Surge Dynamics in a Free-Spool Centrifugal Compressor System

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ABSTRACT

Turbocharger surge has been investigated in a radial impeller-vaneless diffuser free-spool system. Several different aspects are addressed. First, two very different compression systems, one with a large downstream volume and one with the smallest possible downstream volume, are employed to examine stall initiation phenomena as well as the behavior of the compressor characteristics when operating in surge. The measurements show impeller stall at the inducer tips to be a key phenomena in initiating surge. The inducer stall is stationary and asymmetric, due to the presence of the volute, and is most severe near the volute tongue angular position. The compressor characteristic in the large volume system (which gave surge) is observed to be flatter and to lag that in the stabilized small volume system. The difference arises because of the slow development time and differing circumferential extent of the inducer stall present at a given mass flow.

A nonlinear simulation of the system is also presented. The model deviates from previous treatments of unsteady flow in compressor systems in that the assumption of constant rotor speed is relaxed. Including a time lag on the order of the compressor throughflow time, together with proper treatment of speed variations, is shown to dramatically improve agreement with the observed surge behavior.

NOMENCLATURE

\( a \)  
speed of sound

\( A \)  
flow area

\( B \)  
dimensionless parameter; defined in Eq. 1

\( C_x \)  
axial velocity

\( C' \)  
slope of compressor pressure rise characteristic; \( d(\Delta P)/d\Phi \)

\( F \)  
spool inertia parameter; defined in Eq. (8)

\( \omega_H \)  
Helmholtz frequency

\( \gamma \)  
ratio of specific heats; \( c_p/c_v \)

\( I \)  
polar moment of inertia of turbospool

\( L \)  
length

\( m \)  
mass flow

\( m^* \)  
choked flow through impeller eye area \( A_{ref} \) at ambient \( P_0 \) and \( T_0 \)

\( M_{to} \)  
impeller tip Mach index; \( U/a_0 \)

\( N \)  
rotational speed

\( P \)  
pressure

\( r_R \)  
radius

\( T \)  
temperature

\( T' \)  
slope of throttle characteristic; \( d(\Delta P)/d\Phi \)

\( t \)  
time

\( U \)  
impeller blade tip velocity

\( V_p \)  
volume of compression system plenum

\( \Pi \)  
nondimensional pressure; \( P/P_{ref} \)

\( \rho \)  
density

\( \theta \)  
total temperature ratio, characteristic time

\( \Phi \)  
flow coefficient; defined by \( \dot{m}c/(pA U) = C_x U \), where \( p \) is appropriate density; \( C_x U \) in plots

\( \psi \)  
isentropic head coefficient; \([\Pi^{(\gamma-1)/(\gamma-1)}-1]/(\gamma-1)M_{to}^2 \)

\( \Delta P \)  
non-dimensional pressure rise; \( (P-P_0)/(0.5 \rho U^2) \)

\( \phi \)  
inducer eye position

\( \omega \)  
drive

\( H \)  
Helmholtz

\( in \)  
inlet

\( p \)  
plenum conditions

\( ref \)  
reference condition value

\( t \)  
throat

\( 0 \)  
at or relative to ambient condition, equilibrium value

\( 1 \)  
inducer eye position

\( 2 \)  
impeller tip diameter position

\( 3 \)  
volute discharge position

\( \sim \)  
non-dimensional quantity

INTRODUCTION

Most compressors demonstrate marked changes in behavior and flow patterns when operated at flow rates substantially below those for which they were designed. For axial compressors, which

* Work done prior to GE employment at the MIT Gas Turbine Laboratory under Cumming Engine Company and Air Force Research in Aero Propulsion Technology (AFRPT) sponsorship.
are generally much better understood than centrifugal compressors, it is well known that there are two quite different ways in which the machine can adapt to reduced flow. In one of these, the compressor can surge so that an essentially axisymmetric pulsing of the flow occurs. In the other, a rotating stall pattern can be set up so part of the annulus passes little or no flow, with the low flow portion rotating around the circumference. Whether an axial compressor exhibits surge or rotating stall depends not only on the compressor but on the system in which it is installed.

The understanding of stall and surge in centrifugal compressors is considerably poorer than for axial compressors, and it remains an open question whether rotating stall has a role in surge for centrifugal compressors. Rotating stall can occur in centrifugal compressors, as was seen in the pioneering work with centrifugal compressors of Emmons et al. (1955). Evidence of rotating stall has also been found in both the impeller and diffuser by Kammer and Rautenberg (1982) and by Abdel-Hamid et al. (1978). However, the time resolved measurements of the flow in a centrifugal compressor entering surge made by Toyama, Runstadler and Dean (1977) and by Dean and Young (1977) did not show rotating stall prior to the reversal of the overall flow that is characteristic of surge. Instead, they found that surge was preceded by stall in the inlet region of the vane diffuser.

One difficulty in arriving at a general description of centrifugal compressor instability is that the speeds, the overall system, and the compressor geometries cover a very wide range. With a centrifugal stage having a vane diffuser and designed for a pressure ratio of say, 4:1, the element likely to stall at design speed is the vane diffuser. The same machine operated at lower speed so that the pressure ratio is only, say, 2:1 is likely to stall in the impeller inducer. The point is that, as described by Cumpsty (1989), there is an overwhelming effect of matching between components. This difficulty is echoed by the variation in available data; Toyama, Runstadler and Dean (1977) operated at high speed with a vane diffuser, whereas Abdel-Hamid et al. used a compressor at much lower speed with a vaneless diffuser.

The present research is concerned with the type of machine often used in turbocharger systems; a high speed radial impeller with vaneless diffuser surrounded by a volute or scroll. At a given operating speed, there is only one mass flow rate at which the volute produces an axisymmetric pressure field; at all flow rates for which surge is possible, the pressure field from the volute will generally be highly non-axisymmetric (Loretto and Gopalakrishnan, 1986), and this circumferential non-uniformity can be detected upstream of the impeller inducer. At the inception of this research project, it was not appreciated how significant this non-uniformity would be; essentially the rotating stall common in machines which are nominally axisymmetric is replaced by a stationary (casing fixed) flow distortion.

The operation of many centrifugal compressors, including those in turbochargers, is normally limited by the occurrence of surge, which is a self-excited system oscillation. Surge can be classified according to the amplitude of the mass flow fluctuation. "Mild surge" refers to the condition where the annulus average mass flow oscillates but remains in forward flow at all times: the frequency is of the order of the Helmholtz resonator frequency. With low frequency response instrumentation, a compressor in mild surge would appear to be operating stably, although a change may be perceived in the noise. The Helmholtz frequency is thus the inverse of the characteristic time of the compression system up to the point at which it enters deep surge, when an entirely different time scale is relevant. In "deep surge", the mass flow variations are large and the overall flow direction generally reverses for part of the cycle. The frequency is set by the plenum emptying and filling times and is normally well below the Helmholtz frequency. The onset of deep surge terminates the useful operating range of the compressor, so that when a compressor operating map has a line denoted as the "surge line", it usually refers to the condition at which the compressor enters deep surge.

An important non-dimensional parameter in describing the dynamics of compression systems (both axial and centrifugal) is:

\[ B = \frac{1}{2} \frac{U}{\omega_H \sqrt{A_c \lambda}} \]

where \( U \) is the tip speed. \( B \) may be rewritten in terms of Helmholtz frequency as

\[ B = \frac{1}{2} \frac{U}{\omega_H \sqrt{A_c \lambda}} \]

For a given compressor, the system will exhibit a surge as the result of an instability if the value of \( B \) exceeds a critical value. If \( B \) is less than this, the compression system will maintain itself at an overall steady equilibrium point. For axial compressors, this equilibrium point is generally in rotating stall.

The first part of this paper describes experiments which were carried out with the same turbocharger compressor in two different compression systems. One system had the throttle valve immediately downstream of the compressor outlet and thus a small value of \( B \) (0.25, see Table 1). The other system had a large value of \( B \) (0.7), obtained by having a large volume between the compressor outlet and the throttle.

Many compressor systems operate with large values of \( B \), but there were additional reasons for choosing this configuration in the present series of experiments. With large \( B \), surge frequency is low, and the cycle of deep surge can be used to take the compressor essentially quasi-steadily through the stalling process. The long time scale that goes with a large plenum also makes it possible to measure the backflow characteristic of the compressor in deep surge during the plenum blowdown part of the cycle. The large plenum also ensures a wide frequency separation between the phenomena of rotating stall (time scale is on the order of a rotor revolution) and surge. Finally, the large value of \( B \) allows the effect of plenum size on the position of the surge line to be specified.

**TABLE 1**

<table>
<thead>
<tr>
<th>System Parameter</th>
<th>Large B System</th>
<th>Small B System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plenum Volume (m³)</td>
<td>0.21</td>
<td>0.0014</td>
</tr>
<tr>
<td>Stage Inductance L/A (m⁻¹)</td>
<td>350</td>
<td>283.3</td>
</tr>
<tr>
<td>Reference Area (m²)</td>
<td>3.58x10⁻³</td>
<td>3.58x10⁻³</td>
</tr>
<tr>
<td>Helmholtz Frequency @ 48 KRPM (Hz)</td>
<td>7.4</td>
<td>99</td>
</tr>
<tr>
<td>Tip Speed @ 48 KRPM (m/s)</td>
<td>322</td>
<td>322</td>
</tr>
<tr>
<td>B Parameter @ 48 KRPM</td>
<td>2.7</td>
<td>0.25</td>
</tr>
</tbody>
</table>

The reason for operating the compressor with low \( B \), that is with the throttle valve just downstream of the compressor, is that the positively sloped flow characteristics at low flow can be measured using steady state instrumentation. These are generally inaccessible on most stands, because of system instability. The stabilizing aspect
Laboratory Air Supply

SMALL B SYSTEM

Compressor
Turbine
To System Ejector

Throttle

LARGE B SYSTEM

Throttle
Plenum

Fig. 1: Schematic of compression systems examined

of small values of B (i.e. small storage volumes between the
compressor and throttle) is now sufficiently well known to be in
recent textbooks (e.g. Cumpsty, 1989). Linear stability analysis
predicts that the system will be unstable when the slope of the
compressor characteristic \( C' = \frac{\partial \Delta P}{\partial \phi} \) reaches a value

\[
C' > \frac{1}{2B^{2T}}
\]

(2)

where \( T' \) is the slope of the throttle characteristic. In the limit as
\( B \to \infty \), instability occurs when \( C' > 0 \), i.e. at the peak of the total-tostatic pressure rise characteristic. As B decreases, the compression
system can operate stably at increasingly positive values of
compressor slope.

The second part of the paper describes a lumped parameter
analysis of the system dynamics. The overall approach is along lines
somewhat similar to those used by Emmons (1955), Taylor (1958),
Stenning (1980), and others, but a significant difference is that the
effect of speed variation during the surge cycle is now included. This
will be shown to be important in capturing the system dynamics.

Toyama, Rundstadler and Dean (1977) and Dean and Young (1977)
had speculated about the effect of speed variation but were not able to
make any quantitative statements. Speed variations during surge also
occur in jet engines, but the high energy transfer in relation to spool
inertia existing in radial machines makes this effect particularly
important for the turbocharger.

### EXPERIMENTAL FACILITY

Figure 1 shows schematically the arrangement for small and
large B configurations and Table 1 lists the important parameters of
each. Additional details are discussed in Fink (1984, 1988) and
Capece (1982). The test turbocharger was driven by the turbine
using the laboratory compressed air and vacuum capability.

A cross-section of the instrumented turbocharger impeller
and diffuser is shown in Fig. 2. There are no inlet guide vanes. The
impeller blades are radial at exit; most recent impellers have
backswep, but this difference is unlikely to alter the conclusions of
this research because stall is initiated in the inducer. Design
parameters and geometry of the compressor are shown in Table 2.

### TABLE 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total-to-total pressure ratio</td>
<td>3.58</td>
</tr>
<tr>
<td>Corrected mass flow (Kg/s)</td>
<td>.522</td>
</tr>
<tr>
<td>Corrected speed (rev/s)</td>
<td>1150</td>
</tr>
<tr>
<td>Impeller Type</td>
<td>Cummins ST50</td>
</tr>
<tr>
<td>No. of Blades</td>
<td>20</td>
</tr>
<tr>
<td>Inducer Tip Blade Angle (deg)</td>
<td>54.4</td>
</tr>
<tr>
<td>Rotor Exit Blade Angle (deg)</td>
<td>0</td>
</tr>
<tr>
<td>Rotor Tip Diameter (cm)</td>
<td>12.8</td>
</tr>
<tr>
<td>Inducer Hub/Tip Ratio</td>
<td>.428</td>
</tr>
<tr>
<td>Inducer Tip/Rotor Tip Ratio</td>
<td>.576</td>
</tr>
<tr>
<td>Tip Clearance/Inducer Height Ratio</td>
<td>.020</td>
</tr>
<tr>
<td>Diffuser Type</td>
<td>vaneless + volute</td>
</tr>
<tr>
<td>Diffuser Exit/Inlet Radius Ratio</td>
<td>1.619</td>
</tr>
<tr>
<td>Diffuser Width/Inlet Radius Ratio</td>
<td>.092</td>
</tr>
</tbody>
</table>

Fig. 2: Instrumentation meridional positions (in millimeters); steady
state and high response probes at locations shown
corrected flow nondimensionalized by the choked flow through the impeller eye at reference conditions (0.842 Kg/s at 303°K, 1.013 bar). Lines of pressure ratio versus flow for constant corrected RPM, from 25K to 51K, are shown. These represent tip Mach numbers, $M_{t0}$ (based on tip speed and inlet stagnation temperature), from .48 to .98. To the right of the large B surge line, the results are identical for the high and low B systems.

In the large B system, mild surge oscillations were encountered when the compressor was throttled to (time-mean) operation just to the right of the surge line. The oscillation amplitude increased as the mean operating point approached the surge line. To the left of the surge line, the compressor in the large B system operated in a deep surge cycle, with period depending on speed and amount of throttling. At 48K corrected speed, the surge cycle (which lasted on the order of one second) included an audible plenum blowdown with reverse flow in the inlet and large plenum pressure variation. During the blowdown there was a several thousand RPM drop in compressor speed. The surge line was well defined and repeatable within 1% of maximum mass flow at each speed.

Compressor operating points with the small B system are shown to the left of the large B surge line in Fig. 3. With small B, this region of the compressor map is now stable, and the speedlines extend nearly to shutoff. (Near shutoff, the mass flow and pressure ratio exhibit some steadiness, but there is no discernable deep surge.)

Compressor performance in terms of isentropic head coefficient, $\psi_C$, versus flow coefficient, $\phi$, is shown in Fig. 4 for the small B system. When plotted in this fashion, the data nearly collapse into a single curve, although compressibility effects preclude complete similarity of the speedlines. In particular, the peak value of $\psi_C$ occurs at a higher flow coefficient at the higher speeds. In the large B system the instabilities were seen to grow in a region around the peak of the $\psi_C$ versus $\phi$ curve; thus mild (and deep) surge occur at higher values of $\phi$ as corrected speed increases.

Large B System Unsteady Behavior During Surge

Instantaneous compressor flow coefficient, tip Mach number, and isentropic head coefficient are shown for mild surge in Fig. 5. The data are taken at a mean flow coefficient of $\phi = .23$, and a

![Compressor map: large and small B data shown, $M_{t0}$ denotes tip Mach number](image)

Fig. 3: Compressor map: large and small B data shown, $M_{t0}$ denotes tip Mach number

Eight high response pressure transducers (DC coupled Kulite model XT-140-50) were installed in the compressor case, with a ninth located downstream of the compressor in the connecting plenum. Unsteady mass flow in the calibrated inlet tube was measured by a single platinum hotwire mounted six impeller inducer diameters upstream of the inducer leading edge, as well as by a continuity balance on the plenum. Instantaneous rotor speed was detected by a reflective scanner fiber optic probe which senses blade passing frequency. High response micro-miniature K-type thermocouples (Beckman model 300, time constant at roughly one millisecond) were positioned at midspan in front of the impeller inducer and downstream of the impeller. These were used to track the temperature ratio across the impeller during surge. During surge, the flow and head coefficient were calculated using the instantaneous speed measured by the photo optic probe.

The results and discussion of three separate data sets are presented. For the large B system, high response instrumentation was used to measure the instantaneous mass flow, the speed, and pressures in the compressor and downstream plenum during surge. The instantaneous compressor characteristic was computed using the unsteady pressures and taking account of the inertia in the compressor duct (Koff, 1985). The duct inertia required for the correction was measured experimentally by finding the Helmholtz frequency with the duct-plenum system containing a well defined plenum.

With the turbocharger operating in the small B system, time resolved and time averaged data were obtained. Flow asymmetry, and extent of local reversed flow near the volute tongue, were investigated with the small B system.

**Performance Maps**

The measured performance map of total pressure ratio versus mass flow, including data for both the large and small B system, are shown merged together in Fig 3. The abscissa is compressor

![Compressor isentropic head coefficient vs. $C_{p}/U$](image)

Fig. 4: Compressor isentropic head coefficient vs. $C_{p}/U$
corrected speed of 48K, but similar behavior was found at other speeds. The system surge frequency is 7.3 Hz and an audible chugging noise accompanies operation in this mode. The amplitude of the roughly sinusoidal mass flow oscillation is 20% of the mean flow. This is the type of (relatively) small amplitude limit cycle phenomena that Toyama et al. (1977) and Dean (1977) reported during so-called stable operation. As Dean showed by calculation, mild surge limit cycles can occur in the vicinity of the peak due to the curvature of the compressor characteristic.

A deep surge cycle is shown in Fig. 6. The surge is initiated by a one to two percent reduction in throttle area compared to the conditions of Fig. 5. Three different time scales are seen in Fig. 6: a Helmholtz oscillation of approximately 7 Hz, a longer one (roughly 0.65) associated with the plenum blowdown and recovery (deep surge), and a still longer time scale associated with the rotor speed fluctuations.

Four regimes exist in the deep surge cycle. The first is a quiescent period, with the tip Mach number increasing, flow coefficient decreasing, and the compressor moving to the left on its unstalled characteristic. In the second phase, fluctuations grow as the compressor operates in mild surge. In the third phase, the mass flow reverses and the plenum pressure decreases during blowdown. (The flow coefficient is measured by the inlet hot wire, which is insensitive to flow direction. A high flow rate is thus registered during the backflow phase. However, measurements made using the plenum continuity balance confirmed that reversed flow was indeed present.) In the fourth phase, the blowdown finishes and the flow recovers rapidly to the forward direction and then decreases gradually to the flow rate for the quiescent flow. The cycle then repeats. In the backflow region in Fig. 6, high response thermocouples showed the temperature in the inlet duct exceeds the rotor exit temperature. At the end of the blowdown phase of the cycle, the compressor reingests the residual hot gas present in the inlet and returns to operation in forward flow.

Fig. 5: System parameters during mild surge ($\phi_T = .23$); large B system

Fig. 6: Large B system parameters during deep surge ($\phi_T = .225$)

Fig. 7: Effect of operating point on time-resolved behavior during deep surge (large B system); a) ($\phi_T = .19$); b) ($\phi_T = .16$)
Figure 6 is for a throttle area just small enough to yield deep surge. As the throttle was closed, the repetition time for the deep surge became shorter. The latter stages of the surge cycle (the maximum amplitude Helmholtz frequency oscillations followed by deep surge) were altered little but the periods in which the Helmholtz oscillations were small became shorter as the throttle was closed. Figures 7a and 7b show this behavior for throttle settings corresponding to a time-mean, \( \phi \), of 0.19 and 0.16. The time history of flow and pressure rise can be compared directly with the corresponding traces in Fig. 6. (The speed variations are not shown but the trend is the same (Fink, 1988).) The latter part of the surge transient in Fig. 6 can be seen to be similar to the wave forms shown at the lower mass flows.

The (inertially corrected) overall compressor head coefficient is shown versus the instantaneous flow coefficient during a deep surge cycle in Fig. 8. Also shown are the time-averaged data for small \( B \) at flow coefficients below that for surge initiation at large \( B \). The points are 3.2 msec apart in time (2.6 rotor revolutions at 48K corrected speed). The instability growth phase is shown by the cloud of data centered roughly on \( \phi \) = .225, with oscillations developing around the peak of the compressor characteristic. Compressor operation to the right of the peak is essentially quasi-steady. In the forward to backflow transition region (the range \( .1 > \phi > -1.1 \) the data indicate rapid movement of the operating point along a positive sloped characteristic. In backflow, it appears that the compressor operates quasi-steadily on a steady-state backflow characteristic. Most of the decrease in compressor pressure rise occurs during this period. In the forward flow phase, when the plenum is being filled, the compressor also moves relatively slowly along its characteristic to re-enter the instability growth regime which is the precursor to another deep surge.

The different pressure rise of the shutoff points defined by \( \phi = 0 \) for the two curves should be noted. In transition to reverse flow, most of the pressure rise decrease of the compressor in surge occurs well after the overall flow in the compressor has reversed. The differences between instantaneous and quasi-steady compressor performance are also seen by an examination of the transition regions. During the forward-reverse transition, the compressor head exceeds the small \( B \) steady value for the same flow coefficient (.57 vs. .47 at \( \phi = -0.2 \)). In the reverse-forward flow transition, the head is less than the steady state value for more than thirty revolutions of the impeller.

In the reverse-forward transition region, varying inlet gas temperature is a contributor to the lag. Low density (hot) air in the inlet duct from the plenum blowdown is reintegrated by the compressor, resulting in a lower compressor pressure rise. The time of reintegration is roughly the time it takes to empty the inlet tube. Taking ambient pressure in the tube with the measured temperature ratio of 1.27, an average mass flow in transition of 0.3 Kg/sec, and a tube volume of \( 5.9 \times 10^{-3} \) m³, the time to clear the inlet is 19 msec. The data of Fig. 8, however, show this phase to take much longer and hence temperature effects are not solely responsible for the observed lag.

Casing static pressures (in terms of head coefficient) vs time in the compressor during deep surge at 48K are displayed in Fig. 9 with the time interval near flow reversal shown on an expanded scale in Fig. 10. Kulite traces are shown from the two circumferential positions, one near the volute tongue and the other 90 degrees from this. The traces are labeled 1 (ahead of inducer) through 4 (near impeller exit), as shown in Fig. 2, with the plenum Kulite trace marked 5. The bottom two traces of Figs. 9 and 10 show compressor flow coefficient and tip Mach number. The time period of the flow reversal region is approximately .15 seconds.

The link between surge and impeller stalling due to local flow reversal at the inducer tips can be observed in Figs. 9 and 10. At maximum flow during the period of mild surge, the inducer inlet (trace 1) has a value below ambient, indicating forward flow into the inducer. At the flow minima, on the other hand, the inducer inlet static pressure is observed to be above ambient for both circumferential locations, implying local reverse flow at the inducer tips. These local flow reversals in the inducer occur well in advance of overall flow reversal of deep surge.

In the time history shown in expanded scale in Fig. 10, evidence of inducer tip flow reversal occurs first at the circumferential position near the tongue approximately 120 milliseconds (96 rotor revolutions) before overall flow reversal. At 90° from the tongue, flow reversal at the inducer tip is found approximately 90 milliseconds (72 rotor revolutions) before overall.
flow reversal. The instantaneous value of overall mass flow coefficient at which the local flow reversals are seen at the inducer tip is approximately $\phi = .26$ for the near tongue position and $\phi = .225$ for the position away from the tongue; these can be compared with the peak in the compressor characteristic shown in Fig. 4 at $\phi = .225$. This suggests that the characteristic peak is in fact the point on the map where the onset of the tip inducer stall begins over all or most of the circumference. In this compressor, the fluid dynamic circumstance that causes the positively sloped compressor characteristic necessary for surge is an inducer tip stall.

The inducer tip flow reversal is seen to be most severe at the circumferential position corresponding to the volute tongue. The inducer pressure recovery at the circumferential position of the tongue, shown in Fig. 9 by the difference between trace 1 and 2, diminishes when the overall mass flow is a minimum during mild surge. Away from the tongue, on the other hand, there is less evidence of loss in pressure rise in the final few cycles of mild surge. In Fig. 10 the pronounced drop in pressure rise in the impeller inducer at the tongue circumference occurs approximately 13 milliseconds (or, more relevantly, 10 rotor revolutions) before overall reverse flow. The overall flow does not reverse until after the decrease in inducer pressure rise occurs at the tongue.

Evidence that the inducer tip flow is locally reversed at the circumferential position of the volute tongue, while the overall average flow is still forward, can also be seen by examining the pressure levels in traces 1, 2, and 3. When the flow is a minimum during mild surge, these are essentially the same as the values when the overall mass flow has reversed during deep surge. For example, the pressure levels are more or less equal at $t = .135$ seconds in Fig. 10 (in the precursor period to deep surge) and at a later instant of $t = .21$ seconds during plenum blowdown with overall reverse flow. This is not the case for the traces (at these same times) obtained well away from the tongue since, in the precursor period, the inducer is still producing a large pressure rise.

The asymmetric stalling of the impeller at the circumferential location of the volute tongue is linked to the onset of deep surge. This is seen more clearly if the impeller total to static head coefficient of Figs. 9 and 10 is plotted versus overall flow coefficient, rather than versus time. Figure 11 shows such a plot, for a speed of 48K, and includes data from both near and far tongue circumferential positions. Also marked in the plots are the values of overall flow coefficient at which the system enters deep surge. The impeller head at the near tongue circumferential location peaks at $\phi = .23$ compared with the deep surge throttle point of $\phi = .225$. In contrast to the impeller behavior shown in Fig. 11, the vaneless diffuser pressure rise coefficient versus flow coefficient data, as given by Fink (1988), has a positive slope, indicating that the vaneless diffuser has a destabilizing effect on the overall system at all flow rates.

### Rotating Stall

Rotating stall disturbances were observed (with the inducer inlet Kulites) during part of the surge cycle. Static pressure amplitudes associated with this rotating stall were less than 0.01 in terms of head coefficient (Fink, 1988). The signals were strongest in the inducer section, growing weaker as one progressed meridionally back in the machine. Some evidence of this is seen in the pressure...
The asymmetry is also seen in the inducer leading edge wall static pressures. The measured pressures allowed an estimate of the local variation in tip flow coefficient, and Fig. 14 shows the results of this calculation. As the compressor is throttled, the axial velocity at the tip drops to zero at the tongue at a mean flow coefficient of \( \phi = 0.15 \).

The rotating stall obtained at low flow coefficient for the small B system was also of relatively low amplitude, but at very low flow rates (\( \phi < 0.10 \)), the unsteadiness detected was in phase at both circumferential positions, indicating a mild surge and not a rotating stall.

**OVERALL SYSTEM DYNAMICS AND VARIABLE SPEED SURGE MODELLING**

We have not yet discussed the overall system behavior in any detail. In doing this we also present the results of a lumped parameter analysis which shows that the inclusion of a speed variation is needed for proper simulation. Lumped parameter compression system models have been discussed elsewhere (summarized by Cumpsty (1989)), and we enumerate only the major assumptions made:
1) 1-D unsteady incompressible flow in the compressor duct.
2) Isentropic plenum expansion or compression.
3) Density and pressure discontinuity across the compressor which is modeled as an actuator disc. Two types of compressor characteristics are investigated; quasi-steady (a unique actuator characteristic) and unsteady, with the compressor response simulated by a first order time lag on the order of the throughflow time of the compressor.
4) Choked throttle nozzle.
5) Throttle duct length is short (inductance free).
6) Negligible velocity in the plenum.
7) Negligible gas angular momentum in the compressor passages compared to the wheel angular momentum. This last is an additional equation compared to earlier analyses. The equations to be used describe conservation of momentum in the duct, conservation of mass in the plenum, and angular momentum variations in the turbo-spool.

Conservation of momentum in the duct is written as

\[ L_c \frac{d}{dt}(p_2 c_x) = p_{\text{inlet}} - p_{\text{plenum}} - \Delta P_c \]  

(3)

where the subscript 2 refers to conditions downstream of the compressor. Mass conservation in the plenum is given by

\[ \frac{d}{dt}(p_2 V_p) = \bar{m}_c - \bar{m}_t \]  

(4)

where \( \bar{m}_c \) and \( \bar{m}_t \) are the mass flow rates through the compressor and the throttle respectively. Conservation of angular momentum in the turbocharger spool is given by

\[ L_s \frac{d}{dt}(\Phi_c) = \Phi_c \]  

(5)

where \( L_s \) and \( L_c \) are the compressor torque and drive torque respectively. \( \Phi_c \) includes the torque needed to overcome the shroud wall friction.

Equations (3) - (5) are nondimensionalized as follows: mass flows are nondimensionalized by \( p_0 A_c U \), pressure differences by \( \frac{1}{2} p_0 U^2 \), time by the inverse Helmholtz frequency \( 1/\omega_h \), and torques by \( p_0 A_c R_c U^2 \). The density \( p_0 \) is at ambient conditions. The Helmholtz resonator frequency at equilibrium conditions is calculated using \( \omega_h \), the speed of sound in the plenum at the steady state operating point. Recalling that \( B = 0.5 \left( \frac{U}{T_0} - \frac{R_c}{C} \right) \), it can be seen that, for a given system, \( B \) may be used as the non-dimensional wheel speed. Employing this non-dimensionalization, Eqs. (3) - (4) can be written as

\[ \frac{1}{B^2} \frac{d}{dt}(\Phi_c B) = \Delta P_c - \Delta P_p \]  

(6)

\[ \frac{1}{B^2} \frac{d}{dt}(\Delta P_p B^2) = \frac{\tau_p}{T_{po}} (\Phi_c - \Phi_t) \]  

(7)

where \( \tau_p \) is the instantaneous plenum temperature ratio.

If we define a parameter \( F \), based on the spool inertia \( I \), as

\[ F = \frac{2 p_0 L_c A_c R_c^2}{I} \]  

(8)

the equation for the rate of change of the spool angular momentum may be expressed in terms of net torque, \( \Gamma = (\Gamma_d - \Gamma_c) \), as

\[ \frac{d\Gamma}{dt} = FB^2 \Gamma \]  

(9)

If we substitute Eq. (9) into Eqs. (6) and (7) we obtain

\[ \frac{d\Phi_c}{dt} = B(\Delta P_c - \Delta P_p) + FB\Phi_c \Gamma \]  

(10)

and

\[ \frac{d\Delta P_p}{dt} = \frac{1}{B^2 T_{po}} (\Phi_c - \Phi_t) + 2FB\Delta P_p \Gamma \]  

(11)

Equations (9), (10), and (11) are a coupled set of nonlinear equations to be solved for \( B, \Phi_c \), and \( \Delta P_p \), with \( \Delta P_c(\Phi_c) \) and \( \Gamma(\Phi_c) \) the specified compressor pressure and torque characteristics respectively. Further details can be found in Fink (1988).

The simultaneous solution of Eqs. (9) - (11) (using the necessary additional relations between plenum density and pressure, throttle characteristics, etc.) requires compressor rise and torque characteristics, \( \Delta P_c(\Phi_c) \) and \( \Gamma(\Phi_c) \). The flow coefficient used here is defined with the appropriate density \( p \), the impeller eye area \( A_c \), and tip speed \( U \). The data used for \( \Delta P_c(\Phi_c) \) and \( \Gamma(\Phi_c) \) is shown in Fig. 15. In forward flow, the fluid properties used in the definition of these characteristics are those based on inlet condition, but in reverse flow the properties in the plenum are used. The forward flow characteristics were measured with the small \( B \) system, whereas the characteristics in reverse flow were deduced from the deep surge data obtained using the large \( B \) system.

Several classes of disturbances can be examined. The first, and simplest, are those of sufficiently small amplitude that the equations describing the system can be linearized about the equilibrium operating point. A thorough discussion of these is given by Fink (1988), as well as Greitzer (1981). Perhaps the most
important result of the linearized analysis is that, for values of $B$ from 0.5 up, with the present compressor characteristic, neither the wheel speed variation nor the inclusion of the simple model of compressor unsteady response discussed below had a substantial effect on the instability point. Put another way, changes in the rotor inertia by factors of five had less than a one percent effect on the flow coefficient at which the linear system exhibited growing oscillations. Further, for $B = 0.5$, changes in the time constant used in the unsteady response, from zero to twice our best estimate, resulted in less than a four percent change in flow coefficient for linear instability. For $B = 1.0$, the change in flow at instability is roughly two percent. For smaller values of $B$ or much larger time constants, one can calculate larger effects (due to the highly unsteady flow through the compressor), but it is not clear whether the simple model is even appropriate in this regime. In summary, over a wide range of parameters, the point of linear instability can be estimated reasonably well by the simple quasi-steady, constant rotor speed lumped parameter model. This conclusion has also been made by Pinsley et al. (1990) and Gysling et al. (1990), who pointed out the utility of the simple linear models in examining dynamic control schemes to stabilize compression system instability.

Although the linearized analyses give good prediction of the flow at which instability will be encountered, our interest is also in the behavior once the disturbances grow to finite amplitude. In particular, we wish to ascertain whether the different features of the mild and deep surge cycles can be captured by the lumped parameter analyses or, equivalently, what effects must be included to adequately describe the system in the regime where the oscillations are nonlinear. To examine the nonlinear system response, Eqs. (9) - (11) were solved numerically. The equilibrium $B$ value used was 2.7, representative of the experiments at large $B$ when operating at 48 KRPM. The inertia parameter $F$ was set to .043, the value obtained from measurements supplied by the turbocharger manufacturer. Drive torque from the turbine was assumed to be constant. The throttle was set for a specified flow coefficient and a small decrease (1%, say) in compressor mass flow was then introduced.

For an equilibrium flow of $\phi = .235$, which corresponds to operation to the right of the compressor peak, the system response is given in Fig. 16, which shows instantaneous flow coefficient and speed for the system undergoing deep surge. Time is given in terms of Helmholtz resonator periods. The main features are growing oscillations in mass flow followed by a rapid transient to reverse flow and then a plenum blowdown. The speed peaks during the blowdown phase and then drops below the equilibrium value; the speed drops rapidly during the forward flow recovery phase because compressor flow and torque are higher than the equilibrium values. After recovery of the mass flow, there is a long period (an order of magnitude longer than the blowdown phase) in which the compressor speed slowly recovers to the equilibrium value.

It is to be stressed that the compressor characteristic used for the calculation in Fig. 16 is time independent; both torque and pressure characteristic are assumed quasi-steady and continuous in the two flow regimes of positive and negative flow. The deep surge event depends only on a positively sloped compressor characteristic and a $B$ value of sufficient size.

There are three different time scales in Fig. 16: one corresponding to the Helmholtz frequency, the blowdown and fill-up time, and the time associated with the speed variation. The last is due to a new energy storage element in the compression system, which introduces a time scale that is long compared to the first two. Deep surge appears to start when the increasing speed (and $B$) reaches some critical value.

To show that the long period between blowdowns is a result of speed variations only, the system response was calculated with infinite wheel inertia ($F=0$), so that there was constant rotor speed. The infinite inertia results are shown for a throttle setting of $\phi = .235$ in Fig. 17. Compared to Fig. 16, the long quiet period between deep surges is eliminated if the speed is constant. Even when relatively small rotor speed variations are present (five percent, say), the system response is substantially different from that with constant speed.

In Fig. 8, the instantaneous compressor pressure rise trajectory, measured in the large $B$ system was shown to be flatter at flow coefficients below $\phi \sim .2$ than the measured steady state values for the low $B$ system. The instantaneous compressor pressure rise seen with the large $B$ system thus lags that for steady state operation. As discussed earlier, this appears to be due to the differing circumferential extent of impeller stalling seen in the two situations. One might therefore define a time constant which corresponds roughly to a convection time through the impeller and vanless diffuser. If so, we can model the relation between instantaneous and steady state compressor pressure rise coefficients as
\[
\frac{d\bar{\Delta}P_c}{dt} = \frac{1}{\tau_c}(\bar{\Delta}P_{css} - \bar{\Delta}P_c)
\]

where \(\bar{\Delta}P_c\) is the instantaneous compressor pressure rise coefficient, \(\Delta P_{css}\) is the steady state value, and \(\tau_c\) the time constant, is throughflow time nondimensionalized by the Helmholtz resonator period. Equations (9), (10), and (11), along with (12) now form the set of coupled nonlinear equations for the system response.

The compressor throughflow time in Eq. 12 is assumed to be given approximately by

\[
\tau_t = \frac{L_t}{C}
\]

where \(L_t = 0.085\) m is the meridional throughflow length of the impeller and vaneless diffuser and \(C\) is the meridional average flow velocity. The non-dimensional throughflow time constant is thus:

\[
\bar{\tau}_c = \left(\frac{1}{2}\frac{L_t}{L_c}\right)\frac{1}{B_M c}
\]

For the large \(B\) system (at \(\phi = .1, B = 2.7, L_c = 1.27\) m), the numerical value of \(\bar{\tau}_c\) is 0.12, approximately 2.2 rotor revolutions.

For the large \(B\) system, at an initial equilibrium operating point of \(\phi = .235\), the results of the numerical integration with time lag are shown in Fig. 18. The main difference introduced by the time lag (compared with the quasi-steady results of Fig. 16) is the long period of mild surge precursor period to deep surge. Physically, the time lag has the effect of flattening the instantaneous compressor characteristic relative to the quasi-steady curve; this leads to a slower growing instability.

The result of the simulation including the time lag is shown in Fig. 19 with the corresponding large \(B\) experimental results. For a throttle setting of \(\phi = .236\), Fig. 19, the compression system is seen in both the simulation and experiment to enter continuous mild surge. The large amplitude sustained mild surge oscillation in Fig. 7 could not be duplicated without the time lag, i.e. the time lag has the effect of stabilizing the mild surge oscillations.

Reducing the throttle to \(\phi = .235\) causes the compression system to enter deep surge as shown in Fig. 20, which can be compared directly with the simulation in Fig. 18. Good agreement is seen between the theory and experimental data over all four regions of the cycle. A deep surge precursor period of large amplitude mild surge is seen; this was attenuated in the earlier quasi-steady results. The long period between deep surges of the compressor due to speed variations is again visible, as it was in the quasi-steady compressor characteristic computations.
CONCLUSIONS

The conclusions can be divided into three parts.

A. Conclusions from the experimental work specific to this class of machine
1) The stalling element most responsible for surge initiation in this turbocharger compressor is the impeller. This agrees with the conclusion reached by Flynn and Weber (1979).
2) During surge initiation, stall of the impeller occurs first in the inducer at the circumferential location nearest the volute tongue. The tongue-induced stall in the impeller increased in severity at higher values of compressor tip Mach number if the compressor is operated away from the volute match point.
3) The instantaneous compressor characteristic is continuous and exhibits no abrupt stall behavior to initiate surge.
4) When operating in a deep surge cycle, the largest pressure fall-off in the compressor occurs well after the overall flow in the compressor has reversed.
5) Rotating stall in the compressor tested was much weaker than the volute induced pressure non-uniformity, suggesting that a stationary redistribution of flow is usual in this geometry rather than the rotating ones formed in more nearly axisymmetric machines.

B. Conclusions general to radial compression systems
1) The vanless diffuser is a destabilizing element since its characteristic slope is positive. Its characteristic slope is nearly constant near the surge line, however, and it is thus not the component whose performance change is responsible for initiating instability and surge.
2) The position of the pressure rise peak is speed dependent due to compressibility and its effect on the volute matching condition.
3) For a system with large B, surge occurs at the peak of the whole compressor characteristic. Surge oscillations develop about this point, at the Helmholtz frequency, and grow slowly, culminating in deep surge. The frequency of deep surge, which is much lower than the Helmholtz frequency, is set by the plenum blowdown and refilling time.
4) Time averaged characteristics in the low flow range are steeper than the instantaneous ones measured during surge.

C. Conclusions from the compression system modelling
1) For a turbocharger in deep surge, the rotational speed fluctuation can be important. Accounting for the effects of variable impeller speed, even with a quasi-steady compressor characteristic, changes the predicted time behavior of the system substantially from that given by a constant speed model.
2) A precursor period of mild surge before deep surge is shown to be present due to the speed variation. The surge dynamics are affected by the addition of another energy storing element, the compressor rotor. A main effect is to increase the time interval between blowdowns compared to the constant speed case.
3) The magnitude of speed fluctuations present and its effect on the overall dynamics is governed by spool inertia; decreases in inertia make speed effects more important.
4) Inclusion of a simple time-lag model of the effect of unsteadiness in the inducer stall process improves the qualitative and quantitative agreement with experiment. In the computations performed here, the unsteady response was simulated by a first order time lag, with time constant approximately equal to the throughflow time of the compressor.
5) A lumped parameter model with the features discussed in (1) to (4) seems adequate to examine the overall system dynamic behavior.

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