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A Statistical Approach to the Experimental Evaluation of Transonic Turbine Airfoils in a Linear Cascade

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ABSTRACT

In aircraft engine design (and in other applications), small improvements in turbine efficiency may be significant. Since analytical tools for predicting transonic turbine losses are still being developed, experimental efforts are required to evaluate various designs, calibrate design methods, and validate CFD analysis tools. However, these experimental efforts must be very accurate to measure the performance differences to the levels required by the highly competitive aircraft engine market. Due to the sensitivity of transonic and supersonic flow fields, it is often difficult to obtain the desired level of accuracy. In this paper, a statistical approach is applied to the experimental evaluation of transonic turbine airfoils in the VPI & SU transonic cascade facility in order to quantify the differences between three different transonic turbine airfoils. This study determines if the measured performance differences between the three different airfoils are statistically significant. This study also assesses the degree of confidence in the transonic cascade testing process at VPI & SU.

NOMENCLATURE

Variables:

L = loss coefficient

M = Mach number

P = pressure

Pr_s = maximum to minimum static pressure ratio at airfoil exit traverse plane

R = gas constant

s = blade pitch

T = temperature

t = time

u = velocity

x = horizontal (axial) direction

y = vertical (pitchwise) direction

γ = ratio of specific heats

ΔP_T = difference between airfoil upstream total pressure and downstream total pressure, $P_{T1} - P_{T2}$

ρ = density

Subscripts:

1 = upstream blade inlet

2 = downstream of blade

x = upstream of shock

y = downstream of shock

AVG = indicates an average value

ISEN = indicates value calculated with isentropic assumption

S = static conditions

T = stagnation or "total" conditions

INTRODUCTION

Aircraft engine high pressure turbines must operate at very high inlet temperatures and high aerodynamic loading and still provide a high level of efficiency. The resulting turbine airfoils often have high turning angles and supersonic relative exit Mach numbers. Due to cooling and mechanical requirements, large wedge angles and thick trailing edges are usually required. In this environment, typical turbine efficiency loss mechanisms, such as profile loss, trailing edge blockage, secondary flows, and coolant mixing losses, become more severe. In addition, shock structures are formed creating another loss mechanism.

In order to improve efficiency levels in aircraft engine turbines, methods of investigating the various loss mechanisms are required. However, rotating rig tests are prohibitively expensive and analytical methods for cooled, transonic turbines are still being developed. Linear cascade testing offers an economical method of investigating transonic turbine loss mechanisms that are two dimensional in nature including investigating trailing edge shock phenomena, evaluating various airfoil designs, evaluating

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coolant ejection effects, and providing test data for validation of analytical tools.

However, experimental variation in linear cascades can increase to significant levels for transonic and supersonic exit Mach numbers. Defining "significant" is somewhat subjective, but most turbine aerodynamic designers would probably agree that $\pm$ 0.5 points in efficiency is a "significant" level of variation for experimentally evaluating turbine airfoil designs. There have been indications in the literature that "significant" levels of variation in transonic turbine cascade testing are typical. Mee, et.al. (1990) showed analytical predictions of a high level of uncertainty supported to some degree by test data. Also, Kiock, et.al. (1986) showed significant variation in loss measurements for exit Mach numbers greater than .92. Similarly, significant levels of variation have been observed in cascade testing at VPI & SU. If the levels of variation are significant at the exit Mach number of interest, single-sample experiments may lead to misleading conclusions.

The objective of this investigation is to evaluate three airfoil designs at a subsonic exit Mach number and at a supersonic exit Mach number using multiple samples and simple statistical methods. The intent of the airfoil designs is to reduce the downstream shock strength and improve the airfoil's efficiency. A matrix of 216 test points, 108 at subsonic Mach numbers and 108 at supersonic Mach numbers, was designed to evaluate the airfoils. Statistical methods used to interpret the test data include linear regression, Analysis of Variance, and the Duncan significance test. In addition to evaluating the airfoils, quantitative estimates for uncertainty in loss measurements, uncertainty in downstream static pressure measurements, and uncertainty in setting a desired exit Mach number were obtained for the VPI & SU transonic wind tunnel.

DESCRIPTION OF FACILITY

The VPI & SU transonic wind tunnel is a cold-air, blowdown type fed by two storage tanks. The tanks are charged by a four stage reciprocating compressor and the air is passed through a dryer before entering the tanks. A schematic of the facility is shown in Fig. 1. Air flow to the tunnel is controlled by a computer-operated, feedback-controlled, pneumatic valve which sets the cascade upstream total pressure. The computer software requires calibration constants which are determined experimentally for each desired Mach number setting. The facility also includes a storage tank for coolant simulation. Typically, CO_2 is used for the coolant simulation in order to obtain the appropriate coolant to gas density ratio.

For this investigation, the test section consists of eleven aluminum or stainless steel airfoils mounted on 1/2" plexiglass endwalls. The plexiglass endwalls are supported by 1/2" aluminum walls and steel angle iron. A window in the aluminum wall permits shadowgraphs or Schlieren photographs to be taken. Fig. 2 shows the test section, the static pressure tap locations and the traverse planes. The airfoils for this test have a chord of 49.45 mm, an axial chord of 38.10 mm, and a spacing of 37.26 mm. The height of the cascade is 152.4 mm giving an aspect ratio of 4 based on axial chord. The tolerance on the airfoil pitch is ± 0.05 mm and the tolerance on the airfoil contour is also ± 0.05 mm. Station 1 is located at 1.167 axial chord from the airfoil leading edge and station 2 is located at 1.667 axial chord. For this investigation, ΔP_T traverses and wall static pressure measurements were taken only for station 1. The ΔP_T traverse is made with a single traverse probe driven by a stepper motor. The traverse probe is insensitive to the expected angle changes. An upstream total pressure probe in a low Mach number region is used to represent P_{T1}. No tailboard is used since previous studies show the free shear layer gives the best periodicity in this facility. An indication of the periodicity is shown in Figs. 3a and 3b where typical downstream ΔP_T traverse and wall static pressure measurements are given.

Fig. 3a also shows an upstream total pressure measurement over a typical 18 second run time. P_{T1} remains very constant over the 18 second run time indicating the pneumatic valve control system works very well. However, there is still some variation in setting the desired Mach number. One of the objectives of this test was to quantify the uncertainty in setting the cascade isentropic Mach

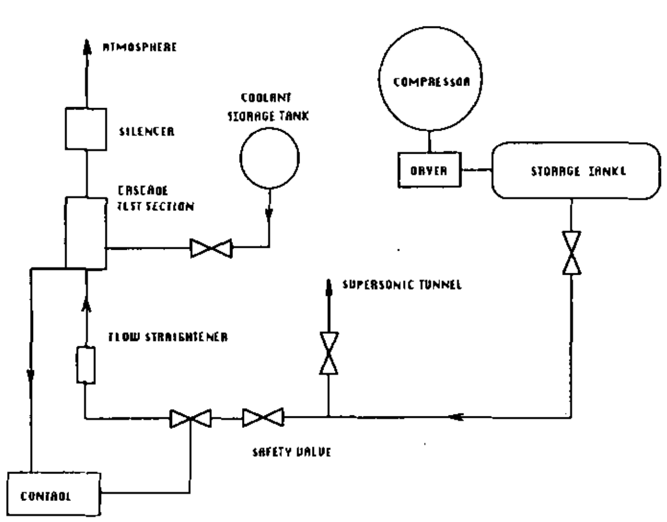

Fig. 1 - Block Diagram of VPI & SU Transonic Cascade Test Facility

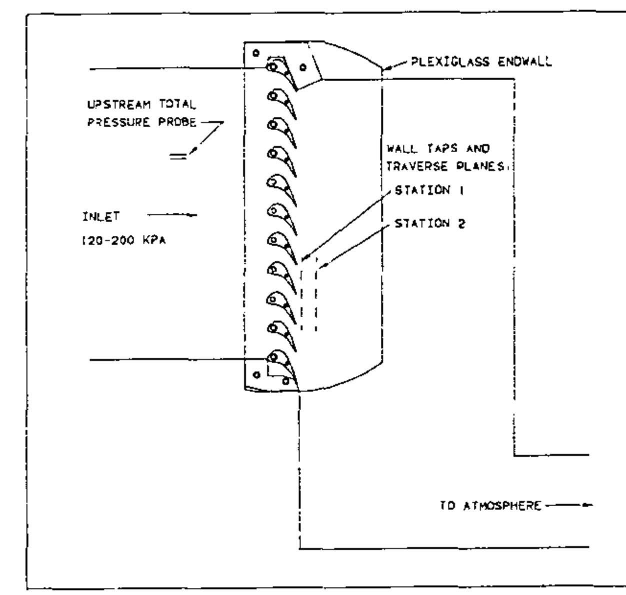

Fig. 2 - Sketch of Cascade Test Section for VPI & SU Transonic Wind Tunnel

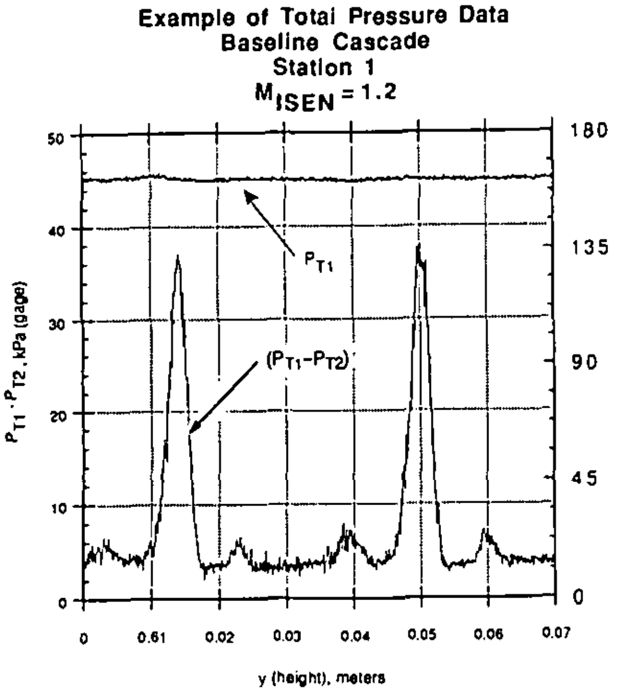

Fig. 3a

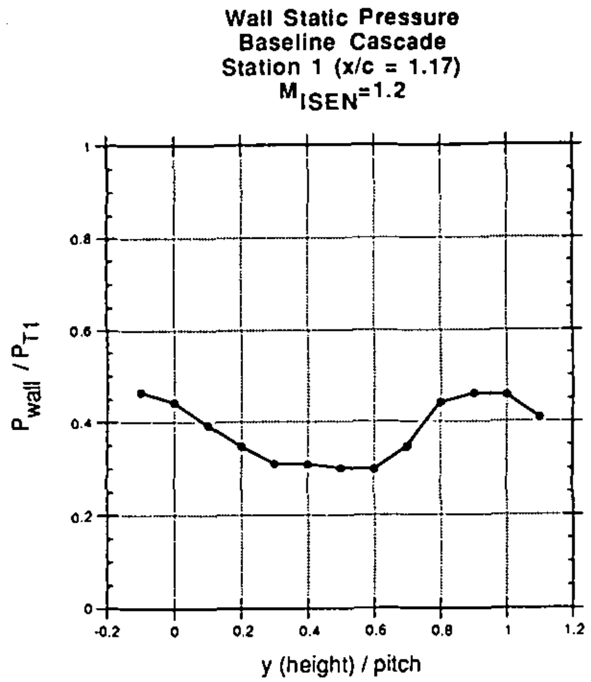

Fig. 3b

Fig. 3 - Examples of Upstream Total, Downstream Total, and Downstream Static Pressure Distributions for a Typical 18 Second Run

number, M_{2ISEN}. The capability of setting M_{2ISEN} is important in order to obtain a complete set of data from the wind tunnel. A complete set of data includes the downstream total pressure distribution, the downstream static pressure distribution, airfoil surface static pressures, and a shadowgraph. Typically, this requires three runs, one for the downstream static and total pressures, one for the airfoil surface static pressures, and one for an unobstructed shadowgraph. Ideally, all three runs are at the same M_{2ISEN}, but in reality, there will always be some variation in M_{2ISEN}.

Two separate data acquisition systems are used during a run. One system uses an electronic pressure scanner (by Pressure Systems, Inc.) to measure endwall static pressures. This system has an automated calibration system. Five sets of static pressure data are taken during a run to provide a static pressure history for the run. The other data acquisition system records ΔP_T and P_{T1} at a frequency of 40 hz over two blade spacings during the 18 second run time. This system is calibrated with a free weight system before each day's testing.

DATA REDUCTION

An isentropic Mach number, M_{2ISEN}, is defined to identify the cascade pressure ratio for a given run. The wall static pressures are arithmetically averaged over space to get P_{S2AVG}. Then, $M_{2ISEN}(t)$ is calculated at each point in time from the isentropic relationship

$$M_{2ISEN}(t) = \sqrt{\frac{2}{(\gamma-1)}\left[\left(\frac{P_{T1}(t)}{P_{S2AVG}(t)}\right)^{(\gamma-1)/\gamma} - 1\right]} \qquad (1)$$

$M_{2ISEN}(t)$ is then averaged over time to give M_{2ISEN}. For the downstream total pressure in supersonic flow, a bow shock is assumed to exist upstream of the probe. The normal shock relation

$$\frac{P_{T,y}}{P_{T,x}} = \left[\frac{\left(\frac{\gamma+1}{2}\right)M_x^2}{1+\left(\frac{\gamma-1}{2}\right)M_x^2}\right]^{\gamma/(\gamma-1)} \left[\frac{1}{\left(\frac{2\gamma}{\gamma+1}\right)M_x^2 - \left(\frac{\gamma-1}{\gamma+1}\right)}\right]^{1/(\gamma-1)} \qquad (2)$$

and the isentropic relation

$$\frac{P_{T,x}}{P_{S,x}} = \left[1 + \left(\frac{\gamma-1}{2}\right)M_x^2\right]^{\gamma/(\gamma-1)} \qquad (3)$$

are solved simultaneously for P_{Tx} and M_x. In order to solve equations (2) and (3), the following assumption is made for $P_{S,x}$

$$P_{S,x} = P_{S2AVG} \; . \qquad (4)$$

The loss coefficient, L, is defined as the mass weighted average of $\Delta P_T/P_{T1}$. The following procedure is used to calculate L. For each of the 800 values of ΔP_T, the quantities $T_{S,x}$, ρ_x, and u_x are determined. $T_{S,x}$ is calculated from the adiabatic relation

$$\frac{T_{T,x}}{T_{S,x}} = \left[1 + \left(\frac{\gamma-1}{2}\right)M_x^2\right]. \quad (5)$$

From the ideal gas law, ρ_x is

$$\varrho_x = \frac{P_{S,x}}{RT_{S,x}}. \quad (6)$$

Finally, u_x is calculated from the definition of Mach number

$$u_x = M_x\sqrt{\gamma RT_{S,x}}. \quad (7)$$

Then, L for a single blade spacing is calculated as

$$L = \frac{\int_0^s \varrho u\left(\frac{\Delta P_T}{P_{T1}}\right)dy}{\int_0^s \varrho u dy} \text{ X } 100, \text{ (L in \%)}. \quad (8)$$

The L values presented later are the average of the loss calculated for two adjacent blade pitches.

DESCRIPTION OF EXPERIMENT

Airfoils

The airfoil shapes that were tested are shown in Fig. 4. The ULTRE airfoil was originally designed with a thinner trailing edge than the baseline and LUT airfoils as indicated in Fig. 4. However, for this investigation, all of the airfoils had a 0.96 mm trailing edge thickness. The airfoils were designed with a 0° inlet angle to accommodate the wind tunnel. The design exit angle is approximately 67° at a M_{2ISEN} of 1.2. The "baseline" airfoil has a converging passage and is typical of present transonic airfoil designs. The "LUT" airfoil (Low Unguided Turning) has reduced unguided turning and a reduced wedge angle but still has a converging passage. The reduced unguided turning reduces the suction side Mach number at the trailing edge and the lower wedge angle reduces the angle between the suction side and pressure side streamlines at the trailing edge. The "ULTRE" airfoil (UNloaded Trailing Edge) has a slight converging-diverging passage to increase the pressure side Mach number and better match the suction side and pressure side static pressures at the trailing edge. The ULTRE also has about the same unguided turning as the LUT but has a larger wedge angle due to the convex pressure side near the trailing edge used to create the C-D passage. A comparison of the Mach number distributions, based on an inviscid analysis, is shown in Fig. 5. The intent of both the LUT and ULTRE designs is to reduce the suction side shock strength and improve the airfoil's efficiency. The parameters of

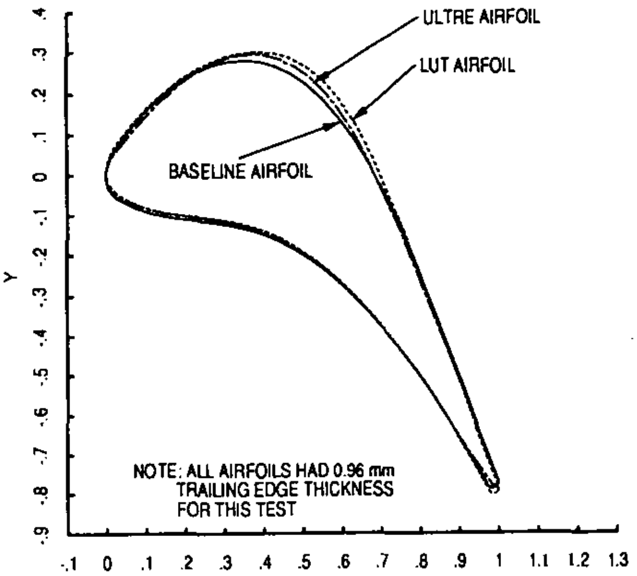

Fig. 4 - Comparison of Airfoil Designs

interest for this study are loss, L, and shock strength, which is indicated by the maximum to minimum downstream static pressure ratio, Pr_s, taken at the endwall.

Test Matrix

In order to limit the size of the test matrix, the experiment was designed around a single subsonic Mach number, $M_{2ISEN} = .82$, and a single supersonic Mach number, $M_{2ISEN} = 1.18$. Since there is some variation in the method of controlling the facility, it is difficult to set the desired M_{2ISEN} exactly. Therefore, the matrix of target Mach numbers was expanded to $M_{2ISEN} = .78$, .82, .86, 1.14, 1.18, and 1.22. This array of target Mach numbers permits interpolation to obtain values for L and Pr_s at the desired

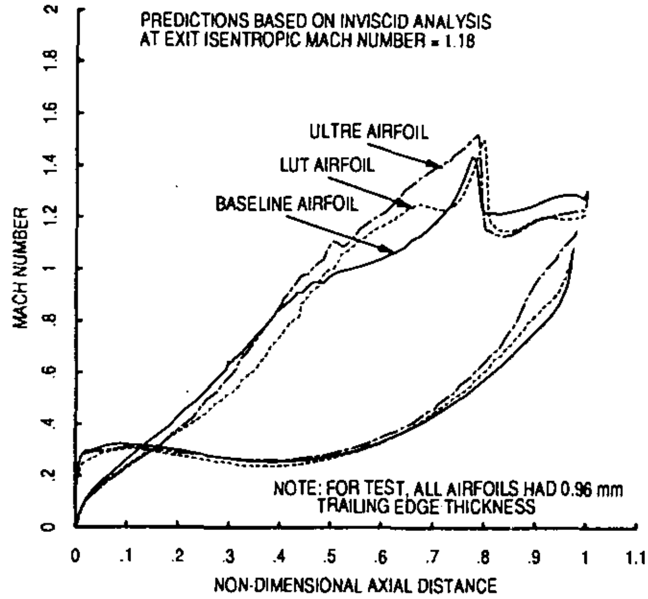

Fig. 5 - Comparison of Predicted Airfoil Mach Number Distributions for M_{2ISEN} = 1.18

TABLE 1 - TEST MATRIX USED FOR THE INVESTIGATION					
		SUBSONIC TESTING		SUPERSONIC TESTING	
AIRFOIL	NUMBER OF REPLICATIONS PER AIRFOIL	TARGET M_{2ISEN}	NUMBER OF REPLICATIONS OF M_{2ISEN}	TARGET M_{2ISEN}	NUMBER OF REPLICATIONS OF M_{2ISEN}
BASELINE	4	0.78 0.82 0.86	3 3 3	1.14 1.18 1.22	3 3 3
LUT	4	0.78 0.82 0.86	3 3 3	1.14 1.18 1.22	3 3 3
ULTRE	4	0.78 0.82 0.86	3 3 3	1.14 1.18 1.22	3 3 3

THE REPLICATIONS FOR THE AIRFOIL TESTING WERE PERFORMED IN A RANDOMIZED SEQUENCE

FOR EACH AIRFOIL TESTING, THE REPLICATIONS OF M_{2ISEN} WERE PERFORMED IN A RANDOMIZED SEQUENCE AND WERE COMPLETED ON THE SAME DAY

M_{2ISEN} of .82 and 1.18. Also, in order to assess the variation in the facility, each of the target Mach numbers was repeated three times in a randomized order.

The test described above was performed for each airfoil design four times for a total of twelve tests. Only one of these tests was run per day. The tests were performed in a randomized order. In the instances where the random sequence of tests called for running the same airfoil on back to back days, the entire cascade was still removed from the wind tunnel and then reinstalled on the next day. However, the cascades themselves were not disassembled due to time constraints (the positioning of the airfoils in the cascade represents another source of variation due to manufacturing and assembly tolerances of the airfoils). A total of 216 data points were taken, 108 at subsonic Mach numbers, and 108 at supersonic Mach numbers. A summary of the matrix is shown in Table 1.

Data Analysis

A linear regression of the loss coefficient, L, and the maximum to minimum downstream static pressure ratio, Pr_s, with M_{2ISEN} as the independent variable was performed for each airfoil for both the subsonic Mach number and the supersonic Mach number cases. A parabolic regression could have been used but the narrow range of Mach numbers justified the convenience of using a linear regression. Also, for this experiment, the emphasis is on only $M_{2ISEN} = 0.82$ and $M_{2ISEN} = 1.18$. Prediction limits and confidence limits were also calculated for each airfoil for 95% confidence. The prediction limits define the uncertainty interval for the next single observation based on the data used in the regression. The confidence limits define the uncertainty interval for the mean line (or the next N observations), as calculated from the N observations used in the linear regression. Detailed explanations of the prediction limits and confidence limits are given by Ott (1988). The uncertainty for the facility is estimated by averaging the uncertainty for the three airfoils as determined by the regression analysis.

In order to compare the airfoils, the Analysis of Variance, ANOVA, is used to determine if a significant difference exists. The ANOVA compares the variation within a subgroup of data to the variation between subgroups. Obviously, the differences between two subgroups must be greater than the variation within the subgroups for the subgroups to be significantly different. The ANOVA provides a mathematical method of evaluating these differences with a given level of confidence. If a significant difference exists, the Duncan method is used to determine which subgroups are significantly different. The Duncan test permits the investigator to mathematically determine the magnitude of the required difference (for a given level of confidence) between the means of the subgroups for the subgroups to be significantly different. There are several other tests similar to the Duncan test. However, the Duncan test is one of the least conservative and is more apt to show a difference between subgroups. For this reason, the Duncan test is frequently used by investigators expecting to observe differences. The ANOVA and Duncan tests were performed for $M_{2ISEN} = 0.82$ and $M_{2ISEN} = 1.18$ where L Pr_s were calculated from the linear regression mean line. A detailed technical discussion of the ANOVA and the Duncan test is given by Ott (1988).

An estimate of the standard deviation in setting M_{2ISEN} is made from the variation in the measured M_{2ISEN}. It is assumed that the facility could be calibrated to set the fixed error between the mean measured M_{2ISEN} and the target M_{2ISEN} to zero for any desired setting. The uncertainty for the facility is estimated by averaging the uncertainty calculated for each airfoil.

UNCERTAINTY ANALYSIS

Ideally, before beginning any experimental effort, an uncertainty analyses would be performed to determine if the proposed apparatus had the capability of meeting the desired level of accuracy. In practice, analytically predicting the uncertainty can be difficult.

Kline and McClintock (1953) described methods of analytically estimating the uncertainty in experiments. Moffat (1981) elaborated further on this topic. Typically, the sources of error are classified as "fixed error" (or "bias") and random error. The random error is defined as "scatter" about some mean value which can be approximated with a normal distribution. The fixed error is the difference between the measured result and an external reference which could be a widely accepted baseline case or a case which can be deduced from conservation laws. Assuming all fixed errors have been accounted for and the experimental process is "zero-centered", the uncertainty for a result R as a function of the uncertainty in N independent variables, x_1, x_2, ...,x_N can be calculated from the equation below

$$\delta R = \sqrt{\left(\frac{\partial R}{\partial x_1}\delta x_1\right)^2 + \left(\frac{\partial R}{\partial x_2}\delta x_2\right)^2 + \ldots + \left(\frac{\partial R}{\partial x_N}\delta x_N\right)^2} \quad (9)$$

The difficulty usually lies in quantifying the derivatives, $\frac{\partial R}{\partial x_i}$, and the variation in the independent variables, δx_i.

Moffat (1981) also describes three levels of replication, the N^{th} order, the first order, and the $zero^{th}$ order. The N^{th} order includes variation between different researchers with physically different facilities and different instrumentation (even if the facilities and instrumentation are of the same type and by the same manufacturer). The first order includes variation due to changes in the instrument calibration, interpretation and interpolation of instrument readings, changes in atmospheric conditions, and other effects which vary in time. For the $zero^{th}$ order of replication, in which the process is steady, there is no change in instrument readings, and the chief source of error is in the inaccuracies of the instrument readings.

For the purposes of evaluating the relative differences in the airfoils within the same facility, the errors on the N^{th} order replication level do not have to be addressed. These include errors from the calibration methods, time response of the probes, geometrical differences in the airfoils, flow exit angle variations, impact of probe on the flowfield, etc. These errors are assumed to be the same for each cascade due to the relative similarity of the cascades. However, in validating analytical tools, it would be desirable to evaluate the magnitude of these errors.

The distinction between the first order replication level and the $zero^{th}$ order replication level is somewhat difficult to define for a blowdown wind tunnel where technically, all measurements are varying in time. For the purposes of this investigation, the zeroth order replication level is defined as testing occurring within the same day, in fact, within a few hours for this experiment. First order replication is defined as testing occurring on different days. Variation associated with the first order replication level include the dismantling and assembly of the cascade into the wind tunnel test section, changes in atmospheric conditions, different tunnel operators, different sequences in starting and running the facility, etc. Unfortunately, these variations cannot be quantified analytically.

At the $zero^{th}$ level of replication, however, there are variations which can be estimated. These include change in total temperature during a run (due to the pressure/temperature/volume effects in the storage tank), observed drifts in the calibration of transducers during a day, unsteadiness in the upstream total pressure, variation in downstream static pressure measurements, interpolations between the 5 sets of static pressure data taken during a run, etc. The resulting predictions for uncertainty to a 95% confidence level are

- Loss Coefficient, ΔL, ± 0.1, (for L in %)
- Isentropic Exit Mach Number, ΔM_{2ISEN}, ± 0.005
- Max to min exit static pressure ratio, ΔPr_s, ± 0.003

In the uncertainty analysis, these variations are independent of Mach number. However, in practice, the uncertainty does vary with Mach number. This may be due to sources of variation at the zeroth level which can not be quantified, including the warming up of the compressor, the warming up of electrical equipment, slight changes in ambient temperature and humidity, etc.

DISCUSSION OF RESULTS

Uncertainty Results

The measured loss for the subsonic conditions for all three airfoils is shown in Fig. 6 with the prediction limits and in Fig. 7 with the confidence limits. Similarly, Pr_s is shown with the prediction limits in Fig. 8 and with the confidence limits in Fig. 9 . A summary of the results is given in Table 2. The Duncan test indicates the differences between the airfoils are statistically significant. Although, for the subsonic case, that the airfoils are "significantly" different is obvious from the graphical presentation of the results. In the subsonic regime, the actual uncertainty for a single observation is estimated to be 0.059 in L and 0.002 in Pr_s. This compares very favorably with the predicted uncertainty of 0.1 for L and 0.003 for Pr_s. This indicates the facility is very well controlled and reasonable conclusions could likely be drawn from single sample experiments.

However, the actual uncertainty in the supersonic regime is much higher than in the subsonic regime. The measured loss in the supersonic regime is shown in Fig. 10 with prediction limits and in Fig. 11 with confidence limits. Pr_s is shown in Fig. 12 with prediction limits and in Fig. 13 with confidence limits. A summary of the results is given in Table 3. The actual uncertainty for a single observation is estimated to be 0.44 for L and 0.021 for Pr_s. These estimates are almost an order of magnitude greater than the subsonic uncertainty estimates and the predicted uncertainty estimates. This indicates some unknown variable or variables are not being controlled.

The uncertainty for the supersonic testing is also indicated in the Duncan test for loss. The Duncan test for loss indicates the difference between the ULTRE and LUT is not significant at the 95% confidence level. For 90% confidence however, the ULTRE and LUT can be shown to be significantly different. The Duncan test for Pr_s indicates the differences between the airfoils are statistically significant at the 95% confidence level.

With this uncertainty in the supersonic regime, it is not unlikely a single sample experiment would give misleading conclusions. Even with multi-sample tests, the conclusions from graphical results such as those in Fig. 10 are not obvious. Applying statistical tools permits the investigator to objectively quantify the differences and uncertainties between subgroups and to assign a numerical value to the level of confidence. The validity of the statistical tools is dependent on the assumed curve used for the regression and on how well the measured uncertainty is

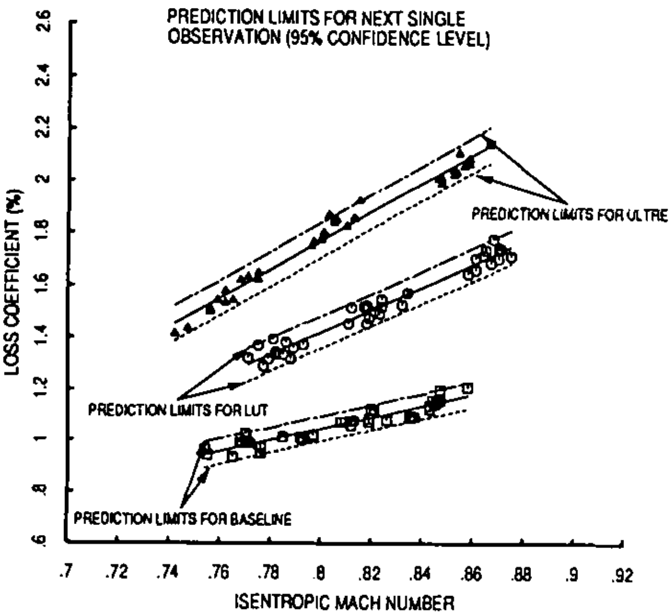

Fig. 6 – Comparison of Measured Loss for Subsonic Exit Mach Numbers with Prediction Limits for Next Single Observation

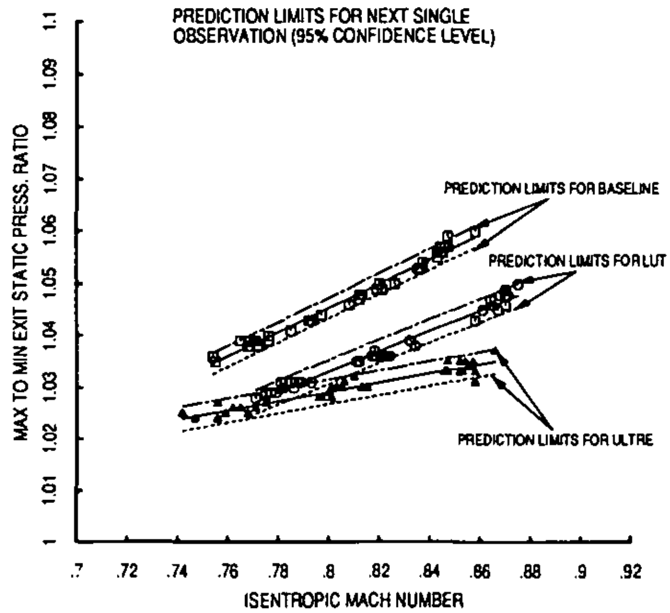

Fig. 8 – Comparison of Measured Downstream Static Pressure Variation for Subsonic Exit Mach Numbers with Prediction Limits for Next Single Observation

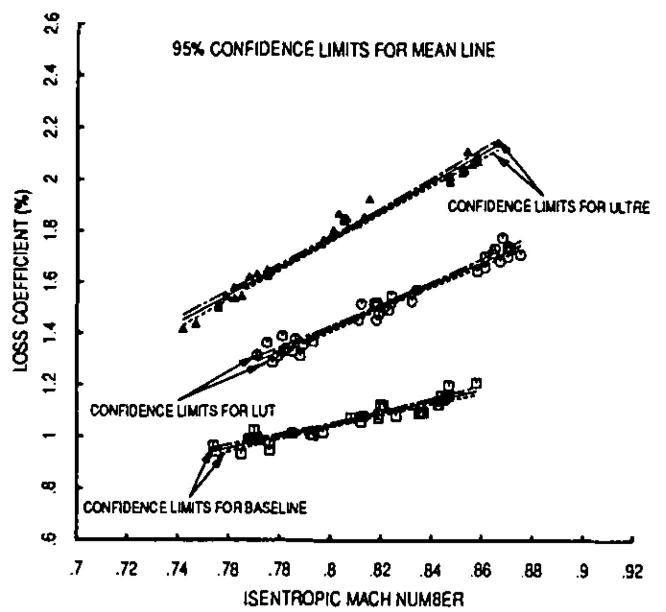

Fig. 7 – Comparison of Measured Loss for Subsonic Exit Mach Numbers with 95% Confidence Limits for Mean Line

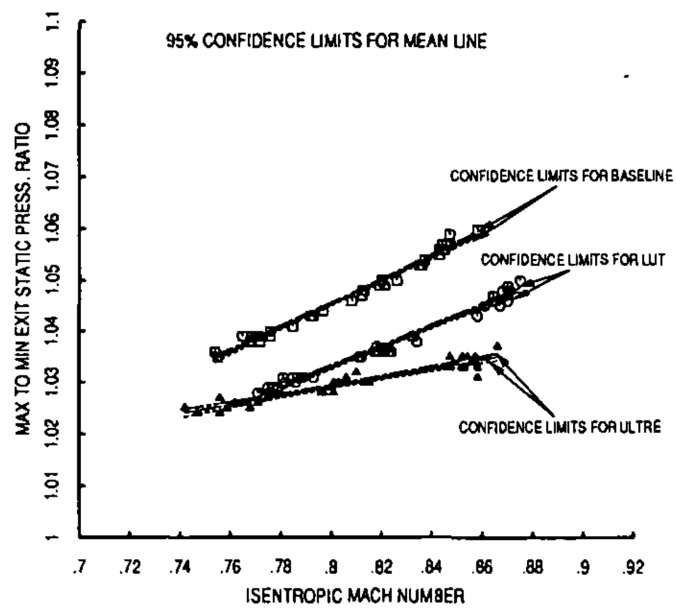

Fig. 9 – Comparison of Measured Downstream Static Pressure Variation for Subsonic Exit Mach Numbers with 95% Confidence Limits for Mean Line

approximated by the probability function used in the statistical methods. The statistical methods used in this study are relatively simple from a statistician's viewpoint. However, as shown in this study, even simple statistical methods permit the investigator to evaluate the relatively small differences in various airfoil designs.

The large increase in uncertainties for L and Pr_s in the supersonic regime naturally leads to speculation and investigation into the source(s) of the variation. Unsteadiness in the flow field may be a source of variation. High speed cinematography has indicated some unsteadiness in the shock structures. The impact of the traverse probe on the flow field may be another possible source of variation. The traverse probe is long and thin to minimize its effect on the flow field, but this geometry can lead to flutter problems when traversing through shocks and wakes. Condensation shocks may also be a source of variation if the dryer system is not working properly. However, this has not been established as of the writing of this paper. Statistical analyses of the test data from this investigation have not shown any consistent significant difference between day to day variation (i.e. first order replication level) and same day variation (i.e. zeroth order replication level). Mee, et. al. (1990) suggested uncertainty may be a function of Reynolds number. This warrants investigation,

TABLE 2 - SUMMARY OF RESULTS FOR M_{2ISEN} = 0.82								
	RESULTS FOR LOSS - 95% CONFIDENCE				RESULTS FOR Pr_s - 95% CONFIDENCE			
AIRFOIL	MEAN L (%)	UNCERTAINTY FOR THIS TEST MATRIX	UNCERTAINTY FOR SINGLE OBSERVATION	DUNCAN TEST*	MEAN Pr_s	UNCERTAINTY FOR THIS TEST MATRIX	UNCERTAINTY FOR SINGLE OBSERVATION	DUNCAN TEST*
BASELINE	1.09	±0.0085 in L ± 0.8% of MEAN	±0.049 in L ±4.5% of MEAN	A	1.050	< 0.001 in Pr_s < 0.1% of MEAN	±0.002 in Pr_s ±0.2% of MEAN	A
LUT	1.51	±0.011 in L ±0.7% of MEAN	±0.062 in L ±4.1% of MEAN	B	1.037	< 0.001 in Pr_s < 0.1% of MEAN	±0.002 in Pr_s ±0.2% of MEAN	B
ULTRE	1.88	±0.012 in L ±0.6% of MEAN	±0.066 in L ±3.5% of MEAN	C	1.031	< 0.001 in Pr_s < 0.1% of MEAN	±0.002 in Pr_s ±0.2% of MEAN	C
AVERAGE OF ALL TESTS	1.49	±0.010 in L ±0.7% of MEAN	±0.059 in L ±4.0% of MEAN	—	1.039	< 0.001 in Pr_s < 0.1% of MEAN	±0.002 in Pr_s ±0.2% of MEAN	—

* DUNCAN TEST INDICATES SIGNIFICANT DIFFERENCE IF LETTERS ARE DIFFERENT

but the facility as presently configured does not have the capability of setting the Mach number and Reynolds number independently. In the interim, the statistical approach as presented in this paper offers a method of quantifying the differences between different airfoils.

Table 4 shows the uncertainty in setting M_{2ISEN}. The uncertainty is based on the standard deviation of the measured values for M_{2ISEN}. It is assumed that the offset between the mean measured value and the target value could be calibrated to zero for any desired setting. The actual uncertainty is about 3 times higher than the predicted uncertainty for M_{2ISEN} over the entire range of Mach numbers. Since the predicted uncertainty is based on the known inaccuracies in the measurements, this implies the actual uncertainty in M_{2ISEN} is due to the repeatability of the control system of the tunnel.

Aerodynamic Results

The results presented in Figs. 11 and 13 and in Table 3 show the LUT and ULTRE designs achieved their objective to some degree by reducing the downstream shock strength. The ULTRE reduced the shock strength, ($Pr_s - 1$), by approximately 18% and the LUT airfoil reduced shock strength by approximately 30%. However, both the LUT and ULTRE designs increased the loss from the baseline airfoil. Supersonically, the LUT airfoil

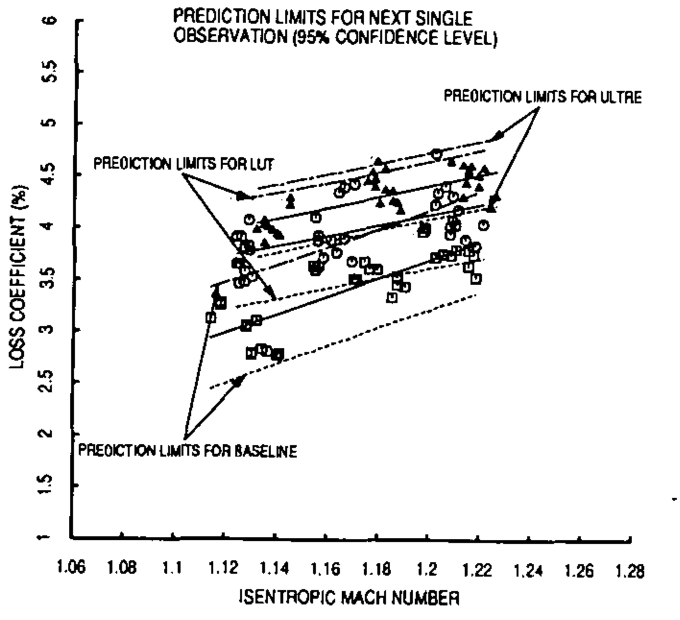

Fig. 10 – Comparison of Measured Loss for Supersonic Exit Mach Numbers with Prediction Limits for Next Single Observation

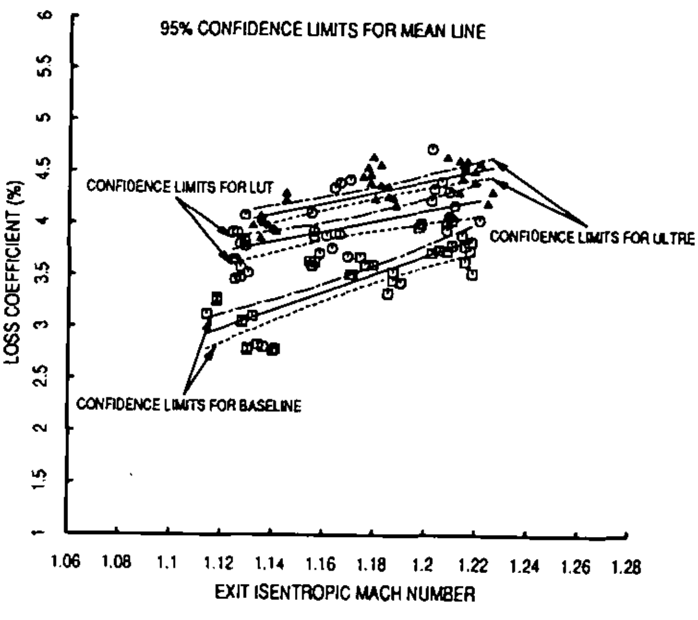

Fig. 11 – Comparison of Measured Loss for Supersonic Exit Mach Numbers with 95% Confidence Limits for Mean Line

TABLE 3 - SUMMARY OF RESULTS FOR M_{2ISEN} = 1.18								
	RESULTS FOR LOSS - 95% CONFIDENCE				RESULTS FOR P_{r_s} - 95% CONFIDENCE			
AIRFOIL	MEAN L (%)	UNCERTAINTY FOR THIS TEST MATRIX	UNCERTAINTY FOR SINGLE OBSERVATION	DUNCAN TEST*	MEAN P_{r_s}	UNCERTAINTY FOR THIS TEST MATRIX	UNCERTAINTY FOR SINGLE OBSERVATION	DUNCAN TEST*
BASELINE	3.52	±0.080 in L ± 2.3% of MEAN	±0.48 in L ±13.6% of MEAN	A	1.330	±0.004 in P_{r_s} ± 0.3% of MEAN	±0.025 in P_{r_s} ±1.9% of MEAN	A
LUT	4.03	±0.096 in L ±2.4% of MEAN	±0.51 in L ±12.7% of MEAN	B**	1.231	±0.004 in P_{r_s} ±0.3% of MEAN	±0.020 in P_{r_s} ±1.6% of MEAN	B
ULTRE	4.31	±0.053 in L ±1.2% of MEAN	±0.32 in L ±7.4% of MEAN	B**	1.269	±0.003 in P_{rs} ±0.2% of MEAN	±0.019 in P_{r_s} ±1.5% of MEAN	C
AVERAGE OF ALL TESTS	3.95	±0.076 in L ±1.9% of MEAN	±0.44 in L ±11.1% of MEAN	—	1.277	±0.004 in P_{rs} ±0.3% of MEAN	±0.021 in P_{r_s} ±1.7% of MEAN	—

* DUNCAN TEST INDICATES SIGNIFICANT DIFFERENCE IF LETTERS ARE DIFFERENT

** DUNCAN TEST FOR 90% CONFIDENCE DID SHOW A SIGNIFICANT DIFFERENCE BETWEEN LUT AND ULTRE

increased L by approximately 14.5% over the baseline and the ULTRE increased L by approximately 22.4% over the baseline. Subsonically, as shown in Table 2 and Fig. 7, the LUT airfoil increased L by 38.5% over the baseline and the ULTRE airfoil increased L by 72.5% over the baseline. While the LUT and ULTRE airfoil designs reduced the suction side shock strength, they have resulted in significant increases in loss from the baseline airfoil.

To understand the differences in the airfoil designs, first consider the geometric features and simple parameters that contribute to the trailing edge shock structure. The most obvious feature is the finite trailing edge thickness. The finite thickness trailing edge will lead to the classical over-expansion and subsequent recompression that creates the pressure side and suction side trailing edge shocks. But even with an infinitely thin trailing edge, the airfoil aerodynamics may still create a shock. Consider an infinitely thin plate with a supersonic suction side Mach number and a sonic pressure side Mach number at the trailing edge. Since there is a static pressure difference between the suction and pressure sides, 1-D compressible flow theory will predict a shock on the suction side at the trailing edge and an expansion around the pressure side at the trailing edge. Also, consider a converging-diverging turbine airfoil design where both the

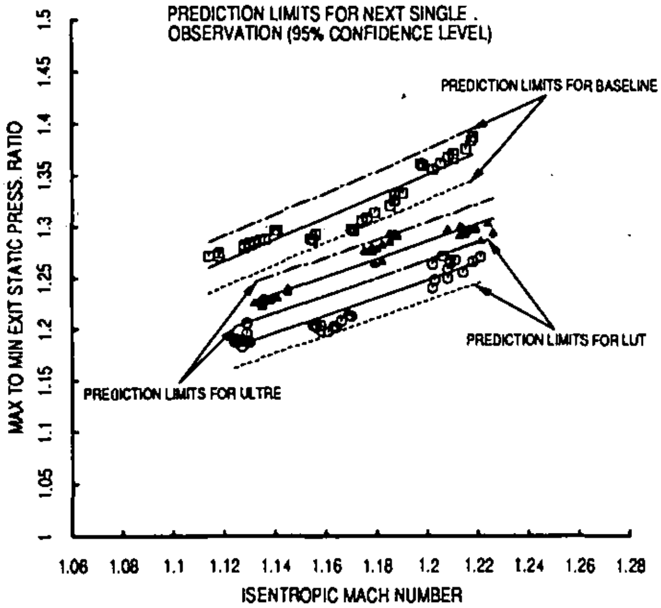

Fig. 12 - Comparison of Measured Downstream Static Pressure Variation for Supersonic Exit Mach Numbers with Prediction Limits for Next Single Observation

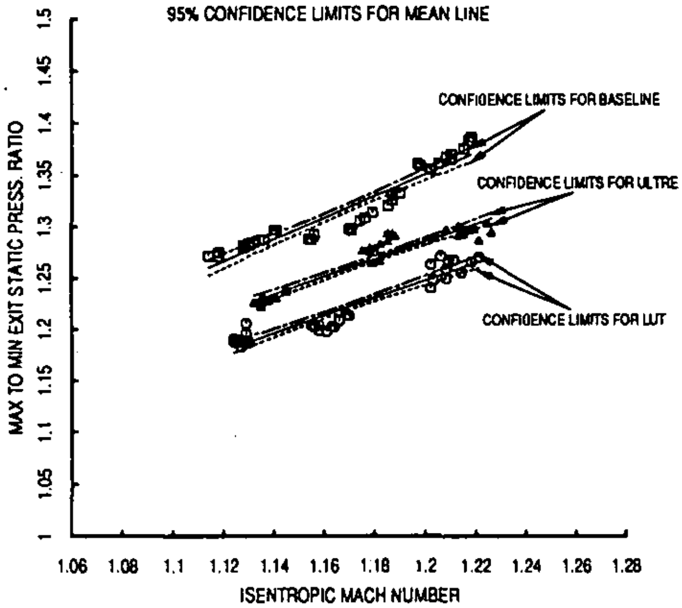

Fig. 13 - Comparison of Measured Downstream Static Pressure Variation for Supersonic Exit Mach Numbers with 95% Confidence Limits for Mean Line

TABLE 4 - SUMMARY OF CAPABILITY OF SETTING M_{2ISEN}								
	RESULTS FOR M_{2ISEN} = 0.82				RESULTS FOR M_{2ISEN} = 1.18			
AIRFOIL	MEAN	STANDARD DEVIATION	UNCERTAINTY FOR 95% CONFIDENCE*	MEAN OFFSET*	MEAN	STANDARD DEVIATION	UNCERTAINTY FOR 95% CONFIDENCE*	MEAN OFFSET*
BASELINE	0.811	0.011	± 0.022	0.009	1.174	0.013	± 0.025	8.006
LUT	0.821	0.007	± 0.014	0.001	1.162	0.005	± 0.010	0.018
ULTRE	0.804	0.006	± 0.012	0.016	1.181	0.004	± 0.008	0.001
AVERAGE OF ALL TESTS	0.812	0.008	± 0.016	0.008	1.172	0.007	± 0.014	0.008

* UNCERTAINTY ASSUMES MEAN OFFSET COULD BE CALIBRATED TO ZERO FOR ANY ONE DESIRED SETTING

suction side and pressure side are supersonic at the trailing edge, the static pressure difference is equal to zero at the trailing edge, and there is zero trailing edge thickness. If there is any wedge angle at all, 1-D compressible flow theory will predict a suction side and pressure side trailing edge shock. Finally, in order to produce the lift to turn the flow, the suction side flow must be accelerated to a lower static pressure than the cascade exit static pressure. Some sort of compression on the suction side is required to match the cascade exit static pressure, most likely in the form of a shock for supersonic flows. The geometric parameter typically used to indicate the suction side curvature downstream of the geometric throat is unguided turning, the difference between surface angle at the geometric throat and surface angle at the trailing edge. In summary, four parameters that have a significant influence on the trailing edge shock structure are the trailing edge thickness, the static pressure difference between the suction side and the pressure side, the wedge angle, and the unguided turning. All of these parameters must be considered when trying to control the trailing edge shock structure on a turbine airfoil.

With the above discussion in mind, compare the airfoil Mach number distributions shown in Fig. 5. With the lower unguided turning, the LUT and ULTRE airfoils have reduced the suction side Mach number from approximately 1.32 for the baseline to about 1.24. Since the exit isentropic Mach number is 1.18, the amount of compression required to match the exit pressure is less for the LUT and the ULTRE resulting in a reduction in the suction side shock strength from the baseline airfoil.

An example of the trade-offs in the parameters controlling the trailing edge shock structure is seen in the comparison of the LUT and the ULTRE airfoils. Both the LUT and the ULTRE have about the same unguided turning. As a result, both have about the same suction side Mach number at the trailing edge. For this investigation, both the LUT and the ULTRE have the same trailing edge thickness. The difference between the LUT and ULTRE airfoils is the ULTRE airfoil has a converging- diverging passage. The pressure side of the ULTRE airfoil becomes convex at about 90% axial chord as shown in Fig. 4. As a result, the pressure side isentropic Mach number on the ULTRE airfoil becomes sonic at about 90% axial chord. While the ULTRE design significantly reduces the static pressure difference between the pressure and suction sides at the trailing edge, the increased wedge angle of the ULTRE airfoil results in an increase in suction side shock strength relative to the LUT airfoil.

The lower unguided turning that resulted in reduced suction side shock strength also increases the airfoil loss. Once again, refer to the comparison of Mach number distributions in Fig. 5. By reducing the unguided turning, the aerodynamic loading on the aft side of the airfoil is reduced. The ULTRE and LUT designs compensate for the reduced aft loading by increasing the forward loading of the airfoil in order to produce the same lift. This results in a higher suction side peak Mach number for the LUT and the ULTRE airfoils as shown in Fig. 5. With the higher peak suction side Mach number and the lower trailing edge suction side Mach number, the suction side diffusion is much stronger on the LUT and ULTRE airfoils than the baseline airfoil. This stronger diffusion essentially manifests itself as a stronger pressure side trailing edge shock which impinges on the suction side. The pressure side trailing edge shock is also closer to being normal to the flow on the LUT and ULTRE airfoils than on the baseline airfoil. In terms of airfoil loss, the benefit of reducing the trailing edge suction side shock strength was more than offset by the increase in the trailing edge pressure side shock strength. This is in agreement with Denton and Xu's (1990) conclusion that reduced suction side curvature downstream of the throat will increase the loss. Denton and Xu reached this conclusion with a simple control volume analysis without regard to the shock/boundary layer interaction. The losses due to the shock/boundary layer interaction will only increase for the LUT and ULTRE airfoils due to the stronger pressure side trailing edge shock impinging on the suction side.

For the off design, subsonic case, the tendency for separation on the suction side will be stronger for the LUT and ULTRE airfoils than for the baseline airfoil because of the stronger suction side diffusion. With the converging-diverging passage of the ULTRE airfoil, separation on the pressure side downstream of the throat is also likely at the subsonic conditions. Therefore, it is not surprising that the LUT airfoil is worse than the baseline subsonically, and the ULTRE airfoil is worse than the LUT. The magnitude of the increases in loss are somewhat surprising though and warrant further investigation.

Qualitatively, the influences of trailing edge thickness, static pressure difference across the trailing edge, wedge angle, and unguided turning can be understood. Quantitatively, the trade-offs between the various airfoil design parameters are still being investigated. As shown in this paper, measurements in supersonic flow are difficult to obtain to the desired level of accuracy. However, accurate experimental measurements are necessary to validate new CFD analysis tools and the subsequent airfoil designs resulting from the improved analytical capability.

CONCLUSIONS

1.) The level of uncertainty in cascade test measurements for loss coefficient and downstream static pressure distribution is significantly greater at supersonic exit Mach numbers than at subsonic exit Mach numbers.

2.) For the VPI & SU facility as configured at the time of this test, the uncertainty for a single sample experiment to a 95% confidence level is estimated to be approximately

$\pm$ 0.06 in L (L in %) at M_{2ISEN} = .82

$\pm$ 0.44 in L (L in %) at M_{2ISEN} = 1.18

$\pm$ 0.002 in Pr_s at M_{2ISEN} = .82

$\pm$ 0.021 in Pr_s at M_{2ISEN} = 1.18

$\pm$ 0.016 in setting M_{2ISEN}.

3.) Multi-sample experiments and simple statistical methods can be used to increase the level of confidence in evaluating airfoils at supersonic exit Mach numbers.

4.) The transonic turbine airfoil shock structure is a function of many airfoil design parameters including trailing edge thickness, pressure difference across the trailing edge, wedge angle, and unguided turning.

5.) Relative to the baseline airfoil, reducing unguided turning and reduced wedge angle will reduce the downstream shock strength.

6.) Relative to the baseline airfoil, the efficiency benefits of reducing the downstream shock strength can be offset by the increase in strength of the trailing edge pressure side shock

SUMMARY

This study illustrates the difficulties in measuring small, but still significant, differences in airfoil efficiency, particularly for transonic and supersonic flow. Ideally, in the experimental process, one would continue to investigate, identify, and control the source(s) of variation until the desired level of uncertainty is reached. In practice, time and funding constraints are likely to limit the level of effort available for refining the experimental process. The statistical approach as shown in this paper gives the investigator a methodology for assessing measured differences relative to the variation in the process. Rigorously, one can argue that the validity of the approach is dependent on how well the true probability function is approximated by the normal distribution used in the data reduction. However, future improvements in turbine efficiency are likely to be incremental in nature requiring measurement of small differences to validate new designs. Methods of assessing measured differences and uncertainty will be required. The authors hope this paper will stimulate further discussion on this topic.

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