



The Society shall not be responsible for statements or opinions advanced in papers or discussion at meetings of the Society or of its Divisions or Sections, or printed in its publications. Discussion is printed only if the paper is published in an ASME Journal. Authorization to photocopy material for internal or personal use under circumstance not falling within the fair use provisions of the Copyright Act is granted by ASME to libraries and other users registered with the Copyright Clearance Center (CCC) Transactional Reporting Service provided that the base fee of \$0.30 per page is paid directly to the CCC, 27 Congress Street, Salem MA 01970. Requests for special permission or bulk reproduction should be addressed to the ASME Technical Publishing Department.

Copyright © 1996 by ASME

All Rights Reserved

Printed in U.S.A.

## WAKE MIXING IN AXIAL FLOW COMPRESSORS

John J. Adamczyk  
NASA Lewis Research Center  
Cleveland OH 44135



### ABSTRACT

Over the years it has been speculated that the performance of multi-stage axial flow compressors is enhanced by the passage of a wake through a blade row prior to being mixed-out by viscous diffusion. The link between wake mixing and performance depends on the ability to recover the total pressure deficit of a wake by a reversible flow process. This paper shows that such a process exists, it is unsteady, and is associated with the kinematics of the wake vorticity field. The analysis shows that the benefits of wake total pressure recovery can be estimated from linear theory and quantified in terms of a volume integral involving the deterministic stress and the mean strain rate. In the limit of large reduced frequency the recovery process is shown to be a direct function of blade circulation. Results are presented which show that the recovery process can reduce the wake mixing loss by as much as seventy percent. Under certain circumstances this can lead to nearly a point improvement in stage efficiency, a nontrivial amount.

### INTRODUCTION

A number of publications have dealt with the passage of wakes through a turbomachinery blade row. One of the first was the publication by Kemp and Sears (1955). In their work they examined the unsteady lift and moment generated by a series of wakes encountering an isolated blade row. Their analysis was based on unsteady thin airfoil theory. Later Kemp and Sears (1956) published a paper in which they derived an expression from which one could estimate the kinetic energy of the unsteady velocity field associated with the vorticity shed off the trailing edge of each blade as a result of a wake encounter.

Ashby (1957) presented a model for estimating the recovery of the total pressure deficit of a wake as it passed through a blade row. Ashby assumed the flow processes involved with wake recovery were quasi-steady. In (1966) Smith published a paper in which he showed that the velocity deficit of a wake could be attenuated as it passed

through a compressor rotor by kinematic flow processes involving the wake vorticity field.

In a publication by Smith (1970), experimental results were shown that indicated that reducing the axial gap between blade rows in a multi-stage axial flow compressor increased the pressure rise and efficiency for a given flow coefficient. Similar findings were reported by Mikolajczak (1977). Smith hypothesized that the increase in performance was due to the kinematics associated with the passage of wakes through blade rows as outlined in his 1966 publication. Smith elaborated further on this subject in his 1993 paper titled "Wake Ingestion Propulsion Benefit", and in a discussion of a paper by Fritsch and Giles (1995).

There are a number of related theoretical analyses which deal with the topic of inlet total pressure distortions which are relevant to the topic of wake recovery. Among these works are those of Hawthorne (1970), Horlock and Daneshy (1971), and Henderson (1982). There were also a number of experimental investigations which examined the passage of a wake through a blade row which are also relevant to the issue of wake recovery as for example the work of Okiishi et. al. (1985).

With the advancements in CFD, a number of researchers have begun examining the wake recovery process numerically. One of the first publications to do so was Fritsch and Giles (1995). Their work and that by Dregel and Tan (1996) raised a number of important issues some of which are addressed in this paper.

The above cited works have provided much useful information. However, there are a number of questions which remain to be answered. For example: is there a performance gain to be had by having a wake mix-out prior to or after it has encountered a blade row, Greitzer (1994)? Are the processes associated with recovery quasi-steady or unsteady? If the processes are unsteady then what is the dependence on wake passing frequency and wake profile? Are the flow processes kinematic as assumed by Smith (1966) or are they dynamic as a model based on the work of Kemp and Sears (1956) would imply? Finally, and perhaps the

most relevant question is whether the recovery of the total pressure deficit of a wake is of sufficient magnitude to warrant further investigation.

These questions along with several others provide the motivation for the present work. The answers to these questions provide a deeper understanding of the impact of unsteady flow processes on the performance of multistage axial flow compressors. The present analysis neglects the effects of viscosity and compressibility and considers the flow to be two dimensional. As such, the analysis provides a meaningful upper bound for the magnitude of the wake recovery process.

## ANALYSIS

### Mixing Loss

Consider a 2 -  $D$  incompressible flow which is varying periodically in time with a period of  $T$ . The velocity field associated with this flow field can be decomposed into a time average component,  $\bar{u}_i(x, y)$ , and an unsteady component,  $u'_i(x, y, t)$ . The coordinate system is oriented such

that the  $x$  axis is normal to the inlet plane of a cascade of airfoils through which the unsteady flow is passing. The time average component is assumed to be spatially periodic in the  $y$  direction over a length  $L$ .  $\bar{u}_i(x, y)$  is further decomposed into a component  $U_i(x)$  which is uniform in  $y$  and a component  $\hat{u}_i(x, y)$  which is periodic in  $y$ . Thus,

$$u_i(x, y, t) = U_i(x) + \hat{u}_i(x, y) + u'_i(x, y, t) \quad (1)$$

A similar decomposition is constructed for the pressure field. The velocity field is nondimensionalized with respect to the magnitude of  $U_i(x)$ ,  $q_{ref}$ , at a location  $x_{ref}$ . All lengths are normalized with respect to the chord length of the cascade through which the flow is passing, while time,  $t$ , is normalized with respect to chord length divided by  $q_{ref}$ . The density of the flow is assumed to be uniform and is nondimensionalized by its value at  $x_{ref}$ . The nondimensionalized density is one and thus will not appear in the equations which follow. The pressure is nondimensionalized by the product of the reference density and  $q_{ref}$  squared.

## NOMENCLATURE

$A_n$	Fourier coefficients	$u_i$	velocity vector
$A_t$	time average operator	$u, v$	$x, y$ velocity components
$A_y$	pitchwise average operator	$W$	wheel speed
$C$	wake velocity deficit	$X$	loss in total pressure
$C_0, C_1, C_2$	constants	$x^i, x, y, \bar{y}$	coordinate directions
$dVol$	differential volumes	$x_{ref}$	$x$ reference location
$f$	variable	$\alpha$	constant
$h_r$	wake pitch	$\beta_r$	flow angle relative to rotor
$K_{in}$	kinetic energy of the first order unsteady velocity field	$\beta_1, \beta_2$	flow angle at stator inlet, exit respectively
$k_n$	reduced frequency	$\Gamma$	airfoil circulation
$L$	stator pitch, pitchwise length scale	$\Delta$	drift function
$L_{en1}, L_{en2}$	length of wake element at stator inlet, exit respectively	$\epsilon$	perturbation parameter
$n$	integer	$\zeta_w$	vorticity
$q$	flow speed	$\psi_w$	stream function
$q_{ref}$	reference speed		
$P$	total pressure	Subscripts	
$p$	pressure	$mix$	mixed out state
$R$	recovery parameter	Superscripts	
$s$	distance along streamline	-	time average state
$T$	time scale	'	unsteady component
$t$	time	-	non axisymmetric component
$U_i$	pitchwise average velocity		

Two averaging operators are defined for use later in this work. The first is the averaging operator in time,

$$A_t f = \frac{1}{T} \int_0^T f dt \quad (2)$$

and the second the averaging operator in  $y$

$$A_y f = \frac{1}{L} \int_0^L f dy \quad (3)$$

where  $f$  is any flow variable.

In the absence of a cascade downstream of  $x_{ref}$  the unsteady flow would be mixed to a uniform flow independent of time by viscous diffusion as it convects downstream. The mixing associated with viscosity results in an irreversible loss in total pressure.

One may estimate the loss in total pressure of an unsteady two dimensional flow due to viscous diffusion by considering the flow through a control volume with the inlet face at  $x_{ref}$ , and the exit at downstream infinity. Across this control volume, mass, specific impulse in the  $x$  direction, and  $y$  momentum are conserved. The equations associated with these conserved variables are:

$$A_y A_t u|_{x_{ref}} = U_{mix} \quad (4)$$

$$A_y A_t (p + u^2)|_{x_{ref}} = p_{mix} + U_{mix}^2 \quad (5)$$

$$A_y A_t uv|_{x_{ref}} = U_{mix} V_{mix} \quad (6)$$

where  $u$  is the  $x$  component of the velocity,  $v$  is the  $y$  component and  $p$  the pressure. The subscript  $mix$  refers to the mixed-out flow state at downstream infinity. These equations are derived by averaging the continuity, and momentum equation in time and space. Flow continuity is expressed by equation (4) which states that the average over both time and  $y$  of the  $x$  component of velocity  $u$  at the inlet to the control volume must equal the mixed-out velocity component  $U_{mix}$ . Equation (5) states that the time average of the specific impulse across the inlet to the control volume is equal to that at the exit, while equation (6) is a similar statement for the  $y$  momentum.

The difference in the time average mass flux of total pressure between the inlet and the exit of the control volume is defined as the loss in total pressure due to viscous diffusion. The loss is given by the expression

$$X = A_y A_t u P|_{x_{ref}} - U_{mix} P_{mix} \quad (7)$$

where  $P$  is the total pressure at the control volume inlet and  $P_{mix}$  is the total pressure at the exit. The total pressure at either location is related to the pressure and velocity field by the equation

$$P = p + \frac{1}{2}(u^2 + v^2) \quad (8)$$

Upon introducing the velocity and pressure decompositions, i.e., equation (1) and the corresponding equation for pressure defined previously, into equations (4)-(6), one obtains an expression for the pressure, and the two velocity components at the exit of the control volume which involves the unsteady and spatially nonuniform components of the velocity and pressure field entering the control volume. These expressions for the flow variables exiting the control volume, when combined with equations (7) and (8), yields the following equation for the loss in total pressure

$$\begin{aligned} X = & A_y A_t (p' u') + A_y (\hat{p} \hat{u}) \\ & + A_y A_t \left[ \frac{1}{2} u (u'^2 + v'^2) \right] \\ & + A_y \left[ \frac{1}{2} \hat{u} (\hat{u}^2 + \hat{v}^2) \right] \\ & + A_y A_t [u' (\hat{u} u' + \hat{v} v')] \\ & - \frac{1}{2 U_{mix}} A_y A_t [\hat{u} \hat{v} + u' v']^2 \end{aligned} \quad (9)$$

The first and second term in equation (9) represent the pressure work at the inlet to the control volume associated with the unsteady and spatially nonuniform components of the flow field. The third term is the flux in kinetic energy at the inlet to the control volume associated with the unsteady velocity field, while the fourth term is the flux in kinetic energy of the spatial nonuniform velocity field at the inlet to the control volume. The remaining terms are the result of coupling between the unsteady and spatial nonuniform velocity components.

Consider two flow situations both of which are depicted in Fig. 1. In the first the unsteady flow associated with a series of wakes is mixed-out across plane 1 prior to entering the cascade. In the second situation the unsteady flow passes through the cascade and is mixed-out across plane 2. The mixing loss at plane 1 is denoted as  $X_1$  and that at plane 2  $X_2$ . The question as to whether compressor performance would improve if a wake were mixed out before or after passing through a blade row, which was raised by Greitzer (1994), is expressed by the statement,

$$\text{is } X_1 - X_2 \text{ positive, negative, or zero?} \quad (10)$$

The answer to question (10) as well as the value of the difference  $X_1 - X_2$ , will be established neglecting the effects of viscosity on the wakes as they pass through the cascade. The resulting estimate is thus an upper bound for the difference and as such provides an estimate of the relevancy of the wake recovery process to compressor performance.

The analysis proceeds by assuming the unsteady flow field to be a small perturbation of order  $\epsilon$  to the time average

flow field. The velocity field is assumed to be represented by a perturbation series in  $\epsilon$  which to  $O(\epsilon^2)$  is

$$u_i(x, y, t) = u_{i0}(x, y) + \epsilon u_{i1}(x, y, t) + \epsilon^2 u_{i2}(x, y, t) \quad (11)$$

A similar expression is assumed for the pressure field. In equation (11),  $u_{i0}$  represents the velocity of the base flow which, based on the stated assumptions, is a potential flow.  $u_{i1}$  is a linear perturbation to the base flow velocity field, while  $u_{i2}$  is the result of nonlinear interactions. The time average of  $u_{i1}$  is zero.

The time average velocity field to  $O(\epsilon^2)$  is

$$\bar{u}_i(x, y) = u_{i0}(x, y) + \epsilon^2 A_t u_{i2}(x, y, t) \quad (12)$$

while the corresponding unsteady velocity component (i.e.  $O(\epsilon^2)$ ) is

$$u'_i(x, y, t) = \epsilon u_{i1}(x, y, t) + \epsilon^2 (u_{i2}(x, y, t) - A_t u_{i2}(x, y, t)) \quad (13)$$

Once again similar expressions can be written for the pressure field.

The mixing planes 1 and 2 are located sufficiently far from the cascade such that the variation with  $y$  of the base flow is of  $O(\epsilon^2)$  or greater. In addition, these planes are located at an axial location such that the variations with  $y$  of the unsteady pressure component  $p_1$  is of  $O(\epsilon^2)$  or greater. That such locations can be established results from the fact that the base flow is a potential flow and the flow is incompressible.

The wake mixing loss at plane 1, located upstream of the cascade,  $X_1$ , is found by introducing the expression for the steady and unsteady velocity components as given by equations (12) and (13) along with the corresponding expressions for the pressure components into equation (9). The wake mixing loss at plane 2, located downstream of the cascade,  $X_2$ , is found by a similar substitution. The resulting expression for the difference  $X_1 - X_2$  is

$$X_1 - X_2 = K_{in_1} R \quad (14)$$

$$= K_{in_1} (1 - K_{in_2}/K_{in_1}) + O(\epsilon^3)$$

$$K_{in_1} = \frac{1}{2} u_0 A_y A_t (u_1^2 + v_1^2)|_{x_1} \quad (15)$$

$$K_{in_2} = \frac{1}{2} u_0 A_y A_t (u_1^2 + v_1^2)|_{x_2} \quad (16)$$

where  $K_{in_1}$  represents the flux in kinetic energy of the unsteady first order velocity component  $u_{i1}$  entering plane 1 and  $K_{in_2}$  the flux in kinetic energy of  $u_{i1}$  entering plane 2.  $R$  is defined as the recovery parameter and is analogous to the definition used by Smith (1993). If the value of  $R$  is one

the unsteady flow exiting the cascade has been completely mixed out by a reversible process prior to encountering the mixing plane at 2. For values of  $R$  less than zero the kinetic energy of  $u_{i1}$  has increased as a result of passing through the cascade. Values of  $R$  between zero and one implies there is a transfer of energy from the unsteady flow to the time average flow state.

Equation (14) states that the recovery process is linked to the change in the kinetic energy of the first order velocity components across the cascade. This result is significant, for it states that the magnitude of the recovery process can be obtained from linear theory while being correct to  $O(\epsilon^2)$ . Note that this is not the case if one is interested in obtaining estimates for the flux in total pressure. Estimates of the flux in total pressure requires the flow field to be evaluated to  $O(\epsilon^2)$ .

Before this section is concluded one additional result will be derived which links the recovery process to the unsteady flow field within the cascade passage. This result comes directly from the kinetic energy associated with the first order velocity component,  $u_{i1}$ . The kinetic energy equation for this component is formed from the vector dot product of  $u_{i1}$  and its momentum equation averaged over time. The kinetic energy for  $u_{i1}$  is,

$$\frac{\partial}{\partial x^i} u_{i0} A_t \frac{(u_{j1} u_{j1})}{2} + A_t (u_{i1} u_{j1}) \frac{\partial u_{i0}}{\partial x^j} = - \frac{\partial}{\partial x^i} A_t p_1 u_{i1} \quad (17)$$

Integrating equation (17) over a control volume comprising one blade passage between plane 1 and 2 yields,

$$\int u_0 A_t \frac{(u_{i1} u_{i1})}{2} |_{x_1} dy - \int u_0 A_t \frac{(u_{i1} u_{i1})}{2} |_{x_2} dy = \int_{Vol} A_t u_{i1} u_{j1} \frac{\partial u_{i0}}{\partial x^j} dVol \quad (18)$$

In deriving this result the divergence theorem is used to convert the volume integral of the first and third term to area integrals. The contribution to the area integrals coming from a blade surface and the periodic surfaces vanishes because of flow tangency and periodicity respectively.

The value of the volume integral of the product  $A_t u_{i1} u_{j1} \frac{\partial u_{i0}}{\partial x^j}$  leads to either the production or reduction of the kinetic energy associated with  $u_{i1}$ . The tensor  $A_t u_{i1} u_{j1}$  represents the deterministic stress as defined by Adamczyk (1985) to  $O(\epsilon^2)$ , while the tensor  $\partial u_{i0}/\partial x^j$  is the rate of strain of the steady flow to  $O(\epsilon)$ . Upon combining equation (14) and (18), the difference  $X_1 - X_2$  becomes

$$X_1 - X_2 = \frac{1}{L} \int_{Vol} A_t \frac{(u_{i1} u_{j1})}{2} \frac{\partial u_{i0}}{\partial x^j} dVol + O(\epsilon^3) \quad (19)$$

Equation (19) links the the deterministic stress,  $A_t u_i u_j$ , directly to the wake recovery process.

The question of whether wake mixing loss is reduced or increased by having the wake pass through a blade row is answered by the sign of the volume integral of  $A_t u_i u_j \frac{\partial u_i}{\partial z}$ . If the sign of the integral is positive,  $X_1 > X_2$ , wake mixing loss is reduced by having the wake pass through a blade row. If the integral is negative, the opposite is true.

Equation (14) will be used in the subsequent analysis to estimate the magnitude of the wake recovery process. The next section focuses on estimating the kinetic energy of the unsteady velocity approaching the cascade,  $K_{in_1}$  in equation (14).

#### Wake Characterization

The origin of the incoming unsteady flow being analyzed is the wakes from an upstream blade row. Since the present analysis neglects the effect of viscosity on the flow within a cascade passage the incoming wakes are modeled as a series of one-dimensional shear flows. The velocity field produced by these wakes is periodic in the tangential direction and thus maybe expanded in a Fourier series in the  $y$  direction. Associated with each Fourier component is a reduced frequency defined by the equation

$$k_n = \frac{2\pi n}{h_r} W \quad (20)$$

where  $n$  is an integer,  $h_r$  is the tangential distance between the wake center lines divided by the airfoil chord,  $W$  is the wheel speed divided by the speed of the oncoming time average flow,  $q_{ref}$  (see Fig. 1). For  $n = 1$ , the reduced frequency is that associated with the wake passing frequency. The reduced frequency parameter is an estimate of the wave length of the incoming disturbance in the stream-wise flow direction relative to the cascade chord length, thus this parameter is a direct measure of the unsteady character of the flow field. Assuming values for the terms in equation (18) typical of those found in core compressors, the value of  $k$  for the first wake harmonic is approximately 5. A value of 0.5 is generally taken to be the limit above which an unsteady flow can no longer be assumed to behave in a quasi-steady manner. A value of 5 is an order of magnitude larger and implies that the unsteady flow associated with the wake recovery process is far from being quasi-steady.

The kinetic energy of the unsteady velocity field associated with a series of wakes each of which has a profile defined by the equation,

$$U_w = C \exp\left[-\alpha \left[\frac{2(y - Wt)\cos\beta_r}{\lambda_w}\right]^2\right] \quad (21)$$

is

$$K_{in_1} \equiv A_t \frac{u_i' u_i'}{2} \Big|_{z_1} = \frac{1}{2} A_t (A_t U_w - U_w)^2$$

$$= \frac{1}{2} \left(\frac{C}{2}\right)^2 \frac{\lambda_w}{(h_r \cos\beta_r)} \left[\sqrt{2} - \frac{\lambda_w}{(h_r \cos\beta_r)} \sqrt{\frac{\pi}{\alpha}}\right] \sqrt{\frac{\pi}{\alpha}} \quad (22)$$

where  $\alpha$  is a constant,  $C$  is the velocity deficit,  $\beta_r$  the relative flow direction at the cascade inlet and the remaining variables are defined in Fig. 1.

The Fourier coefficients of equation (21) are

$$A_n = \frac{C}{2} \sqrt{\frac{\pi}{\alpha}} \frac{\lambda_w}{(h_r \cos\beta_r)} \exp\left[-\frac{1}{\alpha} \left(\frac{n\pi}{2}\right)^2 \left(\frac{\lambda_w}{(h_r \cos\beta_r)}\right)^2\right] \quad (23)$$

The magnitude squared of each Fourier component is a measure of the energy contained in that component.

The ratio of the energy contained in the first harmonic to the kinetic energy of the unsteady velocity field set up by the wakes is plotted in Fig. 2 as a function of  $\lambda_w/(h_r \cos\beta_r)$ , the ratio of wake width to the wake pitch (i.e.  $\alpha = \pi$ , Kemp and Sears 1955). For values of  $\lambda_w/(h_r \cos\beta_r)$  less than 0.2, less than 35% percent of the disturbance energy is contained in the first harmonic, which necessitates the need to consider the higher harmonics in estimating the wake recovery. The results presented in Fig. 2 further substantiate that wake recovery is far from being a quasi-steady process.

In the next section estimates of the kinetic energy of the unsteady velocity field exiting a cascade are presented along with estimates of the recovery parameter.

## RESULTS

The analysis presented previously showed that the magnitude of the wake recovery process can be estimated from linear flow theory. In Verdon's (1987) use of this theory, which forms the basis of this section, each Fourier component of the incoming unsteady velocity field given by equation (11) is analyzed separately. The resulting flow fields are superimposed to yield the flow field generated by the incoming wakes. Verdon's analysis is based on the theory of rapid distortion as developed by Goldstein (1978) and amended by Atassi and Grzedzinski (1989). According to this theory the unsteady velocity field is defined in terms of a potential function and a vector whose curl is equal to the vorticity associated with the incoming disturbance. For a two dimensional incompressible flow this vector can be defined in terms of a stream function.

Far downstream of the cascade the velocity field is a result of the superposition of the velocity field of the wakes and the velocity field associated with the shed vorticity originating from the trailing edge of each airfoil in the cascade. The potential of the velocity field due to the shed vorticity is given in Verdon et al. (1975). The stream function associated with the velocity field of the incoming wakes is given in the appendix.

The velocity field generated by the shed vorticity is proportional to the jump in the potential across the trailing

edge of an airfoil and thus the unsteady circulation about that airfoil. The origin of this velocity field is a dynamic process involving the unsteady pressure field about the cascade. The unsteady velocity field downstream of the cascade associated with the incoming wake vorticity is the result of kinematic flow processes. Both of these vorticity fields contribute to the flux in kinetic energy of the unsteady velocity field exiting the cascade. The relative magnitude of the contribution from each vorticity field will be assessed in the analysis which follows.

In addition a number of questions which were posed in the INTRODUCTION remain to be answered. The answers to these questions will be obtained from a series of simulations from which estimates of the recovery parameter  $R$  (equation (14)) will be derived. The simulations were executed using a wake blade row interaction code called LINFLO developed by Verdon et. al. (1991). This code as previously stated is based on the theory of rapid distortion. LINFLO requires as input the steady potential flow field about a cascade of airfoils. This input was obtained from SFLOW, a code developed by Hoyniak (1994).

All of the simulated cascade configurations have a pitch to chord ratio of six tenths. The location of maximum airfoil thickness is at thirty percent of chord. The incoming flow is at an angle of fifty degrees with respect to the cascade inlet plane. The angle between the incoming wake and the flow entering the cascade is ninety degrees (i.e.  $\beta_1 - \beta_r = 90^\circ$ ), which implies that the angle between the absolute and relative flow direction at the cascade inlet is also ninety degrees.

The first set of results, Fig. 4, is for a cascade of uncambered airfoils whose stagger angle is fifty degrees with respect to the cascade inlet plane (see Fig. 3). The angle of the incoming steady flow is also fifty degrees with respect to the cascade inlet plane, hence the incidence angle is zero. The angle of the steady flow exiting the cascade is approximately fifty degrees with respect to the cascade inlet plane. Because these cascades generate virtually no flow turning the steady aerodynamic force acting on them is near zero as is the time average airfoil circulation. Figure 4 shows the recovery parameter  $R$  as a function of thickness to chord and reduced frequency. The values of reduced frequency span the range of wake passing frequency typical of a core compressor.  $R$  was estimated for discrete values of the thickness to chord ratio of 5, 10 and 15 percent. For a fixed value of reduced frequency, the dependency of  $R$  on thickness to chord ratio is slight. For a fixed value of thickness to chord ratio,  $R$  decreases as the reduced frequency becomes large. For a reduced frequency of ten the recovery is approximately thirty percent with every indication that it will become even smaller as the reduced frequency is increased further.

The results shown in Fig. 4 imply that the magnitude of the velocity field due to the shed vorticity approaches zero as the reduced frequency becomes large. This result is consistent with the findings of Kemp and Sears (1956) and is the result of the unsteady circulation about an airfoil approaching zero for large values of reduced frequency.

The next set of results, Fig. 5, are intended to examine the effect of flow turning due to a combination of airfoil camber and cascade stagger on the recovery process. A finite turning of the time average flow field by the cascade implies a finite time average airfoil circulation. The stagger of the cascades is fixed at seventy degrees with respect to the cascade inlet plane (see Fig. 3). The angle of the steady flow exiting the cascades is near ninety degrees with respect to the cascade inlet plane. The incoming flow is turned through forty degrees.

Figure 5 shows the values of the recovery parameter  $R$  as a function of reduced frequency for a cascade with 5% and 10% thickness to chord ratio. Also shown on this figure are the results taken from the previous figure for the 10% thickness to chord ratio cascade. Recall that the results shown on the previous figure were for zero turning of the steady flow.

At a fixed value of thickness to chord ratio, the dependence of the recovery parameter  $R$  on reduced frequency for the cascades with finite time average airfoil circulation is considerably less than that for the cascade of near zero time average airfoil circulation. Indeed, for the cascades with finite airfoil circulation, it appears that  $R$  becomes independent of reduced frequency as the reduced frequency becomes large. For the cascades with finite airfoil circulation the recovery process appears to be due to kinematic flow processes. If the processes were otherwise the recovery parameter would exhibit a dependency on reduced frequency similar to that shown in Fig. 4. In addition as in the case of the cascades of near zero airfoil circulation, Fig. 4, the effect of thickness to chord ratio on recovery is slight. Finally, note that for the cascades with finite airfoil circulation, the recovery factor is seventy percent, which is a significant value.

The result presented in Fig. 5 suggests that the recovery process is primarily dependent on the net turning of the steady flow and thus the circulation about an airfoil. The next set of results, Fig. 6, are intended to further explore this possibility. The recovery associated with a cascade of airfoils with zero camber, zero stagger, and 10% thickness to chord ratio, see Fig. 3, is compared to that of the 10% thickness to chord ratio cascade of Fig. 5. The angle of the steady flow approaching the zero stagger cascade is fifty degrees with respect to the cascade inlet plane. The flow exits this cascade at nearly ninety degrees with respect to the cascade inlet plane. Thus the flow turning is approxi-

mately forty degrees, the same as it is for the cascade from Fig. 5. The two plots in Fig. 6 show that for reduced frequencies between five and ten the recovery associated with the cascade of zero camber airfoils is only slightly less than that of the cascade of cambered airfoils. Furthermore, the recovery associated with the zero stagger cascade becomes independent of the reduced frequency as the reduced frequency becomes large as was the case in Fig. 5. These results indicate quite clearly that the unsteady wake recovery process in axial flow compressors is primarily a kinematic process related to the flow turning across a blade row and hence the time average airfoil circulation.

#### High Reduced Frequency Limit

The results shown in Figs. 5 and 6 strongly suggest that for large values of reduced frequency the wake recovery parameter asymptotes to a value which is independent of reduced frequency. This observation provides strong motivation for examining the kinetic energy of the unsteady flow field downstream of the cascade as the reduced frequency becomes large.

The unsteady velocity field downstream of the cascade is due to the shed vorticity and that associated with the incoming wake vorticity. The computational results as well as the work of others suggest that as the reduced frequency becomes large the contribution to the downstream unsteady velocity associated with the shed vorticity becomes small. Thus the downstream unsteady velocity field and its kinetic energy are due to the incoming wake vorticity. In the appendix of this paper an expression is given for the stream function associated with the incoming wake vorticity for axial locations far downstream of the cascade. The expression for the stream function can be expanded so as to include only those terms which decay the least with reduced frequency. The algebra involved is lengthy and will not be presented. The resulting expression can be used to obtain an estimate of the kinetic energy of the downstream velocity field in powers of  $1/k_n$ . The kinetic energy of the unsteady downstream velocity field to lowest order is

$$A_t \frac{u'_i u'_i}{2} |_{z_2} = K_{in_1} \left( \frac{\cos \beta_1}{\cos \beta_2} \right)^2 \frac{1}{\sin^2(\beta_r - \beta_1)} \cdot \left\{ 1 + \left( \frac{\cos \beta_1}{\cos \beta_2} \right)^2 \left[ \frac{\sin \beta_2}{\cos \beta_1} + \frac{\cos \beta_r}{\cos \beta_2 \sin(\beta_r - \beta_1)} - \frac{C_1}{\cos \beta_2} - \frac{2h_s}{\cos \beta_2} C_2 \frac{y}{L} \right]^2 \right\}^{-1} + O(1/k_n) \quad (24)$$

where  $K_{in_1}$  is the kinetic energy of the incoming unsteady velocity field,  $\beta_1$ ,  $\beta_2$  the inlet and exit flow angles respectively,  $L$  the cascade pitch to chord ratio.

In deriving equation (24) the drift function  $\Delta$ , which appears in equation (A3) of the appendix was approximated by the expression,

$$\Delta(x_0, \bar{y}) = C_0 + C_1 \bar{y} + C_2 \bar{y}^2 \quad (25)$$

where  $\bar{y}$  is restricted to lie between the periodic boundaries formed by the stagnation streamlines of two adjacent airfoils, and  $x_0$  is located downstream of the cascade. The drift function  $\Delta$ , Verdon (1987), is the integral along a streamline of the time averaged flow field of the inverse of the time averaged flow speed  $q$ ,

$$\Delta = \int \frac{ds}{q} \quad (26)$$

As such the drift function is singular at the stagnation point of a bluff body and along the streamline emanating from the stagnation point. This singular behavior is not accounted for by equation (25). It is assumed that the rapid spatial flow variations associated with the singular behavior of the drift function would be damped if the effects of viscosity were included in the analysis. This damping or smoothing of the flow features represents a loss source which the present analysis does not address.

The value of the constant  $C_1$  is related to the turning of the steady flow by the cascade, while the value of the constant  $C_2$  is a measure of airfoil thickness and the influence of the stagnation region.

Equation (24) shows that for large values of reduced frequency the kinetic energy of the unsteady velocity field downstream of the cascade associated with the incoming vorticity field is independent of the reduced frequency and only depends on the velocity triangle of the steady flow at the cascade inlet and exit, the value of  $C_2$ , which is a function of the thickness to chord ratio, and the pitch to chord ratio of the cascade. This is a rather surprising result given that it came from an unsteady analysis.

For a cascade of thin airfoils at zero incidence,  $C_2$  is zero. For such a configuration the wake recovery parameter is equal to

$$R = 1 - \left( \frac{\cos \beta_1}{\cos \beta_2} \right)^2 \frac{1}{\sin^2(\beta_r - \beta_1)} \cdot \left\{ 1 + \left( \frac{\cos \beta_1}{\cos \beta_2} \right)^2 \left[ \frac{\sin \beta_2}{\cos \beta_1} + \frac{\cos \beta_r}{\cos \beta_2 \sin(\beta_r - \beta_1)} - \frac{C_1}{\cos \beta_2} \right]^2 \right\}^{-1} + O(1/k_n) \quad (27)$$

independent of reduced frequency and also independent of the incoming wake profile. If one were to approximate the constant  $C_1$  by the expression

$$C_1 = \frac{1}{h_s} [\Delta|_{x_0, y_0=h_s} - \Delta|_{x_0, y_0=0+}] \quad (28)$$

one would obtain the result,

$$R = 1 - \left( \frac{L \epsilon n_1}{L \epsilon n_2} \right)^2 \quad (29)$$

where  $Len_1/Len_2$  is the ratio of the length of a wake element upstream of the cascade to its length downstream of the cascade. This result was first derived by Smith (1966). Smith suggested in his 1993 publication that  $C_1$  may be approximated as,

$$C_1 = \frac{4\Gamma}{L} \left(1 + \frac{\cos\beta_1}{\cos\beta_2}\right)^{-2} \quad (30)$$

which is based on the work in an earlier publication, Smith (1955). In equation (30)  $\Gamma$  is the time average circulation about an airfoil. Table 1 contains estimates of the recovery parameter  $R$  derived from equations (27) and (28), equations (27) and (30), and from the simulations. The simulation results are for a reduced frequency of ten.

The estimates of the recovery derived from equation (27) with  $C_1$  estimated either according to equation (28) or (30) are in good agreement with the results derived from the simulations. Furthermore the close agreement of the results obtained from equation (27) shows once again that the flow physics involved in the recovery process is tied to the kinematics of the wake vorticity field.

The estimated levels of the wake recovery parameters show that the potential to reduce wake mixing loss by an unsteady recovery process is large. If the present analysis was appended to the estimates of wake mixing loss such as given by Cumpsty (1989), the estimated adverse effect of separated flow on the performance of an embedded blade row in a multi-stage compressors would be greatly reduced. This reduction in wake mixing loss would have a noticeable impact on compressor efficiency. For example, assuming the wake mixing loss is fifteen percent of the total pressure loss of a blade row, a stage whose loss is split equally between the rotor and stator would have its efficiency increased by nearly a point as a result of reducing the wake mixing loss by seventy percent.

## SUMMARY AND CONCLUSIONS

This work presents an analysis which delineates the unsteady flow process associated with the recovery of the total pressure deficit of a wake as it passes through a blade row. It is shown that wake recovery is related to the change of the kinetic energy of the unsteady velocity field across a blade row and that the change in kinetic energy can be estimated from linear theory. The analysis showed that the estimates of wake recovery based on linear theory were correct to second order and as such is capable of predicting the recovery of wakes of finite depth. It was also shown that the change in the kinetic energy of the unsteady velocity field across a blade row is related to a volume integral of the product of the deterministic stress and the rate of strain of the time average flow field.

The value of the reduced frequency associated with a wake encounter was established to be of the order of five

or greater for a typical core compressor, indicating that the wake recovery process is far from being quasi-steady. The distribution of the energy among the spatial Fourier harmonics which characterize a typical wake was also given.

Simulation for compressor cascade configurations showed that if the reduced frequency based on the wake passing frequency is sufficiently large the recovery process is primarily a function of the flow turning (i.e. airfoil circulation) and independent of the reduced frequency, and thus independent of the wake profile. In addition the results obtained from the simulations strongly suggest that the flow physics associated with the recovery process is tied to the vorticity kinematics of the incoming wakes.

An approximate expression was derived for estimating wake recovery. Its derivation was based on the kinematics of the wake vorticity field. Good agreement was found between the results given by the approximate expression and those derived from the simulations.

Finally, the impact of the recovery process on wake mixing loss is not small. Having the wakes pass through a blade row prior to being mixed-out by viscous diffusion can reduce the wake mixing loss by as much as seventy percent, which translates to nearly a point increase in efficiency of a stage. It is not clear how the effects of viscosity and compressibility would change the significance of the recovery process. An attempt to assess the effects of viscosity and compressibility is currently underway. Preliminary results seem to indicate that for subsonic flows the effect of compressibility would not significantly alter the importance of the recovery process.

From a design perspective it would seem advantageous to minimize the axial gap between blade rows and also to minimize the wake mixing taking place prior to entering a blade row. Reduction of wake mixing between blade rows would necessitate control of the early development of the wake. How this might be achieved requires further study.

It is important that models be developed to account for the effects of wake recovery in simulations of the time average flow field in multistage machinery. This is especially true at off-design conditions where flow separation might be playing a major role. The codes which are currently used to predict the time average flow field do not account for the recovery process and hence may under predict the performance of a machine. The present work shows that the effects of recovery may be accounted for through models for the deterministic stress.

## ACKNOWLEDGMENTS

The author wishes to express his gratitude to Dr. J. M. Verdon and Dr. A. Chuang, United Technologies Research Center for their assistance in familiarizing the author with LINFLO. The author wishes to acknowledge the



assistance of Dr. D. Hoyniak for providing SFLOW. The motivation for the present work stemmed from a question posed to the author by Prof. E. M. Greitzer, Massachusetts Institute of Technology and from the many useful and productive discussions which followed. The author also wishes to acknowledge Dr. C. S. Tan, Massachusetts Institute of

## REFERENCES

- Adamczyk, J.J., "Model Equation for Simulating Flows in Multistage Turbomachinery," *ASME Paper 85-GT-220*, 1985.
- Ashby, G.C. Jr., "Investigation of the Effect of Velocity Diagrams on Inlet Total Pressure Distortions Through Single-Stage Subsonic Axial Flow Compressors," *NACA RM L57A03*, April 1957.
- Atassi, H.M. and Grzedzinski, J., "Unsteady Disturbances of Streaming Motions around Bodies," *Journal of Fluid Mechanics*, Vol. 209, 1989.
- Cumpsty, N.A., "Compressor Aerodynamics," *Longman Scientific & Technical Publishers*, 1989.
- Deregel, P. and Tan, C.S., "Impact of Rotor Wakes on Steady-State Axial Compressor Performance," *Submitted to 1996 IGTI Gas Turbine and Aeroengine Conference*.
- Fritsch, G. and Giles, M., "Second-Order Effects of Unsteadiness on the Performance of Turbomachines," *Journal of Turbomachinery*, July 1995, Vol. 117.
- Goldstein, M.E., "Unsteady Vortical and Entropic Distortions of Potential Flows Round Arbitrary Obstacles," *Journal of Fluid Mechanics* 89, 1978.
- Greitzer, E.M., "Private Communications.", 1994.
- Hawthorne, W.R., "The Flow Through Moving Cascades of Lifting Lines with Fluctuating Lift," *Aeronautical Research Council, Paper No. ARC 32369*, 1970.
- Henderson, R.E., "The Influence of Unsteady Rotor Response On a Distorted Flow Field," *ASME Journal of Engineering for Power*, Vol. 104, July 1982.
- Horlock, J.H. and Daneshyar, H., "Stagnation Pressure Changes in Unsteady Flow," *The Aeronautical Quarterly*, Vol. XXII, August 1971.
- Hoyniak, D. and Verdon, J. M., "Steady and Linearized Unsteady Transonic Analyses of Turbomachinery Blade Rows," Paper presented at the Seventh International Symposium on Unsteady Aerodynamics and Aeroelasticity of Turbomachines, Fukuoka, Japan, September 25-29 1994. Conference Technology for sharing with him the finds of his and his students research on the topic of recovery. Finally, the author benefited greatly during the course of this work from the input provided by the Turbomachinery Flow Physics group at Lewis and is most grateful for their suggestions.
- Proceedings to be published by Elsevier Press, 1995.
- Kemp, N.H. and Sears, W.R., "The Unsteady Forces Due to Viscous Wakes in Turbomachines," *Journal of the Aeronautical Sciences*, Vol. 22, No. 7, 1955.
- Kemp, N.H. and Sears, W.R., "On the Wake Energy of Moving Cascades," *Journal of the Applied Mechanics*, Vol. 23, 1956.
- Mikolajczak, A.A., "The Practical Importance of Unsteady Flow," *Agard CP 177*, 1977.
- Okishi, T.H., Hathaway, M.D., Hansen, J.L., "A Note on Blade Wake Interaction Influence on Compressor Stator Row Aerodynamic Performance," *ASME Journal of Engineering for Gas Turbines and Power*, Vol. 107, No. 2, April 1985.
- Smith, L.H., "Secondary Flow in Axial-Flow Turbomachinery," *ASME Transactions*, Vol. 77, No. 7, 1955.
- Smith, L.H., "Wake Dispersion in Turbomachines," *Journal of Basic Engineering*, ASME Transactions, Vol. 88, Ser. D, No. 3, 1966.
- Smith, L.H., "Casing Boundary Layers in Multistage Axial Flow Compressors," *Flow Research In Blading*, edited by Elsevier Publishing Company, Amsterdam, 1970.
- Smith, L.H., "Wake Ingestion Propulsion Benefit," *Journal of Propulsion and Power*, Vol. 9, No. 1, 1993.
- Verdon, J. M., "Linearized Unsteady Aerodynamic Theory," In *AGARD Manual on Aeroelasticity in Axial-Flow Turbomachines 1, Unsteady Turbomachinery Aerodynamics* (eds. M. F. Platzer & F. O. Carta), AGARD-AG-298, 1987.
- Verdon, J. M., Barnett, M., Hall, K. C., and Ayer, T. C., "Development of Unsteady Aerodynamic Analyses for Turbomachinery Aeroelastic And Aeroacoustic Applications," *NASA CR 4405*, 1991.
- Verdon, J. M., Adamczyk, J. J., and Casper, J. R., 1975, "Subsonic Flow Past an Oscillating Cascade with Steady Blade Loading - Basic Formulation", *Unsteady Aerodynamics*, R. B. Kinney, ed., Proceedings of a Symposium held at the Univ. of Arizona, March 18-20, 1975, Vol. 2.

**Table 1. Recovery Parameter Estimates For a Cascade With 40° Turning**

Thickness/Chord Ratio	High Freq. Limit Eqns. (28), (29)	Smith (1993) Eqns. (28), (31)	Present Simulation Reduced Freq. = 10
10 %	0.67	0.741	0.697
5%	0.71	0.741	0.733
2.5%	0.74	0.741	0.744

**APPENDIX**

The stream function associated with the incoming wake is a particular solution of the equation

$$\nabla^2 \psi = -\zeta_w \quad (A1)$$

where  $\zeta_w$  is the wake vorticity field. Far downstream of the cascade at an axial location  $x_0$  the solution of (A1) is given by the integral,

$$\psi = -\frac{\sqrt{2Kin_1}}{2} \frac{\cos\beta_1}{\sin(\beta_r - \beta_1)} \int_0^L B(\bar{y}) h(x, \bar{y} - y) e^{-i\sigma r} e^{-ik_n(t - \frac{x}{\cos\beta_1})} d\bar{y} \quad (A2)$$

where

$$B(\bar{y}) = \exp\left(-i \frac{k_n \cos\beta_r \bar{y}}{\sin(\beta_r - \beta_1)}\right) \exp\left(ik_n \Delta(x_0, \bar{y})\right) \quad (A3)$$

$$h = \left[ \frac{e^{\bar{\eta}(\gamma+i\delta)}}{1 - e^{(\gamma+i\delta)}} - \frac{e^{-\bar{\eta}(\gamma-i\delta)}}{1 - e^{-(\gamma-i\delta)}} \right] \quad (A4)$$

$$\eta = \frac{\bar{y} - y + x \tan\beta_2}{L} \quad (A5)$$

$$\bar{\eta} = \eta - \text{Integer}(\alpha) + \frac{[\text{sign}(\alpha) - 1]}{2} \quad (A6)$$

$$\gamma \pm i\delta = k_n L \left[ \frac{\cos^2\beta_2}{\cos\beta_1} \mp i \frac{\cos\beta_r}{\sin(\beta_r - \beta_1)} \right] \quad (A7)$$

$$\sigma = -k_n L \frac{\cos\beta_r}{\sin(\beta_r - \beta_1)} \quad (A8)$$

and the drift function  $\Delta$  is defined by equation (27).

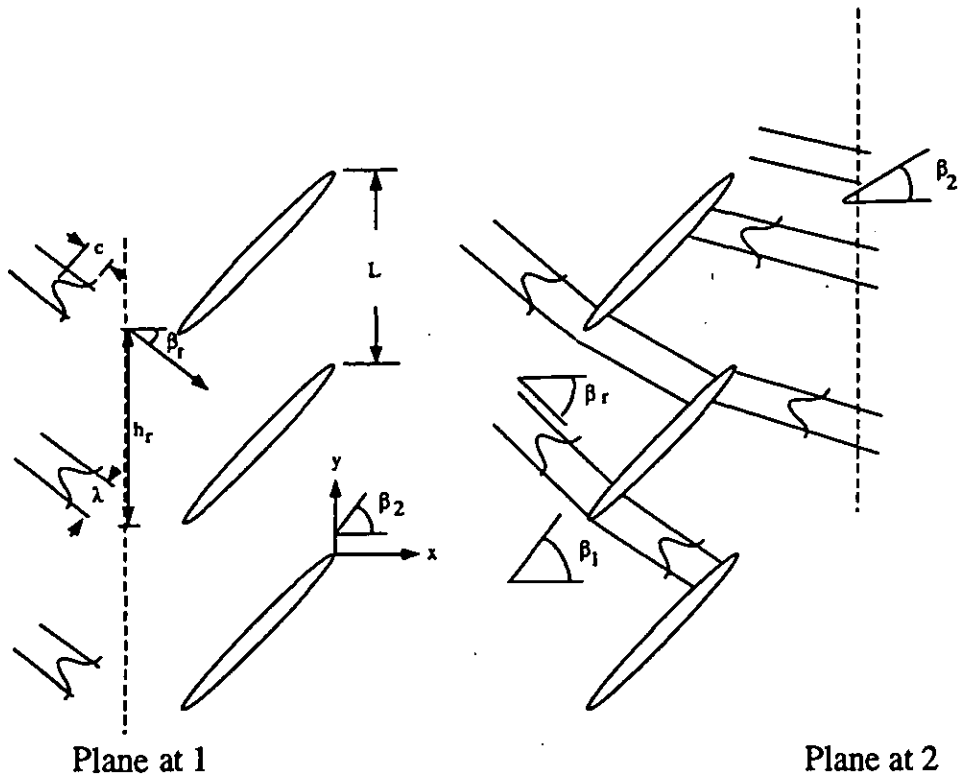


Fig. 1. A sketch of the flow field upstream/downstream of the cascade.

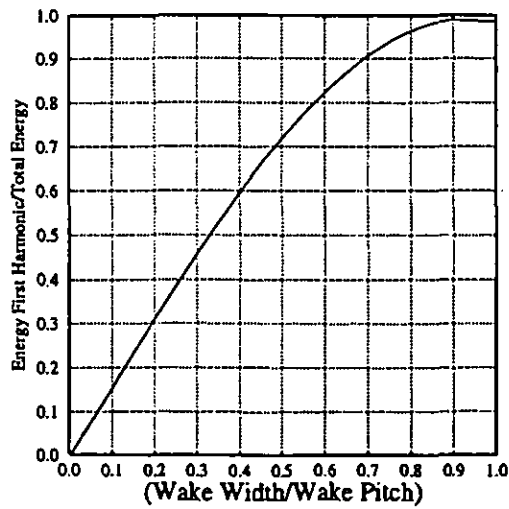


Fig. 2. Energy in first wake harmonic relative to total wake kinetic energy as a function of ratio of wake width to wake pitch (wake width/wake pitch =  $\bar{l} = \lambda_w/h_r \cos \beta_r$ ).

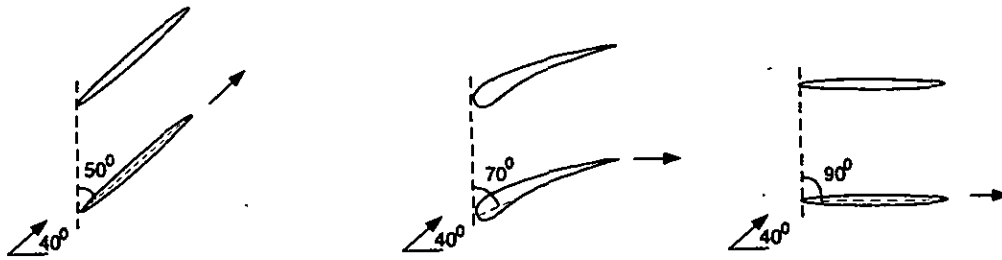


Fig. 3. Simulated cascade configuration.

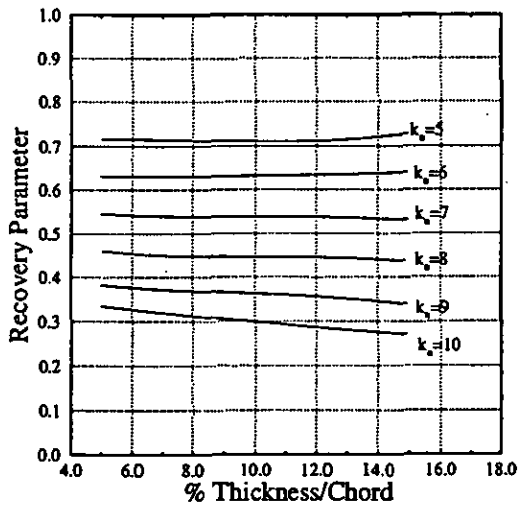


Fig. 4. Wake recovery across a cascade of zero turning as a function of airfoil thickness and reduced frequency.

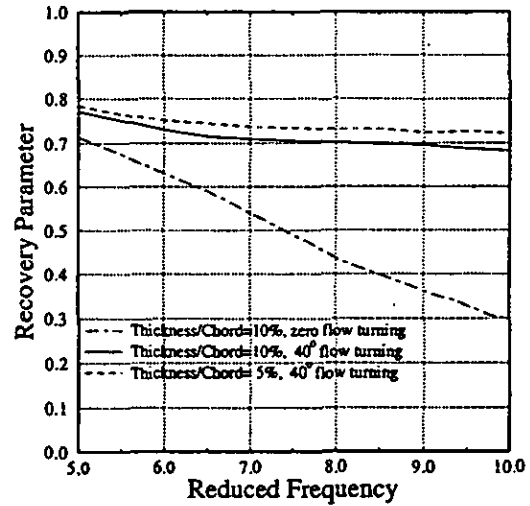


Fig. 5. Wake recovery across a cascade of finite loading as a function of airfoil thickness and reduced frequency.

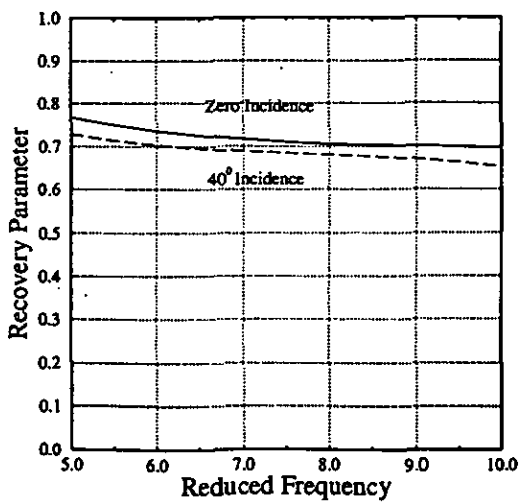


Fig. 6. Wake recovery as a function of reduced frequency across cascades of finite turning.