Sweeping Jet Film Cooling at High Blowing Ratio on a Turbine Vane

Experimental and numerical investigations were performed to study the effects of high blowing ratios and high freestream turbulence on sweeping jet film cooling. Experiments were conducted on a nozzle guide vane suction surface in a low-speed linear cascade at a range of blowing ratios of 0.5–3.5 and freestream turbulence of 0.6% and 14.3%. Infrared thermography was used to estimate the adiabatic cooling effectiveness. Thermal field and boundary layer measurements were conducted at a cross-plane at x/D = 12 downstream of the hole exit. Results were compared with a baseline 777-shaped hole and showed that the sweeping jet hole has improved cooling effectiveness at high blowing ratios (M > 1). The thermal field data revealed that the coolant separates from the surface at high blowing ratios for the 777-shaped hole while the coolant remains attached for the sweeping jet hole. Boundary layer measurements further confirmed that due to the sweeping motion of the jet, the effective jet momentum of the sweeping jet hole remains much lower than that of a 777-shaped hole. Thus, the coolant remains closer to the wall even at high blowing ratios. Large Eddy simulations (LES) were performed for both the sweeping jet and 777-shaped hole to evaluate the interaction between the coolant jet and the freestream in the near hole regions. Results showed that 777-shaped hole has a strong jetting at high blowing ratio that originates inside the hole breakout edges thus causing the jet to blow-off from the surface. In contrast, the sweeping jet does not show this behavior due to its internal geometry and flapping motion of the jet. [DOI: 10.1115/1.4047396]

Keywords: sweeping jet, suction surface boundary layer, large eddy simulation (LES), heat transfer and film cooling

Introduction

Film cooling is commonly used to protect the components in the hot gas flow path where the gas temperature remains well above the material melting point and thus increases the life of the modern gas turbine engines. However, the use of compressor bleed air adds an additional penalty to the overall efficiency of the engine. Therefore, engine designers need to use the coolant efficiently. In an effort to improve the maximum use of the coolant, conventional holes (cylindrical) have been replaced by diffused shaped holes [1] that slow down the coolant before exiting the hole and promote jet attachment (prevent jet liftoff) to improve cooling coverage. Numerous shaped hole designs have been studied extensively to quantify the effect of coolant mass flow rate or blowing ratio (M) [2], coolant-to-freestream density ratio [3,4], freestream turbulence intensity [5,6], and length scales of the approach flow [7]. Additional shaped hole designs have shown improved lateral spreading and improved cooling effectiveness over the conventional cylindrical hole. Among them the laidback fan-shaped hole [8], bean-shaped hole [9], console hole [10], and anti-vortex hole [11] are notable. Most published novel-shaped hole performances were compared against the performance of the cylindrical hole which is not much of a challenge to surpass. Schroeder and Thole [12] developed a baseline-shaped hole commonly known as the 777-shaped hole in an effort to provide baseline flowfield and heat transfer data for evaluating novel shaped hole geometries. A number of studies have been published since then detailing the flow field and heat transfer results [13–18] for the 777-shaped hole.

Previous Work on Sweeping Jet Film Cooling

One of the major challenges associated with the film cooling hole is the detrimental counter-rotating vortex pair (CRVP) that assists the coolant jet liftoff and has a deteriorating effect on film cooling performance. The shaped holes have shown improved cooling coverage due to the reduced strength of the CRVP which is attributed to the formation of unsteady vortices at the exit break out the edge. However, the CRVP still exists for the shaped hole that can regain enough strength to liftoff the coolant jet at high blowing ratio (M = 3) [13]. The sweeping jet (SJ) creates alternating streamwise vortices which interact with the freestream flow to eliminate the CRVP completely [19] that can be created by a unique device devoid of any moving parts called a fluidic oscillator. A common feedback type fluidic actuator consists of an inlet nozzle, a mixing chamber, two feedback channels, and an exit nozzle. The working principle of such a device is explained in detail in Ref. [20]. With the development of additive manufacturing techniques, it is now possible to design such a device for film cooling applications. Thurman et al. [21] first introduced the SJ hole for film cooling application on a flat plate and demonstrated improved film effectiveness in the spanwise direction. Later, a detailed experimental and numerical study was performed by Hossain et al. [22] for sweeping jet film cooling on a flat plate at several freestream turbulence and a range of blowing ratios. Adiabatic cooling effectiveness, convective heat transfer coefficient (HTC), cross-plane temperature field, and the coefficient of discharge were measured and compared with the baseline-shaped hole (777 hole). The flat plate results showed that the SJ hole has higher near hole cooling effectiveness in the lateral direction. This high effectiveness is attributed to the sweeping motion of the jet. The thermal field showed that the coolant distribution for the SJ hole is much more uniformly distributed. Time-resolved flow field data showed that the sweeping motion of the jet creates two alternating streamwise vortices that never exist simultaneously at the same streamwise...
location; thus, they do not interfere with each other. The sense of rotation of these vortices is also opposite to the conventional CRVP. The flat plate results were used to design a much more realistic turbine vane geometry that consists of a row of SJ holes on the suction side of the vane. Experiments were performed in a low-speed linear cascade for a range of blowing ratios (0.5 \( \leq M \leq 1.5 \)) and freestream turbulence (\( T_u = 0.3\% \) and 6.3\%) and compared with the baseline 777 hole. The SJ hole showed higher cooling effectiveness for \( M > 1 \). However, a detailed study of the near hole velocity distribution and thermal field measurements are needed to further understand the effect of the high blowing ratio for the SJ hole.

**Contribution of the Present Work.** The present study reports film effectiveness, thermal field, and boundary layer measurement for SJ hole at high blowing ratio up to \( M = 3.75 \) and high freestream turbulence (\( T_u = 14.3\% \)). Also, large Eddy simulations (LES) are performed for both SJ and 777-shaped hole in order to provide a comprehensive understanding of the flow field near the hole exit that will guide turbine designers to optimize an efficient additively manufactured suction surface film cooling hole design.

**Experimental Setup**

Experiments were performed in a low-speed linear cascade at the Ohio State University Turbine Aerothermodynamics Lab. Figure 1 shows the linear cascade that is an extended section of an existing open-loop wind tunnel. The details of the wind tunnel and the cascade section have been previously reported by Hossain et al. \[22,23\]. Some of the major features are described here briefly. The tunnel is capable of providing a hot flow that can be heated up to 325 K by an inline duct heater. The hot mainstream flow enters into a square section (38 cm \( \times \) 38 cm) followed by a 60 cm circular flow conditioning section. The hot flow then enters into the cascade section followed by a 1.88:1 foam contraction section and a rectangular (38 cm \( \times \) 20 cm) transition section. The cascade test section consists of three nozzle guide vanes with two full passages. The turning section is attached to two adjustable tailboards to ensure periodic flow over the central vane geometry. The tailboards are adjusted until the pressure distribution is matched with an infinite cascade predicted by computational fluid dynamics (CFD) which is shown in Fig. 2. The figure also shows the definition of \( C_p \) and the region where the film cooling measurement was performed by a light-shaded region. One of the tailboards has a 75 mm diameter infrared (IR) viewport where a 5 mm thick Germanium window was used to view the suction surface of the central vane. The upstream flow velocity (\( U_{\infty} \)) and temperature (\( T_{\infty} \)) were measured by a pitot-static probe and a 0.5 mm bead diameter J-type thermocouple, respectively. Both measurements were performed at unity chord (1C) upstream of the vane leading edge.

Two turbulence grids were used to generate freestream turbulence. The locations of the turbulence grids are shown in Fig. 1 where “Turbulence grid 1” (six 25.4 mm square bars) was placed at 11.5C (175 cm) upstream and “Turbulence grid 2” (five 12.5 mm circular rods) was placed at 2C (30.5 cm) upstream of the vane leading edge. The turbulence intensity and length scale were characterized by a constant temperature anemometry (CTA) hotwire at 1C (15.25 cm) upstream of the vane leading edge. The measured turbulence level without the grids was 0.6\% and with the grids was 14.3\% for an average freestream velocity of 9.5 m/s. In the present study, both turbulence grids were used together to ensure isotropic turbulence and thermally uniform flow upstream of the vane. Table 1 shows the details of the upstream turbulence characteristics and spanwise thermal uniformity for each grid configuration.

**Turbine Vane and Hole Geometry.** The nozzle guide vane (OSU vane) geometry studied in this paper was designed at the OSU Turbine Aerothermodynamics Lab. Some of the geometric parameters of the turbine vane are listed in Table 2. The test vanes were manufactured with high resolution stereolithography (commonly known as SLA) with Accura ABS Black (\( k = 0.175 \text{ W/m}^2\text{K} \)) material. The entire vane geometry has two separate modules that include the leading edge module and the trailing edge module. The leading edge module houses the film cooling holes and the coolant supply plenum, and the trailing edge module is attached to the leading edge module through several dowel pins to ensure an aerodynamically smooth surface. Figure 3 shows the schematic of the OSU vane and the location of the film holes. A single row of SJ film cooling holes were used at the suction surface at about 50\% axial chord downstream from the leading edge. The hydraulic diameter (\( D \)) of the metering section of each SJ hole is 2.56 mm with an exit fan angle of 70 deg. The hole pitch (\( P \)) was kept at 6D with an injection angle of 45 deg. Detailed geometric parameters of the sweeping jet hole have been described in Refs. \[22,23\].

A separate nozzle guide vane was manufactured with a single row of the 777-shaped hole for a direct comparison. The 777-shaped hole has a cylindrical metering section with an expansion angle of 7 deg in the forward and lateral directions. Additional details of the geometry of the 777-shaped hole can be found in Ref. \[12\]. The hydraulic diameter of the metering section for both the SJ hole and 777 hole was kept constant (\( D = 2.56 \text{ mm} \)). However, the exit breakout section of the 777-shaped hole has been slightly modified due to the curvature of the vane geometry.

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**Fig. 1** Schematic of the OSU low-speed linear cascade for film cooling experiment

**Fig. 2** The coefficient of static pressure (\( C_p \)) distribution at the midspan of the OSU vane \[23\]
Experimental Measurement

Cooling Effectiveness Measurement. Film effectiveness measurement was performed using IR thermography. The temperature of the suction surface of the vane geometry was measured by a 320×240-pixel resolution FLIR A325sc infrared camera. The camera has an accuracy of ±2% of the measured temperature. The material (Accura ABS black) of the vane was inherently black. Therefore, black paint was not necessary, and the surface emissivity was considered as 0.98. The camera was focused on a 90 mm (spanwise) × 75 mm (streamwise) field of view of the suction surface of the vane geometry through a 5-mm thick Germanium window at an angle of 25 deg which is shown in Fig. 4 by a light-shaded region. Each test starts with setting the freestream flow velocity and temperature at 9.5 m/s and 315 K, respectively. The coolant temperature was maintained around 300 K for each test. Once the surface temperature reaches a steady-state, a set of 100 images are recorded at a frequency of 10 Hz. The images are averaged to get the mean wall temperature (Tw). The freestream temperature (T∞) and the coolant temperature (Tc) are also recorded simultaneously to estimate the adiabatic cooling effectiveness by Eq. (1)

\[
\eta = \frac{T_{\infty} - T_w}{T_{\infty} - T_c}
\] (1)

Velocity and Thermal Field Measurement. The velocity field was measured to characterize the suction surface boundary layer in the centerline (x–y) plane at x/D = 12. CTA measurements were conducted using a single hot film probe. A TSI Model 1127 air calibrator was used to calibrate the hot film with proper reference velocity ranges from 2 to 70 m/s. The upper limit of the velocity range was chosen at a much higher value than the mean freestream velocity so that a single calibration of the probe could be used with the film cooling jet at high blowing ratios. The hot film probe was mounted in a two-axis traverse that can be moved along the spanwise (y–z plane) and wall-normal (x–z plane) direction through one of the side walls of the cascade. Figure 4 shows the boundary layer measurement plane by a dark-shaded region normal to the suction surface. The boundary layer measurement started with moving the hot film probe close to the wall until the measured velocity reached half of the mean freestream velocity of the passage. The probe was then moved in the wall-normal direction with gradually increasing steps starting from 0.05 mm near the wall to 1 mm near the freestream. Velocity data were sampled at a frequency of 20 kHz with a 10 kHz low pass filter for 5 sec at each location, with over 30 points to estimate the mean (\(\overline{u}\)) and root-mean-square (\(u'_{rms}\)) velocity.

The time-averaged thermal field (\(\theta\)) was measured in a spanwise cross-plane (y–z plane) at x/D = 12 which is shown in Fig. 4 by a dark-shaded region. A specially designed 0.05 mm bead diameter T-type thermocouple probe was used for the measurement. The temperature sensor was then connected to a 0.12-mm T-type thermocouple wire and glued to a 1-mm diameter nonconductive wire. The conduction error through the thermocouple wire was corrected by the method described in Ref. [13]. The temperature measurements were performed in a two-dimensional grid extending 16 mm (6.25D) in the wall-normal direction and 20 mm (±7.75D) in the spanwise direction from the hole centerline. Temperature data (\(T\)) were sampled at a frequency of 14 Hz for 10 s at each location of the measurement grid. The coolant temperature (\(T_c\)) and the freestream temperature (\(T_{\infty}\)) were measured simultaneously with two 0.5 mm bead diameter thermocouples (J-type). The thermal field (\(\theta\)) was then estimated by Eq. (2)

\[
\theta = \frac{T_{\infty} - T}{T_{\infty} - T_c}
\] (2)
Test Conditions and Uncertainty Estimation. A set of experiments were performed to evaluate the adiabatic film effectiveness ($\eta$), thermal field ($\theta$), and boundary layer of the sweeping jet film cooling hole at high blowing ratios where the blowing ratio ($M$) is defined in Eq. (3)

$$M = \frac{\dot{m}_c}{A_iU_{local}p_\infty}$$ (3)

Here, $U_{local}$ is the local velocity at the location of the hole exit which was estimated using Eq. (4) and the $C_p$ distribution (shown in Fig. 2) of the vane.

$$U_{local} = U_\infty \sqrt{1 - C_p}$$ (4)

Experiments were also performed for a baseline-shaped hole (777-shaped hole) at two freestream turbulence levels ($Tu$) for a direct comparison. A summary of all the tests is given in Table 3.

The freestream flow velocity was measured by a 745 Pa differential pressure transducer and a pitot-static probe. The accuracy of the pressure transducer was $\pm 1.6\%$. The coolant and the freestream temperature were measured by two J-type (0.5 mm bead diameter) thermocouples with an accuracy of $\pm 0.5\ K$. The accuracy of surface temperature measurement of the IR camera was $\pm 0.5\ K$ based on manufacturer specification. The coolant mass flow was measured by an Alicat (Model FMA-2600) mass flow controller with an accuracy of $\pm 1\%$. The experimental uncertainty was estimated using the method described by Coleman and Steele [24]. The overall uncertainty for the blowing ratio ($M$), cooling effectiveness ($\eta$), thermal field ($\theta$), and streamwise velocity ($u$) are listed in Table 4. Note that the estimated uncertainty reported here is for a 95% confidence interval.

Experimental Results and Discussion

Conduction Correction. The wall thickness of the current vane geometry is 5 mm with a material (Accura ABS Black) conductivity of $k = 0.175\ W/mK$. Thus, the coolant picks up heat as it passes through the coolant supply plenum which affects the final effectiveness estimation. Therefore, a conduction correction was performed by blocking the central three holes, and the coolant was allowed to flow through the remaining holes. Note that the coolant supply plenum is located upstream of the film cooling hole and blocking the central three holes do not significantly alter the backside conduction at the region of interest. The surface temperature was recorded by the IR camera as well as the coolant and the freestream temperature to estimate the cooling effectiveness ($\eta$) using Eq. (1). It is expected that the cooling effectiveness would be zero downstream of the blocked three central holes since no coolant is flowing through these holes. Therefore, any positive value of $\eta$ would indicate the effect of conduction that must be subtracted from the initial effectiveness estimation. Figure 5 shows the initial and corrected span-averaged effectiveness distribution at $M = 1$. The effectiveness contours before and after the corrections are also shown in Fig. 5. It is evident that most of the conduction happens in the near hole region ($0 \le x/D \le 5$) as the coolant supply plenum does not cover all the space underneath the vane surface. A correction coefficient was estimated for both SJ and 777-shaped hole for each blowing ratio and subtracted from the initial effectiveness value.

Adiabatic Film Effectiveness. Figure 6 shows the contours of cooling effectiveness for the SJ and 777-shaped hole at the blowing ratio of $M = 1$ and 3. Results are shown for low freestream turbulence ($Tu = 0.6\%$). The coolant spreading, jet footprint, and shape are very similar for each hole type. This also implies that each hole is receiving a similar amount of coolant that results in excellent periodicity for both hole types (SJ and 777). Figure 6 also shows that the effect of the blowing ratio is significant. At $M = 1$, the cooling effectiveness is very pronounced for both SJ and 777 holes. Several distinguishing features can be identified at this blowing ratio. Due to the sweeping nature, the spreading of the coolant jet for the SJ hole is larger than the 777-shaped hole. The sweeping also helps to improve the near hole effectiveness for the SJ hole. This behavior was also observed previously in flat plate studies performed by Thurman et al. [21] and Hossain et al. [22]. It is important to note that the centerline effectiveness for the 777 hole is higher at this blowing ratio. The majority of the coolant remains in a concentrated stream along the centerline causing a lack of cooling in the region between two adjacent holes. This causes a nonuniform wall temperature distribution in the spanwise direction that ultimately leads to the development of thermal stress. In contrast, the SJ hole shows uniform coverage and higher effectiveness in the lateral direction and regions between two adjacent holes. At $M = 3$, the effectiveness pattern changes dramatically. A significant drop in film effectiveness can be seen for both holes. The centerline effectiveness for the 777-shaped hole shows a clear indication of jet blow-off with very little trace of coolant far downstream of the hole. The SJ hole also shows a significant drop in cooling effectiveness. However, the lateral spreading of coolant is much wider and uniformly distributed in the near hole region ($0 \le x/D \le 10$). In addition, no significant jet liftoff can be seen for the SJ hole. The drop in effectiveness is rather attributed to the enhanced mixing due to the sweeping motion of the jet. The effect of high freestream turbulence ($Tu = 14.3\%$) on film effectiveness is shown in Fig. 7. Film effectiveness drops due to enhanced mixing caused by additional turbulence in the freestream.

![Fig. 5 Effect of conduction on laterally averaged film effectiveness for the sweeping jet hole at $M = 1.0$](image_url)
The coolant distribution for both holes is very similar to the low turbulence case. However, the jet spreading of the 777-shaped hole increases slightly with increasing turbulence at $M = 3$. Similar behavior of higher coolant spreading has been observed and reported by other investigators [14]. In contrast, the jet spreading remains nearly unchanged for the SJ hole at elevated turbulence levels. However, the lateral film effectiveness drops due to the combined effect of the sweeping motion and enhanced mixing. At $M = 3$, the coolant jet lifts off from the surface with very little coolant coverage on the surface for the 777-shaped hole. However, the jet spreading for the SJ hole at this blowing ratio remains much wider and uniformly distributed. This happens due to a relatively low effective jet momentum (fluctuating jet momentum attributed by the sweeping motion) that is being pushed toward the wall immediately after the jet interacts with the freestream. Numerical simulations were performed to further our understanding of the underlying physics of the coolant spreading which is presented in the later sections of the paper.

The span-averaged effectiveness ($\overline{\eta}$) for the SJ and 777-shaped hole is shown in Fig. 8. Results are presented for both low and high freestream turbulence (shown in open and close marker) and for blowing ratio of $M = 1$ and 3 (shown in separate marker). Data were averaged over 20D in the streamwise direction and 18D (covering three holes) in the spanwise direction. At $M = 1$, the span-averaged effectiveness is higher in the near hole region ($0 \leq x/D \leq 10$) for the SJ hole when compared with the 777 hole. This is due to the sweeping motion of the jet that helps the coolant to spread more in the lateral direction thus improving the near hole cooling effectiveness. However, the film effectiveness drops downstream of the hole due to the unsteady nature of the jet that enhances local mixing. The effect of turbulence at this blowing ratio ($M = 1$) is much more severe for the SJ hole than the 777 hole. At high freestream turbulence ($Tu = 14.3\%$), the effectiveness value drops significantly for SJ hole. The difference between the laterally averaged effectiveness for high turbulence ($Tu = 14.3\%$) and its low turbulence ($Tu = 0.6\%$) counterpart (close and open square marker) increases with $x/D$, which implies that the coolant jet is mixing rapidly with the freestream due to elevated turbulence as the jet travels far downstream. In contrast, the 777-shaped hole is reluctant to show this behavior at this low blowing ratio ($M = 1$). The effectiveness value seems to drop slowly with $x/D$ at high turbulence which indicates that the freestream ingestion for the 777 hole at low blowing ratio is not significant. In fact, the span-averaged effectiveness for the 777-shaped hole is higher than the sweeping jet hole at high freestream turbulence which implies that the unsteady sweeping jet (indicated with square marker) is much more sensitive to freestream turbulence at low blowing ratio ($M = 1$) when compared with the steady (indicated with diamond marker) shaped hole.

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lt off and enhanced mixing which is consistent with the observations available in the literature for steady jet [3].

It is important to note that some researchers have reported [5,14] higher effectiveness due to an increased spanwise spreading of coolant at high turbulence which contradicts the result presented in this paper. One of the major differences is that all these studies were performed on flat plates where the turbulence decay (from the measurement location to the hole exit) is much slower. However, the current study was performed on the suction surface of a nozzle guide vane geometry with a relatively fat (large diameter) leading edge. This large curvature of the vane accelerates the freestream flow significantly. Thus, the freestream turbulence decays at a faster rate resulting in a significant drop in velocity fluctuation in the freestream before it reaches the hole exit. Additional velocity measurement in the passage confirmed this claim, which is described in a later section of the paper.

The area-averaged cooling effectiveness ($\bar{\eta}$) is shown in Fig. 9. Results are presented for both SJ and 777-shaped hole (shown in different marker) at a range of blowing ratios ($0.5 \leq M \leq 3.5$) and for low and high freestream turbulence (shown in open and filled marker). Effectiveness data were averaged over three hole pitches (18D) covering a streamwise distance between $2 \leq x/D \leq 20$. Note that the error bars are shown for two cases since the uncertainty of the other cases is very similar. Both holes show expected increasing trend of $\bar{\eta}$ with increasing blowing ratio up to $M = 1$. The difference between $\bar{\eta}$ for SJ and the 777-shaped hole at this range ($M \leq 1$) is not noteworthy and remains well within the experimental uncertainty. However, SJ hole shows a significantly higher value of $\bar{\eta}$ at high blowing ratios ($1 < M \leq 3.5$). More specifically, SJ hole shows as much as 60% higher $\bar{\eta}$ value at $M = 2$ and 56% higher $\bar{\eta}$ value at $M = 3$ when compared with the 777-shaped hole. The additional cooling in the lateral direction at high blowing ratio (shown in Figs. 6 and 7) is responsible for this higher value of $\bar{\eta}$ for the SJ hole while the steady-shaped hole (777-hole) failed to provide sufficient lateral cooling due to jet liftoff at high blowing ratios. This also provides important information about the operating regime of the sweeping jet hole to achieve an improved and more uniform coolant coverage.

**Velocity and Thermal Field.** The measurements of velocity and thermal fields in a cross-plane ($y-z$ plane) at the downstream of the hole exit provide additional information to better understand the coolant distribution for the SJ and 777-shaped hole specifically at high blowing ratio. Data were acquired for the high (14.3%) turbulence case only. Figure 10 shows the non-dimensional thermal field ($\theta$) in a cross-plane at $x/D = 12$ for the SJ hole (left column) and 777 hole (right column). Each contour extends 6D in the lateral direction covering a full hole pitch and 4D in the wall-normal direction. At $M = 1$, the coolant remains attached to the vane surface for both holes. The coolant penetration height remains almost the same (approximately 1.25D) for both holes. However, the lateral spreading is much wider (across the entire pitch) for SJ hole but the lateral spreading of coolant is approximately ±2D for 777 hole. This spreading agrees quite well with the film effectiveness contour shown in Fig. 7 at the corresponding $x/D$ location.

At $M = 3$, the thermal field for the SJ and 777-shaped hole is quite different. The lateral spreading for SJ hole is much wider and covers the entire hole pitch ($\pm 3D$). The contour lines bend outward which is attributed to the sweeping motion of the jet. Similar behavior was observed for the large scale SJ hole in a flat plate study [22]. The $\theta$ value at the core region drops with an increase in blowing ratio. This happens because the vigorous sweeping motion at high blowing ratio ($M = 3$) enhances local entrainment of hot air that warms up the jet core. In contrast, the lateral spreading of the coolant drops significantly for the 777-hole. The contour lines bend inward near the wall. This happens due to the formation of a strong CRVP that brings hot freestream gas toward the core of the jet. This behavior was also observed and reported by other researchers [12,15,16] for the 777-shaped hole. The CRVP also contributes to the jet liftoff that prevents the coolant from spreading at a high blowing ratio ($M = 3$). This results in a lack of cooling in the spanwise direction. By contrast, the SJ hole provides full coverage even at a high blowing ratio which proves the benefit of the sweeping jet hole over the shaped hole.

The time-mean streamwise velocity profiles at $x/D = 12$ are shown in Fig. 11 for a range of blowing ratios ($1 \leq M \leq 4$). Velocity data were taken in a centerline plane for both holes with a hot film probe at a streamwise distance of $x/D = 12$ from the hole exit. The measurement location was chosen because this is the closest distance that can be accessed by an intrusive measurement such as hot film in the current experimental facility. For $M = 1$, the velocity profile for both holes (SJ and 777) monotonically increases with wall-normal distance and matches the freestream at about one hole diameter ($y/D = 1$) above the wall. The thermal field in Fig. 10 also shows that the coolant penetration is approximately the same height ($y/D = 1$) for both holes. The agreement between the thermal field and the velocity profile gives additional confidence to different measurement techniques used in this experiment. As the blowing ratio increases, the velocity profile changes dramatically, and the differences are very pronounced for both holes. The mean velocity profile shows a peak ($>U_{\infty}$) at high blowing ratios ($M \geq 3$).
due to a high jet (coolant) velocity. The location of this peak is different for the SJ and 777 hole. At \( M = 3 \), the peak exists somewhere between \( y/D = 0.25 - 0.75 \). This indicates that the coolant jet remains much closer to the wall for the SJ hole compared with the 777-shaped hole. In addition, the velocity smoothly merges to the freestream velocity at about \( y/D = 3 \) for both holes. The thermal field data (Fig. 10) also shows a similar jet penetration height for both SJ hole and 777 hole. At \( M = 4 \), the peak velocity location for the SJ hole also forms near the wall (approximately \( y/D = 1 \)) for the SJ hole compared to the 777 hole.

One of the possible reasons behind this is that the sweeping jet slows down significantly as it sweeps in the spanwise direction. This creates a lower effective jet momentum that the high-speed freestream flow immediately pushes toward the wall. The close proximity of the coolant to the wall results in improved cooling effectiveness at high blowing ratio. The profiles of streamwise RMS velocity are shown in Fig. 12. Interestingly, the streamwise fluctuation is higher near the wall for the 777-shaped hole compared to the SJ hole at all blowing ratios which is counter-intuitive since the SJ hole is inherently unsteady. One possible reason is that the hot film used in this study was a single element that can only sense the streamwise fluctuation of the velocity. Since the sweeping motion occurs in the lateral direction, the probe failed to detect the transverse component \( (w/U_\infty) \) of the velocity fluctuation which is dominant for the SJ hole. Additional CFD results are shown in the later part of the paper describing this behavior in detail.

### Computational Study

**Details of Computational Domain and Grid.** Large Eddy simulations (LES) were performed to investigate the near hole flow field and heat transfer behavior for both holes. The computational domains for both hole types are shown in Fig. 13. A thin section of the OSU vane geometry was considered that includes a single film cooling hole. The slice thickness extends a full hole pitch (6D) in the spanwise direction. In addition, a periodic section of the cascade passage was considered for the fluid domain. The inlet of the fluid domain extends a single chord (1C) upstream of the vane leading edge and 1.25C downstream of the vane trailing edge. An experimentally measured inlet velocity was applied at the freestream inlet. Note that the inlet total pressure \( (P_t) \) and temperature \( (T_t) \) were also matched with the experiment (see Table 2). A mass flow inlet boundary condition was used at the coolant inlet to match the desired blowing ratio. The turbulence intensity \( (Tu = 14.3\%) \) and the length scale used at the freestream inlet were also measured experimentally. Unsteady inflow fluctuations were imposed using the vortex method described by Mathey et al. [25] The vortex method was used to generate isotropic and homogenous velocity perturbations with predetermined turbulence characteristics at the freestream inlet in conjunction with the mean inlet velocity. A similar method was successfully used and reported by Li et al. [26] in their film cooling study. A periodic boundary condition was applied in the pitch-wise direction \( (y-axis) \), and the walls at the spanwise direction \( (z-axis) \) were considered as symmetric walls with zero normal gradients. The vane surface was considered as a no-slip adiabatic wall. A static pressure outlet boundary condition was used at the outlet.

Computational grids were created using ICEM CFD. A hybrid grid generation approach was applied that includes an unstructured tetrahedral grid in the main domain and a cluster of 20 prism layers at the wall. The grid density was refined near the hole exit to resolve the unsteady flow features and the interaction between the coolant and the freestream. Figure 14 shows a portion of the grid in the centerline plane (Fig. 14(a)) and a close-up view of the hybrid grid near the hole exit (Fig. 14(b)). The non-dimensional wall distance \( (\Delta x^+) \) was kept less than unity which is shown in Figs. 14(c) and 14(d). In addition, the non-dimensional grid distance in the streamwise \( (\Delta x^+) \) and the spanwise \( (\Delta z^+) \) direction were kept approximately 30 and 15 that leads to a total number of grid points for the SJ hole is 53 million and the 777 hole is 48 million, respectively.
Model Setup and Validation. LES calculations were performed using commercial finite volume solver FLUENT. A wall adapting local eddy viscosity (WALE) model proposed by Nicoud and Ducros [27] was used as a subgrid-scale model. This model returns a zero turbulent viscosity for laminar shear flow [28] which allows relatively accurate treatment of the laminar regions in the domain. Several researchers have demonstrated the successful use of this model [29] for similar calculations. Each calculation was performed by setting a freestream inlet velocity of 9.5 m/s and the freestream temperature of 315 K. Two coolant mass flow rates were considered at the hole inlet that corresponds to the blowing ratio of $M = 1$ and 3. A bounded central differencing scheme was adopted with a second-order upwind scheme for spatial discretization. A SIMPLE method was used for pressure-velocity coupling for the convective term, and a second-order implicit discretization scheme was adopted for temporal derivatives. The time-step was set to $\Delta t = 3 \times 10^{-7}$ s for $M = 1$ and $1 \times 10^{-7}$ s for $M = 3$ to keep the CFL number below unity. Each simulation was performed with 10 inner iterations for each time-step.

Calculations were performed on a cluster (8 nodes-224 cores) computer provided by the Ohio Supercomputer Center (OSC) and each LES calculation took approximately 30,000 CPU hours. Data were averaged over 10 flow-through time (FTT) that gives approximately 20 full oscillations for the sweeping jet hole. The model was assessed for both flow field and heat transfer results. The flow field was assessed by comparing the frequency of the oscillating jet emanating from the SJ hole. The predicted oscillation frequency (310 Hz for $M = 1$) agrees well with the measured frequency (282 Hz). The quality of the resolved flow field by LES calculation was also quantified by the power spectral density (PSD) analysis of the upstream velocity fluctuation. Figure 15(a) shows the PSD of a single point velocity fluctuation at 0.25C upstream of the vane leading edge. This confirms that the upstream boundary condition is generating turbulent flow. The PSD distribution is compared with the $-5/3$ slope which matches the inertial subrange to a good extent. This implies that the LES calculation is resolving the inertial subrange properly and capturing adequate dissipative scales in the flow. The heat transfer results were also validated to achieve additional confidence in the numerical calculation. Figure 15(b) shows the span-averaged effectiveness distribution for the SJ hole at $M = 1$. Results are shown for both LES and URANS calculations and compared with the experimental result. It is evident that the URANS result shows a significant underprediction of the film effectiveness while the LES prediction shows a notable improvement over URANS calculation. However, the LES slightly underpredicts the near hole cooling effectiveness. Figure 15(c) shows the time-averaged thermal field at $x/D = 12$ for the SJ hole at $M = 1$. Results are shown for both LES and URANS calculations and compared with the experimental result.

Fig. 14 Computational grid: (a) midplane ($x$–$y$ plane) grid for 777-hole, (b) grid density and prism layer at the hole exit, (c) contour of $y^+$ for SJ hole, and (d) span-averaged $y^+$

Fig. 15 LES Model assessment: (a) power spectral density of velocity fluctuation at 0.25C upstream of the vane, (b) span-averaged effectiveness for the SJ hole at $M = 1$, and (c) time-averaged thermal field at $x/D = 12$ and $M = 1$
It is evident that the thermal field predicted by LES is certainly an improvement over URANS prediction which provides additional assessment and confidence in LES results.

Flowfield and Heat Transfer Results. Although numerical calculations were performed for two blowing ratios ($M = 1$ and 3), results will be discussed for the high blowing ratio ($M = 3$) case in the subsequent sections. The results from the low blowing ratio ($M = 1$) case will be referred to sometimes for comparison. Figure 16 shows contours of the time-averaged momentum flux ($I = \rho_\infty U_\infty^2/\rho_0 U_0^2$) at the centerline plane for both SJ and 777 hole at a blowing ratio of $M = 3$. The contours show the jet penetration height and the local velocity in the near hole region. It is evident that 777 hole shows a strong jetting action at the hole exit that penetrates high into the freestream. This jetting action occurs due to a recirculation zone located at the bottom of the diffuser. This recirculation zone exists on almost every diffused (as well as cylindrical) hole which is extensively reported by several investigators [15,30]. This recirculation zone creates an additional blockage of the coolant jet moves toward the top wall of the metering and diffuser section of the hole (Fig. 16c). This high-velocity jet exits the hole with a high jet momentum that immediately penetrates the freestream and causes jet liftoff resulting in a lack of cooling in the near hole region. In contrast, the SJ hole does not show this type of recirculation in the metering section. In fact, the jet velocity decreases due to the internal geometry and the sweeping motion of the jet. Thus, the coolant jet has a lower effective jet momentum. Once this low momentum jet exits the hole, the high-velocity freestream flow pushes the coolant toward the wall preventing jet liftoff. The contour plot shown in Fig. 16a shows the lower jet momentum that remains close to the wall for the SJ hole. This also causes improved cooling in the near hole region at a high blowing ratio which is shown in Figs. 6 and 7.

The strong jetting action at the hole exit of the 777 hole also leads to a strong shear layer development at the leeward edge of the hole. The strength of this shear layer can be quantified by the turbulent shear stress ($u'v'$). Figure 17 shows the contour of instantaneous (time-resolved) and time-averaged turbulent shear stress at the centerline plane for both SJ- and 777-shaped hole. Results are shown for the blowing ratio of $M = 3$. Instantaneous shear stress contours (Fig. 17a) show the existence of positive $u'v'$ emanating from the upstream edge of the hole and negative $u'v'$ at the downstream edges of both holes. The shear stress results in the development of spanwise vorticity that convects downstream with the flow. The distribution of positive and negative $u'v'$ also take the appearance of the Kelvin–Helmholtz type instability near the hole exit and becomes fully turbulent as it convects downstream. Eberly and Thole [3] and Favwett et al. [31] also observed this Kelvin–Helmholtz type breakdown for the film cooling hole. Note that the $u'v'$ distribution for the SJ hole is much more chaotic than the Kelvin–Helmholtz instability which is attributed to the transverse sweeping motion of the jet.

A significant difference in the magnitude of the positive and negative $u'v'$ in the near hole region can be seen (Fig. 17b) for the SJ- and 777-shaped hole. The magnitude of the time-averaged turbulent shear stress ($\overline{u'v'}$) for the SJ hole is much lower than the 777 hole which indicates that the shear layer at the hole exit is much stronger for the 777 hole. In addition, a large negative $u'v'$ can also be seen at the downstream breakout edge that originates inside the metering section of the hole and covers the entire diffuser section. This

![Fig. 16 Time-averaged momentum flux ratio in the centerline plane at $M = 3$: (a) SJ-hole, (b) 777-shaped hole, and (c) momentum flux contour inside the holes](image_url)

![Fig. 17 Contours of turbulent shear stress ($u'v'$) in the centerline plane at $M = 3$: (a) instantaneous and (b) time-averaged](image_url)
large negative $(-u'v')$ is caused by the recirculation zone inside the hole that also dictates the shear layer rollup as the coolant exits the hole. The coolant jet liftoff at high blowing ratio is attributed to this large negative $(-u'v')$ development at the hole exit of the 777-shaped hole. In contrast, the SJ hole does not show this strong negative $(-u'v')$ inside or at the exit of the hole. The internal geometry and the sweeping motion of the jet do not allow the development of such strong negative $(-u'v')$ that prevents the jet from lifting off at high blowing ratios. Thus, the sweeping film cooling jet improves the cooling performance by keeping the coolant closer to the wall.

Figure 18 shows the contours of turbulence intensity in the centerline plane for the SJ and 777-shaped hole. The maximum turbulence intensity at the hole exit is more than 40% for both holes at this high blowing ratio ($M = 3$). Schroeder and Thole [14] also observed a similar level of turbulence intensity for the 777-shaped hole at $M = 3$, giving additional confidence to the current numerical calculation. The instantaneous turbulence intensity contours (Fig. 18(a)) show high turbulence at the exit of both holes. However, the source of the turbulence is quite different. For SJ hole, the primary source of the high turbulence intensity is the unsteadiness caused by the internal fluid dynamics. In contrast, the inlet breakout edge that causes a large separation region inside the metering section is the primary source of the high turbulence for the 777 hole. The time-averaged turbulence intensity contours (Fig. 18(b)) show similar turbulence for both holes. However, the SJ hole shows slightly higher turbulence far downstream of the hole. This increased level of turbulence is attributed to the sweeping motion of the jet. As the sweeping jet convects downstream, the transverse (spanwise) component ($w'$) of the velocity dominates and becomes a major contributor to the downstream turbulence enhancement.

Figure 19 shows the contours of normalized velocity fluctuation for the SJ and 777-shaped hole in a cross-plane downstream ($x/D = 4$) of the hole exit. The spanwise velocity fluctuation ($w'$) shows the greatest difference between the two-hole types. The magnitude $w'$ is much higher for the SJ hole compared to the 777-shaped hole (Figs. 19(e) and 19(f)). The sweeping motion of the jet increases spanwise fluctuation that leads to enhanced turbulence in the near hole region. In addition, this spanwise motion prevents the formation of the detrimental CRVP and keeps the coolant at close proximity to the wall at a high blowing ratio. The spanwise motion of the jet was not sensed by the single element hotwire probe during the experimental measurement (at $x/D = 12$) leading to the lower streamwise fluctuation measurement for the SJ hole which is shown in Fig. 12.

The sole purpose of the computational study was to complement the experimental results and to extract information at the near hole flowfield and heat transfer behavior for both SJ and 777-shaped hole that are not possible with the current experimental facility. In order to improve the completeness of the current investigation and to estimate the net heat flux reduction, a separate set of simulations was performed with a constant wall heat flux boundary condition to estimate the heat transfer coefficient for the SJ and 777-shaped hole. Figure 20 shows the convective HTC for the SJ and 777-shaped hole at $M = 1$ and 3. It is evident that the heat transfer coefficient increases with the blowing ratio for both holes. However, the HTC distribution is very different when comparing both holes.
The highest value of the local HTC can be seen at the exit of the hole where the jet-to-freestream interaction is the strongest. At $M = 1$, the contour lines are much wider for the SJ hole compared to the 777 hole. At $M = 3$, the HTC distribution changes dramatically. The SJ hole shows an augmented heat transfer in the spanwise direction, and the local augmentation covers the full pitch far downstream of the hole exit ($\alpha D > 10$). This local HTC augmentation is attributed to the sweeping motion of the jet and the spanwise fluctuation of the jet (Fig. 19) that ultimately leads to a deteriorating cooling performance far downstream of the hole exit. In contrast, the 777 hole shows the sign of jet detachment that leads to a local necking in the contour lines at the hole exit. However, the overall magnitude of the HTC increases in the spanwise direction due to enhanced turbulence at high blowing ratio ($M = 3$).

**Conclusion**

The effect of the high blowing ratio for the sweeping jet film cooling hole was investigated experimentally and numerically. Experiments were performed for a row of the sweeping jet holes on the suction surface of a turbine nozzle guide vane (OSU vane) geometry in a low-speed linear cascade. Film effectiveness, thermal field, and velocity boundary layer data were acquired at a range of blowing ratios and two freestream turbulence ($Tu = 0.6\%$ and $14.3\%$). Results were compared with a baseline 777-shaped hole at the corresponding conditions. LES were also performed for both holes with a WALE model to predict the unsteady flow and heat transfer behavior of the coolant jet. A summary of the major findings is described below.

The SJ hole shows higher film effectiveness ($\eta$) in the near hole region at high blowing ratios ($1 \leq M \leq 3.5$) compared to the baseline 777 hole. At $M = 3$, the coolant jet lifts off from the surface with very little coolant coverage on the surface for the 777-shaped hole. However, the jet spreading for the SJ hole at this blowing ratio remains much wider and uniformly distributed. The effect of elevated burning on $\bar{\theta}$ at high blowing ratio ($M = 3$) is nearly negligible for the SJ hole. However, the 777 hole shows a significant drop in $\bar{\theta}$ with increasing turbulence at high blowing ratio. The area-averaged effectiveness ($\bar{\eta}$) for the SJ hole is significantly improved at high blowing ratios ($1 \leq M \leq 3.5$). The SJ hole shows such as 60% higher $\bar{\eta}$ value at $M = 2$ and 56% higher $\bar{\eta}$ value at $M = 3$ compared to the 777-shaped hole.

The thermal field revealed that the lateral spreading of coolant is much wider for the SJ hole while the coolant spreading drops significantly for the 777-shaped hole at $M = 3$. The contour lines bend inward near the wall due to the formation of a strong CRVP that slowly brings hot freestream gas toward the core of the jet resulting in a lack of cooling in the spanwise direction for the 777 hole. Velocity boundary layer measurement shows that the mean velocity profile forms a peak at high blowing ratios and the peak exists in the wall-normal direction somewhere between $y/D = 1.5–2$ for 777 hole and $y/D = 0.25–0.75$ for the SJ hole. This indicates that the coolant jet remains much closer to the wall for the SJ hole compared to the 777 hole.

Numerical results show a local recirculation zone inside the diffuser section of the 777-shaped hole at high blowing ratio ($M = 3$) that creates a high-velocity jet with a high jet momentum which then immediately penetrates the freestream and causes the jet liftoff resulting in a lack of cooling in the near hole region. In contrast, the jet velocity for the SJ hole is reduced due to the internal geometry and the sweeping motion of the jet. Thus, the coolant jet exits the hole with a lower effective jet momentum and the high-velocity freestream flow pushes the coolant toward the wall preventing jet liftoff. A large negative turbulent shear stress ($\langle u'v' \rangle$) exists at the downstream breakout edge of the 777 hole that originates inside the metering section of the hole. This causes the local recirculation zone inside the hole and also dictates the shear layer rollover as the coolant exits the hole. However, the SJ hole does not show this strong negative ($\langle u'v' \rangle$) inside or at the exit of the hole that prevents the jet from lifting off at high blowing ratios. The sweeping motion of the jet emanating from the SJ hole increases spanwise fluctuation that leads to enhanced turbulence and local heat transfer augmentation at the near hole region at a high blowing ratio.

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**Nomenclature**

- $h =$ convective heat transfer coefficient, W/m² K
- $u =$ streamwise velocity
- $v =$ wall-normal velocity
- $w =$ spanwise velocity
- $x =$ streamwise direction
- $y =$ surface normal direction
- $z =$ spanwise direction
- $D =$ throat hydraulic diameter, 2.56 mm
- $M =$ blowing ratio, ($\rho U_2/\rho U_{\infty}$)
- $P =$ vane pitch or static pressure or hole pitch (6D)
- $T =$ temperature
- $\bar{m}_l =$ coolant mass flow rate, kg/s
- $C_{op} =$ pressure coefficient, $\frac{P_{local} - P_{in}}{\frac{1}{2} \rho_{in} U_{\infty}^2}$
- $U_{\infty} =$ freestream velocity at 1C upstream of the vane
- $is =$ momentum flux ratio, ($\rho U_2^2/\rho U_{\infty}^2$)
- $DR =$ density ratio ($\rho_{in}/\rho_{\infty}$)
- $Tu =$ freestream turbulence intensity at 1C upstream
- $Re_{D} =$ hole Reynolds number

**Greek Symbols**

- $\alpha =$ thermal diffusivity ($k/\rho c_p$)
- $\eta =$ film effectiveness
- $\theta =$ non-dimensional temperature (thermal field)
- $\kappa =$ thermal conductivity, W/m K
\[ \nu = \text{kinematic viscosity, m}^2/\text{s} \]
\[ \rho = \text{density, kg/m}^3 \]
\[ \rho_c = \text{density of coolant, kg/m}^3 \]

**Subscripts**
- \( c \) = coolant
- \( ex \) = exit
- \( t \) = total or throat
- \( w \) = wall
- \( \infty \) = freestream

**Superscripts**
- \( ' \) = fluctuation
- \( - \) = (overbar) average/time-averaged
- \( \bar{\text{area-averaged}} \)

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**References**


