92-GT-20

THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
345 E. 47 St., New York, N.Y. 10017

BREAK

Pressure Distributions in a Single and Two Versions of a Double Volute of a Centrifugal Pump

W. de OJEDA, Research Assistant
R. O. FLACK, Professor
Department of Mechanical and Aerospace Engineering
University of Virginia
Charlottesville, VA 22903

S. M. MINER, Assistant Professor
Department of Mechanical Engineering
U.S. Naval Academy
Annapolis, MD 21402

ABSTRACT

Pressure measurements were recorded around the impeller and along the casing wall of a centrifugal pump, 0.60 (1583 US units) specific speed, assembled with a single volute/single discharge, and two versions of a double volute/single discharge. The latter comprised a splitter positioned in the second half of the discharge (i) midway between the impeller and casing, and (ii) along a spiral symmetric to the first–half casing section. The objective of such double volute casings is to reduce forces on the impeller and thus provide longer lives. Flow rates tested ranged from 20% to 105% of design. A repeated pattern consisted of pressure increasing from the first cutwater to the splitter leading edge at which the pressure drops and thereafter increases to the discharge. This pattern was noted at all flow rates with the symmetric volute geometry and only at flow rates higher than 60% for the centered splitter. By integration of the pressures static forces were found. Time averaged static forces ranged from 6.2 N at design to 33.0 N at 20% flow for the single volute. Both double volute configurations showed considerable thrust reduction throughout but for a few exceptions. Reductions ranged from 26% at 30% flow to 62% at 90% flow for the center splitter, and from 52% reduction at 20% flow to 72% at 80% flow for the symmetric splitter. For comparison of performance of the different configurations, at flow rates above 85% of design the head was 8% and 9% less for the double volutes than for the single volutes. At flows below 40% of design the head was 3% and 4% higher for the double volutes than for the single volute.

NOMENCLATURE

b	passage width
C_r	absolute radial velocity
g	gravitational constant
H	developed head
Q	flow rate through pump
R_2	outer impeller radius
U_2	impeller peripheral speed
W	impeller width including shrouds
θ	circumferential position
ν	kinematic viscosity
ω	rotational speed (rad/sec)

INTRODUCTION

Understanding and predicting the hydraulic and dynamic characteristics of centrifugal pumps requires detailed knowledge of the flow field. Even at design flow the interaction between the impeller and volute is significant, taking the form of asymmetries in the flow field due to a non–uniformity in the static pressure and exit velocity distribution around the impeller discharge. These asymmetries give rise to an unbalance force, minimum when operating at design but which may increase by three and higher orders of magnitude at off–design. The resulting shaft deflections cause excessive metallic contact between it and the wearing rings and bearings inducing wear and leakage. Additional fatigue through increased deflections have been known to cause shaft failure. This paper presents pressure measurements of the flow field in the volute of a laboratory centrifugal pump which was modified to fit two versions of a double volute. This data details the asymmetries in the pressure distribution for three different configurations, and was used to calculate a resultant static force.

Binder and Knapp (1936) measured velocities and static pressures in a double suction centrifugal process pump and a single suction single volute centrifugal process pump. Circumferential traverses were performed. Static pressures and velocities were integrated to find the static force acting on the impeller. The maximum force was at 19% capacity and the minimum was at 100% capacity.

Bowerman and Acosta (1957) studied the effect of volute shape on the hydraulic performance of a laboratory double discharge pump. Flow passages were rectangular cross sections and head capacity curves were found for different volutes. Circumferential static pressure profiles and velocity profiles were obtained.

Iversen et al. (1960) measured pressures in the volute of a centrifugal process pump and also measured the force on the impeller. They integrated the pressures to obtain static forces but did not account for momentum nonuniformities. As a result, these forces did not agree with directly measured forces.

Worster (1963) also measured the static pressures in a centrifugal process pump. He studied the effect of tongue

Presented at the International Gas Turbine and Aeroengine Congress and Exposition
Cologne, Germany June 1-4, 1992

length on the hydraulic performance and pressure distribution. Pressures were strongly dependent on the circumferential position.

Kanki et al. (1981) used pressure sensors on two impellers and in double volute/single discharge and vaned diffusers. They measured both static and fluctuating pressures for concentric and eccentric impellers and studied flow rates from 17% to 120% of the design values. At 100% flow they measured a nearly uniform pressure distribution in the double volute pump. At 17% flow, however, very low pressures at the tongues were seen. Large circumferential variations of pressures were observed for the vaned diffusers. They also directly measured forces using an overhung rotor with strain gages mounted to the shaft.

Stepanoff (1957) was one of the first to report experimental results for impeller forces. He measured the static deflection of the impeller running at on– and off–design flow rates for a single volute/single discharge centrifugal process pump. By calibration of the shaft with known static weights he was able to determine the load on the shaft during operation.

Agostinelli et al. (1960) tested 16 different pumps with different specific speeds. Single and double volute–single discharge pumps were tested. Calibrated strain gages on the bearing housing were used to measure two components of force. Results are presented for flow rates from 0 to 170% of design conditions. The thrust proportionality factor originally proposed by Stepanoff as a function of capacity was revised by the authors to also be a function of specific speed.

Biheller (1965) used the same test rig as that by Agnostinelli et al. (1960) to test 18 different pumps. The objective was to determine the effect of impeller/case concentricity on the radial forces. Eleven had single volutes; three had fully concentric volutes; and four had semiconcentric casings. Pumps with both shrouded and unshrouded impellers were used. A semi–empirical equation curve fitting the data for static forces for the three different types of pumps was presented.

Domm et al. (1966) and Hergt and Krieger (1969) also used a pump test rig to measure static forces. The impeller was on an overhung rotor and the two support bearing pedestals were instrumented with calibrated strain gages. The authors took data for normal and eccentric operation of impellers in logspiral volutes and pumps with guidevanes and presented the data in nondimensional form.

Uchida et al. (1971) used an overhung rotor with strain gages on a pedestal to measure static and dynamic forces. They tested a pump with six different tongue shapes and sizes. Static force measurements are presented in raw data form (amplitude and direction) and the authors do not correlate their data.

Grein et al. (1975) used calibrated strain gages mounted on a shaft to measure the static forces on the impeller. They studied two pumps (vaned diffuser and pump–turbine).

Meier–Grotrian (1973) used an overhung rotor with strain gages on two bearing pedestals to measure radial forces. He tested one single laboratory volute and separately tested four different impellers. He constructed the volute so that its shape could be parametrically varied. He tested four different volute shapes.

Schwarz and Wesche (1978) studied six different volute/impeller combinations. All pumps had double entries and single volutes. They measured the impeller load with pedestals which were instrumented with strain gages.

Chamieh et al. (1982) and Jery et al. (1985) developed a facility and expanded the data base of impeller forces in volute pumps. Testing was conducted by systematically varying orbit/pump speed ratio, pump speed and flow coefficient in several impeller/volute combinations. The time averaged force on the impeller was determined; however, details of the flow field were not measured.

In summary, a number of researchers have studied the pressure and forces in different volute geometries in the past. Both single and double volute configurations were examined. Some fundamental data is available to assist in the understanding of the developed forces on the impellers. However, one full set of data is not available for any geometry. Thus for the current research a rig was configured to provide such data. Namely, a centrifugal pump that can be run with either single or double volutes was designed. The rig was designed so that static pressure data around the volute and impeller periphery can be collected. The rig was also designed so that velocity data from non–intrusive laser velocimetry can be obtained. The two sets of data will provide the two components of force generation: non–uniform pressures and momentum flux distributions, thus providing insights into how the forces can be reduced. In this paper the pressure data is reported for a single volute and two different double volutes. In a future paper the velocity measurements (and analysis thereof) will be presented.

EXPERIMENTAL APPARATUS

Pump

The pump used for this study is documented in Hamkins and Flack (1987) and Miner et al. (1989). The flow width (b) is 24.6 mm and the impeller peripheral radius is 101.6 mm. The dimensionless specific speed $(\omega Q^{1/2}/(gH)^{3/4})$ of the pump is 0.60 (1583 US units) which corresponds to a dimensionless design flow coefficient $(Q/(2\pi R_2 b U_2))$ of 0.063. The Reynolds number $(C_{r2}R_2/\nu)$ at design conditions is 4.0 x 10^4. Figures 1 and 2 depict the impeller and volute geometry. The impeller is a four bladed geometry with 16° logarithmic spiral blades and the volute is also a logarithmic spiral with an 83° angle. The single volute geometry along with window locations for later laser velocimeter measurements are shown in Fig. 2. The pump is constructed of Plexiglas and the casing walls are 50.8 mm thick.

The flow loop is shown in Fig. 3. This is a closed loop system fed from a 2000 liter reservoir tank. The flow straightener in the 76.2 mm dia pipe upstream of the pump inlet provides a swirl free inlet flow to the pump. Static pressure taps are located 5 pipe diameters upstream of the impeller inlet and just beyond window 11 in the discharge.

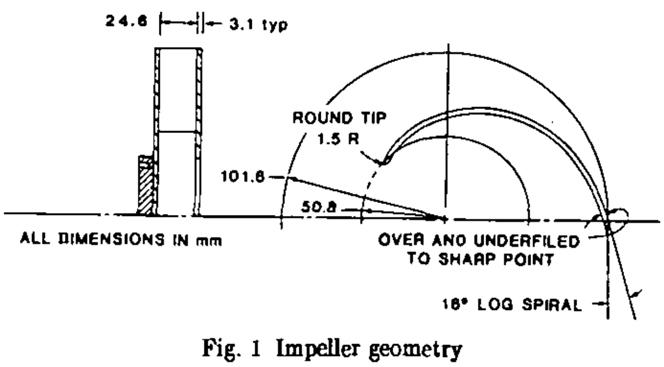


Fig. 1 Impeller geometry

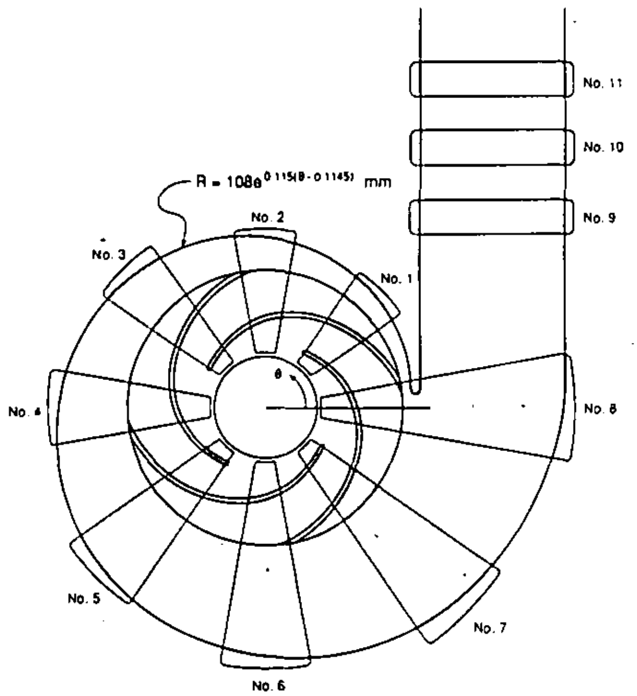


Fig. 2 Volute and window locations

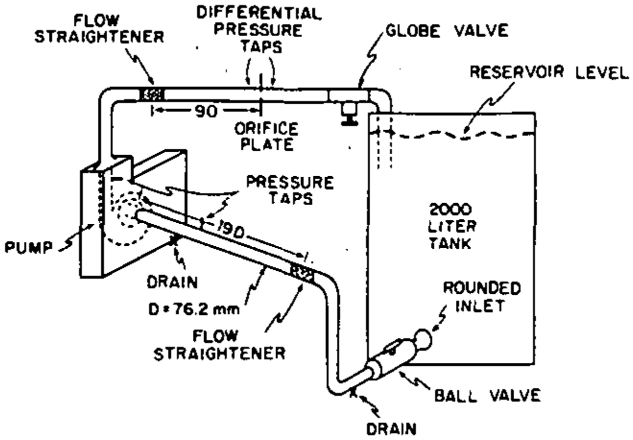


Fig. 3 Flow loop

Yaw probe traverses in the inlet pipe indicated that the flow was swirl free three diameters upstream of the pump at shut off conditions. The second flow straightener, 9 pipe diameters upstream of the orifice flow meter, conditions the flow for the orifice plate. The glohe valve is used to adjust the flow. The running speed for the pump is 620 RPM yielding a tip velocity (U_2) of 6.60 m/s. The design point for the present pump design is 6.3 ℓ/s at 0.222 x 10^5 N/m². This design poiot is based on an estimate of the impeller–volute matching point. The matching point is the condition which causes the average angular momentum of the flow at the impeller exit to equal the average angular momentum in the throat of the volute.

Double Volute Configurations

Static pressures around the impeller discharge and along the casing were measured in this pump assembled under three different configurations, a single volute/single discharge and two versions of a double volute/single discharge. Figs. 4.a and 4.b show the double volute geometries used. The splitter was simply positioned midway between the impeller and the casing in configuration I. In configuration II, the splitter was shaped as a logarithmic spiral, identical to the casing, providing a symmetrical flow discharges. The design concept of the second version is to equalize the flow patterns and thus pressures for the two discharge regions. As a result ideally the impeller force is zero.

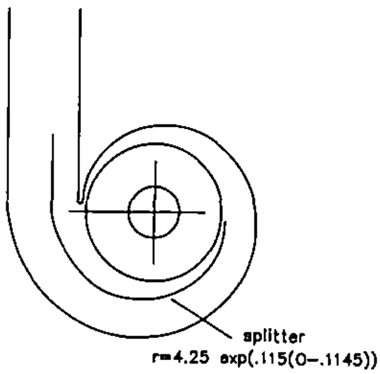


Fig. 4.a Double volute version I

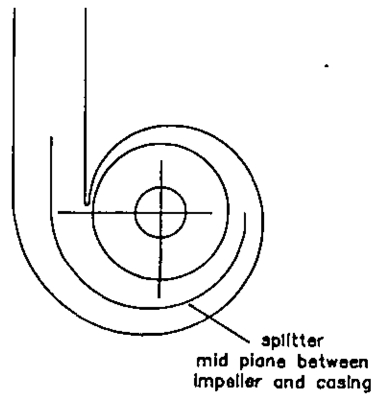


Fig. 4.b Double volute version II

Pressure Tap Locations

Fig. 5 shows tap locations; these span from just before the cutwater located at 5.6 degrees around the impeller and along the casing wall each positioned at 45 degrees intervals. The base line for angle measurements is also depicted on this figure. Local developed pressure data presented in the next section are local developed static pressures and not local developed heads (in which velocities are included). With the pump power shut off the local developed pressures are of course all identically zero.

Uncertainties

Pressure measurements were recorded with water manometers to within 3 mm of water. These measurements represented time averaged quantities, independent on the blade passage. This uncertainty corresponds to $\pm$ 29.13 N/m². Forces were computed assuming a uniform pressure distribution around each pressure tap; under this assumption, the uncertainty in the resultant thrust added to $\pm$ 0.1 N. The typical uncertainty in flow rate was calculated as 1.3%.

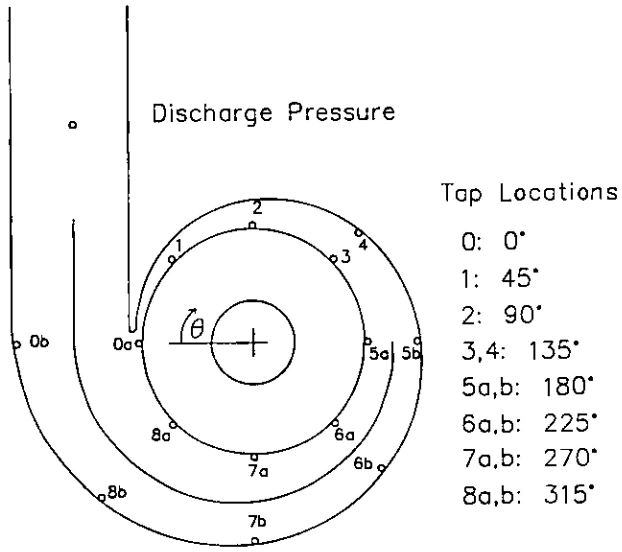


Fig. 5 Static pressure tap locations

RESULTS

Pump Performance

Pump performance depicted as variation of total developed head versus flow rate for all three configurations is shown on Fig. 6. The single volute configuration peaks at 77% flow, whereas both double volute configurations peak at 55% flow. Double–volute configurations exhibit decreased performance for flow rates above 55% of design. Increased friction losses from the added boundary layers would account for this decrease. However, for flow rates below 55% the total head measured with the double–volute configurations was greater. The single volute configuration would be expected to experience greater recirculation losses in the impeller and volute in this regime. For example, Miner et al. (1989) showed evidence of recirculating flow in the pump passages at 40% of design for the single volute configuration based on measured outward radial flow along the pressure surface and inward radial flow along the suction surface in the impeller.

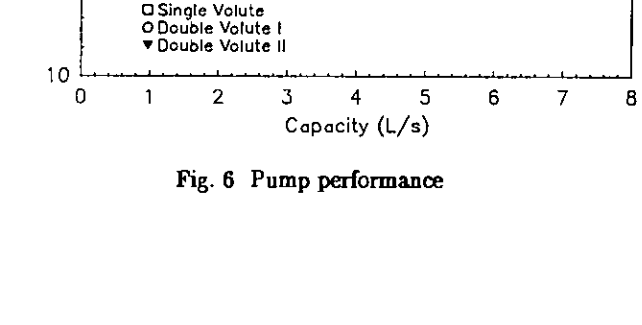


Fig. 6 Pump performance

For flow rates larger than 80% of design, the decrease in total head averaged 8%; the second version of the double volute was less than the first in this range by one percent. For flow rates below 40% of design the increased performance averaged 3% and in this case the second version of the douhle volute led the first.

The variation of pump efficiencies for these configurations are also shown in Fig. 6. The efficiency lines were calculated using Euler's theoretical head assuming no pre–rotation in the fluid, which was documented in earlier tests. The purpose of these efficiencies is simply to compare the operation of the three configurations tested, not to document the total efficiency of the pumps. The maximum efficiency exhibited in both double volute configurations were both within 1% of 67% of the same flow rate (55% of design flow). The single volute exhibited a 69% peak efficiency at 77% of design flow.

As with total head measurements, single volute pump efficiencies are greater for high flow rates averaging 6%. Similarly, for low flow rates, the double volute configurations averaged 4% increased efficiencies over the single volute configuration.

Knapp (1941) also showed that variations of casing type did not affect peak efficiencies. The relative decrease and increase of head performance of the double volute with respect to the single volute matched that described above in the first group of pumps tested by Knapp. It was not observed, however, in a latter group of pumps tested with higher specific speeds.

Pressure Measurements

Pressures were measured around the impeller and around the casing wall as it is shown on Figure 5. Recorded data from both versions of double volute configurations are compared with single volute data for flow rates ranging from 1.3 ℓ/s to 6.6 ℓ/s. High flow rates are designated as those encompassing 80, 90, 100 and 105% of design flow. Low flow rates include 20, 40, and 60%; the design condition was also included in the latter group for comparison purposes.

Double Volute Version I. Fig. 7.a shows pressure data collected around the impeller from the single volute and double volute version I configurations for high flow rates. The right most points (labeled at location D) correspond to pressures measured at discharge (window 11). Pressures exhibit a uniform distribution for the highest flow rates, 100 and 105%. For lower flow rates the pressures become considerably larger towards the discharge. The double volute configuration exhibits an increased pressure around the impeller from the first cutwater (located at 5.6 degrees) up to the second cutwater (located at 180 degrees) at which the pressure drops and increases thereafter with magnitudes comparable to those measured with the single volute. The early increase of pressures would balance the higher pressures measured towards discharge.

Both effects, the earlier increase in pressure and the sudden pressure drop at the splitter cutwater, are not so noticeable at lower flow rates, 40 and 60%, and vanish at 20%. Fig. 7.b shows that at 40 and 60% the pressures up to the splitter cutwater are only slightly higher and, instead of dropping, level across it. The 20% flow pressure distribution is relatively uniform before and after a manifested discontinuity centered around 135 degrees from the first cutwater.

Pressures measured around the impeller were integrated to compute a time averaged static force and results are presented on the following section. Other pressures were

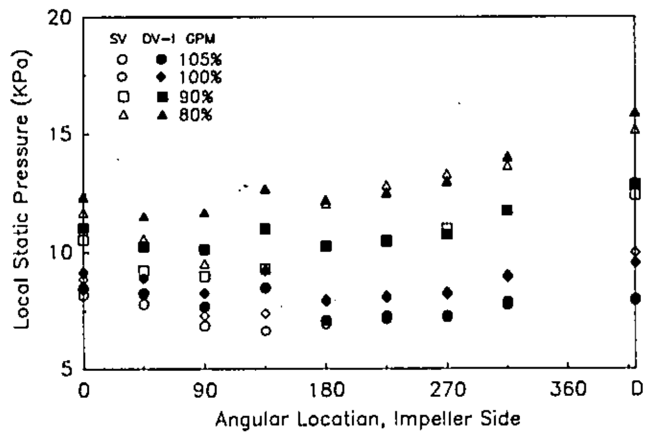


Fig. 7.a Impeller side pressure variation for single and double volute (I) configurations

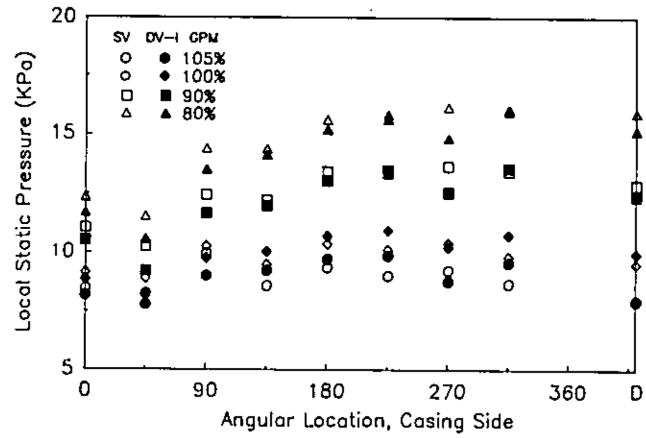


Fig. 8.a Casing side pressure variation for single and double volute (I) configurations

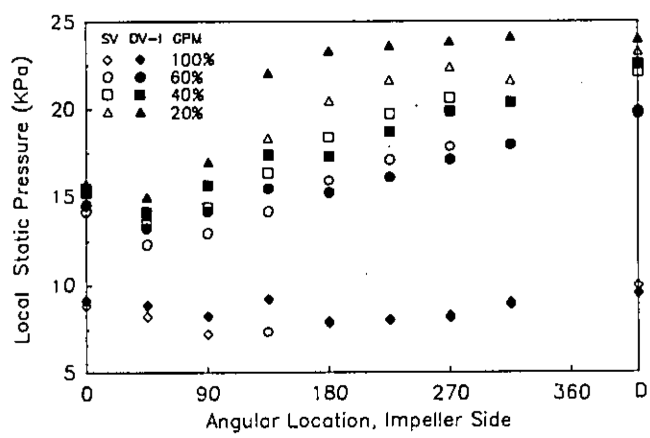


Fig. 7.b Impeller side pressure variagion for single and double volute (I) configurations

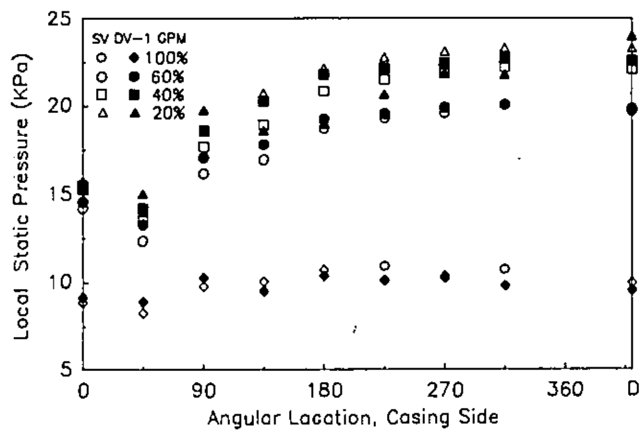


Fig. 8.b Casing side pressure variation for single and double volute (I) configurations

measured and are shown on Figs. 8.a and 8.b. These correspond to measurements made along the casing for high and low flow rates regimes. Recorded measurements for this version of double volute and single volute yielded comparable magnitudes for the range of flow rates excluding the lowest, at 20% of design, at which large variations were noted around the second cutwater.

Double Volute Version II. This second version of a double volute was built to attain two symmetrical discharges, hence the splitter and the outer casing have same geometries and cutwaters are 180 degrees apart. The effects caused by the splitter cutwater in this configuration are similar to those found in the earlier one. In the present case, however, the effects are present for all flow rates.

At high flow rates, Fig. 9.a, measurements showed an increased pressure from the first to the second cutwater at which a sudden drop is observed. This pressure drop is greater than in the former configuration. In addition, the second version of the douhle volute yielded lower pressures after the second cutwater than those with the single volute.

Fig. 9.b shows data gathered from low flow rates, with same characteristics as those descrihed above. At 20% flow, the pressure drop across the splitter cutwater is greatest and increases thereafter always lower than those measured with the single volute.

This second configuration produced a greater symmetry in the flow field than the former geometry. At all flow rates the pressures around the impeller comprised two distinct curves each expanding from one cutwater to the next, with greater resemblance to one another particularly at midrange flows, from 40 to 80%. The increased symmetry obtained here would yield a greater force balance around the impeller when compared to the former configuration.

Pressure measurements taken at the casing wall are shown on Figs. 10.a and 10.b for high and low flows respectively. As with the previous configuration pressures measured do not exhibit large variations when compared to the single volute.

Howard et al. (1987) studied the interaction of an impeller with a single and two versions of a double volute. As compared to those presented above, the pressures at the impeller discharge exhibited a great uniformity for all

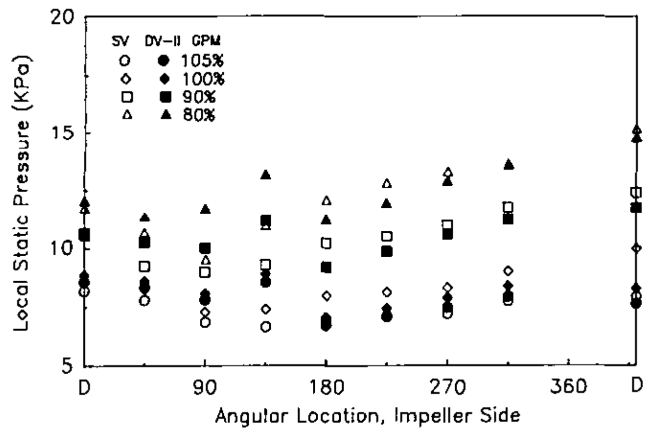


Fig. 9.a Impeller side pressure variation for single and double volute (II) configurations

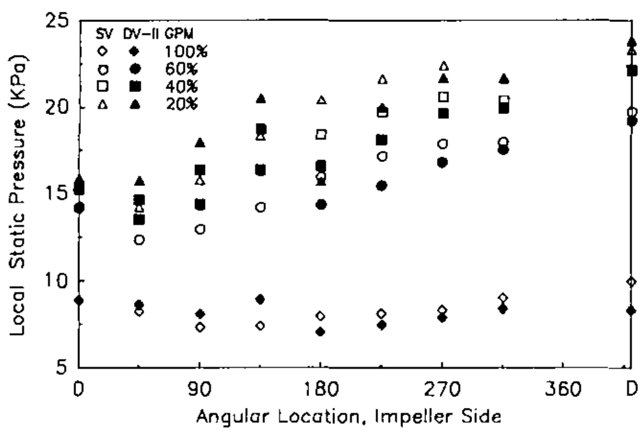


Fig. 9.b Impeller side pressure variation for single and double volute (II) configurations

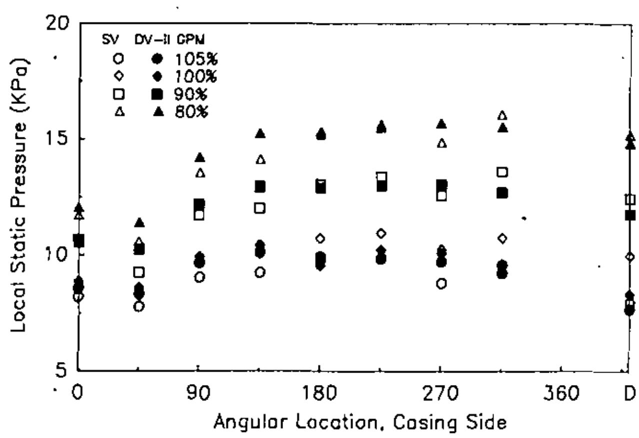
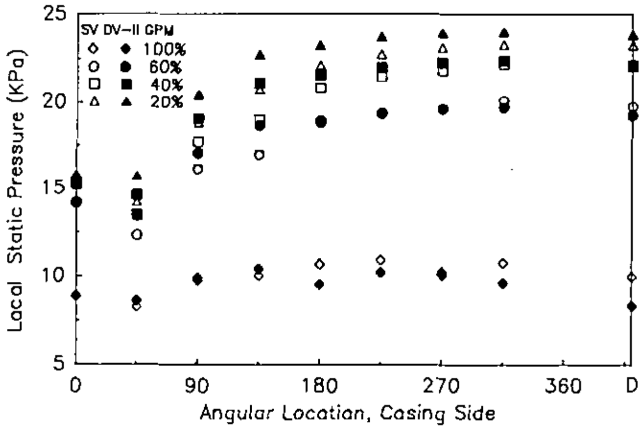


Fig. 10.a Casing side pressure variation for single and double volute (II) configurations



10.b Casing side pressure variation for single and double volute (II) configurations

configurations through a wide range of flow rates. No quantitative improvement is shown, and pressure profiles do not give an indication as to the effects of tongue clearance or splitter geometry. The pump, however, used in this reference was of low specific speed and the recorded pressures of comparatively low magnitude.

Measurements by Knapp (1941) showed similar pressure distributions as those obtained here for single and double volute pumps operating at low capacities. The double volute configuration of this reference, nonetheless, exhibited discontinuities in the pressure distribution profile manifested in a quick drop and rise in midsection of both volutes which contrasted to the steady increase in pressure along each volute as documented in this paper.

Force Calculations

Forces were computed by integrating the recorded pressures around the impeller. One must remember that the total force is due to an integration of both pressures and momentum fluxes; thus, the forces presented here include only half of the contribution. Forces due to momentum fluxes will be published in a subsequent paper. Figures 11 and 12 show respectively magnitude and direction of the force computed for varying flow rates in all three configurations tested. In the single volute configuration, forces ranged from 6.1 N at 0.3° at design flow to 33.0 N at 74.7° at 20% design flow.

Both versions of the double volutes yielded a strikingly significant reduction of radial thrust at flow rates lower than 90% of design. The first version of double volute showed a consistent thrust reduction throughout the range of flows except at 20% of design. The reduction ranged from approximately 26% at 30% flow to 62% at 90% flow. In the vicinity of 20% flow the thrust pattern is reversed and the splitter developed a greater unbalance. This phenomena is understood by observing the pressure distribution around the impeller from Fig. 7.b; contrary to higher flow rates, the pressure after the splitter leading edge remained uniform.

A consistently greater reduction was encountered with the second version, and in this case for all flow rates below 90% of design. The magnitude in which the thrust was reduced varied from approximately 52% at 20% flow to 72% at 80% flow.

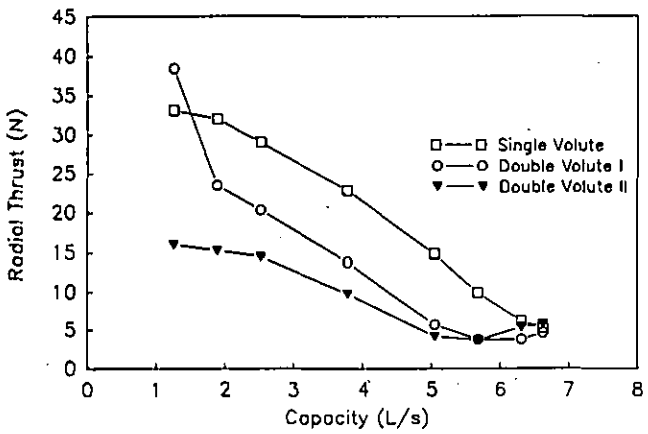


Fig. 11 Radial thrust from integratinn of pressures

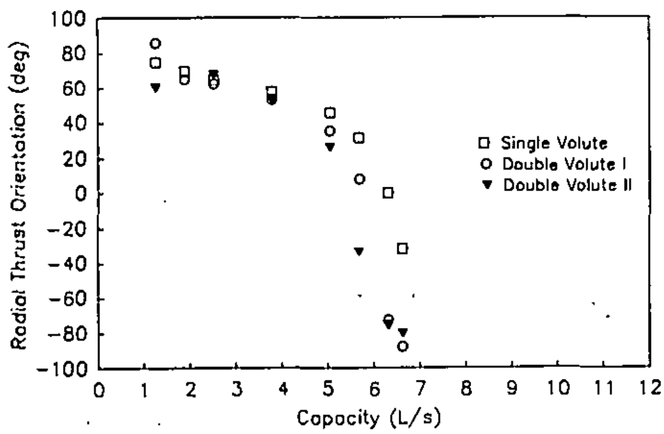


Fig. 12 Orientation of the radial thrust

The direction of the radial thrust in the single volute configuration varied according to operating capacities from 75° to −32° (in the standard XY coordinate system) at 20 and 105% flows, respectively. The angle varied gradually for increasing capacities from shut–off up to 60% flow and exponentially thereafter. For flows helow 60% of design, data from all three configurations overlap except at 20% flow which exhibit $\pm$ 10° variation. At higher flow rates, angles differ considerably. At design the variation is greatest, the orientation of the radial thrust is displaced 75° from the single volute in both double volute configurations.

Both versions decreased the radial thrust on the impeller by creating a repeated flow pattern over each half of the impeller discharge. The greater success of the second version is attrihuted to its physical symmetry. The non–symmetric geometry and in particular the increased clearance at the splitter tongue in the first version did not yield the repeated flow pattern of the second version. Large impeller forces should he avoided as they reduce the life of a pump. Namely, when such forces load a bearing, the bearing eventually fails. The actual levels required for failure are dependent on the particular bearings, pump designs, fluids used, duty cycle and other factors. This and other data has shown that pumps that operate at off design conditions have more such prohlems due to the larger forces. As shown in this paper, by adding the splitter, forces are reduced, but one loses efficiency or head because of the added friction.

The influence of the tongue clearance and position on performance and pressure uniformity was studied by Worster (1963). The author experimented with a single volute/single discharge pump and attrihuted the greatest efficiency at design to reduced tongue clearance. Uchida et al. (1971) also gave experimental evidence of higher efficiencies with small tongue areas. No direct correlation however of tongue geometry was given with pressure distribution or developed thrust. In the case of douhle volutes the reduced clearance allowed the current authors to gain repeatahility in the circumferential pressure distribution while the maximum efficiency and that of design were reduced.

Empirical models have been developed to predict radial thrust due to both pressure and momentum unhalances in centrifugal pumps. Probahly the most referenced model is that documented by Stepanoff (1957). This model, although it follows the general trend of the pressure forces calculated here, predicts lower total forces than the current pressure forces at high flows and predicts higher total forces than the current pressure forces at low flow rates (a zero net thrust is assumed at design in the empiricism). Agostinelli et al. (1960) revised this model to include a specific speed parameter. The total predicted forces were closer to the pressure forces measured specifically at low capacities and at design, where the force is no longer assumed zero for this empiricism. Imaichi et al. (1980) calculated the total radial force exerted on an impeller in a logarithmic volute with two different tongue geometries. Their analytical model showed that for increased tongue length and reduced tongue radius a larger thrust is developed at design although at high and low capacities the thrust is reduced. This particular geometry characterized the pump used here where a nonzero radial thrust was ohtained at design. Figure 13 shows predictions of Stepanoff and Agostinelli with measured data. Force components were nondimensionalized dividing by $2HR_2W$, where W is the impeller width including the shrouds.

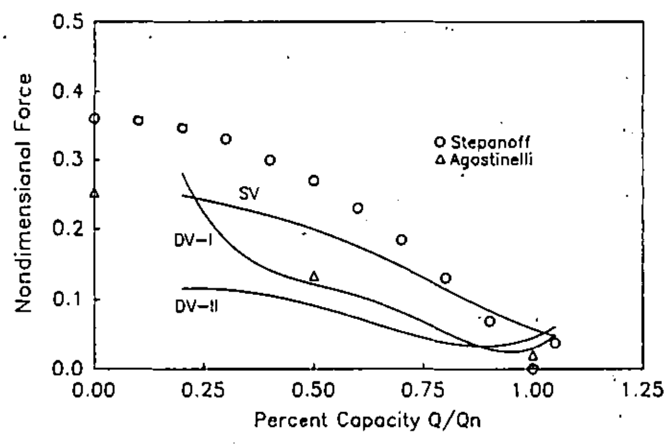


Fig. 13 Comparison of nondimensional forces with empirical models

CONCLUSIONS

Measurement of pressure profiles around the impeller discharge and along the casing wall was made in three configurations: single volute/single discharge, and two versions of douhle volute/single discharge accomplished by inserting a splitter positioned in the second half of the discharge midway between impeller and casing wall or symmetric to the first half casing wall. Time averaged static forces were calculated hy integrating the pressure profiles. Important conclusions are:

1. At flow rates above 80% of design the first double volute resulted in a developed head of 8% (on the average) less than the single volute; the second double volute resulted in a developed head of 9% less than the single volute. At flow rates below 40% of design the first double volute resulted in a developed head of 3% (on the average) greater than the single volute; the second double volute resulted in a developed head of 4% more than the single volute. At high flow rates the decreased performance of the double volute is attributed to increased boundary layer friction. At low flow rates the increased performance is attributed to better control of the recirculation regions.

2. A repeated pattern consisted of pressure increasing from the first cutwater to the splitter leading edge at which the pressure dropped and thereafter increased till the discharge. The pattern was noticeable at higher flow rates in the first version of the double volute; the second version exhibited this pattern throughout the range of operation.

3. The repeatability in pressure distribution is attributed to the same geometry of the wall casing and splitter. The increased clearance at the leading edge of the first splitter version does not yield the necessary pressure drop across the cutwater for the lower flow rates and hence does not yield the repeatability encountered on the second version.

4. Pressures recorded along the casing wall for both double volute configurations did not vary considerably from those measured in the single volute.

5. Integrating pressures around the impeller showed the double volutes to be most effective in reducing pressure generated radial thrust. The symmetric configuration exhibited greatest consistent reduction throughout operating capacities.

ACKNOWLEDGEMENT

This research was sponsored by the Rotating Machinery and Controls (ROMAC) Industrial Research Program at the University of Virginia.

REFERENCES

Agostinelli, A., Nobles, D., and Mockridge, C.R., 1960 "An Experimental Investigation of Radial Thrust in Centrifugal Pumps," Journal of Engineering for Power, Trans. ASME, Vol. 82, No. 2, pp. 120–126.

Biheller, H.J., 1965 "Radial Force on the Impeller of Centrifugal Pumps with Volute, Semi–Volute, and Fully Concentric Casings," Journal of Engineering for Power, Trans. ASME, Vol. 87, No. 3, pp. 319–323.

Binder, R.C. and Knapp, R.T. 1936 "Experimental Determinations of the Flow Characteristics in the Volutes of Centrifugal Pumps," Trans. ASME, Vol. 58, pp. 649–661.

Bowerman, R.D., Acosta, A.J., 1957 "Effect of the Volute on Performance of a Centrifugal–Pump Impeller," Trans. ASME, Vol. 79, No. 5, pp. 1057–1069.

Chamieh, D.S., Acosta, A.J., Brennen, C.E., Caughy, T.K., and Franz, R., 1982 "Experimental Measurements of Hydrodynamic Stiffness Matrices for a Centrifugal Pump Impeller," Workshop on Rotordynamic Instability Problems in High Speed Turbomachinery, Texas A&M University, NASA CP 2250, pp. 382–398.

Domm, U., Hergt, P., and Radestock, J., 1966 "Einige Anwendungen der Singularitätenmethods der Radialmaschine und Vergleiche mit Messungen," Wissenschaftlicher Verein für Strömungsmachinen, Budapest.

Grein, H., Bachmann, P., Barp, B., Annen, H., and Schweizer, F., 1975 (January) "Radial Forces on Hydraulic Turbomachinery," Sulzer Technical Review, pp. 37–41.

Hamkins, C.P., Flack, R.D., 1987, "Laser Velocimeter Measurements in Shrouded and Unshrouded Radial Flow Pump Impellers," Journal of Turbomachinery, Trans. ASME, Vol. 109, No. 1, pp. 70–78.

Hergt, P. and Krieger, P., 1969 "Radial Forces in Centrifugal Pumps with Guide Vanes," Proceedings of the Institution of Mechanical Engineers, Vol. 184, Pt. 3N, pp. 101–107.

Howard, J.H.G., Abramian, M., Herman, P., "Experimental Investigation of Impeller and Volute Flow Fields for a Low Specific Speed Pump with Single and Double Volutes," Proceedings of the 1987 ASME/JSME Thermal Engineering Joint Conference.

Imaichi, K., Tsujimoto, Y., Yoshida, Y., 1980 "A Two Dimensional Analysis of Interaction Effects of Radial Impeller in Volute Casing," IAHR–AIRH Symposium, Tokyo.

Iversen, H.W., Rolling, R.E., and Carlson, J.J., 1960 "Volute Pressure Distribution, Radial Force on the Impeller, and Volute Mixing Losses of a Radial Flow Centrifugal Pump," Journal of Engineering for Power, Trans. ASME, Vol. 82, No. 2, pp. 136–144.

Jery, B., Brennen, C.E., Caughy, T.E., and Acosta, A., 1985 "Forces on Centrifugal Pump Impellers," Proceedings of the Second International Pump Symposium, Texas A&M University, College Station, TX pp. 21–29.

Kanki, H., Kawata,Y., and Kawantani, T., 1981. "Experimental Research on the Hydraulic Excitation Force on the Pump Shaft," ASME Paper No. 81–DET–71.

Knapp, R.T., 1941 "Centrifugal–Pump Performance as Affected by Design Features," Trans. ASME, Vol. 63, pp. 251–260.

Meier–Grotrian, J., 1973 "Untersuchungen der Radialkraft auf das Laufrad einer Kreiselpumpe bei verschiedenen Spiralgehauseformen," VDI Berichte, No. 193, pp. 159–164.

Miner, S.M., Beaudoin, R.J., Flack, R.D., 1989 "Laser Velocimetry Measurements in a Centrifugal Flow Pump," Trans. ASME, Journal of Turbomachinery Vol. 111, No. 3, pp. 205–212.

Schwarz, D., and Wesche, W., 1978 "Radial Thrust on Double Entry Single Stage Centrifugal Pumps," Sulzer Technical Review, No. 2, pp. 63–68.

Stepanoff, A.J., 1957 "Centrifugal and Axial Flow Pumps. Theory, Design and Application," 2nd Ed., Wiley, New York.

Uchida, N., Kensaku, I., and Shirai, I., 1971 "Radial Forces on the Impeller of a Centrifugal Pump," Bulletin of Japan Society of Mechanical Engineers, Vol. 14, No. 76, pp. 1106–1117.

Worster, R.C., 1963 "The Flow in Volutes and its Effect on Centrifugal Pump Performance," Proceedings of the Institution of Mechanical Engineers, Vol. 177, No. 31, pp. 843–875.