



Comparison of Theoretical Methods Describing the Heat Transfer in Vertical Tube Condensers Under Conditions Corresponding to the Condensation of Flue Gas From a Biomass Boiler

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A theoretical and experimental study was conducted on the condensation of water vapor in a vertical tube condenser under conditions corresponding to the condensation of flue gas from a biomass boiler, where flue gas is considered a mixture of water vapor and a high content of noncondensable gas (NCG). Four fundamental theoretical methods were identified for determining the heat transfer coefficient and condenser heat output—empirical correlations, heat and mass transfer analogy, diffusion layer theory, and boundary layer theory. These methods were compared in terms of their usability in the design of flue gas condensers in energy systems and experimentally verified. Experiments were carried out in a 1.5 m long vertical double-pipe condenser with a condensing gas flowing in a downward direction through an inner tube with a diameter of 25 mm. The mass concentration of NCG in a mixture with water vapor ranged from 11 vol% to 86 vol% and the inlet Reynolds number of the condensing gas ranged from 1936 to 14,408, which corresponds with the conditions of condensing flue gas from biomass boilers. The boundary layer theory is highly complex and impractical for the calculation of heat exchangers. Empirical correlations have a wide dispersion of the result, because they consider only the fundamental parameters of the process. Nevertheless, heat and mass transfer analogy and diffusion layer theory seem to be the most suitable for flue gas condensers since they capture the physical essence of the phenomena. [DOI: 10.1115/1.4067072]

Keywords: condensation, energy systems, heat and mass transfer, heat exchangers

Introduction

In recent years, condensers have played a significant role in recent years in the search for energy savings in the power and process industry. During biomass combustion, the water vapor content in the flue gas is approximately 10–25% for air combustion but it can reach up to 45% for oxyfuel combustion [1], so a significant portion of energy is carried out from the boiler in the form of latent heat which decreases the boiler efficiency [2]. Flue gas condensers are capable of recovering the latent heat of water vapor in flue gas, which could increase the efficiency of the entire energy system [3]. Furthermore, this solution is economically viable [4]. Nevertheless, the potential of this solution is constrained by the necessity of

maintaining a specific temperature level for the extracted heat, which is largely dependent on the heating water [5]. This issue can be addressed by incorporating a heat pump [6]. During oxyfuel combustion of biomass [1], which is one of the modern methods still only laboratory-verified used to capture CO₂ from combustion processes, condensers play a crucial role during the separation of CO₂ from the flue gas. After the condensation of the majority of the water vapor, the remaining flue gas consists mainly of CO₂, which can then be stored or utilized [7]. The process of vapor condensation from flue gas obtained by burning biomass can be theoretically considered as water vapor condensation in the presence of a high concentration of noncondensable gas (NCG). Vertical tubes or spray condensers are suitable for this application [8,9]. The advantage of tube condensers is that the cooling water does not mix with the condensate and is not polluted by the substances that might be present in the flue gas [8]. Condensation in such condensers is affected by the content of NCGs, which might reduce the heat transfer [10].

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The condensation of water vapor in the presence of NCG is a widely studied topic. In 1929, it was discovered that even a small amount of NCG has a significant influence on heat transfer during the condensation process. It was evaluated that heat transfer decreased by more than 50% at an NCG concentration of 0.5% [11]. Therefore, the presence of NCG influences the design and operation of condensers in a wide range of applications. Condensers in cooling technology, steam turbines, in geothermal power plants, and various applications in the chemical and process industry are a well-developed and crucial part of technologies. However, in these areas, the concentration of condensable gases is quite small, while the condensation of water vapor in the presence of a high concentration of NCG is still not fully explored.

To predict the heat transfer process for the condensation of water vapor in the presence of an NCG, several theoretical methods have been developed. Empirical correlations represent the simplest option to determine the condensation heat transfer coefficient (HTC). However, empirical equations are limited by a range of usage validity. A review of empirical correlations is presented in the works of Huang et al. [12] or Dehbi [13]. Analytical methods that offer more physical investigation of the problem are another option of determining the condensation HTC. Colburn and Hougen developed the heat and mass transfer analogy method [14]. In their study, they presented methods for calculating the condenser size sufficient to condense a defined amount of water vapor from the gas–water vapor mixture. Siddique et al. [15] conducted an analytical study of water vapor condensation in the presence of air, helium, and hydrogen. The theoretical model was developed according to the heat and mass transfer analogy and the results were compared with the experimental data. It was found that air is more inhibiting than hydrogen or helium and that the heat transfer characteristics of hydrogen or helium with water vapor are almost identical. Kim et al. [16] developed a theoretical model based on the heat and mass transfer analogy and compared the data with experiments of condensation of water vapor in the presence of nitrogen at high pressures. They concluded that with increasing system pressure, the condensation HTC increased. However, as the nitrogen mass fraction increased, the condensation HTC decreased. They also concluded that the proposed model could predict the HTC with higher agreement with experiments than any other correlation examined and is suitable for evaluating condensation heat and mass transfer at high pressure. Maheswari et al. [17] developed a theoretical model to study the local HTC for condensation inside a vertical tube. The predicted HTC was in close agreement with the data obtained from several selected experiments. For low Reynolds numbers, the thermal resistance in the gas–water vapor mixture was higher than the resistance in the condensation film. Other authors developed theoretical models based on the heat and mass transfer analogy, usually for the vertical tube and small non-condensable concentrations. Studies using the heat and mass transfer analogy generally show sufficient accuracy in predicting the local and overall HTCs compared to experimental data [18–21].

Diffusion layer modeling represents another analytical approach. This method introduces the use of Fick's law of molecular diffusion to formulate the condensation flow of water vapor toward the phase interface. The method was first introduced by Peterson et al. [22] in 1993. Other studies continued Peterson et al.'s work and presented few adjustments or improvements to the theory. Anderson et al. [23] and Herranz et al. [24] adjusted the equation for thermal condensation conductivity in cases of high-temperature gradients where the density of the gas diffusion layer could be variable. Subsequently, Liao [25] conducted a generalization of this method for the mass flowrate with a correction to the equation for the thermal condensation conductivity. This seemed to be a more accurate solution than expressing molar flowrate in calculating the local condensation flowrate. Another work was done by Kuhn et al. [26] who compared three theoretical models with experimental data. Theoretical models were based on the degradation factor method and diffusion layer theory expressed in the molar and mass transfer relation forms. The results from all three theoretical approaches corresponded

quite well with the experimental data. However, both models based on diffusion layer theory have higher accuracy than the results from the degradation factor method. Other studies [23,27,28] also use this method to determine heat and mass transfer.

Flue gas condensers for biomass boilers are a relatively novel technology, and the calculation methods for this application are not yet well developed. While there are general methods for condensers, these were originally designed for other applications, so it is necessary to verify their use for the specific conditions of flue gas condensation in a biomass boiler. The condensing gas flow with a high content of NCG, a low velocity, and a low value of the Reynolds number are the typical conditions for flue gas condensers in energy systems, resulting in a significant drop in the HTC, which has a substantial impact on the design and operation of the condensers. This work focuses on investigating condensation under these conditions, since they are still not fully resolved. The current state-of-the-art focuses primarily on the conditions that occur during loss of coolant accidents (LOCA) in the nuclear industry [29]. The scope of this paper is defined by the actual parameters that occur in the flue gas condensers. The aim of this work is to analyze current theoretical methods suitable for heat transfer modeling of water vapor condensation with high content of NCG in a vertical tube. Then to compare the results of these methods with experimental results to evaluate the suitability for modeling of heat transfer in vertical tube condensers under conditions corresponding to the condensation of flue gas from a biomass boiler. On the basis of this comparison, the most suitable method for designing a vertical tube condenser for condensing flue gas from a biomass boiler will be determined. This will facilitate the development of more efficient and precise designs for such devices.

Theoretical Methods for Determining the Heat Transfer Coefficient

Experimental methods (empirical correlations) and analytical methods (heat and mass transfer analogy and diffusion layer method) are included in the analysis due to the relatively high reliability, simplicity, and the possibility of examining the physical essence of the problem. Boundary layer theory (a purely theoretical method) would offer the highest accuracy, however, its implementation is very complex and impractical. Therefore, this type of method is not involved.

Empirical Correlations. Empirical correlations, such as the correction and degradation factors, represent the simplest way to determine the condensation HTC during gas–water vapor condensation α_{con} . It is usually best to consider most of the crucial parameters in the empirical equations since the condensation depends on several factors such as geometry, the range of bulk NCG concentration Re_G , the Reynolds number of condensing gas Re_G , condensing gas pressure p_G , and type of the NCG. However, in general only a few parameters are considered in the equations.

Several empirical equations were chosen depending on their validity range to compare the agreement of the results with experimental values (see Table 1). Empirical correlations are often mentioned in published studies and are considered to be established with sufficient agreement with experimental results.

Heat and Mass Transfer Analogy. The analytical methods were developed on the basis of the work of Colburn and Hougen [14] and Peterson et al. [22] with respect to other articles dealing with this method [26,30,31]. The validity of analytical models is primarily dependent on the fulfillment of the assumptions used in the models, and it is consistent with the parameters of the experiment. This method is based on the analogy between momentum, heat, and mass transfer [14]. It uses experimentally established data as well, although to a lesser extent than empirical correlations. The calculation model is made for a vertical tube, wherein the

Table 1 Overview of experimental equations for condensation HTC determination

Source	Suggested equation	Validity
Akaki et al. [20]	$\alpha_{con} = 0.33 \left(\frac{p_a}{p_i}\right)^{-0.67}$; $\alpha_{con} = 2.11 \times 10^{-4} Re^{0.8} \left(\frac{p_a}{p_G}\right)^{-0.99}$	for $650 < Re_G < 2300$; for $2300 < Re_G < 21,000$
Uchida et al. [38]	$\alpha_{con} = 380 \left(\frac{W_v}{1-W_v}\right)^{0.7}$	$0.09 < W_v < 0.77$; $1 < p_G < 9$ bar; $10 < T_G - T_{F,W} < 140^\circ C$
Vierow [39]	$F = \frac{\alpha_{con}}{\alpha_{Nu}} = (1 + aRe_G^b)(1 - cW_a^d)$	$0.063 > W_a \rightarrow a = 2.88 \times 10^{-5}$; $b = 1.18$; $c = 10$; $d = 1$ $0.6 > W_a > 0.063 \rightarrow a = 2.88 \times 10^{-5}$; $b = 1.18$; $c = 0.94$; $d = 0.13$ $W_a > 0.6 \rightarrow a = 2.88 \times 10^{-5}$; $b = 1.18$; $c = 1$; $d = 0.22$
Kuhn et al. [26]	$F = \frac{\alpha_{con}}{\alpha_{Nu}} = (1 + aRe_G)(1 - bW_a^c)$	$W_a < 0.1 \rightarrow a = 7.32 \times 10^{-4}$; $b = 2.601$; $c = 0.708$ $W_a > 0.1 \rightarrow a = 7.32 \times 10^{-4}$; $b = 1$; $c = 0.292$
Siddique et al. [36]	$Nu(x) = 6.123 Re^{0.223} \left(\frac{W_{a,w} - W_{a,b}}{W_{a,w}}\right)^{1.144} Ja^{-1.253}$	$0.1 < W_a < 0.95$; $445 < Re_G < 22,700$; $0.004 < Ja_G < 0.07$

condensing gas enters the condenser in the upper part and flows inside the inner tube, while cooling water enters the condenser in the lower part and flows upward within the annulus [12,26]. It is considered that the condensate forms a film on the cooled tube surface, while the condensing gas flows in the tube core (see Fig. 1).

Heat released from the condensing gas is transferred through the gas–water vapor boundary layer to the phase interface and is equal to the heat transferred through the condensate film and the wall to the cooling water. The total heat flux from the condensing gas \dot{q}_o corresponds to the sum of the latent and sensible heat flux.

$$\dot{q}_o = \dot{q}_{sen} + \dot{q}_{lat} = \alpha_{sen} \cdot (T_G - T_{F,SAT}) + \dot{m}_{con} h_v \quad (1)$$

The condensate film with thickness increases as the water vapor condenses. As condensation proceeds, NCG accumulates at the phase interface, resulting in a decrease in the partial pressure of the water vapor and its saturation temperature at the phase interface. As the temperature and the partial pressure of water vapor at the phase interface temperature are unknown, they must be determined iteratively from the heat balance at the gas-liquid interface. The heat transfer coefficient α is derived from the Nusselt number Nu which characterizes heat transfer in the condensation.

$$\alpha = \frac{Nu \cdot \lambda}{L} \quad (2)$$

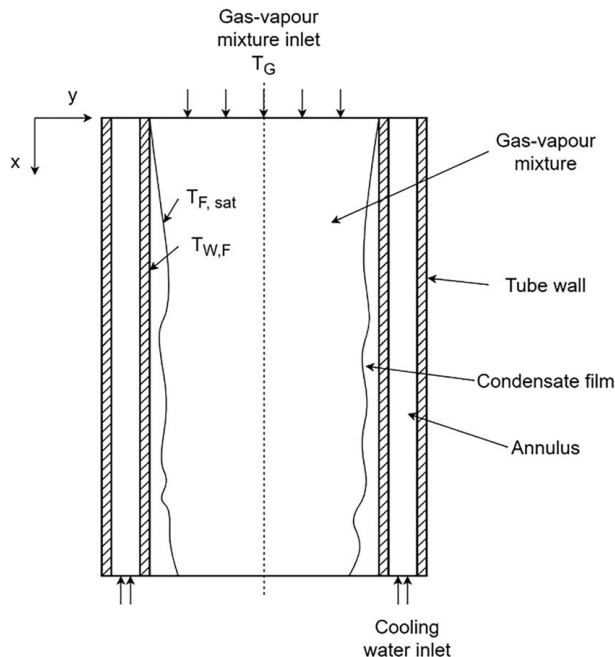


Fig. 1 Schema of the physical model

where λ is the thermal conductivity of the fluid and L is the characteristic length. The mass transfer coefficient β is derived from the Sherwood number Sh which characterizes the mass transfer of condensing water vapor through NCG.

$$\beta = \frac{Sh \cdot D}{L} \quad (3)$$

where D is the mass diffusivity and L is the characteristic length.

The flowchart diagram is shown in Fig. 2. In the solution procedure, it is necessary to ascertain the geometry of the condenser, the inlet mixture temperature, pressure, noncondensable fraction, inlet vapor flowrate, the coolant flowrate, and inlet temperature. An estimation of an exit coolant temperature $T_{K,out}$, an inner wall temperature $T_{W,F}$, and an interface noncondensable mass fraction W_F is the first step. The corresponding interface saturation pressure p_F and temperature $T_{F,sat}$ are obtained using the Gibbs–Dalton ideal gas mixture relation and the assumption that steam is at saturation conditions. After calculation of the friction correction factor and the interface shear stress, the condensate film thickness and the condensation HTC are determined. By balancing the mass transfer (the

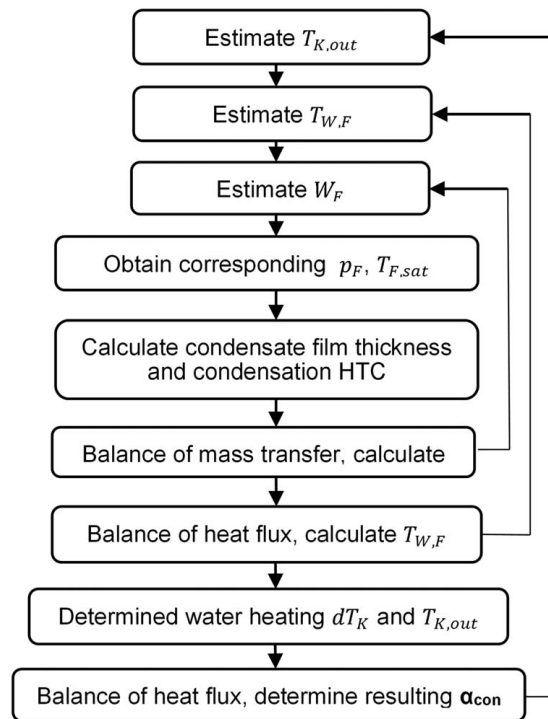


Fig. 2 A flowchart diagram of the calculation procedure for the heat and mass transfer analogy method

diffusion of water vapor through the inert gas toward the condensation surface), the interface mass fraction W_F can be obtained. This is then compared with the estimated value and iterated. By balancing the heat flux between the condensing gas and the cooling water, an improved inner wall temperature $T_{W,F}$ is obtained, and the value is then iterated. Additionally, the outside wall temperature and the mean cooling water temperature T_K are determined. An exit coolant temperature $T_{K,out}$ is derived from the cooling water heating dT_K , then the cycle is iterated. All system parameters are determined based on this balance, the primary result is the condensation HTC α_{con} , which is obtained from the heat flux balance and the temperature distribution.

This method provides results in good agreement with the experimental results, thereby offering an opportunity to study the physical process underlying the phenomenon. The primary disadvantage is the quite complicated calculation that includes the necessity of iteration. There are also some limitations due to the validity conditions of the equations used in the method, but they are relatively low compared to empirical correlations.

Diffusion Layer Theory. The diffusion layer theory, which was first presented by Peterson in 1993 [22], is also based on a heat and mass transfer analogy, however, it calculates differently few parameters and develops its own parameter such as the calculation of the condensate flowrate is based on the Fick's law and the modified Clausius–Clapeyron equation, which defines the effective condensation thermal conductivity k_{con} . The overall energy balance at the phase interface is then calculated with the adjusted equation.

$$\dot{q}_o = \alpha_{con}(T_G - T_{F,sat}) + \alpha_{sen}(T_G - T_{F,sat}) \quad (4)$$

In Eq. (1), the latent heat is calculated from the condensation flowrate which is based on the concentration gradient. Meanwhile, in Eq. (4), the condensation HTC is calculated using the condensation thermal conductivity which is based on Fick's law and the relationship between saturation pressures and saturation temperatures. Thus, the driving force of the latent heat is converted to the temperature difference which is used in Eq. (4). According to the diffusion layer theory, the condensation HTC α_{con} is then calculated using the analogy between heat and mass transfer according to the equation.

$$\alpha_{con} = \frac{Sh_G k_{con}}{L} \quad (5)$$

where

$$k_{con} = \frac{1}{T\Phi} \frac{D_G h_v^2 M_v^2 p_G}{RT_G^2} \quad (6)$$

with the gas–vapor logarithmic mean vapor concentration ratio Φ defined as

$$\Phi = \frac{Y_{G,avg}}{Y_{v,avg}} = - \frac{\ln\left(\frac{(1 - Y_{G,b})}{(1 - Y_{G,i})}\right)}{\ln\left(\frac{Y_{G,b}}{Y_{G,i}}\right)} \quad (7)$$

The rest of the theoretical method remains the same as in the heat and mass transfer analogy method presented in this work.

Experiments

Experiments were performed to evaluate selected theoretical methods. Air was chosen as NCG due to its ease of preparation in an exact ratio with water vapor, which allows to analyze a wider range of the operation parameters. Two series of experiments were conducted on the test rig: experiments with the condensation of pure water vapor to correctly determine the cooling side HTC and experiments with the condensation of an air–water vapor mixture.

Experimental Apparatus. A schematic diagram of the experimental apparatus is shown in Fig. 3. The system is an open loop comprised of three sections: the main test section; the water vapor, and the air supply section; cooling water section.

The main test section comprises a vertical double-pipe heat exchanger consisting of two concentric stainless-steel tubes. The mixture of water vapor and air enters the heat exchanger at the top and is directed vertically downward through a calming section before flowing into the inner vertical tube. The cooling water flows in an upward direction into the annulus. The heat exchanger is in counter-current configuration. The inner tube of the heat exchanger is 2000 mm long with an inner diameter of 25 mm and a wall thickness of 1.5 mm. The outer tube is 1500 mm long with an inner diameter of 36 mm and a wall thickness of 2 mm. The tube material is stainless steel 1.4301 (AISI 304). The annulus is made of two concentric tubes and is 4 mm wide. Stainless pins are used as spacers at three circumferential positions to keep the annulus concentric.

The water vapor is produced in a steam generator, which produces steam steadily at a nominal power of 20 kW. Air is extracted from the laboratory hall and blown to the mixing point by a compressor. The inlet air humidity was measured separately at the beginning of each test. The air volume flowrate is measured by a rotameter VA40/V/R (A.P.O.-ELMOS, Czech Republic) with current output calibrated with a deviation of 2.5%. The pressure of the air is measured by a U-tube water manometer. Tap water is used as a cooling medium in the condenser. Microfiber insulation and rubber foam are wrapped around the heat exchanger to prevent potential heat losses. Four T-type thermocouples were placed in condenser tapings measuring the inlet and outlet temperature of cooling water, and another two T-type thermocouples were installed to measure the inlet and outlet temperature of the water–vapor–air mixture. All thermocouples were calibrated with a deviation of $\pm 0.1\%$. The cooling water flowrate is measured using a FLONET FN2010.1 (Elis Pilsen Ltd., Czech Republic) with a current output calibrated with a deviation of 0.6%. A laboratory balance with 1 g is used to determine the amount of condensate. Datalogger DAS 240 BAT with the DASLAB software is used for data collection.

Experimental Procedure. The range of experiments was designed with regard to operating conditions typically encountered with flue gas condensers from biomass boilers. The mass flowrate of water vapor is set at the beginning, and the desired concentration of NCG in the mixture is controlled by modifying the air mass flowrate. The velocity of the condensing gas is determined by the control valve of the steam generator and the air inlet mass flowrate. The temperature and pressure of the supplied air are measured before entering the mixing section. The condensate is collected in a condensate tank for each specific measurement. Any excess condensing gas in the heat exchanger outlet is released into the surroundings. All experiments were carried out under atmospheric pressure.

Experiment Evaluation. From the experimentally determined overall HTC of the condenser, it is possible to calculate the HTC on the condensing side with knowledge of the cooling side HTC. To enhance the precision of the outcome, the experimental determination of the HTC was conducted for our specific heat exchanger. Consequently, the HTC of the cooling water side was ascertained through experimentation with pure water vapor, whereas the HTC of the condensing side was determined in accordance with the conventional condensation theory [30]. The condenser heat transfer rate \dot{Q}_{exp} was calculated as

$$\dot{Q}_{exp} = \dot{M}_K c_{p,K}(T_{K,out} - T_{K,in}) + \dot{Q}_{loss} \quad (8)$$

The overall HTC of the condenser k was calculated as

$$k = \frac{\dot{Q}_{exp}}{A T_{ln}} \quad (9)$$

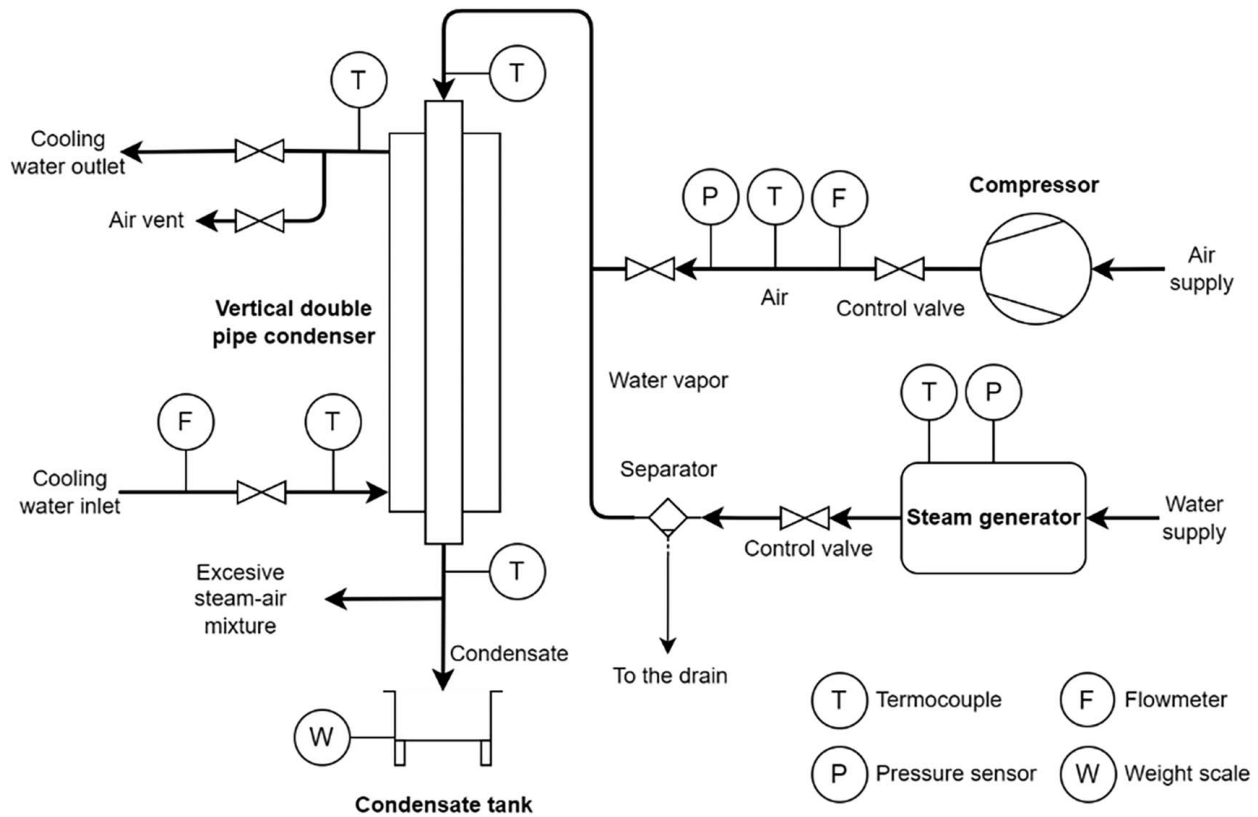


Fig. 3 Schematic diagram of the experimental setup

where A is the heat transfer area and the T_{ln} is logarithmic temperature difference

$$T_{ln} = \frac{(T_{G,in} - T_{K,out}) - (T_{G,out} - T_{K,in})}{\ln \frac{(T_{G,in} - T_{K,out})}{(T_{G,out} - T_{K,in})}} \quad (10)$$

The cooling side HTC α_K was determined from the equation

$$\frac{1}{k} = \frac{d_i}{d_o} \frac{1}{\alpha_K} + \frac{d_i \ln \frac{d_i}{d_o}}{2\lambda} + \frac{1}{\alpha_F} \quad (11)$$

where the condensation HTC for pure water vapor α_F was determined with respect to the film roughness and the shear stress of the water vapor according to the theory stated in the VDI Heat Atlas [30]. The condenser was thermally insulated and heat loss Q_{loss} was controlled by measuring the surface temperature of the condenser. It achieved a maximum of 0.2% of the heat performance in all measurements, so it has a negligible effect on the calculation.

The condensation HTC for experiments with the air–water vapor mixture was evaluated on the same basis as in pure water–vapor experiments. The condenser heat transfer rate was calculated according to Eq. (8), the overall HTC of the condenser according to Eq. (9) with Eq. (10). The condensation HTC α_{con} was determined based on Eq. (12) where the cooling HTC α_K for a given mass flow was used from the previous experiments and Eq. (11).

$$\frac{1}{k} = \frac{d_i}{d_o} \frac{1}{\alpha_K} + \frac{d_i \ln \frac{d_i}{d_o}}{\lambda} + \frac{1}{\alpha_{con}} \quad (12)$$

For the comparison of theoretical methods, the error range for each method was determined as follows:

$$\text{error} = \frac{(\text{calculated value} - \text{experimental value})}{\text{experimental value}} \cdot 100\% \quad (13)$$

Uncertainty Analysis. The uncertainty analysis of the experimental results was conducted according to the procedure described in Refs. [32] and [33]. The type B uncertainty of an individual measured quantity is determined as a maximal error of measurement source dividing a factor of the probability distribution which is $\sqrt{3}$ for the normal distribution. The uncertainties of measured quantities are transferred via relationships to the quantities calculated according to the rules for the evaluation of standard uncertainty through functional relationships with uncorrelated variables (Eqs. (8)–(12)) [33]. The expanded uncertainty is obtained by multiplying the final standard uncertainty by a coverage factor which is 1.96 for a 95% level of confidence.

The maximum expanded uncertainty based on a 95% confidence level was 12% for the condensation HTC. This assumes that the HTC of pure water vapor condensation can be determined with a maximum error of 15%, which has a low effect on the calculated HTC on the cooling side. The general uncertainty of the analysis was given primarily by the uncertainty of the measurement of air and water vapor flows and temperatures.

Results and Discussion

Experimental Results. Experiments with a total of 45 runs were carried out. The influence of the Reynolds number of the condensing gas and the inlet NCG concentration on the condensation HTC was evaluated. The range of the main parameters in the experiments is shown in Table 2. The inlet temperature of the condensing gas was kept at a saturation point ranging from 60.9 °C to 97.2 °C, so condensation begins right at the inlet of the condenser. The

Table 2 Overview of the scope of the experiments

Inlet bulk air concentration	11–86 vol%
Inlet condensing gas Reynolds number	1936–14,408
Inlet condensing gas velocity	1.7–11.8 m/s
Inlet cooling water temperature	11.8–15.1 °C

cooling water HTC was determined experimentally during the pure water vapor condensation tests. The range of the cooling water HTC was between 5477 W/m² K and 7675 W/m² K. The inlet water vapor mass flowrate in the condensing gas ranged from 1.1 kg/h to 6.7 kg/h.

The dependence of the condensation HTC on a bulk inlet air concentration is shown in Fig. 4. A decrease in the condensation HTC with an increase in the inlet NCG concentration is observed for all ranges of the Reynolds number of the condensing gas. As the Reynolds number of condensing gas increases, higher values of the condensation HTC are observed. The water vapor concentration in real flue gas from moist biomass might vary in the range of 10 vol% up to 45 vol%, depending on the combustion air excess used. A higher moisture content in the flue gas leads to an increase in the condensation HTC and therefore to a higher condenser heat output. When the Reynolds number of the condensing gas inlet was between 2112 and 3080, a decrease in condensation the HTC with higher air bulk inlet concentration is not so significant. The results of the experiments agree with the results of other experimental studies [26,31,34,35] on a vertical tube condenser. The condensation HTC decreases as the bulk inlet NCG concentration increases and increases as the Reynolds number of the condensing gas inlet increases.

Evaluation of the Theoretical Methods. The results obtained from the theoretical methods were compared with experimental results with the NCG concentration in condensing gas up to 45 vol%. Figure 5 shows a comparison of the condensation HTC determined from experiments and from the empirical correlations. It is evident that empirical correlations give a large variance of the results compared to the experiments. The results obtained from the Akaki's correlation (see Table 1) show a fair trend with the error range from -26% to 65% which is better than the results from other correlations. The Uchida's correlation underestimates the HTC condensation with the error range from -73% to 9%. Uchida's correlation does not consider the velocity or the

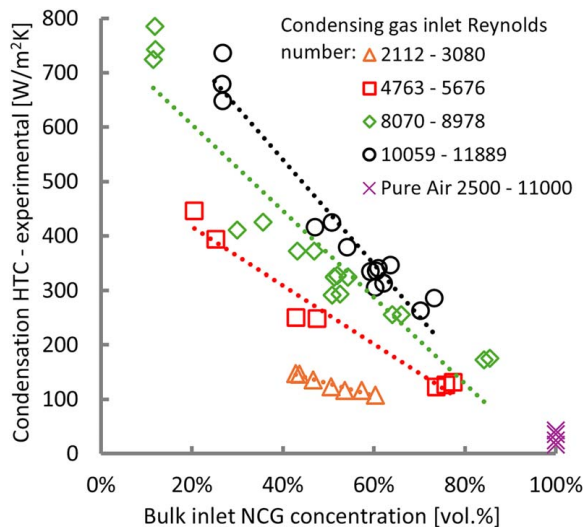


Fig. 4 Experimental dependence of the HTC condensation on the bulk inlet NCG concentration and the condensing gas inlet Reynolds number

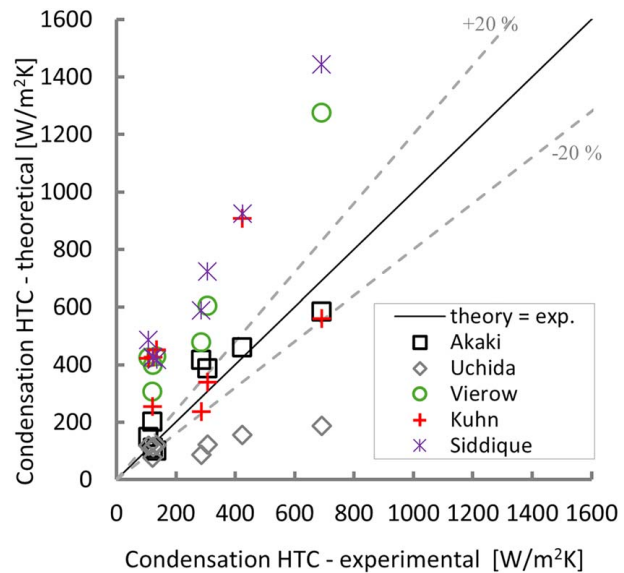


Fig. 5 Comparison of empirical correlations with the experimental results

Reynolds number of the condensing gas and other parameters in the empirical equation, so the deviation of the results might be caused by that. The equations developed by Siddique, Kuhn, and Vierow have a considerable wider error range from -17% to 311%. Most of the results obtained from the correlations tend to overestimate the condensation HTC, even though Kuhn's and Vierow's correlations take into account some of the most important physical parameters. The results from the experimental equations often do not fulfill the range of $\pm 20\%$ error between theory and experiment, which is generally considered as a satisfactory result and their error is higher and exceeds the acceptable range.

In Fig. 6, there are shown predicted results of the condensation HTC obtained from the heat and mass transfer model and the diffusion layer model. The results agree well with the experiment for both analytical models with a deviation of within $\pm 20\%$ for all selected runs. The heat and mass transfer analogy model overpredicts the condensation HTC for higher results of condensation HTC and underpredicts the condensation HTC slightly for low Reynolds numbers. In Ref. [22], a comparison of empirical equations

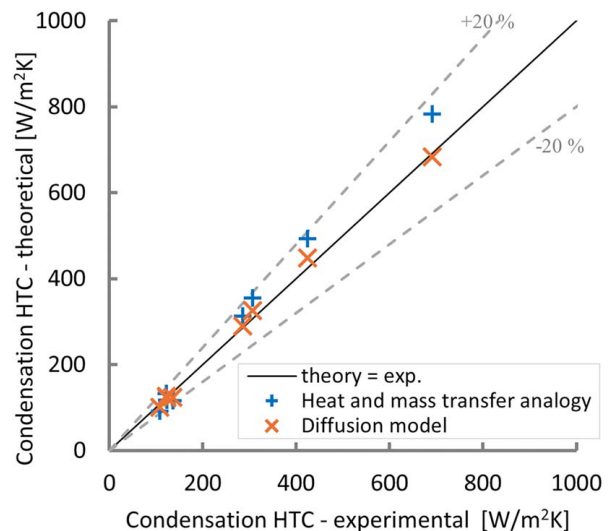


Fig. 6 Comparison of analytical methods with the experimental results

and the heat and mass transfer analogy was conducted. The results showed that the heat and mass transfer model is more accurate than empirical equations, which is also in agreement with the presented analysis. The results of the diffusion layer model are in close agreement with those of the experiments for all selected measurements. The model overestimates the condensation HTC, however, the error is relatively minimal.

A comparison of the results from the analytical models and experiments shows that including all the major parameters of the heat and mass transfer process leads to a robust theoretical model as well as a smaller error of the results. The utilization of the empirical correlations could produce an error in the results, even though the range of validity conditions of the correlations agrees with experiments performed, leading to a faulty design of the heat exchanger whose thermal rating would be significantly different from the design one. However, the physical nature of the phenomena is not taken into account using these equations, so further investigation is not possible, although it might be desirable.

A summary of the standard deviation (STD) between the theoretical and experimental results is shown in Table 3. The results predicted by the analytical methods are in significantly better agreement with the experimental results than all empirical correlations.

The STD between the results of the heat and mass transfer analogy and the diffusion layer method, and the results of the experiments is 12% and 5%, respectively. The STD of empirical correlations for the same experimental results is between 30% and 116%. The lowest STD of 30% is observed for the Akaki's and Uchida's equations. The STD of other empirical correlations from Vierow, Kuhn, and Siddique is notably high between 83% and 116%. Although these empirical correlations are considered relatively reliable in the literature, their usability is limited, e.g., for condensation of vapor in a high content of NCG. Vierow's equation was introduced at air concentrations between 0% and 14% and even though the correlations were adjusted for a higher NCG coefficients, it may be the reason of a deviation. The same applies for the Kuhn's and Siddique's correlation, which are stated based on the experiments conducted with NCG concentrations from 1–40% and 10–35%, respectively.

The results are in agreement with Ref. [27] where the degradation factor and diffusion layer methods were compared. The diffusion layer method achieved a lower STD of 8.4% compared to an STD of 17.6% for the empirical correlation. Similar results were obtained in Ref. [18] where the heat and mass transfer model was developed for a vertical tube. The relative error of the model was 18.7%. The STD of the empirical correlation presented in Ref. [36] was 51%, which also shows a lower degree of accuracy of the empirical equation for the condensation HTC. The high error of the results of the empirical equations was observed in the work of Lee et al. [37], wherein the differences between the HTC from the empirical correlation according to Dehbi [13] and the experiments were in the range of 28–101%. Furthermore, Lee et al.'s work conducted a comparison with the correlation of Uchida et al. [38] and for various inlet parameters it was observed that the correlation can lead to an error. This means that even though the results of the Uchida et al. correlation show a fair

Table 3 Overview of the STD for the analyzed theoretical methods

Theoretical method	Relative STD
Heat and mass transfer analogy	12%
Diffusion layer model	5%
Akaki correlation	30%
Uchida correlation	30%
Vierow correlation	88%
Kuhn correlation	116%
Siddique correlation	83%

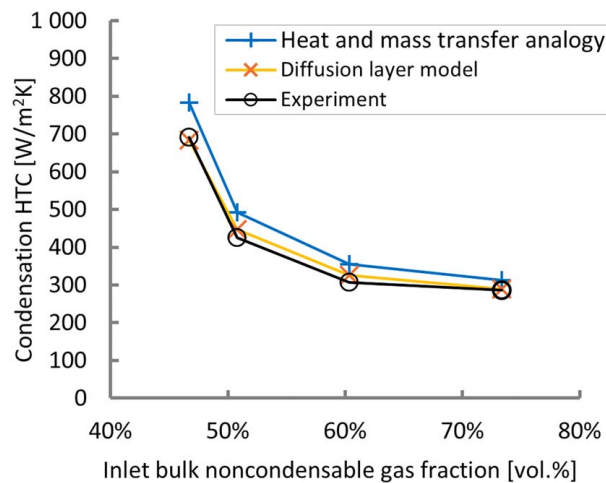


Fig. 7 Results for the condensation HTC and bulk inlet NCG concentration (Reynolds number between 10,167 and 14,408)

agreement with the experiments, this may not always be the case. One of the most significant works on empirical equations for the prediction of condensation HTC was made by Dehbi [13]. It was observed that the condensation HTC is strongly dependent on various inlet parameters such as the effect of the vapor fraction in the condensing gas, the effect of the total pressure of the condensing gas, the effect of wall temperature subcooling, and the effect of a constant NCG density. Therefore, when one of these crucial parameters is not included in the empirical equation or the validity of the empirical equation is not satisfied, a significant error can be obtained.

Figures 7 and 8 show the comparison of the condensation HTC predicted by the heat and mass transfer analogy model and the diffusion layer model, and the experimental condensation HTC of the condensing gas Reynolds number between 10,167 and 14,408 and below 5000. The results predicted by the analytical models are in good agreement with the experimental values for both extreme values of the Reynolds number for the NCG fraction and the condensing gas.

The local condensation HTC calculated from theoretical methods is shown in Fig. 9 for one specific measurement. The highest values of the condensation HTC are observed at the condenser inlet, although as the condensing gas flows downward, the local HTC decreases significantly. A low decrease in the condensation HTC is predicted from the equations of Akaki and Uchida. On the

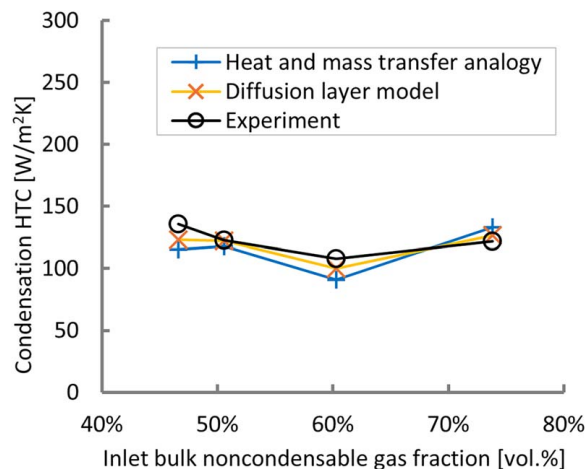


Fig. 8 Results for the condensation HTC and bulk inlet NCG concentration (Reynolds number below 5000)

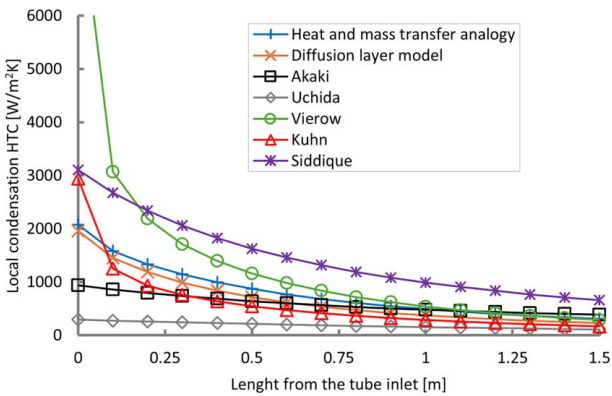


Fig. 9 Course of the local condensation HTC along the tube length

other hand, the equations proposed by Kuhn, Vierow, and Siddique give a steeper decrease in the condensation HTC along the tube length. Analytical models show a slight decrease in the condensation HTC. The decrease of the condensation HTC and heat transfer is caused by the lower water vapor content in the condensing gas and lower Reynolds number of the condensing gas, since the local Reynolds number of the condensing gas decreases as the vapor condenses. The local condensation HTCs are in good agreement with experimental and theoretical results of the works that also studied the local condensation HTC in the vertical tube [17,18,37]. A strong decrease of the condensation HTC is frequently observed at the condensing gas inlet, whereas as condensation proceeds, the condensation HTC does not vary as much. During the condensation of water vapors with high content of the NCG, the decrease of the condensation HTC at the tube inlet may not be so strong, given that the heat transfer is already significantly reduced by the high NCG content. This conclusion also supports the fact that the condensing gas does not vary so much along the tube length since only a small local condensation rate occurs.

Conclusions

The calculation methods for flue gas condensers in biomass boilers are not yet sufficiently developed. While general calculation methods for condensers are available, they were originally designed for other applications. This work investigates the theoretical methods applicable for the calculation of the heat transfer for water vapor condensation with a high content of NCG corresponding with conditions during the condensation of flue gas from a biomass boiler in a vertical tube heat exchanger. Four available methods were identified (empirical correlations, heat and mass transfer analogy, diffusion layer theory, boundary layer theory). The first three of them were evaluated as applicable to the practical design of heat exchangers for the utilization of waste heat from the flue gas of biomass combustion. Boundary layer theory was evaluated as very complex and impractical for the calculation of heat exchangers.

Experiments were conducted to investigate the condensation of water vapor in the presence of NCG in the vertical double-pipe heat exchanger. The experiments were performed with various air inlet concentrations ranging from 11 vol% to 86 vol% and Reynolds numbers of the condensing gas ranging from 1936 to 14,408 with respect to the real conditions observed in the relevant applications.

The five different empirical correlations proposed by various authors to predict the condensation HTC were analyzed. Although the condensation of water vapor in the presence on NCG is a long-studied topic, empirical correlations with the validity of the operating conditions of flue gas condensers are scarce. The results of the empirical correlations exhibited only minimal concordance with the

experimental results. The highest level of agreement was found from the Akaki's and Uchida's correlation with STD of 30% and the lowest level of agreement was observed from Kuhn's correlation with STD of 116%. Therefore, while the empirical correlation for the condensation HTC may be useful for an approximate design of condensers, its error might be quite high. This is probably a consequence of the limited inclusion of the physical nature of the phenomena, as the correlations do not consider the influence of some important parameters of the heat and mass transfer process.

Analytical methods (the mass transfer analogy and the diffusion layer theory) are more complex than empirical correlation. However, they are more reliable and facilitate the modeling of the physical nature of the problem. The predicted results from these methods agree well with the experimental results. The STD for the heat and mass transfer analogy and for the diffusion layer theory is 12% and 5%, respectively. A small error of the analytical models is probably due to the involvement of all important parameters of the process. The range of the validity of these calculation models is quite wide, hereby enabling their use in the investigation of relatively unexplored topics. Therefore, analytical models are more suitable for the prediction of the condensation HTC and can be recommended for the design of condensers.

A decrease in the condensation HTC is observed as the concentration of water vapor in the condensing gas decreases. An increase in the Reynolds number of condensing gas has a positive effect on the condensation HTC, and it is higher when the Reynolds number of the condensing gas is high. All theoretical methods show a stable consistent decrease in the HTC condensation along the tube length with a strong decrease at the condenser inlet. The differences in the decrease of condensation HTC are considerable. At the same time, the results obtained from the analytical models correspond to the average values of the results obtained from empirical correlations.

These results can be used in the search for a suitable design procedure for flue gas condensers, for their more efficient or accurate realization. Further research should focus on experiments with real flue gas from a biomass boiler and analysis of the composition of the flue gas condensate and its effect on heat transfer.

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Conflict of Interest

There are no conflicts of interest.

Data Availability Statement

The datasets generated and supporting the findings of this article are obtainable from the corresponding author upon reasonable request.

Nomenclature

p	= pressure, Pa
A	= heat exchanger area, m^2
D	= diffusion coefficient, m^2/s
L	= characteristic length, m
M	= molar weight, g/mol
R	= molar gas constant, 8.31451 J/K/mol
T	= temperature, K
Y	= molar fraction
W	= mass fraction
\dot{m}	= local mass flux, kg/m^2s
\dot{q}	= heat flux, W/m^2
\dot{M}	= mass flowrate, kg/s

\dot{Q} = heat transfer rate, W
 c_p = specific heat capacity, J/kgK
 h_v = latent heat of condensation, kJ/kg
 k_{con} = effective condensation thermal conductivity, W/m²K
 d_i = tube inner diameter, m
 d_o = tube outer diameter, m
 HTC = heat transfer coefficient, W/m²K
 Ja = Jacob number
 Nu = Nusselt number
 Re = Reynolds number
 Sh = Sherwood number

Greek Symbols

α = heat transfer coefficient, W/m²K
 β = mass transfer coefficient, m/s
 λ = thermal conductivity, W/mK

Subscripts

a = air
 b = bulk
 i = interphase
 o = overall
 t = turbulent
 v = vapor
 F = film
 G = condensing gas
 K = cooling water
 W = wall
 con = condensation
 exp = experimental
 in = inlet
 lat = latent
 out = outlet
 sen = sensible
 theor = theoretical
 F,SAT = film saturation
 SAT = saturation

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