MEASUREMENTS OF TIME-VARYING JET ISOPLETHS USING DUAL LIGHT SHEET PLIF

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ABSTRACT
Mass transfer analogies have long been used to experimentally determine the distribution of cooling fluid through pointwise sampling for turbine blade, nozzle or combustor film cooling. The behavior of a turbulent jet or plume flowing into its surroundings cannot be fully understood from point measurements alone, however. Full-field measurement of the instantaneous distribution of cooling fluid can reveal the structure and mechanisms governing cooling performance. This paper describes an improved dual light sheet PLIF (Planar Laser Induced Fluorescence) technique developed for full field concentration measurements. An analytical model of laser light sheet / fluorescent dye interaction was formulated and used to evaluate the light sheet attenuation corrections. With the more common single light sheet technique, these corrections lead to substantial concentration uncertainty which can be substantially reduced by using a dual light sheet. The dual light sheet technique was used to study the time-varying position and area of concentration isopleths for a round jet issuing into quiescent surroundings. Results show that, although concentrations at any point vary widely with time, the area within a given concentration isopleth remains virtually constant.

NOMENCLATURE

A Isopleth area, m²
a(C) Dye attenuation function, 1/mol·cm.
B Jet shape factor.
C(x,y) Dye concentration distribution, mol/l.
d Discharge tube nozzle diameter, cm.
F(x,y) Fluorescence distribution.
F(C) Conversion function relating image intensity to dye fluorescence, 1/m²·watt·mol.
I(x,y) Light sheet intensity distribution, watts/m²·steradian.
I ave Isopleth area root mean square fluctuation intensity.
r Distance from light sheet focal point, mm.
R 1/2 Distance from light sheet focal point to jet center line, mm.
U Jet Velocity.
x Position to left or right of jet centerline, mm.
y Position measured from jet discharge tube nozzle in jet flow direction, mm.
δ Jet half width, mm.
η Film cooling effectiveness.
θ Angle above or below light sheet horizontal center line.

Subscripts
area Refers to isopleth area.
aw Denoted value at adiabatic wall.
dilute Refers to a dilute, uniform concentration dye image.
jet Refers to a jet image.
s Denotes value at jet source.
θth Denotes threshold concentration level.
w Denotes value at a wall.
0 Denotes base or reference position.
1 Denotes light sheet #1 (from left in image).
2 Denotes light sheet #2 (from right in image).
1/2 Denotes jet velocity half-width.
1/2C Denotes jet concentration half-width.

Superscript
" Denotes fluctuating quantity
— Denotes mean quantity

INTRODUCTION
Modern gas turbine engines operate with high peak combustion temperatures to improve cycle efficiency. These peak temperatures are generally above allowable metal temperatures, so components such as turbine vanes, blades, endwalls and combustors must be protected using some sort of active cooling.

One method used is discrete hole film cooling. Cool air flows through holes in the surface and into the boundary layer. If properly designed, this film of cool air will reduce the heat transfer to the surface. Since the cooling air is supplied by the compressor and bypasses...
the combustion process, it is very costly in terms of cycle efficiency to
design, it is very costly in terms of cycle efficiency to
determine the distribution of the fluid, cooling fluid through pointwise
sampling. The heat transfer literature typically reports only the mean
jet fluid concentration at the surface. Pedersen et al. (1977) and Gold-
stein and Stone (1994) are good examples of this technique. This sur-
face concentration is a measure of film cooling performance and is
defined as the adiabatic wall film cooling effectiveness,
\[
\eta = \frac{T_w - T_a}{T_s - T_a} = \frac{C_w}{C_s}
\]
(1)

See Goldstein (1971) for a discussion of how \( \eta \) is used in film cooling
design.

Work has also been done to study the velocity and tempera-
ture/concentration fields of the fluid. Cooling jets using single point mea-
surement techniques, as in Honami et al. (1992) and Campbell and
Moffat (1994). By taking measurements at many points and treating
the measurements as a mean plus a fluctuating quantity, a model of the
mean jet(s) can be obtained. However, it can be misleading to model
jet behavior based on this point measurement of mean flow and time
averaged concentration behavior. For example, the “flapping” and
“wandering” behavior of real jets and plumes produces low average
concentrations at regions which actually experience short periods with
high concentration followed by longer periods with little or no jet
fluid. Full-field measurements of the instantaneous distributions of
cooling fluid can reveal the structure and mechanisms governing
cooling performance. It is very desirable to make full field concentra-
tion measurements and study the behavior of jets and plumes on both
an instantaneous and time-varying basis.

**PLANAR LASER INDUCED FLUORESCENCE**

Various flow visualization techniques can be used to make a jet
or plume visible. However, any technique which makes the jet flow
opaque provides little information on internal flow structure and al-
most none on concentration distribution. Techniques which make a
thin cross-section of the jet material visible are much better suited to
measuring concentration distributions. This can be accomplished by
seeding the jet with a dilute tracer which is visible only when illumi-
nated and then illuminating a cross-section of the jet using a thin sheet
of light. With suitable care, this technique can provide instantaneous,
full-field concentration measurements for a jet or plume cross-section.

Long et al. (1981) used gas jet seeded with small (1 \( \mu \)m diame-
ter) aerosol particles to Mie scatter a laser light sheet. Scattered light
intensity was used as a quantitative measure of fluid concentration.
Fluorescent materials have advantages over Mie scattering particles
for fluid concentration measurements. As discussed by Shlien (1988),
fluorescent materials are detectable at extremely low concentrations
so the seeded flow can be almost completely transparent. When stimu-
lated by appropriate wavelength light, the materials fluoresce at a
different wavelength, allowing the experimenter to separate dye flu-
orescence from scattered laser light by simple filtering. Also, the flu-
orescence intensity variation can be lineal over a three order of magni-
tude concentration range. The combination of laser sheet lighting and
dilute fluorescent dyes leads to the Planar Laser Induced Fluorescence
(PLIF) technique, applicable to gas or liquid mixing experiments as
well as combustion studies. An overview of PLIF techniques can be
found in Hanson (1986) or in Kychakoff et al. (1987). The technique
has been applied to channel flow by Walker and Tiederman (1989), to
mixing layers by Koochesfahani and Dimotakis (1985,1986), and to
turbulent jets by Shlien (1988), Dimotakis et al. (1983), Dahm and

The PLIF technique was used in this study to learn more about
the concentration isopleth behavior of round jets entering quiescent
fluid as a stepping stone toward more complex jet flow studies. The
development of the dual light sheet PLIF technique was inspired by
the need to reduce attenuation correction uncertainties and thus im-
prove the measurement accuracy over the broad range of concentra-
tions produced by a jet mixing with surrounding fluid. This paper
provides analytic descriptions of dye fluorescence, light sheet intensity,
and dye attenuation of laser light. The attenuation corrections are
discussed and the dual light sheet technique is presented as a means to
avoid the correction ambiguities. The power of the technique is illus-
trated by new results on the behavior of concentration isopleths for a
jet mixing with its surroundings.

**EXPERIMENT**

Figure 1 shows a water flow apparatus designed to study water
jet flows into a quiescent tank. The apparatus has the capability to
study single or dual parallel jets issuing into fluid with or without a
cross flow. In this study, only results for a single jet with no cross
flow and no wall interaction are discussed. The jet discharged through
an 8.8 mm internal diameter tube inserted 40 mm below the water
surface of a glass visualization tank. The discharge tubes were thin
wall brass tubing with 9.5 mm outside diameter. Tank dimensions
were 600 mm by 1200 mm with 400 mm water depth. Water jet flow
was controlled by an on/off solenoid and regulated by a needle valve
with constant head provided by a source tank 3 meters above the
visualization tank surface. The mean jet velocity was set at 18 cm/sec
for all experiments in this study. This relatively low flow velocity
was chosen so that camera shutter times could be sufficiently long for
adequate exposure while still “freezing” the flow. The Reynolds
number based on mean velocity and tube internal diameter was only
1600, so natural flow from the discharge tube was an initially laminar
jet. To artificially trigger turbulence inside the jet tube and provide a
uniform, turbulent flow profile, a turbulence generator was installed
inside the discharge tube for some tests. Shown schematically in Fig.
1, the turbulence generator was made by drilling eight 1.3 mm diame-
ter holes in a round plastic plug which was pressed into the jet tube,
25 mm from the discharge end. This provided a nearly uniform, tur-
bulent flow profile at the jet exit (Campbell, 1991). The density dif-
fERENCE between the jet and the surrounding fluid was negligible, so
the jets were purely momentum driven, not affected by buoyancy.

For visualization, the jet fluid was dyed with disodium fluo-
rescein at a concentration of 1x10^4 mol/l. When stimulated with
blue-green light from an argon ion laser, disodium fluorescein emits
light in the 530 nm to 580 nm band with peak power around 550 nm
(yellow-green light). As well as light sheet intensity and dye concen-
tration, the fluorescence of disodium fluorescein is affected by tem-
perature and by solution pH if it is below 8.0. To minimize tempera-
ture effects, both the dyed source water and the clear visualization
tank water were kept at room temperature (22 ± 2°C). Both the jet
and tank fluid were held at a pH of 9.0 using a dilute buffer solution of
ammonium hydroxide (2.4x10^{-2} mol/l) and boric acid (1.2x10^{-1}
MEASURING CONCENTRATION FROM IMAGES:
ANALYTICAL MODELS

The fluorescent light intensity at any point in a PLIF experiment depends on dye concentration, dye fluorescent response function, and local laser illumination intensity. Thus

$$F(x,y) = f(C) \cdot C(x,y) \cdot (l_1(x,y) + l_2(x,y))$$

where $F$ is the measured fluorescence intensity in the image, $l_1$ and $l_2$ are the illumination intensity for the two laser light sheets and $C$ is the concentration of fluorescent dye at that point. The transfer function $f(C)$ could be a nonlinear function but, over the range of illumination intensities and dye concentrations used in this study, a simple calibration constant was sufficient, (see Shlien (1988)).

For a constant laser light sheet thickness, the intensity of each light sheet, $l_1$ and $l_2$, decreases along a radial path according to

$$dl = \frac{l(r,\theta) - a(C) \cdot C(r,\theta) \cdot l(r,\theta)}{r}$$

The first term simply describes light sheet intensity reduction as the sheet expands radially away from its source. The second term describes attenuation due to the dye absorbing light sheet energy in proportion to the local intensity, $l$, and local concentration, $C$, multiplied by an absorption function, $a(C)$. With intensity specified as $l_0$ at some location $r_0$, Eq. (2) can be integrated to yield

$$I(r,\theta) = \frac{l_0(r,\theta) \cdot r_0}{r} \exp \left(-\int a(C) \cdot C(r,\theta) \cdot dr \right)$$

Walker (1987) reported that the absorption function $a(C)$ for fluorescein could be replaced by a constant, $a = 0.85 \times 10^4$ l/(mol·cm), based on absorption experiments with a single line, 488 nm collimated argon-ion laser beam. Walker measured attenuation over a path length of 102 mm with concentrations ranging from $C = 8 \times 10^{-5}$ to $1.2 \times 10^{-5}$ mol/l.

For this study, the absorption function was explicitly measured by filling the visualization tank with various concentrations of disodium fluorescein and illuminating it with a single light sheet. With a uniform dye concentration, Eq. (3) can be integrated along a ray and combined with Eq. (1) to yield an expression for the reduction in fluorescence intensity, $F$, with distance, $r$, along a ray through the uniform concentration, $C$:

$$\frac{F \cdot r}{F_0 \cdot r_0} = \exp(-a(C) \cdot C \cdot (r - r_0))$$

For a range of fluorescein concentrations ($0.6 \times 10^{-5}$ to $1 \times 10^{-5}$), the fluorescence reduction due to absorption consistently matched the expected exponential decay law. Figure 3 shows an example of the attenuation data for $C = 0.9 \times 10^{-6}$. For each fluorescence reduction curve the slope is a measure of the absorption function, $a(C)$, at one dye concentration. For the range of dye concentrations considered in this study, it was found that the absorption function varied slightly with concentration. The data can be represented by...
to within ±1%, where $C_{ref} = 2.46 \times 10^{-6}$ mol/l.

The average value of the absorption function over the concentration range tested was found to be $1.37 \times 10^6$ mol/cm, which is 60% higher than the value measured by Walker (1987). No explanation of this difference has been found. Walker's collimated-beam absorption experiments had local laser intensities 200-600 times higher than the light sheet intensities used here, which may have caused the difference. The high absorption function value shows that illumination reduction due to absorption is a significant concern in correctly measuring image concentration from fluorescence images. In this study the analytical model was used to design the experiment such that attenuation could be neglected. The higher value of a measured here allowed for conservative estimates of the attenuation.

Equations (5) and (3) provide an analytic model of illumination intensity reduction for a laser sheet illuminating an experimental system containing disodium fluorescein. This model can be used to calculate actual concentration from observed fluorescence in an experiment with corrections for light sheet expansion and dye absorption. The procedure is an adaptation of that used by Dahm and Dimotakis (1987) with a collimated beam. However, it becomes more complicated for a light sheet because the experimenter must accurately establish the light sheet intensity distribution, $I_d(\theta)$ at some position $r_o$. Because of minor optical imperfections, misalignments, and performance drifts, the light sheet intensity distribution is not Gaussian like the input laser beam and must be measured experimentally along the side of the experiment where the light sheet enters the test section. Then the measurement of concentration from fluorescence involves calculating integrated attenuation corrections for each point across the experiment to account for all light sheet expansion and attenuation due to dye concentrations along the light path from the light entry point to the point considered.

To illustrate the extent of the correction factors and the potential for uncertainty, calculated intensity distributions for light sheets passing through a uniform concentration fluorescein solution are shown in Fig. 4. The illumination variation for a single light sheet depends on the degree to which the light sheet expands as it crosses the test section, (thus, the distance from light sheet focal, $R_o$), and on the quantity of dye each ray passes through, (thus, the dye concentration, $C$, and path length, $\Delta r$). For an experiment typical of those in this study, with a jet about 10 mm diameter of $1 \times 10^{-4}$ mol/l fluorescein, the attenuation is about 12%. This is the worst case attenuation; as the jet spreads the light sheet path length increases as $\delta_{sec}$, but mean concentration is proportional to $1/\delta_{sec}$, so the $C \Delta r$ term in Eq. (5) goes like $1/\delta_{sec}$ and decreases as the jet expands.

Lower dye concentrations give a reduced attenuation but the corresponding reduction in fluorescence intensity lowers the signal to noise ratio in image processing. The light sheet attenuation can become very large in an experiment with a longer path length (71% attenuation for 100 mm path length through $1 \times 10^{-5}$ mol/l fluorescein, for example). Because the illumination correction required for a single light sheet becomes large and is integrated along the ray path, they represent a significant uncertainty in measured concentration.

The uncertainty in measured concentration can be substantially reduced by using dual, opposed light sheets to illuminate a single plane in the experiment. By lighting the experiment from both sides simultaneously, the drop in illumination intensity for each light sheet is largely compensated by the rise in intensity for the other sheet. Fig. 4 also shows the intensity distributions calculated with the analytic model for the same experimental situations when illuminated by dual, opposed light sheets. For a 10 mm wide jet of $1 \times 10^{-5}$ mol/l fluorescein the maximum attenuation was reduced from 12% to 2.6%. This is small enough to be negligible compared with other sources of error. Even greater reductions in uncertainty result when the dual light sheet technique is used with more dilute dye solutions.

By reducing the illumination intensity corrections to a negligible quantity, the dual opposed light sheet PLIF technique also simplified experimental analysis. With the single light sheet technique, it was necessary to accurately measure the illumination intensity distribution at one edge of the experiment and to calculate an integrated correction across each image depending on the distribution of fluorescence intensity in that image. With the dual light sheet technique, no attenuation correction was required and it was possible to measure the light sheet intensity distribution across the full experimental field using images of the experimental volume filled with a dilute, uniform concentration of dye. Figure 5 shows the illumination intensity distribution measured by photographing the experimental volume filled with a $0.2 \times 10^{-4}$ mol/l dye solution. As indicated, the light sheet intensity distribution produced by the combination of fiber optic cables and cylindrical lenses was nonuniform and asymmetric along the direction parallel to the jet axis. In the direction perpendicular to the jet axis, the light sheet is uniform, with less than 2% variation due to light sheet spreading and attenuation. This 2% variation was ignored and dilute dye fluorescence images taken shortly after each experiment were assumed to be a direct measure of light sheet intensity distribution for that experiment.

For jet mixing experiments, the peak illumination intensity of the dual light sheet was positioned 15 jet diameters downstream of the jet origin. Higher illumination intensity for the region where dye had been diluted by mixing with ambient fluid kept the dye fluorescence and film exposure more consistent across the image, allowing accurate measurement across a wider range of source material concentration. Figure 6 shows the raw fluorescence image of an initially laminar jet illuminated by this laser light sheet, showing an increasing fluorescence along the initial, uniform core as the jet material approaches the zone of highest illumination intensity. To convert the raw fluorescence image to a concentration map, advantage was taken of the negligible attenuation losses in the dual light sheet PLIF system. Rather than using complex integration along rays to account for attenuation, the jet image fluorescence was simply normalized by multiplying the dilute image concentration, $C_{dilute}$ divided by the observed light sheet intensity distribution, $F_{dilute}(x,y)$ as shown in Eq. (7).

$$
C_{jet}(x,y) = F_{jet}(x,y) \frac{C_{dilute}}{F_{dilute}(x,y)} \frac{f(C_{dilute})}{f(C_{jet})}
$$

The factor, $f(C_{dilute})/f(C_{jet})$, was included to allow for changes in camera aperture or exposure time between the experiment and the dilute concentration image. The ratio could be easily obtained by comparing fluorescence of the jet core region, (where the jet concentration was known), with fluorescence for the same region of the dilute dye image. Figure 7 shows the resulting full field concentration...
map obtained by applying Eq. (7) to the image of Fig. 6.

As a test of this image processing technique a laminar jet was produced using a saline solution for the jet material and omitting the jet turbulence generator. The higher density jet fluid resisted transition to turbulence due to vertical flow acceleration, giving a long jet core with a uniform, known dye concentration. Figure 8 shows the uniform concentration region along the laminar jet core. The raw fluorescence image shows nonuniform intensity with fluorescence varying in proportion to the light sheet intensity. The calculated concentration profile should be uniform and shows that the calculated concentration matches the actual jet concentration uniformly within ±7.5%. Thus the dual light sheet PLIF technique provides a relatively simple method of obtaining and calibrating full field concentration images with an overall accuracy within ±7.5%.

This analytical framework has not addressed fluorescence trapping issues. Since the fluorescence band of the dye overlaps the absorption band, a significant amount of the fluorescence produced by the dye in the light sheet can be absorbed by non-illuminated dye in front of the light sheet, between the light sheet and the detector. With a fixed light sheet, there is no way to evaluate the amount of dyed jet fluid between the light sheet and the camera at any instance. The best the experimentalist can do is to estimate the importance of this effect analytically (assuming a mean jet behavior). This issue is important for all PLIF techniques, not just the single and dual sheet techniques discussed here.

ISOPLETH BEHAVIOR FOR A ROUND JET INTO QUIESCENT SURROUNDINGS

Instantaneous full-field concentration measurements allow new ways to examine mixing processes such as those of jets or plumes. For example, Fig. 9 shows a contour plot of the instantaneous concentration field of a round turbulent jet flowing into quiescent surroundings. The lines in this figure represent lines of constant concentration, or isopleths. The area within any isopleth is a measure of the area in the image plane containing source material at a greater concentration than the isopleth concentration value. Similarly, the area between any two isopleths is the area in the jet plane with concentration in the range specified by the two isopleth values. The average of many concentration images would give the mean jet concentration map as shown in Fig. 10. This mean concentration map could be measured using conventional point averaging techniques. It can also be described mathematically given the well defined Gaussian profile of a typical jet, measured jet spreading rates, and conservation of momentum. The following equation describes the mean isopleth area of a Gaussian profile jet:

$$A_h = \frac{2B_1}{B_0^{3/2}B_2} \left( C_{1,\text{h}} \right) d \int \frac{1}{\xi \ln \left( 1/\xi \right)} d\xi \left( 8 \right)$$

where $A_{1,\text{h}}$ is the mean area inside the threshold concentration, $C_{1,\text{h}}$, $d$, and $B_2$ are the source concentration and diameter, respectively. The various $B_1$ is are shape constants describing the spreading rates of the momentum and concentration jets. A derivation of Eq. (8) is given in Appendix A.

Because real jets and plumes "flap" and "wander", the time-averaged concentration profiles tends to significantly underestimate the maximum distance from the source at which fractional concentrations of source material may occur. The full field concentration measurement provides a unique means to measure the instantaneous area within a given isopleth or between a pair of isopleths. Such information is of significant value in answering questions about jet mixing where the average distribution is not adequate. It is possible to measure the fluctuation in concentration isopleth areas in a manner similar to the more familiar fluctuating concentrations and velocities at a point. If the mean isopleth area for a given concentration range is expressed as $A$, then the fluctuating area can be defined as

$$A' = A - \bar{A} \left( 9 \right)$$

and the isopleth are fluctuation intensity can be defined as

$$i = \frac{\sqrt{A'^2}}{A} \left( 10 \right)$$

The mean area and fluctuation intensity can thus be defined and evaluated for any fractional concentration of original source material.

Figures 11 and 12 presents data on the mean isopleth area and fluctuation intensity for isopleths at 10% of jet-fluid-concentration intervals. In Fig. 11 the areas between isopleths is being plotted, rather than the total areas inside isopleths. Noting the logarithmic scale in Fig. 11, it can be observed that the mean isopleth cross sectional areas increases almost geometrically for more dilute concentrations of source fluid. This is well predicted by Eq. (8). Equation (8) deviates from the measured data only at higher concentrations (i.e. near the jet source) where the isopleth area is a very small fraction of the total observed jet isopleth area and the assumption of a Gaussian jet profile breaks down.

It is interesting to note that the isopleth area fluctuation intensity, shown in Fig. 12, remains relatively constant (in the range of 0.1 to 0.3) over the 10% to 90% dilution range, even though the mean area within those isopleth ranges increases by two orders of magnitude.

FILM COOLING

To date, the dual light sheet technique has only been applied to single free jets with no cross-flow. One simple light sheet geometry that could be used to study film cooling would be to arrange dual light sheets parallel to and almost touching the film cooled surface, as shown in Fig. 14. This arrangement minimizes the distance the light sheets pass through the dyed fluid. If the film cooling holes had diameters much larger than the light sheet thickness, then the concentration field measured with the light sheets almost on the surface would be the instantaneous film cooling effectiveness distribution. The light sheets could also be held at various heights above the surface to measure the mean and fluctuating distribution of film cooling fluid in the boundary layer.

CONCLUSIONS

Point measurements give insufficient information to properly model the instantaneous concentration distribution of a jet or plume flowing into its surroundings. To properly assess the volume occupied by a certain range of source fluid concentration, and how that volume varies with time, an instantaneous, full-field concentration measurement is required. PLIF concentration measurement systems provide such a measurement but significant uncertainties are produced using typical single light sheet systems. An improved dual light sheet
PLIF system has been described which substantially reduces the concentration measurement uncertainty.

The dual light sheet PLIF concentration measurement system was used to study the time-varying position and area of concentration isopleths for a round jet issuing into quiescent surroundings. Results show that, although source fluid concentrations at any point vary widely with time, the area within a given concentration isopleth remains virtually constant. Full-field measurement of the instantaneous distribution of cooling fluid can be used to reveal the structure and mechanisms governing cooling performance.

REFERENCES


APPENDIX A—MEAN ISOPLETH BEHAVIOR

If the time averaged mean concentration field is considered, there will be Gaussian profiles of concentration across a jet.

\[ C = C_0 \exp \left( -B_0 \frac{x^2}{\delta_{2C}} \right) \]  

(A1)

where

\[ B_0 = \ln(2) \]

By mass conservation,

\[ C_x U_j \frac{\pi d^2}{4} = B_1 C_0 U_0 \frac{\pi \delta^2_{2C}}{4} \]  

(A2)

where \( B_1 \) is the shape constant of the profile. Then, by momentum conservation,

\[ \rho U_j \frac{\pi d^2}{4} = B_2 \rho U_0 \frac{\pi \delta^2_{2C}}{4} \]

or

\[ U_0 = \frac{U_j d_j}{B_2^{1/2} \delta_{2C}} \]

(A3)

Using (A3) in (A2)

\[ C_0 = \frac{B_0^{1/2} \left( \frac{\delta_{2C}}{\delta_{2C}} \right) d_j}{B_1 \delta_{2C}} \]  

(A4)

Turbulent free jets spread linearly with distance from a virtual origin, so the half width is described by

\[ \delta_{2C} = B_3 (y + y_0) \]  

(A5)

If a mean isopleth is plotted at some threshold concentration \( C_n \) as shown in Fig. 13 then \( x_n \) can be determined from Eq. (A1), i.e.

\[ x_n = \delta_{2C} \left( -\ln \left[ \frac{C_n \delta_{2C}}{C_x B_3 (y + y_0)} \right] \right)^{1/2} \]  

(A6)

and the area is, using Eq. (A5)

\[ \Delta A_n = \frac{1}{2} \int_0^{x_n} dy = \frac{\sqrt{2} \delta_{2C}}{B_0^{1/2}} \left( -\ln \left[ \frac{C_n B_3 (y + y_0)}{C_x \delta_{2C}} \right] \right)^{1/2} dy \]  

(A7)
If the virtual origin is defined such that \( C_0 = C_e \) at \( y = 0 \), then after considerable rearrangement it is found that:

\[
\alpha_{AB} = \frac{2R_e}{R_e^{1/2}B_1B_2} \left( \frac{C_e}{C_f} \right) d_\zeta \int \xi \left[ \ln \left( \frac{1}{\xi} \right) \right]^{1/2} d\xi \quad \text{(A8)}
\]

For a given flow the "combined shape factor," (i.e. the first fractional term to the right of the equal sign) can be determined by measuring a given isopleth area in a jet. The integral has no simple closed form solution, but it is very easy to compute numerically or fit with a simple power law function.

Fig. 1: Jet flow apparatus and illumination light sheet geometry.

Fig. 2: Laser and fiber optic system to produce two laser sheets.

Fig. 3: Light sheet attenuation in disodium fluorescein (\( r_0 = 144 \text{ cm} \), \( C = 0.9 \times 10^{-4} \text{ mol/l} \), \( a = 1.3 \times 10^5 \text{ mol/cm} \)).
Fig. 4: Intensity variation as a function of optical depth for single and dual light sheets. $R_o = 2 \text{ m}, \Delta r = 0.05 \text{ m}$.

Fig. 5: Fluorescence distribution for dilute uniform concentration dye with dual light sheets.

Fig. 6: Uncorrected instantaneous fluorescence image of an axisymmetric jet illuminated by dual light sheets.

Fig. 7: Instantaneous concentration distribution of jet in Fig. 6, obtained using Eq. (7).

Fig. 8: Laminar dye column fluorescence, light sheet intensity and calculated dye concentration.

Fig. 9: Typical instantaneous concentration plots for a single, turbulent jet into quiescent surroundings (same isopleth levels as Fig. 11).
Fig. 10: Mean concentration isopleths for a single, turbulent jet into quiescent surroundings.

Fig. 11: Mean isopleth area for a single, turbulent jet. Note that the small $C > 1.0$ is caused by camera errors, and the $C < 0.1$ region is not valid because not all of the jet fluid at $C < 0.1$ could be contained in the field of view.

Fig. 12: Fluctuating isopleth area for a single, turbulent jet.

Fig. 13: Schematic of mean isopleth area calculation.

Fig. 14: Dual light sheet PLIF for measuring film cooling effectiveness.