Le. The graphs depict similar behavior to the original problem [16].

Tables 1–3 present the comparison between local similar skin friction, Nusselt and Sherwood numbers of Farooq et al. [16] and present non-similar results.

Table 1 Comparison of local similar and non-similar C_{fx} (local skin friction) for different values of λ versus N when $M = \text{Ec} = N_b = \gamma = N_t = 0.1$, $\xi = \text{Sc} = 1$, $\text{Pr} = 3$

Parameters		Farooq et al. [16]	Present	
λ	N	$C_{fx}\,(\text{Re}_x^{(1/2)})$		Difference (%)
1	1	−0.4909596405	−0.6647215909	26.14
	2	0.1349248240	−0.1404157755	196.08
	3	0.7207345529	0.3502839440	105.75
2	1	0.1691179725	−0.1207441687	240.06
	2	1.3206151204	0.8362957739	57.91
	3	2.4177911886	1.7301144394	39.74

Table 2 Comparison of local similar and non-similar Nu (local Nusselt number) for different values of N_b versus Ec when $\text{Pr} = 3$, $N_t = M = 0.1$, $N = \lambda = \text{Sc} = \xi = \gamma = 1$

Parameters		Farooq et al. [16]	Present	
Ec	N_b	$\text{Nu}\,(\text{Re}_x^{-(1/2)})$		Difference (%)
1	0.1	−0.5659884824	−0.3587657268	57.75
	0.2	−0.5433373428	−0.3413113332	59.19
	0.3	−0.5260558981	−0.3335272975	57.71
2	0.1	−0.5594207277	−0.1566273066	257.16
	0.2	−0.5080076972	−0.1506193302	237.27
	0.3	−0.4827801575	−0.1455588629	231.67

Table 3 Comparison of local similar and non-similar Sh (local Sherwood number) for different values of Sc versus N_t when $N_b = Ec = M = \gamma = 0.1$, $Pr = N = \lambda = \xi = 1$

Parameters		Farooq et al. [16]	Present	
Sc	N_t	$Sh\,(Re_x^{-(1/2)})$		Difference (%)
3	0.1	−1.8928151531	−3.0470646222	37.88
	0.3	−1.5128377645	−2.9675413434	49.02
	0.5	−1.1221686851	−2.8326909229	60.38
5	0.1	−2.5964060620	−4.1192374623	36.96
	0.3	−2.3333398044	−4.0680085443	42.64
	0.5	−2.0125506640	−3.9544145612	49.10

5 Conclusion

The authors agree with the concerns of Pantokratoras [17] on Farooq et al. [16]. Therefore, in this corrigendum, the non-similar model is developed in which the physical parameters are independent of "x". Non-similar solutions depict similar graphical behavior by comparing it with the published article. The important thermal and mass transport quantities such as Nusselt number and Sherwood number have been computed for the non-similar model.

Acknowledgment

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Conflict of Interest

There are no conflicts of interest.

Data Availability Statement

The datasets generated and supporting the findings of this article are obtainable from the corresponding author upon reasonable request. Data provided by a third party are listed in Acknowledgment.

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