VORTEX SIMULATION OF ROTOR/STATOR INTERACTION IN TURBOMACHINERY

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ABSTRACT

A new numerical method is presented in this paper to simulate rotor/stator interaction in turbomachinery by use of a vortex method based on a Lagrangian frame. The algorithm takes the result from steady solution as input, which can give an initial description of the unsteady disturbance flow field. To calculate the unsteady response to these disturbances, the Lagrangian vortex method is used to capture the convective process, and the deterministic vortex scheme to approximate the viscous diffusion process. The application of Baldwin-Lomax turbulence model in deterministic vortex scheme to approximate the viscous diffusion process. The agreement between the computational and experimental results is generally good. The sweeping characteristic of wakes, the influence of unsteadiness on incidence and the decaying features of unsteady velocities, pressure are included in the paper.

INTRODUCTION

It is well recognized that turbomachinery flow fields are inherently unsteady due to relative motion between stationary and rotating blade rows. The purposes of studying this kind of unsteadiness are manifold, including deeply understanding the real flow process, assessing the influences of commonly adopted time-averaged procedure on steady values and probing the influence of unsteadiness in mixing in turbomachines. As this unsteadiness causes unsteady gasdynamic force to act on blades, obtaining this information will lay a foundation for gas-solid coupling unsteady computation to approach related problems, such as flutter. It is well known that the location and behavior of boundary layer transition are of significant influence on turbomachine performance, and the unsteadiness will affect the transition process. This shows another important role of the present study. From all we mentioned above, one sees that simulation of this unsteadiness is of great importance in improving our knowledge on turbomachinery gasdynamics and thus improving our design level.

Due to the great importance of the problem, many researches have been conducted, such as Erdos and Alzner (1977), Hodson (1984), Rai (1985), Giles (1990), Chen et al. (1985), He (1990). These works have yielded useful and significant results that provide insight into the problem, however, there is much work to do to improve these methods and to deepen our understanding of the problem studied.

The vortex method presented in this paper is based on a Lagrangian frame. Fluid carrying continuously distributed vorticity is divided into many discrete vortex particles which are traced in the Lagrangian manner. More strictly speaking, the vortex method adopted here is not a usual one, rather, it is a disturbance vortex method, i.e. for a single-stage compressor, if a relative steady solution of upstream rotor is known, and if an absolute steady solution of downstream stator is also known (these solutions can be easily obtained by use of maturely developed code, such as Denton’s code), then the unsteady effects of rotor on stator can be worked out through the moving vortex particles in an absolute system. It is worth noting that using the vortex method to cope with the interaction problem is convenient. The authors’ original idea for developing this method is based on the considerations that for modern transonic compressors, if the velocity is sufficiently high at the interface between rotor and stator, then the effects of upstream unsteadiness on the downstream is much stronger than that of the downstream on the upstream. As the circulation distribution of vortex particles represents the circumferential non-uniformity of upstream flow, the unsteady mechanism can be directly and conveniently described by the process of vortex particles moving downstream.

The vortex method has been successfully used to simulate many unsteady flows, such as shear layer evolution, trailing vortex roll up, vortex shedding behind blunt bodies and wake development. However, as far as we know, this method has not been applied to simulate rotor/stator interaction. In view of its inherent advantages, we tried to explore its ability in dealing with the very complicated interaction problem. The results presented below are encouraging.

ANALYSIS

GOVERNING EQUATIONS

The vorticity dynamic equation for two dimensional, viscous compressible flow is:

\[ \frac{D \zeta}{Dt} = \nabla \times \mathbf{u} \times \mathbf{u} \]

\[ \nabla \cdot \mathbf{u} = 0 \]

Where \( \zeta \) is the vorticity, \( \mathbf{u} \) is the velocity vector, and \( Dt \) is the material time derivative.
\[
\begin{align*}
\frac{\partial \omega}{\partial t} + (V \cdot \nabla) \omega = -\omega (V \cdot V) + \nu \nabla^2 \omega 
\end{align*}
\]  
(1)

Now, divide a variable into two parts, i.e.

\[ q = \bar{q} + q' \quad (q = \omega, V) \]  
(2)

where \( \bar{q} = \frac{1}{T} \int_0^T q \, dt \), and \( T \) is a time period in a rotor / stator interaction problem. When the convergent solution is approached, the time-averaged value \( \bar{q} \) is no longer a function of time. \( q' \) a disturbance variable. According to the definition above, one sees that:

\[ \int_0^T q' \, dt = 0 \]  
(3)

Substituting \( \omega \) and \( V \) of Eq.(2) by \( \omega = \bar{\omega} + \omega' \), \( V = U + u' \), respectively, one obtains:

\[ \frac{\partial \bar{\omega}}{\partial t} + \frac{\partial \omega'}{\partial t} + (U \cdot \nabla) \bar{\omega} + (U \cdot \nabla) \omega' + (u' \cdot V) \bar{\omega} + \nu \nabla^2 \omega' = 0 \]  
(4)

where \( \bar{\omega} = \nabla \times U \) is time-averaged vorticity, and \( \omega' = \nabla \times u' \) disturbance vorticity.

If the Mach number of time-averaged velocity is not very high, say, less than 2 or 3, then we can assume that the volume expansion corresponding to disturbance velocities be zero:

\[ \nabla \cdot u' = 0 \]  
(5)

When the Mach number increases and the circumferential non-uniformity increases, the error resulting from this assumption will increase, however, the benefit of employing this assumption in saving computer time is obvious. Further discussion about this assumption can be referred to Wu and Chen (1997). And more importantly, the tenability of this method is not affected by whether or not this assumption is made.

According to the assumption, the third and the fourth terms in the right-hand side of Eq. (4) can be neglected. Performing time-averaging operation to Eq. (4), and applying Eq. (3), one obtains:

\[ (U \cdot \nabla) \bar{\omega} + (u' \cdot V) \omega' = -\bar{\omega} (V \cdot U) - \omega' (V \cdot U) \]  
(6)

Subtracting Eq. (6) from Eq. (4), applying Eq. (5) and the feature that any time-averaged quantity does not vary with time, we obtain the final equation for disturbance vorticity:

\[ \frac{d \omega'}{dt} = -(u' \cdot V) \bar{\omega} - \omega' (V \cdot U) + (u' \cdot V) \omega' + \nu \nabla^2 \omega' \]  
(7)

where \( \frac{d}{dt} \) stands for material derivative. In the case of laminar flow, the viscosity is \( \nu = \nu_f \), for turbulent flow \( \nu = (\nu_f + \nu_t) \), the turbulent viscosity \( \nu_t \), was predicted by Baldwin-Lomax (1978) turbulence model.

The correlation term in right-hand side of Eq. (7) \( (u' \cdot V) \omega' \) stands for time-averaged quantity of the production of the two disturbance variables. It can be obtained by storing this production at each time step and then performing time-averaging operation.

**INITIAL AND BOUNDARY CONDITIONS**

The initial condition can be specified in a similar manner as Chen et al. (1985). We take a single-stage compressor as an example to describe the method (Fig.1). At an initial time \( t_0 \), as we don’t know the values of disturbance quantities inside the stator passage, so we set them equal zero except that of upstream boundary of stator computation domain. As mentioned above, if the relative steady solution of rotor is known, which can be written as \( \bar{\omega}_r (\bar{x}, \lambda t) \), where \( \bar{x}(x, y) \) stands for the coordinate of a point in the flow field, \( \lambda \) is the angular velocity of rotor, then the initial condition can be written as:

\[ q'(x, y, t_0) = \begin{cases} \bar{\omega}_r (x, y) - \bar{\omega}_r (x) & \bar{x} = \bar{x}_1 \\ 0 & \bar{x} \neq \bar{x}_1 \end{cases} \]  
(8)

where \( \bar{\omega}_r (x) \) is the circumferential averaged value of \( \bar{\omega}_r \), and \( \bar{x}_1 \) is the coordinate of upstream boundary of stator computational domain.

On the solid wall, the no-penetration and no-slip conditions should be satisfied:

\[ u' \cdot n = 0 \]  
(9)

\[ u' \cdot t = 0 \]  
(10)

where \( n, t \) are, respectively the normal and tangent unit vectors on the solid wall.

At upstream boundary, one can obtain the upstream boundary condition for disturbance variables at time \( t \), according to the same idea as obtaining Eq. (8):

\[ q'(x, y, t) = \bar{\omega}_r (x, y - W t) - \bar{\omega}_r (x) \]  
(11)

where \( t = t_0 + k \Delta t \), \( \Delta t = \frac{T}{M} \), time period, \( \tau = \frac{L}{W} \), \( \tau \) is rotor blade spacing, \( W \) is the moving velocity of rotor blades, and \( M \) is the number of time steps in one period \( T \).

On geometric boundaries in a single passage, if the rotor and stator have the same numbers of blades, the simple periodic condition holds:

\[ q(x, y, t) = q(x, y + P_s, t) \]  
(12)

where \( P_s \) is the stator blade spacing. If the rotor spacing differs from that of stator (Fig.1), one has to use the 'phase shift periodic boundary condition:

\[ q(x, y, t) = q(x, y + P_r, t + \Delta t) \]  
(13)

where \( \Delta t = (P_r - P_s) / W \).

As this boundary condition is rather complicated when the vortex method is used, we will not discuss it here in detail. The reader who concerns this can refer to Wu and Chen (1997).
TURBULENCE MODEL

Baldwin and Lomax turbulence model is used to calculate the turbulence viscosity coefficient, but this model faces two difficulties when it is used to compute unsteady rotor/stator interaction: one is the much higher computed viscosity in the outer region than that in the boundary regions according to the original notation of Baldwin and Lomax model; the other is the difficulty in continuously tracing moving wake centerlines, which is often encountered by other numerical methods.

To overcome the first problem, we redefine $y$ according to the method of Valkov et al. (1995) as the smallest distance at which $F(y)$ meets the condition:

$$\frac{dF}{dy}|_{y = y_{\text{min}}} = 0 \quad (14)$$

Based on this definition, the different effects of boundary layer and wakes can be distinguished.

In this paper, as the vortex method is a Lagrangian frame, which has the advantage to trace the trajectories of particles, so Baldwin-Lomax turbulence model can be conveniently used in wakes. But $F(y)$ and $\mu_{\text{BLM}}$ are defined as follows:

$$F(y) = y_0|y| \quad (15)$$

$$\mu_{\text{BLM}} = (\sqrt{u'^2 + v'^2})_{\text{max}} - (\sqrt{u'^2 + v'^2})_{\text{min}} \quad (16)$$

![Fig. 2 The centerlines and half-width lines of wakes](image)

The center point of the wake moving into the inlet of stator computational domain at any time is defined as the point where the direction of disturbance vorticity changes. The centers of wake transported in the stator passage at different time in a period are located. All the center points in one period consist the centerline of wakes. Fig. 2 shows three positions of wakes, for each position, the upper line is the centerline of wake, the lower line is a half-width line of wake.

In the outer layer of the boundary, the total viscosity is:

$$\mu_{\text{tot}} = \mu_i + \mu_w \quad (17)$$

where $\mu_i$ is the turbulence viscosity coefficient of boundary layers. $\mu_w$ the eddy viscosity of wakes. In the inner layer of the boundary, the effect of wakes is relatively weak, so the wake viscosity is negligible.

COMPUTATION PROCEDURE

The disturbance vorticity field can be represented by a set of discrete vortex particles:

$$\omega'(r, t) = \sum_{j=1}^{N} \Gamma_j \cdot f(x_j(t)) \quad (18)$$

where $N$ is the number of vortex particles, $r$ vector radius, $x_j(t)$ and $\Gamma_j$ the location and circulation of $j$th vortex at time $t$, respectively. $f_\sigma = \frac{1}{\sigma^2} f\left(\frac{\sigma}{r}\right)$, where $\sigma$ is the radius of vortex particle, $f(r)$ shape function, in the present study, a Gauss core of second order is adopted, i.e. $f(r) = \frac{1}{\pi} e^{-r^2}$.

The disturbance velocity field induced by the vortex particles are evaluated as follows:

$$\tilde{u}_v'(r, t) = \sum_{j=1}^{N} \Gamma_j \cdot K_\delta (r - x_j(t)) \quad (19)$$

where $K_\delta = K \cdot f_\delta$, $K(r) = \frac{-(y, x)}{2\pi |r|^3}$, and * stands for convolution operator.

According to Batchelor (1967), the disturbance velocity associated with expansion rate $H_{\text{m}}$ is:

$$\tilde{u}_v'(\tilde{x}, t) = \frac{1}{2\pi} \int H_{\text{m}} \frac{r}{r^3} dS' \quad (20)$$

According to Eq.(5), the expansion rate of disturbance velocity $\nabla \cdot \tilde{u}' = H_{\text{m}} = 0$, so $\tilde{u}_v' = 0$.

The disturbance velocity field consisted of $\tilde{u}_v' + \tilde{u}_v''$ usually can not meet boundary condition, so a potential disturbance velocity is added to ensure a no-penetrable condition on solid walls, i.e.

$$(\tilde{u}_v' + \tilde{u}_v'') \cdot n = -\tilde{u}_v' \cdot n \quad (21)$$

The potential disturbance velocity is the gradient of a potential function:

$$\nabla \phi(x) = \tilde{u}_v' \quad (22)$$

and the potential function is the solution of Laplace equation:

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad (23)$$

under the boundary condition of Eq.(21) on solid walls and unsteady boundary condition on geometrical boundaries of Eq. (12) or Eq. (13).

After $\Delta t$, a vortex particle originally at $\tilde{x}_1$ moves to $\tilde{x}_2$, according to the relation:

$$\tilde{x}_2 = \tilde{x}_1 + (\tilde{\Omega} + \tilde{u}') \Delta t \quad (26)$$

where the disturbance velocity $\tilde{u}'(\tilde{x}_1)$ is determined by the induced velocity and potential velocity at point $\tilde{x}_1$.

The circulation variation of disturbance vorticity caused by convection and viscous process is prescribed as follows:

$$\Gamma_j'(t + \Delta t) = \Gamma_j' + F[\nabla \cdot \tilde{u}'] \cdot \tilde{\Gamma}_j - \left[\nabla \cdot (\tilde{\Omega} + \tilde{u}') \cdot \tilde{\Gamma}_j + \nabla \tilde{u}' \cdot S_j \right] \cdot \Delta t \quad (24)$$

where $F[\cdot]$ stands for integration scheme, $\tilde{\Gamma}_j$ time-averaged circulation, $S_j$ a vortex element area. The correlation $\nabla [u' \cdot \nabla] \Gamma$ can be obtained through summing up $u' \cdot \nabla \Gamma$ of each time step and then performing time-averaging operation.

The contribution of viscous term on disturbance vorticity can be obtained by use of the deterministic vortex method. See below.
VISCOSITY TERM

Under the research frame employed by the present paper, it is assumed that once a periodic convergent solution is reached, the trajectories of vortex particles remain unchanged from period to period. Therefore it is not suitable to simulate viscous diffusion process by employing the random walk method, instead, the deterministic vortex method is employed. Here the method proposed by Fishelov (1990) was used, and the operator on vorticity can be written in the discrete form:

\[ \nabla^2 \omega_i = \sum_{j=1}^{N} \Delta f_{ij} (x_i - x_j) \nabla^2 \omega_i (t) \quad (25) \]

where \( \Delta = \nabla^2 \). This is our work form for viscous term.

VORTICITY GENERATION ON SOLID WALLS

The no-slip condition on solid walls can be satisfied if a set of vortex sheets is generated on solid walls for every time step. The strength of the vortex sheet must meet the condition:

\[ \Gamma' = \overrightarrow{\dot{u}'} \cdot \tau \quad (26) \]

where \( \overrightarrow{\dot{u}'} \) stands for the sliding velocity on solid walls, \( \tau \) a unit vector tangent to the solid wall. The vortex sheet is divided into several small elements, once they enter into main flow region, they are converted into vortex particles, while keeping their vorticity strength unchanged.

RESULTS AND DISCUSSIONS

To check the present method, we calculated the unsteady flows due to rotor/stator interaction in the first stage of NASA-67 compressor. The calculated results--unsteady velocity correlations \( \overrightarrow{u'} \cdot v' \), \( u'^2 \), \( v'^2 \)--are compared with experimental data (Hathaway (1986)) at four axial positions: 0, 20, 50 and 100 percent of chord, see Fig.3, Fig. 4 and Fig. 5. These contours show that the prediction and experimental data are in general agreement, especially \( u'^2 \) and \( v'^2 \). Although \( u'^2 \) is not very good near the stator leading edge, satisfactory agreement between the computational and experimental results at 20, 50 and 100 percent of chord is obtained. From the prediction we can conclude that the present method is reliable.

Table 1. Airfoil Mean section Characteristics of a Single-Stage Compressor

<table>
<thead>
<tr>
<th></th>
<th>Rotor</th>
<th>Stator</th>
</tr>
</thead>
<tbody>
<tr>
<td>stage pressure ratio</td>
<td>1.66</td>
<td>0</td>
</tr>
<tr>
<td>( U_{in} ) (m/s)</td>
<td>44.1</td>
<td>0</td>
</tr>
<tr>
<td>blade number</td>
<td>23</td>
<td>40</td>
</tr>
<tr>
<td>chord length (m)</td>
<td>0.1432</td>
<td>0.08115</td>
</tr>
<tr>
<td>solidity</td>
<td>1.491</td>
<td>1.442</td>
</tr>
<tr>
<td>radius ratio</td>
<td>0.41</td>
<td>0.51</td>
</tr>
<tr>
<td>aspect ratio</td>
<td>1.8</td>
<td>2.4</td>
</tr>
<tr>
<td>inlet angle (deg)</td>
<td>43.24</td>
<td>47.28</td>
</tr>
<tr>
<td>outlet angle (deg)</td>
<td>41.63</td>
<td>0</td>
</tr>
</tbody>
</table>

Based on the method, we calculate the unsteady flows in a high loading transonic compressor stage. The main features of the stage are listed on Table 1. The flow field on the stream surface at middle span was simulated. An H-type grids employed with the node number (47+43)*33, and 47, 43, represent axial grids of rotor and stator, respectively; 33, circumferential grid. The Reynolds number is \( 0.9 \times 10^7 \) based on the stator chord length and stator inlet velocity. One time period is divided into twenty time steps.

To see the convergence history, Fig. 6 and Fig. 7 show the static pressure and Mach number contours, respectively, as a function of rotor blade passing cycle. The pressure is normalized with respect to the inlet steady total pressure. We see from the figures that after three cycles the periodic repetition of pressure and Mach number contours appears from cycle to cycle, with the averaged relative error for total mesh points less than \( 3 \times 10^{-2} \).

To study the interaction of rotor wake with stator blades, Fig. 8 shows the disturbance vorticity contours at different time instants in a period. We see from the figures that when a wake is sweeping the stator blades, a complete wake is cut into several separate segments. So long as the vortex particles move into the stator passage, there appears a tendency that the particles are pushed towards the pressure surface of stator blades. There seem to be two processes during the particles moving downstream, i.e. (1) decentralization-centralization, Fig. 8a-f, with two vorticity concentration areas formed (see A and B) at circle 4.5, the strengths of which are -2.6 and -1.1, respectively; (2) concentration-decentralization, Fig. 8g-j, with vorticity concentration areas decentralized and broken near the exit of stator blades, where the pressure tends towards uniform distribution. The latter process indicates that as an interaction between wakes, vortices and boundary layers, a strong mixing process occurs, resulting in a mixing of main flow with fluid carrying highly concentrated vorticity. This coincides with the observation of Kerrebrock and Mikolajczak (1970).

An important problem much concerned by engineers is the influence of unsteadiness on incidence. Fig. 9 shows the variation of disturbance incidence with time in a period. The distribution is similar to a sine shape. The maximum disturbance is as high as 2.84 degrees, while the minimum is -0.374 degrees.

To study the decaying characteristic of unsteadiness, the amplitudes of the first harmonic of unsteady streamwise and transverse velocities at four axial location are shown in Fig.10 and Fig.11. The reference velocity is the steady velocity at the stator leading edge. The percent of the pitch is defined as 0 percent on the pressure side and 100 percent on the suction side. And the percent of the chord is defined as 0 percent at leading edge and 100 percent at trailing edge. So \( x / c = -.015 \) stands for a location of 1.5 percent axial chord upstream of stator; \( x / c = 0.168 \) and \( x / c = 0.75 \), two axial positions between the leading and the trailing edges; \( x / c = 1.018 \) represents a location of 1.8 percent axial chord downstream.

The maximum amplitude of streamwise disturbance velocity occurs at about 10 percent pitch near the pressure surface and the axial position \( x / c = 0.168 \), with magnitude as high as 18 percent of \( U_0 \). This is the result of the chopping of rotor wakes by the stator leading edge. Along the stator chord, the streamwise velocity decreases gradually. At \( x / c = 0.75 \), the amplitude is smaller than that at \( x / c = 0.168 \). And at the position \( x / c = 1.018 \), the amplitude of streamwise disturbance velocity is smallest of the four axial positions, where the disturbance velocity tends towards uniform distribution. The transverse disturbance velocity has the same characteristic as the
streamwise one. But the largest amplitude of the transverse velocity fluctuation is about 8 percent of $V_0$, occurring at 30 percent pitch near pressure surface and the axial position $x/c = 0.168$. From Fig. 10 and Fig. 11, we can see that the influence of unsteadiness are obvious near the leading edge of the stator blade and then decay rapidly along the stator chord. In addition, we also can see that the unsteadiness is stronger in the main flow region rather than near the pressure or suction surfaces. This is different from the turbulence fluctuation, which is much stronger in boundary layer near solid surfaces.

The amplitude of the first harmonic of the unsteady pressure on the stator blade surface is shown in Fig. 12 and Fig. 13. The amplitude is normalized by the inlet steady total pressure. For this configuration, the unsteady pressure on the pressure surface decays along the stator chord. This effect is much more significant from leading edge to 10 percent of stator’s axial chord. From this position to the trailing edge, the decaying trend becomes slower, and the minimum occurs at the trailing edge. On the suction surface of the stator blade, the amplitude decays gently. There is a little increase from the leading edge to 40 percent of chord, after this position, the amplitude begins to decrease. These decaying trends of unsteady pressure on solid walls agree with Ho and Lakshminarayana (1996), who observed the decay of unsteady pressure.

CONCLUDING REMARKS

The unsteadiness caused by wakes and other vortices in turbomachinery is simulated by employing a Lagrangian-disturbance vortex method proposed in the present paper. The Baldwin-Lomax turbulence model is applied in boundary layer and wakes with some development to overcome the difficulty encountered by other numerical methods, such as the much higher computed viscosity in the outer region than that in the boundary layers, and the difficulty in continuously tracing moving wake centerlines. Numerical results show that the predicted results are in good agreement with the experimental data. The calculated results also show that variety of unsteady flow phenomena, such as sweeping wakes and other vortices, and decaying of unsteadiness can be well simulated with good convergence property.

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Fig. 3 Circumferential distribution of $u'v'$ ($m^2/s^2$) for NASA-67 compressor

Fig. 4 Circumferential distribution of $u^2$ ($m^2/s^2$) for NASA-67 compressor

Fig. 5 Circumferential distribution of $v^2$ ($m^2/s^2$) for NASA-67 compressor
Fig. 6 Time history of Pressure at 50% chord of stator

Fig. 7 Time history of Mach Number at 50% chord of stator

Fig. 8 Disturbance vorticity contours at different instants in one period
Fig. 9 Variation of disturbance incidence with time in one period

Fig. 10 First harmonic of the streamwise disturbance velocities

Fig. 11 First harmonic of the transverse disturbance velocities

Fig. 12 First harmonic of unsteady pressure on pressure surface

Fig. 13 First harmonic of unsteady pressure on suction surface