A DESIGN STUDY OF RADIAL INFLOW TURBINES WITH SPLITTER BLADES IN THREE-DIMENSIONAL FLOW

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Abstract

An inverse design technique to design turbomachinery blading with splitter blades in three-dimensional flow is developed. It is based on the use of Clebsch transformation which allows the velocity field to be written as a potential part and a rotational part. It is shown that the rotational part can be expressed in terms of the mean swirl schedule (the circumferential average of the product of radius and tangential velocity) and the blade geometry that includes the main blade as well as the splitter blade. This results in an inverse design approach in which both the main and the splitter blade geometry are determined from a specification of the swirl schedule. Previous design study of a heavily-loaded radial inflow turbine, without splitter blades, for a rather wide variety of specified mean swirl schedules result in a blade shape with unacceptable non-radial blade filament; the resulting reduced static pressure distribution yields an "inviscid reverse flow region" covering almost the first half of the blade pressure surface. When the inverse design technique is applied to the design study of the turbine wheel with splitter blades, the results indicate that the use of splitter blades is an effective means for making the blade filament at an axial location more radial as well as a potential means for eliminating any "inviscid reverse flow" region that may exist on the pressure side of the blades.

1 Introduction

In the two papers on the "Theory of Blade Design for Large Deflections" published by Tan, et al. (1984), a new technique was presented for designing the shape of turbomachinery blades in three-dimensional flow. Applications of this theory to the design of radial turbomachinery blading have been reported by Borges (1986), Ghaly (1986), Hawthorne and Tan (1987), Zangeneh (1988), and Yang, Tan, and Hawthorne (1993). The experimental data reported by Borges (1986) and Zangeneh (1988) showed that the radial inflow turbines designed by the new technique gave an improvement in efficiency over a rather wide operating range. Studies on the influence of the loading distribution, stacking position, etc., upon the resulting blade shape and pressure distribution were reported by Ghaly (1986). A concise but general exposition of the theoretical basis for the present design technique was presented by Hawthorne and Tan (1987). Yang (1993) applied the theory successfully to the design study of turbomachinery bladings in transonic flow regimes. In this paper, we describe:

- the extension of the theory to include the design of turbomachinery blade-rows with main blades as well as splitter blades;
- the application of the method to the design study of a highly-loaded radial inflow turbine wheel with splitter blades.

The design study carried out in (2) above is built upon the work by Yang (1991, 1993), which show that for a wide variety of specified mean swirl distributions the resulting inverse design calculations for a heavily-loaded radial inflow turbine invariably yields a blade geometry with unacceptable non-radial blade filaments (i.e., unacceptable large blade lean angle). Furthermore, the resulting reduced static pressure distribution is always such that there exists a fairly large region of "inviscid reverse flow" on the pressure surface of the (designed) blade. A key objective of the present work is to assess if the incorporation of splitter blade in the design of the radial turbine wheel would result in a blade geometry that has nearly radial blade filament as well as a negligibly small region of "inviscid reverse flow".

The paper is arranged as follows. In the next section we describe a concise theoretical development for inclusion of splitter blade in the inverse design technique presented by Tan et al. (1984), Hawthorne & Tan (1987) and Yang et al. (1993). We then discuss the design criteria and the manner in which the mean swirl distribution is prescribed for the situation where an impeller has main blades as well as splitter blades. This is followed by representative results from the design calculations. Finally we state the summary and the conclusions.

2 Theoretical Development

As the theoretical formulation and the key results have been presented (Tan et al., 1984; Hawthorne & Tan, 1987; Yang et al., 1993), we will confine the theoretical development to those aspects pertinent to the inclusion of splitter blades in the inverse design technique.

It has been proposed (Tan et al., 1984; Hawthorne & Tan, 1987; Yang et al., 1993) that a useful design specification for the inverse design of blading in three-dimensional flow is the mean swirl distribution \( \rho \psi \) (\( \psi \) is the tangential velocity); it is given by:

\[
\rho \psi(r, z) = \frac{B}{2\pi} \int r \psi(r, \theta, z) d\theta
\]

where the over bar "-" defines a tangential mean, \( B \) is the number...
of blades, and \((r, \theta, z)\) is the usual right-handed cylindrical coordinate system. We will use subscript \((r, \theta, z)\) to denote vector component in the radial, tangential and axial direction respectively. As \(2\pi rV_\theta\) is the circulation around the axis, the swirl distribution is thus related to the bound circulation \(\Gamma\) on the blade. Thus it is natural to use surfaces of singularities to represent the blade surfaces (for the main blade as well as splitter blade):

\[
\alpha_j(r, \theta, z) = \theta - f_j(r, z) = \pm \frac{2\pi n_j}{B} \text{ for } n = 0, 1, 2, 3, \ldots, (B - 1) \quad (2)
\]

where \(f\) is the angular coordinate of a point on the blade surface, \(B\) is the number of blades, and \(n\) is an integer. The subscript \(j\) can assume a value of 1 or 2; the value of 1 pertains to the main blade while the value 2 pertains to the splitter blade.

Assuming the flow is steady and that far upstream the approaching flow is axisymmetric, irrotational and reversible (i.e. a homentropic flow), then the absolute vorticity \(\Omega\) bound to the blades can be written in the form

\[
\Omega = \sum_{j=1}^{2}(\nabla r V_\theta)_j \times \nabla \alpha_j \delta_\phi(\alpha_j) \quad (3)
\]

where \(\delta_\phi(\alpha)\) is the periodic delta function (Lighthill, 1958) given as

\[
\delta_\phi(\alpha) = \frac{2\pi}{B} \sum_{n=\infty}^{\infty} \delta(\alpha - \frac{2\pi n_j}{B}) = \sum_{n=\infty}^{\infty} e^{inB\alpha} \quad (4)
\]

Use of Clebsch’s transformation (Lamb, 1932) allows the (absolute) velocity \(\vec{V}\) to be written as (Tan etc, 1984; Hawthorne & Tan, 1987; Yang etc, 1993)

\[
\vec{V} = \vec{\Phi} + \sum_{j=1}^{2} r V_\theta \nabla \alpha_j - S(\alpha_j) \nabla r V_\theta_j \quad (5)
\]

where \(S(\alpha)\) is the periodic sawtooth function (Lighthill, 1958), given as

\[
S(\alpha) = \sum_{n=\infty}^{\infty} \frac{e^{inB\alpha}}{inB} \quad (6)
\]

We can decompose the velocity as a sum of a mean part \(\bar{V}\) and a periodic part \(\tilde{V}\) as

\[
\vec{V} = \bar{\bar{V}} \vec{\Phi} + \sum_{j=1}^{2} r V_\theta \nabla \alpha_j - S(\alpha_j) \nabla r V_\theta_j \quad (7)
\]

\[
\tilde{V} = \bar{\bar{V}} \vec{\Phi} - \sum_{j=1}^{2} S(\alpha_j) \nabla r V_\theta_j \quad (8)
\]

The continuity equation can be written as

\[
\nabla \cdot \bar{\bar{V}} = -\bar{\bar{W}} \cdot \nabla \ln \rho \quad (9)
\]

where \(\bar{\bar{W}}\) denotes the relative velocity and \(\rho\) the density. Taking the pitch average of the above equation, we obtain

\[
\nabla \cdot \bar{\bar{W}} = -\bar{\bar{W}} \cdot \nabla \ln \rho \quad (10)
\]

We introduce an artificial density (Ghaly, 1986) \(\rho_a(r, z)\) that satisfies

\[
\nabla \cdot \rho_a \bar{\bar{W}} = 0 \quad (11)
\]

to account for compressibility effects. Use of Eq.(10) in Eq.(11) yields

\[
\bar{\bar{W}} \cdot \nabla \ln \rho_a = \bar{\bar{W}} \cdot \nabla \ln \rho \quad (12)
\]

We use Stokes stream function, \(\psi\), to describe the mean flow. Thus the velocity components in the mean flow are given as:

\[
\rho_a \bar{W}_r = -\frac{1}{r} \frac{\partial \psi}{\partial z} \quad (13)
\]

\[
\bar{W}_\theta = \sum_{j=1}^{2} \frac{1}{r} \left( \frac{\partial \psi}{\partial r} \right)_j \quad (14)
\]

\[
\rho_a \bar{W}_z = -\frac{1}{r} \frac{\partial \psi}{\partial r} \quad (15)
\]

The governing equation for \(\psi\) can be shown to be

\[
\left[ \frac{\partial^2 \psi}{\partial z^2} - \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \psi}{\partial r} \right) \right] = 0 \quad (16)
\]

A governing equation for the periodic velocity \(\tilde{V}\) is

\[
\nabla \cdot \tilde{V} = -\bar{\bar{W}} \cdot \nabla \ln \rho + \bar{\bar{W}} \cdot \nabla \ln \rho \quad (17)
\]

Use of Fourier Series for representing the \(\theta\)-dependence of the velocity (since the flow has inherent blade-to-blade periodicity) allows us to write

\[
\bar{V} = \sum_{n=\infty}^{\infty} \bar{V}_n(r, z)e^{inB\theta} \quad (18)
\]

Substituting the above equation and Eq.(8) into Eq.(17), the result is

\[
\nabla^2 \bar{V}_n = \frac{2}{inB} \nabla^2 r V_\theta_j - e^{inB\theta} \left( \frac{\partial^2 \psi}{\partial r^2} + \frac{\partial^2 \psi}{\partial z^2} \right) + \left( -\bar{\bar{W}} \cdot \nabla \ln \rho + \bar{\bar{W}} \cdot \nabla \ln \rho \right) FT(n) \quad (19)
\]

where \(\nabla^2 \bar{V}_n = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \bar{V}_n}{\partial r} \right) + \frac{\partial^2 \bar{V}_n}{\partial z^2} - \frac{n^2 \pi^2}{B^2} \) and \(FT(n)\) denotes the Fourier coefficients of the \(n^{th}\) mode.

The condition of no normal velocity on the blade surface yields

\[
\bar{W}_n \cdot \nabla \alpha_j = 0 \quad (20)
\]

where \(\bar{W}_n = \frac{1}{2}(\bar{W} + \bar{W}^*)\) is the velocity at the blade (superposition \(+/-\) denotes flow variables pertaining to the blade pressure/suction surface). Upon expanding the above equation in cylindrical coordinates, we have

\[
\bar{W}_r \frac{\partial f_j}{\partial r} + \bar{W}_z \frac{\partial f_j}{\partial z} = \sum_{k=1}^{2} \left[ r V_\theta_k \right] - \omega + (\bar{\bar{V}})_{nj} \cdot \nabla \alpha_j \quad (21)
\]

In the absence of splitter blades, all the above equations reduce to the equations for the situation with no splitter blades. The effect of blade thickness can be included in the above formulation as described by Yang etc (1993); this is not repeated in the formulation above as the key objective here is to explore the aerodynamic influence of the use of splitter blades in a highly-loaded radial inflow turbine.

The procedure for determining the blade shape is an iterative one (Tan etc, 1984; Hawthorne & Tan, 1987; Yang etc, 1993). A first guess for the blade shape is used to compute the velocities, and the blade shape is updated by using the blade boundary condition in Eq.(21). The solution for Eq.(21) specifies the integration constant for \(f_j\), which implies that values of \(f\) have to be specified along a line on the surface stretching from the hub to the shroud for both the main blade and the splitter blade. This specification has been referred to as the "stacking condition" (Tan etc, 1984; Hawthorne & Tan, 1987; Yang etc, 1993).
2.1 Boundary Conditions

When the relative flow is subsonic, both the governing equations for the mean and the periodic flow are of the elliptic type. Thus, their solution requires the specification of a boundary condition at: (a) an upstream boundary; (b) a downstream boundary; (c) the shroud and the hub surfaces; and (d) at the blade surface.

For practical implementation, we will have to truncate the Fourier series by taking a finite n in Eq.(18). Spatial discretisation on (r,z)-plane is based on finite-element technique(Yang etc, 1993).

3 Design Criteria and The Mean Swirl Distribution \( rV_\theta \)

In this section we state the criteria used for assessing the aerodynamic goodness of results from design calculation for the main and splitter blades. Then we briefly review the technique for the specification of \( rV_\theta \)-distribution.

3.1 Design Criteria

The parametric influence of loading distribution, stacking position, lean in stacking angle, slip factor, number of blades, and hub and shroud profile geometry on the blade shape and the resulting flow field has been explored(Yang etc, 1991, 1993). However, those results show that for a rather wide choice of \( rV_\theta \) distributions, the resulting blade shapes are such that the blade filaments at constant z section are highly nonradial; this aspect of the blade shape is unacceptable from structural/stress consideration. Furthermore, it is desirable to have a region of "inviscid reverse flow" on the pressure surface which may imply flow separation for real fluids.

In the present investigation, the inclusion of splitter blades in the design is proposed as a means to eliminate or reduce the "inviscid reverse flow regions" and to arrive at designs in which the blade filaments at constant z section is more radial. In view of these, we have adopted the following measures for assessing the goodness of the resulting design calculation:

- the curvature of the blade shapes at constant z sections
- the extent of the region of "inviscid reverse flow"

However, the extent of the reverse flow region can be considered as secondary as the reverse flow region may not necessarily result in an unacceptable loss level (Yang etc, 1991, 1993).

The inclusion of splitter blades in the design procedure results in several additional degrees of freedom. These are:

- the length of the splitter blades
- \( rV_\theta \) distribution on the splitter blades
- stacking position of the splitter blades
- lean in stacking angle of the splitter blades
- the relative angular distance between the main blades and the splitter blades at the stacking position (i.e. only the value at the stacking position; it does not apply to the entire splitter blade)

However, some of the above degrees of freedom are constrained by the limitation that the relative angular distance between the main and splitter blades has to be within the values of 0 and \( \frac{2\pi}{3} \). This implies that the resulting splitter blade shape cannot cross the main blade, which arises because of physical consideration. However, it is desirable to have a relative angular distance that has a value between 0.4 and 0.6 when expressed as a fraction of the local pitch (which is equal to the angle between main blades, i.e., \( 2\pi/3 \)) so as to avoid flow choking.

3.2 Specification of \( rV_\theta \) distribution

The various aspects pertaining to the specification of \( rV_\theta \) distribution have been presented in detail (Yang etc, 1991, 1993). We will simply summarise the key aspects here for ease of reference.

The swirl distribution \( rV_\theta \) cannot be specified arbitrarily(Tan etc 1993; Hawthorne & Tan, 1987; Yang etc, 1993); it has to satisfy boundary conditions that involve its value and its normal derivative along the boundary of the blade region. Thus there are two conditions along each boundary so that the generation of a smooth \( rV_\theta \) distribution from a fourth order equation such as the following biharmonic equation is an appropriate one:

\[
\nabla^4 (rV_\theta) = R(r,z) \tag{22}
\]

where

\[
\nabla^4 = (\frac{\partial^4}{\partial r^4} + \frac{4\partial^2}{\partial r^2} + \frac{\partial^4}{\partial z^4})
\]

The forcing function \( R(r,z) \) can be used as a means to control the swirl distribution. However by taking \( R(r,z) \) to be identically zero (Yang etc, 1993) the swirl distribution is determined upon specifying the value of \( rV_\theta \) along the hub and the shroud, the leading edge and the trailing edge. Thus the problem is reduced to specifying the swirl distribution along the boundary. A way to implement this is as follows:

The hub or the shroud region is divided into three sections (see Fig. 1) (Yang etc, 1993): from the leading edge (L.E.) to a point called \( A \), from point \( A \) to a second point called \( B \), and from point \( B \) to the trailing edge (T.E.). The maximum value of \( \rhoV_\theta \) is specified at point \( A \); \( s \) is used to denote the meridional distance along the quasi-streamline coordinate. To control the shape of the loading, we specify the magnitude of \( \rhoV_\theta \) along the second section (from point \( A \) to point \( B \)). The Kutta-Joukowski condition requires that \( \rho\omegaL = 0 \) at the trailing edge, and \( \rho\omegaD \) at the leading edge is specified, which would be zero for zero incidence angle. A typical \( rV_\theta \) distribution along the hub of the main and the splitter blades is shown in Fig. 1, where lowercase \( a \) and \( b \) denote the points for the splitter blade and the upercase \( A \) and \( B \) for the main blade. In general:

Section 1 : \( rV_\theta(s) \) is cubic in \( s \) and we need to specify

\[
\begin{align*}
(rV_\theta)_{L.E.} & = (rV_\theta)_{L.E.}^s, (rV_\theta)_{T.E.} = (rV_\theta)_{T.E.}^s \quad \text{at L.E.} \\
(rV_\theta)' & = (rV_\theta)'_{max}, (rV_\theta)'' = (rV_\theta)''_{A-B} \quad \text{at } A
\end{align*}
\]

Section 2 : \( rV_\theta(s) \) is quadratic in \( s \) and the dependence on \( s \) is chosen such that

\[
\begin{align*}
(rV_\theta)_{A} & = (rV_\theta)_{L.E.}^s, (rV_\theta)'_{A} = (rV_\theta)'_{A} \quad \text{at } A \\
(rV_\theta)_{B} & = (rV_\theta)_{T.E.}^s, (rV_\theta)''_{B} = (rV_\theta)''_{B} \quad \text{at } B \\
(rV_\theta)' & = (rV_\theta)'_{T.E.}, (rV_\theta)'' = 0 \quad \text{at } T.E.
\end{align*}
\]

in addition we need to specify

\[
(rV_\theta)'_{T.E.} = 0 \quad \text{at } T.E.
\]

where subscript \( A, B, L.E. \), and \( T.E. \) denote the respective value at location \( A, B, L.E. \), and \( T.E. \).

The choice of the location of point \( A \) (the maximum loading position), the location of point \( B \) (the extent of finite loading), \( (rV_\theta)_{max} \) (the maximum loading), and \( (rV_\theta)_{A-B} \) (the shape of the loading) allows one to produce a wide variety of swirl distributions.
3.3 Division of \( (r \theta V_r) \) between Main and Splitter Blades

The net specified \((r \theta V_r)\) has to be shared between the main and the splitter blades; in particular one needs to assign a priori a fraction of \((r \theta V_r)\) at the leading edge to the splitter. For the results presented here this fraction is the ratio of the mean length of the splitter blade to the sum of the mean length of the blade and the splitter (the mean length is taken to be the average of the meridional distances along the shroud and the hub of the blades). Such a simple linear division would yield a vanishing value of \((r \theta V_r)\) for a splitter blade of zero length and a value identical to that of the main blade for a splitter length equals that of the main blade. In addition the above division of \((r \theta V_r)\) at the leading edges and the manner in which \((r \theta V_r)\) is prescribed along the hub and the shroud would result in a leading distribution which is approximately the same for both the main and the splitter blade.

3.4 The Stacking Conditions

As it is the net \((r \theta V_r)\), and not the individual \((r \theta V_r)\) on each blade, that appears on the RHS of Eq.(21) therefore both the main blade and the splitter would assume identical blade shape if one were to neglect the blade-to-blade effects (last term on the RHS of Eq.(21)). Thus if indeed the blade-to-blade variation is relatively small, the camber distribution of the main and the splitter blade would approximately be similar for identical stacking specifications. Based on this heuristic argument the same stacking conditions for the main blade and the splitter have been chosen for the present study. However the angular distance between the main and the splitter blades at the stacking positions need not be necessarily set to \( \pi \) (i.e. the splitter is exactly halfway between the main blades at the stacking position) so that it can be considered as an additional degree of freedom; its choice can influence the relative mass flow between the two different passages: one between the suction side of the main blade and the pressure side of the splitter blade and the other between the suction side of the splitter and the pressure side of the main blade. When the relative angular distance \( \sigma \), expressed as fraction of the angle between the main blades, is less than or greater than 0.5, the splitter is closer to the suction side or the pressure side of the main blade respectively. Only the position of the splitter blade relative to the main blade is specified at the stacking position.

3.5 Distribution of \( \frac{\mathbf{W}_s}{r} \) and the Wrap Factor

Upon neglecting the last term in Eq.(21), the blade boundary condition reduces to:

\[
\frac{\partial f}{\partial z} = \frac{1}{V_s} \frac{\mathbf{W}_s}{r}
\]  

This suggests that if we integrate the value of \( \frac{\mathbf{W}_s}{r} \) with respect to the distance along the hub and the shroud from the stacking position to the trailing edge and take the difference between them, we would get some guidance on how to make the blade filament at constant \( z \) more radial. This integrated value will be referred to as the Wrap Factor given as:

\[
\text{Wrap Factor} = \left( \int_{\text{trailing edge}}^{\text{stacking axis}} \frac{\mathbf{W}_s}{r} ds \right)_{\text{hub}} - \left( \int_{\text{trailing edge}}^{\text{stacking axis}} \frac{\mathbf{W}_s}{r} ds \right)_{\text{shroud}}
\]  

The values of \( \frac{\mathbf{W}_s}{r} \) and the distances along the hub and the shroud are precisely known once we specify the geometry and the \( r \theta V_r \) distributions along the hub and shroud. The Wrap Factor can be used as a measure of the degree of curvature in the blade filament; this can be done a priori without having to implement the design calculation.

If the values of the mean meridional velocity on the hub and the shroud are the same at the same axial location and if the last term in Eq.(21) is indeed small, then one would expect to have a Wrap Factor of value close to zero for highly radial blade on cross section at constant \( z \). Otherwise, a value of the Wrap Factor different from zero would result in an optimally radial blade filament. This Wrap Factor will be used as a guide for specifying \( r \theta V_r \) distributions along the hub and the shroud for the design examples presented in the next section.

4 Numerical Results From a Design Study

We now present a sample of results from a design study of a radial inflow turbine with splitter; this turbine has been proposed for application in a helicopter power plant (Gvinskas et al, 1984). The design specifications for the turbine are: mass flow rate of 2.37 kg/sec, power of 1105 kW, inlet stagnation pressure of 1.637 x 10^6 N/m^2, inlet stagnation temperature of 1607 K, wheel speed 64,000 rpm, rotor tip diameter of 0.2038 m. The absolute flow angle at the inlet is about 75° and the corresponding absolute Mach number was computed to be about 0.98. The radial inflow turbine wheel will be designed to yield a blade camber distribution that will result in the complete removal of swirl at the outlet. Thus, the flow will leave the turbine wheel axially at the exit.

For the results to be shown in the following, lengths are in units of impeller tip diameter and \( r \theta V_r \) in units of its value at the blade leading edge unless indicated otherwise.

The calculation is implemented with a grid resolution of 49 elements spanning hub-to-shroud region, 35 Fourier collocation points between adjacent blades, and 145 elements extending from inflow to outflow boundary (Fig. 2). A 4-node isoparametric elements for interpolation is used. For a blade number of 14 and a wide variety of specified \( r \theta V_r \) distributions as well as stacking conditions (Yang et al, 1993), the three-dimensional inverse design method was used to shape the blade filament (Fig. 3) and a relatively large region of "inviscid reverse flow" on the blade pressure surface (Fig. 4). Design calculations have been carried out (Tjokroaminata, 1992) to elucidate the influences of splitter length, \( r \theta V_r \) distribution, stacking conditions, finite incidence angle, blockage and slip factor on the blade shape as well as on the extent of "inviscid reverse flow" region on the pressure surface of the blades. Here we will present sample of results to demonstrate the potential benefits of the use of splitters in radial inflow turbine design.

For the design examples reported here the splitter length has been taken to be 0.61 of the length of main blade. In the first case (which will be referred to as Case A) the \( r \theta V_r \) along the hub and the shroud are as shown in Fig. 5a and 5b respectively. The maximum value of \( \frac{\partial \mathbf{W}_s}{r} \) on the hub is chosen to be -1.75 while the maximum value of \( \frac{\partial \mathbf{W}_s}{r} \) on the shroud is -3.0. The specified \( r \theta V_r \) distribution along the hub and shroud is rather similar to that reported by Zangeneh (1988) which yields a blade shape with a lean angle of no more than 10°. Shown in Fig. 6a and 6b are the corresponding swirl distributions on the main and splitter blades. The guideline (that the stacking position should be chosen to be near where the loading is maximum and preferably slightly downstream of it) reported by Yang et al (1993) the stacking axis is selected to be at a location 1/3 chord downstream from the leading edge; this corresponds to the location of the maximum loading. The radial distribution of resulting main blade camber along constant \( z \) section is shown in Fig. 7. Indeed this new \( r \theta V_r \) distribution gives a significant improvement in the blade camber distribution. The lean angle along constant \( z \) section has been reduced to 25.8° from a value of 56.3° (results of Fig. 3). These results appear to indicate that for designing blade with more radial filament along constant \( z \) section, one should specify the maximum loading along the shroud to have a value more negative than that along the hub. Indeed, such a specification would tend to yield a more optimal value for the Wrap Factor alluded to in the above.
The inverse design theory and the accompanying computational procedure have been extended to include the design of splitter blades. In particular, the present study has focused on blade design that attempts to make the blade filament on constant s section more radial as well as to reduce the extent of region of "inviscid reverse flow" on the pressure side. Based on the results from these parametric design calculations, we deduce that:

1. The use of splitter blades gives more flexibility for tailoring the loading on the main blades and this has resulted in the reduction or elimination of "inviscid reverse flow" region on the pressure side of the blades. Its use has also resulted in blades with nearly radial blade filament.

2. The extent of the "inviscid reverse flow" region is more influenced by the specified loading than by the stacking position.

3. A number referred to as the Wrap Factor can be constructed to give an indication of the curvature of the blade shape at constant s section. The use of this number can reduce the number of design calculations aimed at obtaining more radial blade shape.

4. In general, the main blade filament near the trailing edge can be made more radial by increasing the Wrap Factor while that near the leading edge can be made more radial by imposing a non-zero incidence angle.

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References


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**Fig. 1** Schematic plot of $rV_0$ distribution along the hub of the main and splitter blades; A and B (a and b) demarcates region of different dependence of $\frac{\partial V_0}{\partial s}$ on s for main blades (for splitter blades).

**Fig. 2** Grid used for design calculation (splitter length of 0.61).
Fig. 3  Radial distribution of blade camber on s-section at \( \frac{r}{r_{TE}} = 0.05, 0.25, 0.5, 0.75, 1.00 \) from design calculations in (Yang et al., 1993); \( r_{TE} \) denotes value of \( r \) corresponding to blade trailing edge.

Fig. 5a  The distribution of \( r\theta, \bar{W}_\theta \), and \( \frac{\partial r\theta}{\partial s} \) along the hub for Case A (stacking at 1/3 chord).

Fig. 4  The \( M_{rel} \) on the pressure side indicating presence of inviscid reverse flow over nearly first half of blade from design calculations in (Yang et al., 1993).

Fig. 5b  The distribution of \( r\theta, \bar{W}_\theta \), and \( \frac{\partial r\theta}{\partial s} \) along the shroud for Case A (stacking at 1/3 chord).
Fig. 6a  The distribution of $r V_\theta$ on main blade for Case A (stacking at 1/3 chord).

Fig. 7  Radial distribution of main blade camber on z-section at $z/z_{chord} = 0.05, 0.25, 0.5, 0.75, 1.00$ for Case A (stacking at 1/3 chord).

Fig. 6b  The distribution of $r V_\theta$ on splitter blade for Case A (stacking at 1/3 chord).

Fig. 8  The distribution of $r V_\theta$, $W_\theta$, and $\partial V_\theta/\partial s$ along the shroud for Case B.
Fig. 9a. The distribution of $rV_{\theta}$, $\frac{\partial V_{\theta}}{\partial s}$, and $\frac{\partial^2 V_{\theta}}{\partial s^2}$ along the hub for Case C.

Fig. 10a. Radial distribution of main blade camber on $z$-section at $\frac{r_{za}}{r_{za}} = 0.05, 0.25, 0.5, 0.75, 1.00$ for Case B.

Fig. 9b. The distribution of $rV_{\theta}$, $\frac{\partial V_{\theta}}{\partial s}$, and $\frac{\partial^2 V_{\theta}}{\partial s^2}$ along the shroud for Case C.

Fig. 10b. Radial distribution of main blade camber on $z$-section at $\frac{r_{za}}{r_{za}} = 0.05, 0.25, 0.5, 0.75, 1.00$ for Case C.
Fig. 11 Radial distribution of splitter blade camber on z-section at $\frac{r}{r_{as}} = 0.05, 0.25, 0.5, 0.75, 1.00$ for Case C.

Fig. 13a $M_{rel}$ on the pressure side of splitter blade for Case B.

Fig. 12 $M_{rel}$ on the pressure side of main blade for Case C.

Fig. 13b $M_{rel}$ on the pressure side of splitter blade for Case C.
Fig. 14a  The distribution of $r\dot{V}_\theta, \frac{\partial V_\theta}{\partial s},$ and $\frac{\partial^2 V_\theta}{\partial s^2}$ along the hub for Case D (nonzero $\frac{\partial V_\theta}{\partial s}$ at L.E.).

Fig. 14b  The distribution of $r\dot{V}_\theta, \frac{\partial V_\theta}{\partial s},$ and $\frac{\partial^2 V_\theta}{\partial s^2}$ along the shroud for Case D (nonzero $\frac{\partial V_\theta}{\partial s}$ at L.E.).

Fig. 15  Radial distribution of main blade camber on s-section at $\frac{s}{s_0} = 0.05, 0.25, 0.5, 0.75, 1.0$ for Case D (nonzero $\frac{\partial^2 V_\theta}{\partial s^2}$ at L.E.).