MODELLING OF BY-PASS TRANSITION WITH CONDITIONED NAVIER-STOKES EQUATIONS AND A K-E MODEL ADAPTED FOR INTERMITTENCY

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ABSTRACT

Turbomachinery flows are characterized by a very high intensity turbulent mean part. As a consequence, laminar flow in boundary layer regions undergoes transition through direct excitation of turbulence. This is the so-called bypass transition. Regions form that are intermittently laminar and turbulent. In particular in accelerating flows, as on the suction side of a turbine blade, this intermittent flow can extend over a very large part of the boundary layer. Classical turbulence modelling based on global time averaging is not valid in intermittent flows. To take correctly account of the intermittency, conditioned averages are necessary. These are averages taken during the fraction of time the flow is turbulent or laminar respectively. Starting from the Navier-Stokes equations, conditioned continuity, momentum and energy equations are derived for the laminar and turbulent parts of an intermittent flow. The turbulence is described by the classical k-ε model. The supplementary parameter introduced by the conditioned averaging is the intermittency factor. In the calculations, this factor is prescribed in an algebraic way.

NOMENCLATURE

\[\begin{align*}
C_f &= \frac{\tau_w}{(\rho U^2)} / 2 \quad & \text{skin friction.} \\
f_\mu, f_\delta & \quad & \text{damping functions.} \\
I & \quad & \text{intermittency function.} \\
k & \quad & \text{turbulence kinetic energy.} \\
Re_\theta &= U_\infty \theta / \nu \quad & \text{Reynolds number.} \\
Re_\tau &= U_\infty \tau / \nu \quad & \text{momentum thickness Reynolds number.} \\
Tu & \quad & \text{turbulence intensity (\%).} \\
\bar{u} & \quad & \text{global time average.} \\
\bar{u}_t & \quad & \text{average during the turbulent state.} \\
\bar{u}_l & \quad & \text{average during the laminar state.} \\
u', \nu'' & \quad & \text{fluctuating velocity components.} \\
u_* & \quad & \text{friction velocity.} \\
u^+ & \quad & \text{velocity in wall units.} \\
y^+ & \quad & \text{distance in wall units.} \\
\gamma & \quad & \text{intermittency factor.} \\
\delta_t & \quad & \text{displacement thickness.} \\
\epsilon & \quad & \text{turbulence dissipation.} \\
\mu & \quad & \text{dynamic viscosity.} \\
\rho & \quad & \text{density.} \\
\theta & \quad & \text{momentum boundary layer thickness.}
\end{align*}\]

Sub- and Superscripts

\[\begin{align*}
l & \quad & \text{laminar state.} \\
t & \quad & \text{turbulent state.} \\
tot & \quad & \text{sum of molecular and turbulent quantities during the turbulent phase.} \\
tr & \quad & \text{transition.}
\end{align*}\]

Marks

- conditioned Favre-averaging.
- conditioned Reynolds-averaging.

CONDITIONED AVERAGING

We define an intermittency function \(I(x,y,z,t)\) that takes the value 1 in a turbulent region and the value 0 in a non-turbulent, say laminar, region. The time-averaged value of this function during some time inter-
val T is defined as the intermittency factor
\[ \gamma = \frac{1}{T} \int_0^T I(x, y, z, t) dt = \gamma(x, y, z, t). \]

The time interval T is to be chosen such that it is large with respect to the time scales of the turbulence, but still small with respect to the time scales of the mean flow.

We consider first the turbulent conditioned mean value of a quantity. For the time being we only study Reynolds-averaging. Afterwards, we verify the results for Favre-averaging. As an example we take the velocity component in x-direction: u. This quantity can be decomposed in mean and fluctuating components as follows:
\[ u = \bar{u} + u', \]
\[ \bar{u} = \bar{u}_1 + u_1' \quad \text{for } I=1, \]
\[ u = u_0 \quad \text{for } I=0. \]

Laminar fluctuations are neglected here. The turbulent mean value and fluctuation satisfy
\[ \overline{u u'} = 0, \quad \overline{u} = \gamma \bar{u}_1 = \frac{1}{T} \int_0^T I u dt. \]

The laminar mean value and the global mean value satisfy
\[ (1 - \gamma)u = (1 - \gamma)u_1. \]

Further, we derive the equations for the conditioned averages of a space or time derivative quantity. The turbulent conditioned average of a space derivative term \( \frac{\partial u}{\partial x} \) is defined by \( I \frac{\partial \bar{u}}{\partial x} \). During the turbulent phase we decompose by
\[ u = \bar{u} + u' \quad \text{and} \quad \frac{\partial u}{\partial x} = \frac{\partial \bar{u}}{\partial x} + \frac{\partial u'}{\partial x}. \]

We accept that the quantity \( \frac{\partial u}{\partial x} \) is uncorrelated, like \( u' \), such that the time average during the turbulent phase of this quantity is zero. As a consequence the contributions of the turbulent phase in the integral defining the mean value is \( \gamma \frac{\partial \bar{u}}{\partial x} \). Further, there are contributions coming from the fronts between turbulent and laminar zones. Fig. 1 shows schematically the passage of an upgoing front, i.e. a front where the state changes from laminar to turbulent.

During the passage of the front, the space derivative is seen as
\[ \frac{\partial u}{\partial x} = \frac{u_1 - \bar{u}_1}{\delta x_1} - \frac{u_0'}{\delta x_1}, \]

where we accept that the front is spaced over a distance \( \delta x_1 \). We take now as convention that we consider the front passage as part of the turbulent phase. The contribution of an upgoing front to the integral defining the turbulent mean value is given by
\[ \frac{1}{T} (u_1 - \bar{u}_1) \frac{\delta t_1}{\delta x_1}, \]

where \( \delta t_1 \) is the time interval during which the front passes. We consider the mean value of \( u_1' \) even during this small time interval as being zero. The quantity \( \frac{\delta t_1}{\delta x_1} \) represents the passage velocity \( c_{x_1} \) of the upgoing front in x-direction.

Similarly the contribution from a downgoing front is given by
\[ \frac{1}{T} (u_2 - \bar{u}_2) \frac{\delta t_2}{\delta x_2}. \]

The contribution from one passage of a turbulent zone is
\[ \frac{1}{T} (u_2 - u_1) \frac{1}{c_{x_2}} - \frac{1}{c_{x_1}}. \]

By integrating over many passages, a sum of terms of form (1) appears. The interpretation of this sum is straightforward. We consider the definition of the intermittency factor on the position \( P(x, y, z) \) and on a position \( P' \), an infinitesimal distance \( \delta x \) further in x-direction. When the upgoing front passes at time \( t_1 \) at the position \( P \), it passes at time \( t_1 + \frac{\delta x}{c_{x_1}} \) at the position \( P' \). Similarly, the downgoing front passes at times \( t_2 \) and \( t_2 + \frac{\delta x}{c_{x_2}} \). Neglecting higher order variations of \( c_{x_1} \) and \( c_{x_2} \) during the passage, the fraction of time turbulent flow is seen at point \( P \) is given by
\[ \gamma = \frac{\Sigma (t_2 - t_1)}{T}, \]

while at point \( P' \) it is
\[ \gamma + \delta \gamma = \frac{\Sigma (t_2 - t_1)}{T} + \frac{\delta x}{T} \Sigma (\frac{1}{c_{x_2}} - \frac{1}{c_{x_1}}). \]

Figure 1: Upgoing front
Hence
\[ \frac{\partial \gamma}{\partial z} = \frac{1}{T} \Sigma \left( \frac{1}{c_{z_1}} - \frac{1}{c_{z_2}} \right). \]
This results in the rule for a space derivative
\[ I \frac{\partial u}{\partial x} = \gamma \frac{\partial \bar{u}}{\partial x} + (\bar{u}_i - u_i) \frac{\partial \gamma}{\partial x}. \] (2)
This rule is valid for every other space direction.

The laminar conditioned mean value is simply
\[ \frac{(\bar{u}_i - u_i)}{\delta t_1} \]
and \[ \frac{(u_i - \bar{u}_i)}{\delta t_2}. \]
The sum of the expressions (2) and (3) gives
\[ \frac{\partial u}{\partial x} = \frac{\partial (\gamma \bar{u}_i + (1 - \gamma)u_i)}{\partial x} = \frac{\partial \bar{u}_i}{\partial x}, \]
which, of course, should be the result.

Following a similar reasoning, conditioned mean values for a time derivative quantity can be constructed.

For \( I = 0 \), we have:
\[ u' = \bar{u} - u = \bar{u} - u_i = \gamma \bar{u}_i + (1 - \gamma)u_i - u_i = \gamma (\bar{u}_i - u_i). \]

Hence
\[ (u')^2 = (1 - \gamma)^2 (\bar{u}_i - u_i)^2 + u_i^2 + 2(1 - \gamma)(\bar{u}_i - u_i)u_i \text{ for } I = 1 \]
\[ = \gamma^2 (\bar{u}_i - u_i)^2 \text{ for } I = 0 \]
\[ (\bar{u}_i)^2 = \gamma (1 - \gamma)^2 (\bar{u}_i - u_i)^2 \]
\[ + \gamma u_i^2 + (1 - \gamma)^2 (\bar{u}_i - u_i)^2 = \gamma (\bar{u}_i)^2 + \gamma (1 - \gamma)(\bar{u}_i - u_i)^2. \] \( \text{(5)} \)
Similarly
\[ \bar{u}'u' = \gamma \bar{u}_i u_i + \gamma (1 - \gamma)(\bar{u}_i - u_i)(\bar{u}_i - u_i). \]

**CONDITIONED FLOW EQUATIONS**

It is immediately clear that the rules for conditioned mean values and derivatives go over to Favre-averages (Vandroumme, 1991). We define mean and fluctuating parts of density:
\[ \rho = \bar{\rho}_i + \rho'_i \text{ for } I = 1, \text{ where } \bar{\rho} = \gamma \bar{\rho}_i, \]
\[ \rho = \rho_i \text{ for } I = 0. \]
Hence
\[ \bar{\rho} = \gamma \bar{\rho}_i + (1 - \gamma)\rho_i. \]
Further, a turbulent Favre-average for velocity is defined by
\[ I \bar{\rho} = \gamma \bar{\rho}_i. \]
The global Favre-average follows from
\[ \bar{\rho} \bar{u} = \bar{\rho} \bar{u}_i = \gamma \bar{\rho}_i \bar{u}_i + (1 - \gamma)\rho_i u_i. \]

We derive now the conditioned turbulent mean mass equation. The unaveraged equation is
\[ \frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0. \]
According to the rules for derivatives, we obtain as turbulent conditioned mean equation
\[ \gamma \frac{\partial \bar{\rho}_i}{\partial t} + (\bar{\rho}_i - \rho_i) \frac{\partial \gamma}{\partial t} + \gamma \frac{\partial \bar{\rho}_i}{\partial x} + (\bar{\rho}_i \bar{u}_i - \rho_i u_i) \frac{\partial \gamma}{\partial x} \]
\[ + \gamma \frac{\partial \bar{\rho}_i}{\partial y} + (\bar{\rho}_i \bar{u}_i - \rho_i v_i) \frac{\partial \gamma}{\partial y} = 0. \]
This equation can also be put into conservative form as
\[
\frac{\partial \tilde{p}_t}{\partial t} + \frac{\partial \tilde{p}_t}{\partial x} + \frac{\partial \tilde{p}_t}{\partial y} = \rho_t \frac{\partial \tilde{u}}{\partial t} + \rho_t u \frac{\partial \tilde{u}}{\partial x} + \rho_t v \frac{\partial \tilde{v}}{\partial y}.
\]  
(6)

The laminar equation simply is
\[
\frac{\partial \alpha}{\partial t} + \frac{\partial \rho_t u \alpha}{\partial x} + \frac{\partial \rho_t v \alpha}{\partial y} = 0.
\]  
(7)

By summing (6) and (7) multiplied by \((1 - \gamma)\), we obtain
\[
\frac{\partial (\gamma \tilde{p}_t + (1 - \gamma) \rho_t \tilde{u}_t)}{\partial t} + \frac{\partial (\gamma \tilde{p}_t + (1 - \gamma) \rho_t \tilde{v}_t)}{\partial y} = 0.
\]

This equation represents a global mean mass equation.

The conditioned turbulent mass equation used in the calculation is
\[
\frac{\partial \tilde{p}_t}{\partial t} + \frac{\partial \tilde{p}_t \tilde{u}_t}{\partial x} + \frac{\partial \tilde{p}_t \tilde{v}_t}{\partial y} = \rho_t \frac{\partial \tilde{u}}{\partial t} + \rho_t u \frac{\partial \tilde{u}}{\partial x} + \rho_t v \frac{\partial \tilde{v}}{\partial y}.
\]

Similarly, the momentum equations can be treated.
We write the momentum equations in compact form as
\[
\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u u_j}{\partial x_j} + \frac{\partial \rho}{\partial x_j} = \tau_{ij} \frac{\partial \rho}{\partial x_j},
\]
where the summation convention is used. The terms \(\tau_{ij}\) denote the molecular stress components. During the turbulent phase, the Favre- and Reynolds-decompositions are
\[
\rho u_i = \rho (\tilde{u}_i + u_i),
\]
\[
\rho u_i u_j = \rho (\tilde{u}_i + u_i) (\tilde{u}_j + u_j),
\]
\[
p = \langle p \rangle + p',
\]
\[
\tau_{ij} = \tau_{ij}^\text{Favre} + \tau_{ij}^\text{Reynolds}.
\]

The turbulent conditioned equations are
\[
\frac{\partial \tilde{p}_t \tilde{u}_i}{\partial t} + \frac{\partial \tilde{p}_t \tilde{u}_i \tilde{u}_i}{\partial x_j} + \gamma \frac{\partial \tilde{p}_t \tilde{u}_i u_j}{\partial x_j} = \rho_t \frac{\partial \tilde{u}_i}{\partial t} + \rho_t u_i \frac{\partial \tilde{u}_i}{\partial x_j} + \gamma \frac{\partial \tilde{u}_i u_j}{\partial x_j}.
\]

The usual eddy viscosity modelling approximations are now introduced:
\[
\begin{align*}
\tilde{p}_t \tilde{u}_i u_j & = -\tau_{ij} \tilde{e}_i \tilde{e}_j + \mu \tilde{S}_{ij}, \\
\tau_{ij} & = \frac{\mu \tilde{S}_{ij}}{\mu}.
\end{align*}
\]

where \(\tilde{e}_i\) is the turbulence kinetic energy during the turbulent phase, \(\mu\) is the eddy viscosity and \(\tilde{S}_{ij}\) is the shear tensor based on Favre-averages during the turbulent phase. We neglect here front contributions to the turbulent mean shear stress.

For instance, the resulting momentum-x equation is:
\[
\frac{\partial (\tilde{p}_t \tilde{u}_t)}{\partial t} + \frac{\partial (\rho_t \tilde{u}_i \tilde{u}_i)}{\partial x} + \frac{\partial (\tilde{p}_t \tilde{u}_i \tilde{v}_i)}{\partial y} = \frac{\partial (\tilde{u} + \mu_t) \tilde{S}_{xz}}{\partial x} + \frac{\partial (\tilde{u} + \mu_t) \tilde{S}_{xy}}{\partial y}.
\]

The momentum y-equation is similar. The energy equation and the Reynolds stress equation, which forms the basis of the \(k\) and \(\epsilon\)-equations, can be treated in the same way. The result is, in each case, an equation which is similar to the global averaged equation supplemented with source terms due to the front passages.

The resulting energy equation is:
\[
\frac{\partial (\tilde{p}_t \tilde{E}_t)}{\partial t} + \frac{\partial (\rho_t \tilde{H}_t \tilde{u}_t)}{\partial x} + \frac{\partial (\tilde{p}_t \tilde{H}_t \tilde{v}_t)}{\partial y} = \frac{\partial (\tilde{E}_t + \tilde{S}_{tx} \tilde{u}_t + \tilde{S}_{ty} \tilde{v}_t - \tilde{q}_t)}{\partial x} + \frac{\partial (\tilde{E}_t + \tilde{S}_{xz} \tilde{u}_t + \tilde{S}_{yz} \tilde{v}_t - \tilde{q}_t)}{\partial y} + (\rho_t \tilde{E}_t - \tilde{p}_t \tilde{E}_t) \frac{\partial \tilde{\gamma}}{\partial t} + (\rho_t \tilde{H}_t \tilde{u}_t - \tilde{p}_t \tilde{H}_t \tilde{u}_t) \frac{\partial \tilde{\gamma}}{\partial x} + (\rho_t \tilde{H}_t \tilde{v}_t - \tilde{p}_t \tilde{H}_t \tilde{v}_t) \frac{\partial \tilde{\gamma}}{\partial y}.
\]

The mean total energy \(\tilde{E}_t\) and mean total enthalpy \(\tilde{H}_t\) during the turbulent phase are given by:
\[
\tilde{E}_t = \tilde{e}_t + \frac{1}{2} (\tilde{u}_t^2 + \tilde{v}_t^2) + \tilde{k}_t,
\]
\[
\tilde{H}_t = \tilde{E}_t + \frac{\tilde{p}_t}{\rho_t} + \frac{(\tilde{u}_t^2 + \tilde{v}_t^2)}{3},
\]
where $\bar{e}_i$ is the mean internal energy.

For the Reynolds stress equations, the source terms largely compensate each other. For the present research, we decided therefore to neglect these source terms. So, we model the turbulence by the classical (low Reynolds number) $k$- and $\varepsilon$-equations, but written for the turbulent conditioned averaged values. These equations, written in the Yang-Shih variant (Yang and Shih, 1992), are:

$$
\frac{\partial \bar{p}_t}{\partial t} + \frac{\partial \bar{p}_t \bar{u}_i}{\partial x} + \frac{\partial \bar{p}_t \bar{u}_i}{\partial y} =
\frac{\partial (\bar{\mu} + \mu_k)}{\partial x} \frac{\partial \bar{u}_i}{\partial x} + \frac{\partial (\bar{\mu} + \mu_k)}{\partial y} \frac{\partial \bar{u}_i}{\partial y} + \bar{P}_k - \bar{e}_i,
$$

$$
\frac{\partial \bar{p}_t \bar{e}_i}{\partial t} + \frac{\partial \bar{p}_t \bar{e}_i \bar{u}_i}{\partial x} + \frac{\partial \bar{p}_t \bar{e}_i \bar{u}_i}{\partial y} =
\frac{\partial (\bar{\mu} + \mu_k)}{\partial x} \frac{\partial \bar{e}_i}{\partial x} + \frac{\partial (\bar{\mu} + \mu_k)}{\partial y} \frac{\partial \bar{e}_i}{\partial y} + [C_{\varepsilon} \bar{P}_k - C_{\varepsilon} f_2 \bar{p}_t \bar{e}_i] \frac{1}{\bar{T}} + \varepsilon,
$$

where

$$
\bar{P}_k = \left\{ \mu \left[ \frac{\partial \bar{u}_i}{\partial x} + \frac{\partial \bar{u}_j}{\partial x} \right] - \frac{2}{3} \delta_{ij} \frac{\partial \bar{u}_i}{\partial x} \right\} \frac{\partial \bar{u}_i}{\partial x},
$$

$$
\mu_k = C_{\mu} f_\mu \bar{p}_t \bar{u}_i T, \quad T = \frac{\bar{k}_t}{\bar{c}_i} + \sqrt{\frac{\bar{\nu}}{\bar{c}_i}},
$$

$$
\varepsilon = \frac{\bar{\mu} \mu_k}{\bar{c}_i} \frac{\partial^2 \bar{u}_i}{\partial x \partial x} \frac{\partial^2 \bar{u}_i}{\partial x \partial x} R_v = \frac{\bar{k}_t}{\bar{c}_i} \frac{\bar{\nu}}{\bar{c}_i},
$$

$$
f_\mu = 1 - \exp(a_1 R_v + a_3 R_v^3 + a_5 R_v^5),
$$

$$
f_2 = 1 - \exp(-\frac{R_v^2}{36}),
$$

The following model constants are used:

$$
C_{\mu} = 1.44, \quad C_{\varepsilon} = 1.92, \quad a_k = 1, \quad a_\varepsilon = 1.3, \quad C_\mu = 0.09, \quad a_1 = -1.5 \times 10^{-6}, \quad a_3 = -5.10^{-7}, \quad a_5 = -1.10^{-10}.
$$

At the wall, $\bar{k}_t = 0$ and $\bar{e}_{tw} = 2\nu \left( \frac{\partial \bar{k}_t}{\partial y} \right)^2$ are imposed.

RESULTS

The above equations have been used to calculate an intermittent flow on an adiabatic flat plate with no pressure gradient ($\frac{\partial^2}{\partial x^2} = 0$). Two test cases with different turbulence levels have been chosen to compare computational results with experimental data. The intermittency $\gamma$ is prescribed algebraically according to Dhawan and Narasinha:

$$
\gamma(x) = 1 - \exp \left\{ -\bar{\sigma}(Re_x - Re_{str})^2 \right\}, \quad (8)
$$

where $\bar{\sigma}$ and the point of transition are given by empirical correlations. Mayle (1991) gives

$$
\bar{\sigma} = 1.5 \times 10^{-10} T u_x^2 \quad (9)
$$

$$
Re_{str} = 400 T u_x^{-\frac{1}{4}}. \quad (10)
$$

Hourmouzias (1989) has a similar expression for the determination of the transition point.

$$
Re_{str} = 460 T u_x^{-0.68} \quad (11)
$$

In comparison with the experimental data, both correlations (10) and (11) predicted a transition point upstream of the measured one, with (11) being closest. Therefore (11) has been used knowing transition occurs more downstream in reality.

The equations are solved in their steady state form by a relaxation procedure. A vertex-centered finite volume discretisation combined with an upwind TVD formulation is used. Full details of the numerical method are given by Steelant and Dick (1993). To obtain stability of the relaxation method, a careful treatment of the source terms is necessary. The negative source terms have to be linearized and put on the left hand side of the equations. In the turbulent flow equations, only the source due to $1.44$ is used. This term is zero before the transition point and is activated after this point. In order to avoid singularity, a lower bound for $\gamma$ is taken to be 1%.

The first test case has been carried out by Kuan and Wang (1990). The free stream has a turbulence level of 1.1% and a velocity $U_{\infty} \equiv 13.8m/s$.

Figure 2 shows the skin friction coefficient in function of $Re_x$ where the dotted lines represent the laminar and turbulent values. Curve (a) represents the experimental data. Curve (b) is the result obtained with global averaged Navier-Stokes equations and the $k-\varepsilon$ model, without taking into account the intermittency. As is well known, these method gives a too early and too rapid transition. Curve (c) is the result obtained with the present method. The transition point is predicted too early but the transition length is in accordance with the experimental data. Figure 3 shows the evolution of the profile of the global streamwise velocity fluctuation $u'$ during transition for different positions along the plate. The global streamwise Reynolds normal stress is, with the usual approximation, given by (see (5))

$$
\bar{u}'^2 \approx \bar{k}_{\varepsilon} = \gamma \bar{k}_t + \frac{1}{2} (\gamma (1 - \gamma) [(\bar{u}_t - u_i)^2 + (\bar{\sigma}_t - v_i)^2],
$$

where $\bar{K}_t$ is the turbulence kinetic energy during the turbulent phase. Experiments are represented by square boxes. Upstream of the transition point, the experimental data show already appreciable levels of $u' = \sqrt{\bar{u}'^2}$, indicating the importance of laminar fluctuations. Since
we neglect these, we underpredict seriously the levels of \( u' \) in the beginning of the transition. Further in the transition phase the double peak is well represented and corresponds well with the experimental data. Once the maximum in the skin friction is reached, the double peak disappears and the \( u' \) profile tends towards that of a fully developed turbulent one.

Velocity profiles at onset, in the middle and at the end of transition are shown in figure 4. As transition is predicted too early (fig. 2), we can expect that the profile in the middle of the transition tends more to a turbulent profile than the experimental values.

The second test case has been experimentally investigated by Rolls-Royce and is known under the acronym T3A. The corresponding \( T_u = 3\% \) and \( U_{\infty} = 5.4m/s \). A full description of the experimental setup can be found in Savill (1990). In fig. 5 we can conclude again that the transition point occurs too early while the transition length has approximately the correct value. Concerning the streamwise fluctuation \( u' \) (fig. (6)), the same remarks can be drawn as in the previous case: the fluctuations are underpredicted in the beginning of the intermittency zone while the levels correspond better further downstream. Velocity profiles for this test case are shown in fig. (7).
Figure 4: Velocity profiles for different positions on the plate (KW).

Figure 6: Streamwise fluctuations for different positions on the plate (T3A).
CONCLUSIONS

Conditioned averaged Navier-Stokes equations have been deduced to model the transition zone. Therefore intermittency factor \( \gamma \) has been incorporated in the weighting functions. Two flat plate flows with high turbulence levels (\( Tu = 1\% \) and \( 3\% \)) were calculated and compared with experiments.

Regarding the global parameter \( C_f \), the correlations of Mayle and Hourmouziadis predict transition point upstream of the experimental value. On the other hand, the calculated turbulent spot formation rates according to Mayle agree well with experiments for both test cases.

The level of turbulence kinetic energy is significantly higher in the transition zone than in fully turbulent flow. This raise in turbulence kinetic energy is well reproduced due to the influence of the intermittency factor \( \gamma \). Without \( \gamma \), it is not possible to obtain these high levels. Only in the beginning of the intermittency zone, \( k \)-levels are low as laminar fluctuations, which can be very important prior to transition, were neglected.

AKNOWLEDGEMENTS

The research reported here was granted under contract 9.0001.91 by the Belgian National Science Foundation (N.F.W.O.) and under contract IUAP/17 as part of the Belgian National Programme on Interuniversity Poles of Attraction, initiated by the Belgian State, Prime Minister's Office, Science Policy Programming.

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