Wave Engine Aerothermodynamic Design

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ABSTRACT

A method for aerothermodynamic preliminary design of a wave engine is presented. The engine has a centrifugal precompressor for the wave rotor which feeds high and low pressure turbines. Three specific wave engine designs are presented. Wave rotor blades are naturally cooled by the ingested air; thus combustion temperatures can be as high as 1900 K. Engine pressure ratios of over 25 are obtained in compact designs. It is shown that no nozzles at the end of the rotor blade passages yield the highest cycle efficiencies which can be over 50%. Rotor blades are straight and easily milled, cast or fabricated.

NOMENCLATURE

A Flow area
a $p_4/p_5=p_5/p_6=a=0.95$ (in this paper)
B Blade and port angles measured from axial direction
Cpr Compressor
c Velocity of sound
cp, cv Specific heats at constant pressure and constant volume
D Effective rotor blade passage width, $s \cdot \cos(B_{i})-t$
H Blade height
HPTb High pressure turbine
k Specific heat ratio, $cp/cv=1.4$
L Blade length
Lx Blade length projected on axial direction (rotor axial length)
Lb Distance from blade inlet to intersection of reflected shock with hot gas, cool air interface
Lf Distance from blade inlet to intersection of waves emanating from downstream edges of ports 5 and 6
LPTb Low pressure turbine
M Flow Mach number (relative to rotor if state is on rotor)
Mx Shock Mach number relative to flow
P Power
p Pressure
$P_o$ Stagnation pressure
R Specific gas constant
Rtr Wave rotor
s Blade pitch in peripheral direction
t Blade thickness or time
T Absolute temperature
$T_o$ Absolute stagnation temperature
U Flow velocity (relative to rotor if state is on rotor)
Vw Rotor pitch-line speed
W Mass flow rate
Y Peripheral distance at pitchline
$Y_n$ Width of port n in peripheral direction at pitch line

η Cycle efficiency
$\eta_{sc}$ Compressor polytropic efficiency
$\eta_t$ Turbine efficiency
ρ Density

Subscripts
a Ambient state, absolute
c Blade passage
e Exit of blade or rotor
i Inlet of blade or rotor
1,2,...,n State or region number

INTRODUCTION

Principles of shock compression in time dependent flow have been applied for over half a century. Some applications are the buzz bomb, shock tubes and tunnels, and wave rotors. On a wave rotor the time variable for an observer rotating with the blades is the peripheral distance divided by the peripheral speed. The observer or blades are exposed to different conditions at various times as stator ports and end walls are passed.

Brown Boveri began development of a wave rotor or pressure exchanger, Seippel (1946) and later Keller (1984). It is currently used for supercharging IC engines. Pearson (1985),

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the cool air temperatures. Heating of the air
revolution. Therefore, the blade temperatures
cool air and hot combustion gas flow in each
progresses. Thus, Fig. 2 is the conventional
gible at wave rotor speeds. A typical wave
and cooling of the gas by the blades is negli-
gible at wave rotor speeds. A typical wave
rotor or gas generator is shown in Figs. 1
and 2. The vertical or Y direction of Fig. 2
also represents the time variable; an observer
on the rotor passes the ports as time, Y/Vw,
progresses. Thus, Fig. 2 is the conventional
x-t diagram of time dependent flow.
Wave engines behave like the Brayton cycle when viewed from the stator, i.e.,
adiabatic compression, constant pressure
combustion and adiabatic expansion. Wave
engines can develop large pressure ratios.
With high pressure and combustion inlet tem-
peratures of 1900°C cycle efficiencies of over
50% may be attained.
Possibly an even greater advantage over
steady flow turbomachinery is the considerably
lighter weight of the wave machines. This
light weight is due to two factors:
1. Shock compression occurs in much smaller
distances than does conventional steady flow
compression.
2. Compression pressure ratio across a
single shock is much greater than in a steady
flow diffuser for the same change in subsonic
velocities. A shock wave travels with super
sonic speed relative to the flow into which it
propagates. However, the flow on both sides
of the shock in this paper is subsonic.
High speed rotation results in short axial length rotors and compact machines.
Short rotors permit close control over total
axial expansion and leakage between rotor and
stator end walls. With the small surface area, frictional effects are less important,
and wave machines retain their high efficiency
as size is reduced. In addition wave rotors
are not subject to surge as are conventional
rotating compressors.
Wave machines have simple blade shapes
and only one rotor for much of the compression
and expansion. Precompression pressure ratios
of 2.0 to 3.0 and final expansion accom-
plished on a separate compressor and high and
low pressure turbines. This configuration is
shown in Fig. 3.

WAVE ENGINE OPERATION
The wave engine considered in this paper
is shown in Figs. 1, 2 and 3. The wave rotor
is operated at nearly constant speed, because
peak sizes are determined by arrival of shock
waves or interfaces between hot and cold
gasses. Off-speed operation results in degra-
dation of performance, because the shock waves
and interfaces no longer coincide with the
location of the stator port edges. The free-
pellet turbine rotor may operate at any speed
dicted by the application. Pressure rise in the
wave rotor is accomplished with a four
shock system shown in Fig. 2. These shocks
are as follows:
1. Shock "a" is generated by stopping the
scavenge air flow at state 2 by the blade
passages rotating past the stator end wall.
2. Shock "b" is generated by opening the
blade passages to the hot gas flow at state 5.
3. Shock "c" is reflected by opening the
blade passages to back pressure at state 6.
4. Shock "d" is generated by stopping the
air flow from the rotor into port 6 by the
blade passages rotating past the end wall.
This shock is intersected and weakened by
expansion waves which result when the flow is
stopped by the blades rotating past the down-
stream edge of port 5.

AEROTHERMODYNAMIC DESIGN
Calculation Procedures and Assumptions
Leakage from the wave rotor is neglected.
The rotor axial length is short compared to
earlier machines. Thus rotor clearances are
easily controlled as the materials expand
thermally.
Nozzles are formed by bending the blades
near their downstream ends as shown in Figs. 1
and 2. Nozzle length is assumed negligible
compared with the blade length. It shown in
this paper that straight nozzles without noz-
zeles yield the highest cycle efficiency and
the most compact rotor in the range of useful
combustion temperatures. Then this assumption
is not needed.
A single wave is used to represent an
expansion fan or waves between states 7 and
10. It is assumed that the single wave propa-
gates at the arithmetic average of the speeds
of sound before and after the expansion fan.
This approximation is satisfactory, because
these waves are weak. Flow balances between
states 3, 5, and 6 assure that the calculated
location of the interface "j" is correct. The
downstream end of the interface is the loca-
tion of the downstream edge of the high pres-
sure air port 6. The aerothermodynamic condi-
tions at which Wg = Wc also result in Ws and
Wc calculated within 0.4% of Ws, the flow
carried into the second stage compression by
the rotor.
Single wave representation of the exp-onen-
fan states between states 10 and 14 is used to
determine the separation of the outlet flow to
the high and low pressure turbines. A flow
balance from state 10 to states 12 and 15
assures that the wave r and interface geometry
and aerothermodynamic properties are consist-
ent. Because the pressure ratios between
states 10-11, 11-13, and 13-14 are larger than
those between 7-8 and 8-10, the calculated total flow into ports 12 and 15 is up to 2.5% larger than the flow carried by the rotor at states 3 or 10. However the simplicity afforded by single wave representation more than compensates for these small errors.

Compression waves generated by non-instantaneous opening of a blade passage are assumed to coalesce immediately to a shock. This phenomenon is discussed later.

The polytropic precompression efficiency is taken as 0.85. Expansion from the wave rotor is isentropic. The power turbines following this expansion are taken as isentropic, because the turbines do not affect the compression process.

**First Stage Compression**

Large changes in velocity result in large shock pressure ratios. However, mechanical simplicity and low losses dictate that the engine be designed for subsonic velocities in rotor and stator.

The velocity at state 2 relative to the rotor, which generates the first shock, is limited by either choked flow of hot gas leaving the blade nozzles, $M_{14e}=1$, or sonic velocity of the flow at state 2, $M_2=1$. $M_{14e}=1$ determines $M_{14}$ upstream of the nozzle exit through the nozzle area ratio $A_e/A_c$, Eq. (A-8).

If blade end nozzles are unchoked, $M_2=1$ is taken at design point operation. Then

$$M_2 = M_{14e}(c_{14}/c_2) \quad (1)$$

Once $M_2$ is known, state 3 is obtained by the shock Eqs. (A-12) to (A-14), Appendix A, noting that $M_3=0$. In either case $c_{14}/c_2$ must be determined by the rotor compression process through states 2, 3, 5 and 10 and isentropic expansion to state 14, $p_{14}=p_2=p_1$. This procedure is described in the next two Sections.

**Second Stage Compression**

The second stage compression results from heat addition in the combustor to flow from the high pressure air port 6. This heated flow then reenters the rotor at port 5. Above a minimum pressure $p_5$ and rotor speed $V_w$ the rate of increase of $W_5$ with $p_5$ is greater than the rate of increase of $W_6$ with $p_6$ smaller, $a$. The fractional pressure drop in the combustor loop is $1-a$. At low pressure $p_5$ the flow off the rotor is larger than the flow onto the rotor. Because $W_6$ flows through the combustor and becomes $W_5$ the pressures $p_5$ and $p_6$ increase until at steady state the flows onto and off the rotor become equal. Flow $W_5$ increases more rapidly with pressure $p_5$ because the flow per unit area is greater at lower Mach numbers; $M_5>M_7$ or $M_{ea}$ because of the temperature rise in the combustor. The tangential momentum of flows off the rotor accelerates...
the rotor, as does the inflow of tangential momentum at port 1.

The largest velocity \( U_5 \) relative to the blades yields the largest shock pressure ratio, \( p_5'/p_3' \). This relative velocity is obtained from the minimum absolute velocity \( U_{bg} \), when \( U_{bg} \) is axial. Thus the velocity in port 5 is set axial. For zero angle of attack to the blades of the relative velocity \( U_5 \), the rotor pitch line speed \( U_0 \) is set equal in magnitude to the tangential component of the relative velocity, \( U_5 \sin(B_i) \). Later the blade inlet angle \( B_i \) is set and the stator inlet angle \( B_1 \) at state 1 is calculated for zero angle of attack for \( U_{bg} \).

First, for each value of \( c_5/c_6 \) selected the procedure for second stage compression
solution is as outlined below for each \( A_5/A_6 \):

1. Select a value of \( M_4 \).
2. Calculate \( c_4/c_3 \) and \( p_4/p_3 \) with wave Eqs. (A-12) to (A-14).
3. Vary \( p_7/p_4 \) until \( A_5/A_6 \) becomes the value
   selected. Note that \( p_5/p_4 = p_6/p_4 = 1/a \) where
   \( a = (p_2/p_1) \) is the fractional pressure drop in the combustor loop. For the nozzle flow
   \( p_5/p_4 = (p_7/p_4)' \) a
4. Vary \( p_7/p_5 \) until \( M_5 \) is the same
   calculated from states 8-5-4-7-9 as when
calculated from states 8-9. The wave Eqs. (A-9)
to (A-14) are used.
5. Calculate state 9' by noting that the
   nozzle exit pressure from state 9' is the same
   as that of the nozzle exit pressure from state 7,
   or \( p_{9gr} = p_{97} = p_6 \).
6. Locate points where shocks "c" and inter-
   face "j" reach the upstream and downstream
   ends of the blade passages respectively. Also
   locate the point where shock "h" reaches the
downstream end of the blades. These points
   relative to the leading edge of port 5 are
   used to locate port edges and determine the
   flow of gas onto and off the rotor at ports 5
   and 6. Waves downstream of state 9' are weak;
   thus, changes in pressure between state 9' and
   the trailing edge of the exit port 6 are
   neglected.
7. Vary the selection of \( M_4 \) in Step 1 above
   until the flows \( W_5 \) and \( W_6 \) become equal.

Port Sizes and Locations

To determine port widths note that in the
time that it takes waves and/or interfaces to
travel down and back up the blade passages the
rotor pitch line travels a distance \( Y = t \cdot Vw \).
For example, to locate the point on the inlet
air port 1

\[
t_1 = Y_1/V_w = t_w = L/U_2 + L/(M_2 \cdot c_2 - U_2)
\]

or for the width of port 5

\[
t_5 = Y_5/V_w = Lb/U_5 + Lb/(M_5 \cdot c_5 - U_5)
\]

\[
Y_5 \cdot c_3/(L \cdot V_w) = c_5 \cdot c_2 \cdot Lb/[1/M_5 + 1/(M_5 \cdot c_5 - U_2)]
\]

\[
Lb/L, Y_6 \cdot c_5/(L \cdot V_w), \text{ and other non-dimensional port widths are obtained in a similar manner.}
\]
The angle of the blades \( B_5 \) has no effect on the
width of  a port, because the relative velocities are in the same direction as the
blade lengths, \( L \) and \( Lb \). After all port sizes are calculated as if the blades were in the
axial direction, the entire exit stator with
its ports is rotated a distance \( L \cdot \sin(B_i) \)
outside to the direction of rotation, designated as \( Y_0 \) or offset in Table 2.

Flow and Flow Continuity

Flow \( W_5 \) may be less than, equal to, or
greater than \( W_7 \) depending upon whether ports 5
and 6 are shorter or longer than the wave and
interface geometry of Fig. 2. If the port
width \( Y_6 \) is shorter, interface "j" and cool
air enters region 10 decreasing the final
compression temperature \( T_{10}, \) and \( p_{10} \).
In this case \( W_5 \) becomes less than \( W_7, \) net work de-
creases faster than the heat added; the cycle
efficiency decreases. If ports 5 and 6 are
longer than shown in Fig. 2, \( W_5 \) becomes greater-

\[
W_5 = \rho_5 \cdot U_5 \cdot \cos(B_i) \cdot Y_5 \cdot H = W_6
\]

\[
W_6 = \rho_6 \cdot U_7 \cdot \cos(B_1) \cdot Y_7 \cdot H + \rho_9 \cdot U_9 \cdot \cos(B_1) \cdot Y_9 \cdot H
\]

where \( \rho_9 \) is assumed nearly constant until
the interface reaches the downstream end of
the blade passages, because waves in this
region are weak. After the expansion fan
emanating at the closing of port 5 and the
shock emanating at the closing of port 6
intersect, the final mixed state 10 tempera-
ture is calculated with the energy equation.
Weber (1978) shows that the state 10 tempera-
ture is also given by

\[
T_{10} = T_3 + k \cdot (T_{05} - T_{06})
\]

Flow at states 3 and 10 is trapped between
stator end walls. Because flow \( W_5 = W_6, \) flow
\( W_{10} = W_7. \) Specific volumes at states 3 and
10 must be equal, because \( W_{10} \) and \( W_7 \) are
carried in the same area at the same velocity.
Because the gases are assumed perfect, \( p_6/p_{10} = T_3/T_{10}. \) This process may be considered the
equivalent of constant volume combustion.
Expansion

After the flow has been trapped between end walls at state 10, expansion from the rotor occurs. To determine the width of ports 12 and 15 the single wave approximation of an expansion fan is used as shown in Fig. 2. Wave Eqs. (A-9) to (A-11) are used to obtain a relation for \( P_{10}/P_2 = P_{10}^e/P_4^e \) expansion. Mach number \( M_1 \) is varied until \( P_{10}/P_2 = (P_{10}/P_2)^{\text{compr.}} \) i.e., this procedure yields the \( M_1 \) for which the rotor expansion pressure ratio equals the rotor compression pressure ratio. Eqs. (A-9) to (A-11) also yield the pressures \( P_{11}/P_{10} \) the temperatures \( T_{12}/T_{10} \) and \( T_{15}/T_{10} \) or \( T_{15}/T_{a} \). Smallers \( \phi_0/\phi_e \) and lower combustion temperatures result in choking of the nozzle from state 14. Larger \( \phi_0/\phi_e \) and higher combustion temperatures result in \( M_2=1 \). Rotor inlet temperature is calculated, depending upon precompression from

\[
T_5/T_a = (c_5/c_3)^2 \cdot (c_3/c_1)^{2 \cdot (T_1/T_a)} \tag{9}
\]

Cycle Efficiency, Power and Rotor Size

Cycle efficiency is obtained by evaluating \( \eta = \text{net work/heat added} \).

The net power may be written as

\[
P = W_1 \cdot c_0 \cdot (T_5 - T_6) \cdot \eta \tag{10}
\]

\[
W_1 = W_3 = \rho_3 \cdot W \cdot H \cdot L \cdot \cos(B_i) \tag{11}
\]

With the equation of state the power density becomes

\[
P/(HL \cdot a \cdot c_a) = k/(k-1) \cdot P_1/P_a \cdot P_3/P_2 \cdot c_a/c_1 \cdot c_2/c_3 \cdot c_4/c_3 \cdot M_3 \cdot \cos(B_3) \cdot \sin(B_3) \cdot T_5/T_a \cdot (1 - T_6/T_5) \cdot \eta \tag{12}
\]

If the precompression pressure and temperature ratios and the combustion temperature are constant, the only variable is the blade angle \( B_i \). Thus the power density is maximum at \( B_i = 45^\circ \).

A sample calculation leading to the last equations is shown in Table 1 for \( c_3/c_5 = 0.5 \). Other values of \( c_3/c_5 \) yield results plotted in Figs. 4 and 5.

Design speed \( \dot{W} = c_3 = M_3 \cdot c_4/c_3 \cdot \sin(B_3) \).

After all non-dimensional port widths and power density are calculated, power and \( B_i \) are selected. Blade height, \( H \), pitch, \( s \), and thickness, \( t \), are selected so that the blades are rigid and the free flow area is relatively large (84% for designs 2 and 3 of Table 2). With an 0.079 cm thick shroud around the blades the entire structure is very rigid. No critical speed should exist between zero and design speed. The two end wall widths between ports 1 and 5 and ports 6 and 12 are arbitrarily taken as 4 s.

DISCUSSION

It is found that extremely long rotor periphery is required for the smaller \( \phi_0/\phi_e \) considered. This is due to the fact that the nozzle acts as a flow restriction. Also area ratio \( \phi_0/\phi_e = 1 \) is selected for rotor design, because it yields maximum cycle efficiency for the combustion temperatures of interest as shown in Fig. 4. Power density decreases slightly at the larger values of \( \phi_0/\phi_e \) in Fig. 5, but it is near its maximum value at \( \phi_0/\phi_e = 1 \), and in the short blade lengths.

Table 2 presents three designs for a 500 hp net engine. The first has maximum power density and smallest rotor volume at blade angle \( B_1 = 60^\circ \). This design results in an axial rotor length \( L_x = 3.19 \) cm for the highest temperature ratio \( T_5/T_a = 6.5 \). It requires a precompression stagnation pressure ratio of 3.7, although only the static pressure ratio of 2.63 must be produced. The second design has \( B_1 = 48^\circ \). This design has a smaller power density and larger rotor volume. However, it has a shorter axial length \( L_x = 2.59 \) cm. It requires a precompression stagnation pressure ratio of only 3.18 with the same static pressure ratio as the first design.

The prototype wave engine built by GPC (Weber, 1978 and Coleman, 1984) had a rotor blade area ratio \( \phi_0/\phi_e = 0.5 \) and a \( B_1 = 48^\circ \). At low combustion temperature this flow restriction resulted in the expansion waves from closing of flow from port 5 intersecting the interface "i" before it reached the downstream ends of the blades. Flows \( W_5 \) and \( W_6 \) became less than \( W_3 \) for temperature ratios \( T_5/T_a < 3.55 \) for \( \phi_0/\phi_e = 0.5 \). The GPC engine was operated at \( T_5 = 1123 \) K or \( T_5/T_a = 3.74 \). The pitchline speed was 108 m/s which was 56% of the design speed of 194 m/s. The flow ratio \( W_5/W_1 \) was 0.52. The over scavenging flow was not reported. However, the temperature ratio \( T_5/T_a \) was large enough to permit the flow \( W_6 \) to be equal to \( W_3 \). The pitchline speed of operation was too low, so that the shock wave system fell inside ports 5 and 6. Thus, there was no flow onto the rotor downstream of the point where shock "i" reached the hot gas port 5. The calculations of this paper are consistent with the measured \( W_5/W_1 \).

![Fig. 4 CYCLE EFFICIENCY](http://manufacturingscience.asmedigitalcollection.asme.org/GT/proceedings-pdf/GT1991/78989/V001T01A002/2400329/v001t01a002-91-gt-004.pdf)
Losses

Shock losses are included in the shock equations of Appendix A. Fractional pressure loss \( \frac{P_5 - P_6}{P_a} \) is taken as 0.05 in the combustor loop for reentry of high pressure air as hot gas.

Because blade passages do not open instantaneously, initial compression waves may not coalesce to a shock between blade inlet and exit. For rotor speeds of Table 2 the opening time of a blade passage varies from 20 to 30 \( \mu \text{s} \). The ratio of opening time to wave travel time down the passage length is 0.09 to 0.18. The GPC rotor (Weber, 1978 and Coleman, 1984) had a blade length to passage width, \( L/D \), of 5.6. Designs of Table 2 have \( L/D \) of 8 to 16. It is not certain how far the time of wave coalescence requires the location of port 6 to be moved in the direction of rotation.

Measurements on the GPC rotor indicated that shock "b" of Fig. 2 impinged on the downstream end wall almost one pitch beyond predictions using instantaneous opening. Data of Kántorwik, A. at Cornell Aeronautical Labs indicate similar results with \( L/D=6 \). Thus port 6 should be moved in the direction of rotation.

The largest loss can result from end wall and peripheral leakage from the rotor. Leakage is minimized by small end wall and peripheral clearances. Short axial length and small diameter rotors permit designs of small clearance, because expansion is less for a given temperature rise of the rotor. The short axial length rotors are a direct result of high speed rotation (\( \text{Vw}=190 \) to 275 m/s). For high speed rotation the peripheral width of a port increases, and more flow enters the rotor before shocks “b” and “c” return to the port inlet. This result is shown in Table 1 where the dimensionless parameters are \( \frac{Y_{15} Y_{15}^{1/2}}{V_a L} \). These parameters result from the shock and flow equations of Appendix A.

Mixing at the gas-air interfaces increases the temperature of the air being compressed and decreases the temperature of the expanding gas. However, in a private communication Berchtold, consultant to Brown Boveri, stated that this effect is insignificant in the pressure produced by the wave rotor. Measurements by Thayer (1981) indicated that the width of the mixing region was about 1.2 times the rotor passage width. This result suggests that much of the mixing is caused by the vortex which is rolled up by the flow jetting into a blade passage as it begins to open. The vortex resulting from this gradual opening is expected to be about one passage width in diameter.

Entry and exit losses from the blades are not included. Exact information on the pressure loss in the combustor reentry loop is not available. Consequently this pressure drop (taken as 5% in this paper) can be adjusted to include rotor inlet and exit losses.

Flow rates in ports 1, 5, 6, 12 and 15 may be calculated from a modified Eq. (6) and the information in Table 2 or Eq.

\[
W_6 = k \cdot \frac{L_6 \cdot p_5 / p_a \cdot c_a / c_6}{1 - \frac{M_6 \cdot p_a}{c_6}} (13)
\]

CONCLUSIONS

Wave rotor designs presented in Table 2 represent extremely compact, high pressure ratio, high temperature ratio engines. The advantages of wave rotors discussed in this paper are incorporated in the wave rotor designs presented. The wave rotor is coupled with a centrifugal precompressor and two radial flow turbines. Design temperature ratios of most interest are \( \frac{T_5}{T_a} = 5.5 \) to 6.5 or combustion temperatures of 1600 to 1900\( ^\circ \text{K} \). Rotor axial lengths are small; end wall clearances can also be small, resulting in better control of leakage. Rotor blades are straight without bends for outlet nozzles. Therefore, rotors are easily fabricated, milled or cast even with a peripheral shroud around the blade tips.

All designs presented require the precompressor and wave rotor to be powered by the low pressure turbine and a small fraction of the high pressure turbine. Slightly below design speed the rotor will produce power because the momentum outflow from the rotor and inflow to the rotor produces thrust in the direction of rotation.

APPENDIX A

SUMMARY OF COMMONLY USED GAS DYNAMIC EQUATIONS

Equations referred to in this paper are summarized below. In addition to the equation of state for a perfect gas,

\[
p = \rho RT \quad (A-1)
\]

the specific heats may be written

\[
c_p = \frac{dh}{dT} \quad (A-2)
\]

\[
c_p - c_v = R \quad (A-3)
\]

\[
k = c_p / c_v \quad (A-4)
\]

Specific heats are assumed constant. The speed of sound is

\[
c^2 = kRT \quad (A-5)
\]
The following adiabatic and/or isentropic, one-dimensional, steady flow equations are used between states to calculate states in regions of Fig. 2 where no waves are crossed:

\[ T_0/T_1 = 1 + (k-1)/2 \cdot M^2 \]  

\[ p_0/p = (T_0/T_1)^k/(k-1) \]  

\[ A/A^* = 1/M^2 [(k+1)+1+(k-1)/2] \cdot M^2 \]  

\[ T_2/T_1 = (c_2/c_1)^2 \]  

\[ p_2/p_1 = (c_2/c_1)^{2k/(k-1)} \]  

The following adiabatic and/or isentropic, one-dimensional, steady flow equations are used when expansion waves are crossed between regions of Fig. 2 where shock waves are crossed between regions of Fig. 2. Numbers 1 and 2 indicate states before and after the wave.

\[ c_2/c_1 = 1 + (k-1)/2 \cdot (U_2-U_1)/c_1 \]  

The + or - sign is taken dependent upon the wave direction, noting that the speed of sound, temperature and pressure decrease after passage of an expansion wave.

The following one-dimensional flow equations are used in calculation of states when shock waves are crossed between regions of Fig. 2. Numbers 1 and 2 indicate states before and after the wave.

\[ T_2/T_1 = 1+2(k-1)/k \cdot M^2 \]  

\[ (U_2-U_1)/c_1 = \pm 2/(k+1) \cdot M^2 \]  

\[ p_2/p_1 = 1 + 2k/(k+1) \cdot M^2 \]  

### TABLE 1. EFFECT OF Ae/At on CYCLE EFFICIENCY AND POWER DENSITY

<table>
<thead>
<tr>
<th>Ae/At</th>
<th>Cycle Efficiency (%)</th>
<th>Power Density (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>c3/c5=0.5</td>
<td>0.5000</td>
<td>0.6000</td>
</tr>
<tr>
<td>M4</td>
<td>0.7456</td>
<td>0.7350</td>
</tr>
<tr>
<td>p4/p5</td>
<td>1.6716</td>
<td>1.6574</td>
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<tr>
<td>p5/p6=.95</td>
<td>3.0932</td>
<td>3.0380</td>
</tr>
<tr>
<td>c4/c3</td>
<td>1.1990</td>
<td>1.1950</td>
</tr>
<tr>
<td>p7/p4</td>
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<td>1.5780</td>
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<td>Mx4</td>
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<td>1.2229</td>
</tr>
<tr>
<td>c7/c4</td>
<td>1.0854</td>
<td>1.0568</td>
</tr>
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<td>p0/p1</td>
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TABLE 2 ENGINES (pa=1 atm, Ta=294 K, 500 HP, H=1.588 cm, 10 =0.051 cm)

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REFERENCES