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TIME-FREQUENCY ANALYSIS OF COMPRESSOR ROTATING STALL BY MEANS OF WAVELET TRANSFORM

Shengfang Liao and Jingyi Chen
Institute of Engineering Thermophysics
Chinese Academy of Sciences
Beijing, China

BREAK

ABSTRACT

In this paper, an application of Wavelet Transform, which is a newly developed time-frequency technique of signal processing, is demonstrated in analyzing compressor rotating stall signals. In contrast to conventional signal processing methods, e.g. Fourier Transform, Wavelet Transform is very suitable for analyzing transient processes as rotating stall inception in compressors. In this study, some typical rotating stall signals are processed via Morlet's wavelet. It is concluded that Wavelet Transform has a great advantage in detecting rotating stall inceptions, which are usually very weak and embedded in relatively stronger noises. In the diagrams resulted from the transform, every emergence of precursor as well as full stall signals of a certain frequency is illustrated versus time.

NOMENCLATURE

$\psi(t)$	mother function of wavelet
$\psi_{a,b}(t)$	wavelet function
$s(t)$	signals
$W(b,a)$	continuous wavelet transform
$\hat{\psi}(\omega)$	Fourier transform of $\psi(t)$
a	parameter of dilation or compression
b	shift of time
t	time
ω	angular frequency

INTRODUCTION

Unsteady phenomena in both axial and radial compressors, including surge and rotating stall, have been found an obstacle in improving compressor performance. Various studies were documented on this topic in the past decades. Especially in recent years, the hot-spot of study was focused on stall inception and active control of surge. The study on stall inception was first reported by McDougal(1989) and latter detailed by Day(1991, 1993). The concept of active control of surge was first proposed by Epstein and his coauthors(1986) and realized in laboratory by Ffowcs Williams and Huang (1989). There are also many other authors engaging in this topic, which have been shedding light on tackling this problem.

In the developing process of rotating stall, some precusors have been detected before the emergences of fully developed stall in various published papers. It is clear that there are two types of precursors: one is modal wave, which is progressively amplified pertubations with the same frequencies as fully developed stall(McDougall, 1989, Garnier et al, 1990); the other is more sudden growth of small localized disturbances (Day, 1991).

In the previous studies, the pressure or velocity signals were processed by means of traditional processing technique such as frequency-spectral analysis, matched filtering and so on. The common-used Fourier analysis has its drawbacks in applying to non-stationary signals as stall inception. It ommits time characteristics of signals. The amplitude of every frequency is actually the sum of the amplitudes of this frequency in the whole time domain. So it is very difficult to detect local weak precursors emhedded in relatively stronger noises. The fundamental problem of Fourier-like analysis is that the basis functions of Fourier transform extend over infinite time and therefore can not match to the abrupt signals. A possible alternative is short-time Fournier analysis. But its effectiveness is limited by its non-adapted window size. The method of matched filter is effective only if the shape, or at least the bandwidth of the original signals is well known(details refering to Tuteur, 1987).

In the present study, a newly developed method, i.e.

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wavelet transform, is adopted in analyzing rotating stall inceptions. The wavelet transform is based on some short-time oscillatory functions called wavelet, whose time and frequency size are self-adapted to the whole range of frequency through dilation or compression of the time scale rather than a frequency change. This allows us to build a time-frequency analysis without losing local details, which perhaps are relatively weak. It also can provide a real-time monitoring method by its fast algorithm, which is beyond this paper. It was first proposed in geology and well developed and used in speech, radar, sonar processing and image compression. Here, an attempt to apply this method to some typical rotaing stall inceptions is presented.

WAVELET TRANSFORM

DEFINITION Wavelet transform is based on some time-dependent oscillating elementary functions $\psi_{a,b}(t)$ called "wavelets"(details can be found in Chui, 1994). Then continuous wavelet transform of a signal $s(t)$ can be written as

$$W(b,a) = \int_{-\infty}^{\infty} s(t)\psi_{a,b}(t)dt \quad (1)$$

or

$$W(a,b) = \int_{-\infty}^{\infty} \hat{s}(\omega)\hat{\psi}_{a,b}(\omega)d\omega$$

The wavelet family $\psi_{a,b}(t)$ can be defined by its "mother function" $\psi(t)$ as

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi(\frac{t-b}{a}) \quad (2)$$

In the wavelet $\psi_{a,b}$, there are two parameters, a and b. The parameter a is a dilation (or compression) of the time scale. Larger a represents low frequency and vice vesa. The parameter b is a translation along the time axis.

Wavelets must satisfy several restrictions. One is the so-called admissibility condition

$$\int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|}d\omega < \infty \quad (3)$$

This formula is equivalent to the following

$$\hat{\psi}(0) = 0 \quad (4)$$

or

$$\int_{-\infty}^{\infty} \psi(t)dt = 0 \quad (5)$$

Another property of wavelets is that they should be oscillatory and restricted in an as small as possible time-bandwidth, or say, they must have a finite support in time domain. That is, they must have non-zero value only in a certain range of time (t_{min}, t_{max}). These restrictions are the reason they are called "wavelet".

In this paper, we choose Morlet's wavelets(Morlet et al, 1982)(see Fig.1)

$$\psi(t) = e^{-t^2/2}e^{i\omega t} \quad (6)$$

In Fig.1, the upper plot is the real part of (6) with different value of a, the lower plot is their frequency spectrum. The central frequency of wavelet $\psi_{a,b}(t)$ is $f = \frac{\omega}{2\pi a}$. When $\omega \geq 5.33$, this wavelet satisfy all above restrictions(Grossmann, A and Morlet, J., 1984). If wavelets are confined in (t_{min}, t_{max}), then (1) can be rewritten as

$$W(a,b) = \int_{b-t_{min}}^{b+t_{max}} s(t)\psi_{a,b}(t)dt \quad (7)$$

or

$$W(b,a) = \int_{\omega_{min}}^{\omega_{max}} \hat{s}(\omega)\hat{\psi}_{a,b}(\omega)d\omega$$

Hence, $W(b,a)$ provide information about the signal on a time interval (t_{min}, t_{max}). With varying parameter a, we can dilate or compress the time interval of concentration. Therefore the time window can be dilated in low frequency range and compressed in high frequency range.

From the definition of (1), we can conclude that wavelet transform is a filtered version of initial signal $s(t)$ with the filter chosen to conform with certain addmissibility restrictions.

For computing the wavelet transform, a relatively large parameter a is selected and then decreased in step. The integration is performed with the translation of b.

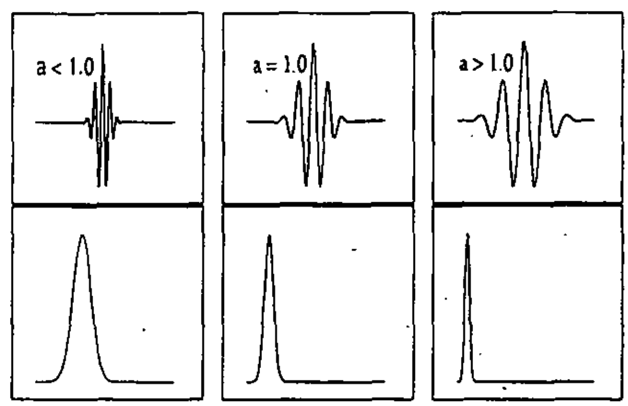

Fig.1. Wavelets and their frequency spectrum

UNDERSTANDING THE RESULTS OF WAVELET TRANSFORM In order to display the results of (1) in a way which allows us to analyze signals visually, it is convenient to plot the results in a "time and scale" half-plane (b,a) $(a > 0)$. Where a-axis faces downward and then higher frequencies are above lower frequencies. We also can say that the figure is a "time and frequency" plot. Because $W(b,a)$ is a complex value, it can be written as

$$W(b,a) = |W(b,a)|e^{i\varphi(b,a)} \quad (8)$$

The modulus $|W(b,a)|$ is plotted as a density picture according to its magnitude. The higher density correspond to larger value and vice versa.

The phase $\varphi(b,a)$ is defined in $[0, 2\pi)$. It is also plotted by the density of black dots on the picture. In a period, from

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left to right, the density of phases increase, corresponding to the regular increases of $\varphi(b,a)$ from 0 to 2π. When the phases reach 2π, they are wrapped around to 0. That causes abrupt changes between 2π and 0, and forms highly visible lines, corresponding to constant phases. The constant phase lines play an important role in representing signals, because they are independent on the magnitude of $W(b,a)$ and the choices of wavelets. It also can be proven that the constant phase lines will converge to the points of abrupt changes and so can be employed as a detector of abrupt changes in real-life signals (Grossmann et al, 1987).

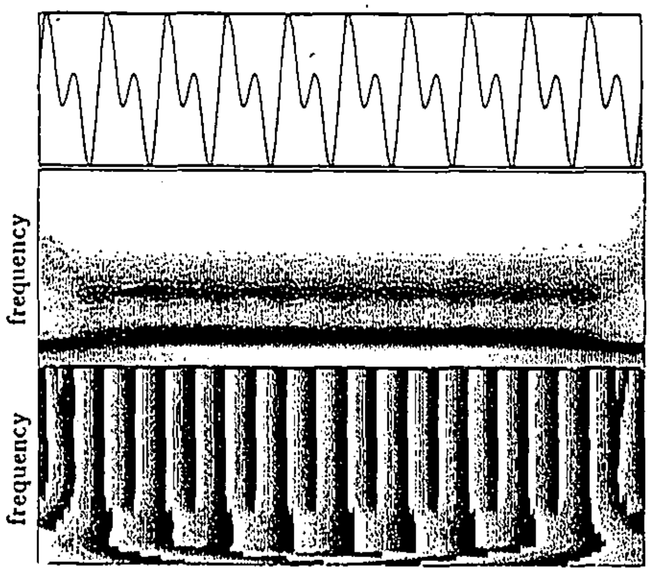

Fig.2 An example of wavelet transform

To illustrate the comments above, an example is shown in Fig.2. The signal is consisted of a sine wave and a double fequency sine wave (the top figure). In the middle figure, two ranges of modulus are clearly shown. The bottom figure is phase of the signal where two clusters of constant phase lines are shown, corresponding to the peaks of the signal. By the definition (1) and (2), although in some areas of time-frequency plane, the modulus is zero, there are still phases. So the lower frequency phase lines will extend to high frequencies. Some authors filtered the phase plots with the magnitude ones (Grossmann et al, 1987). The more extensive and detailed referneces about phase plot can be found in two conferences about wavelets(Combes et al ed., 1987 and Meyer, ed., 1989).

ILLUSTRATION OF IMPLEMENTATION In order to illustrate the apitude of wavelet transform, an artificial signal including three spikes (the top figure of Fig.3) is created and embeded in an computer-generated white noise modulated by an high frequency sine wave, which can be seen as background signals of rotating blades in compressors. The resulted signal is shown in the second figure of Fig.3. The signal-noise ratio is low for the first two spikes, respectively 0.5 and 1.0.

The results of wavelet transform are plotted in the third figure of Fig.3. From the results, three areas, which comform with the locations of three spikes, are marked intensly. It is commonly difficult for Fourier transform to detect such short-time and low signal-noise ratio signal. This example also shows us that, although the signals are mixed with large magnitude noise, the wavelet shows advantageous robustness in detecting the real signals.

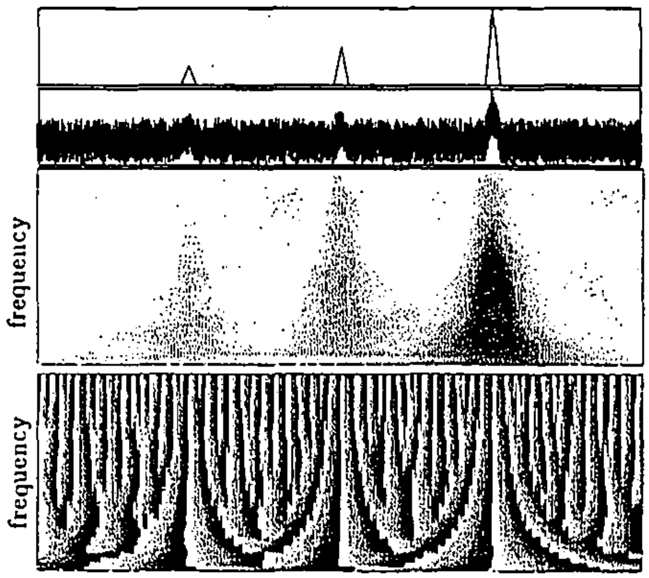

Fig.3 An example of wavelet transform of a artificial signal

The bottom figure shows the phases defined in (8). From the constant phase lines, the locations of three spikes are more clearly marked. It is also concluded that, on the phase plot, the resolution of the locations of abrupt changes is the same, in spite of different magnitude of these local events. It also shows that the converging constant phase lines detect the abrupt changes of frequency rather than those of magnitude.

ROTATING STALL

EXAMPLE 1 In order to verify the effectiveness of wavelet transform in analyzing rotating stall signals, the experimental signals are adopted from a previous study written by one of the authors and his co-authors(Fig.7 in Chen et al, 1994). The experiments were conducted in a high speed centrifugal compressor at 12000 rpm in University of Hannover, Germany. The signals selected from that paper were analyzed with concern of the inceptions of rotating stall in detail. Here, the validity of the data for analysis is assumed.

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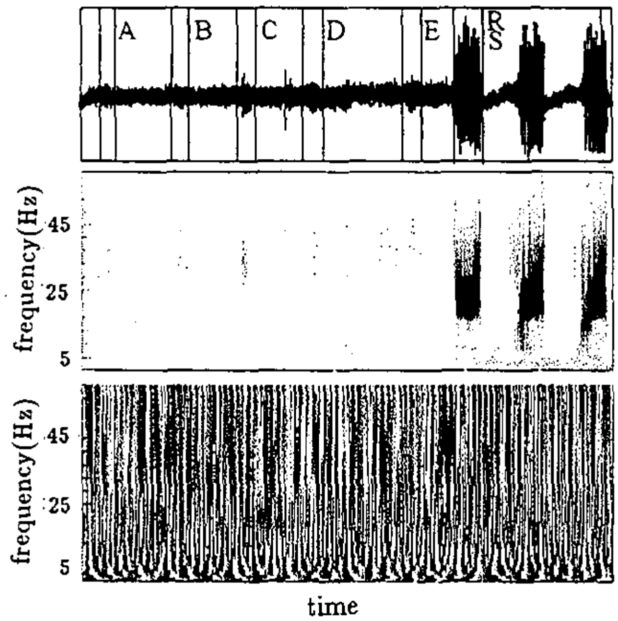
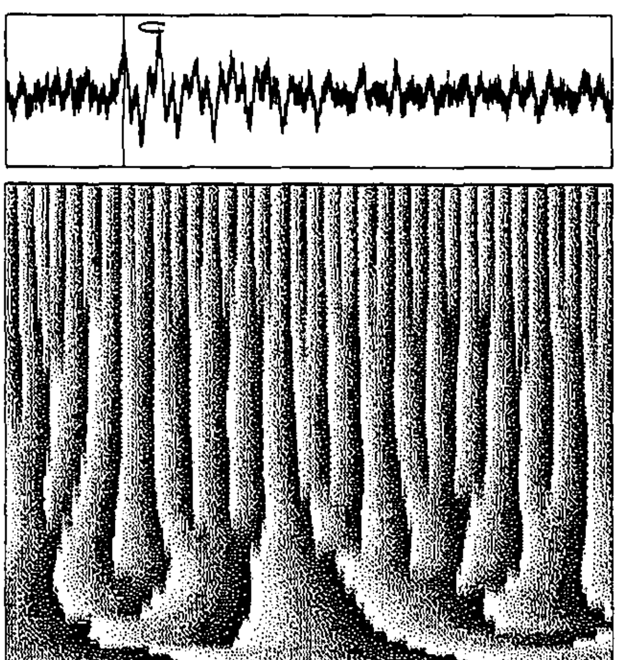
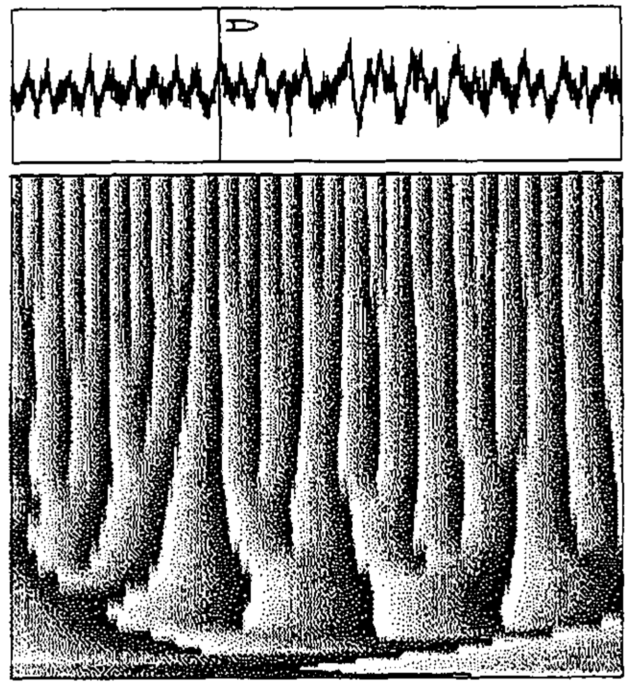


Fig.4 Wavelet transform of signal from a centrifugal compressor

The original signal and results are shown in Fig.4. The top plot is the original static pressure signal. In the middle plot (modulus of $W(b,a)$), three intermittant fully developed rotating stall are located clearly by the three intensively black patch on the right of this figure. Before the fully developed rotating stall, five clusters of precursor (denoted in



(a) Segment C



(b) Segment D

Fig.5 Zoom in segments of signal and their phases

the original signal as A, B, C, D, E) are also distinctly located by the five relatively less intensive stripes on the left. By reading the scale of the frequency axis, it is also clear that the frequencies of the precursors are relatively higher than that of the fully developed rotating stall.

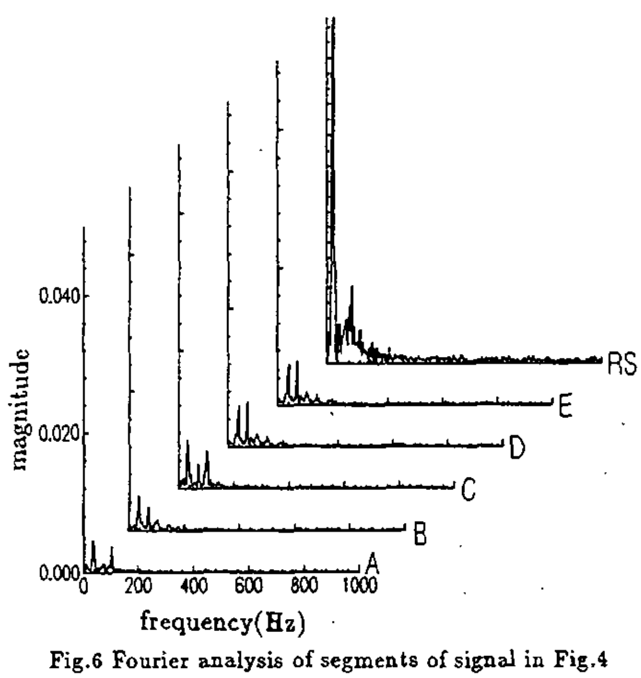


Fig.6 Fourier analysis of segments of signal in Fig.4

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The bottom plot of Fig.4 is the phase of wavelet transform. Without zooming in the figure, it is seemingly difficult to distinguish the constant phase line. Fig.5 shows the initial signals of segment C and D and the phases of these two segments of precursors by zooming into these segments. The constant phase lines acurately point to the peak of the precursors. In the marked positions (respectively in (a) and (b)), there are constant phase lines converging to the two locations of the two segments of precursor signals, which is similar to the case of Fig.3. With reference of the results from Fig.3, these locations indicate the abrupt changes of frequency. So it can be stated that they indicate the onset locations of precursors of rotating stall whose frequencies are different from the background frequencies of the signal. The converging phase lines are distinguishable especially in (b) of Fig.5, while they are not very clear in (a), but still detectable. It has a great advantage that the constant phase lines can locate the abrupt changes of frequency in a signal without concerning how weak the signal is. This results can be employed in analyzing the abrupt inceptions of rotating stall, in which the modulus patterns can be adopted to locate roughly the locations of the precursors and the phase patterns can accurately indicate the transition of the frequency.

In order to corroborate the authenticity of the results of wavelet transform, Fourier transforms are performed on the five precusor segments marked by wavelet transform. The results are shown in Fig.6. In this figure, several prevailing frequencies of those segments are detected, which comform with the results by wavelet transform and by Chen et al (1994). The conclusion that the frequency of rotating stall is lower than those of the precursors, is also valid.

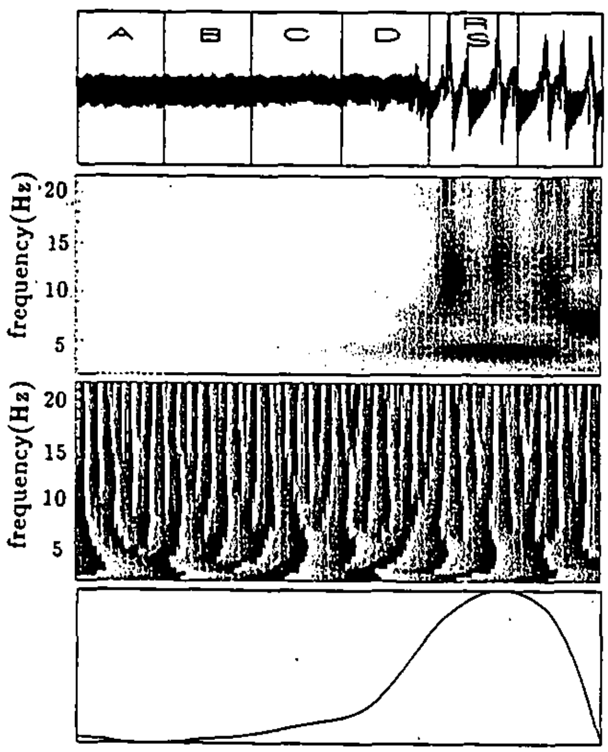

Fig.7 Wavelet transform of a signal an axial compressor

In this analysis, the experiences of fast and efficiently locating the precursors of rotating stall by wavelet transform are indeed exciting. Locating the precursors by performing Fourier transform on segments of signals are difficult and time-consuming, in which some relatively weak localized precursors are often missed. However, in the time-frequency diagrams of wavelet transform, the localized events can be more easily detected.

EXAMPLE 2 In this subsection, a typical signal of rotating stall of an axial compressor is provided by Prof. Xu (1995). The test rig is Deverson rig at Whittle Laboratory, Cambridge University. The inception of rotating stall includes a modal wave.

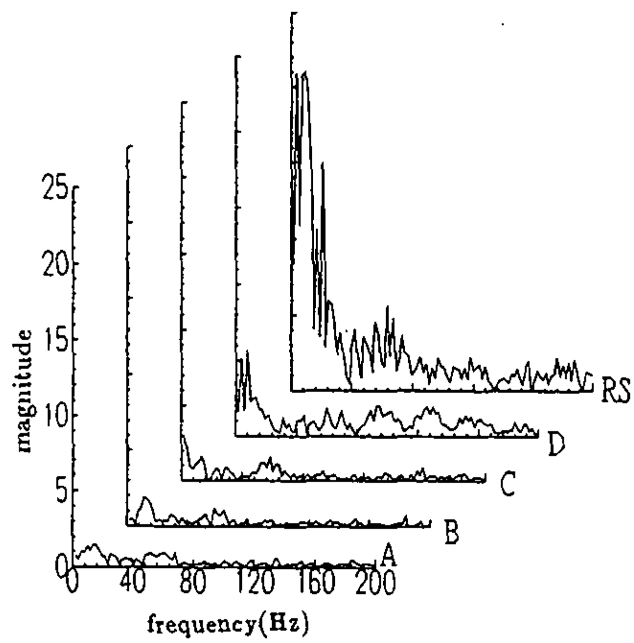

Fig.8 Fourier analysis of segments of signal in Fig.7

Fig.7 shows the original signal and the results of wavelet transform. On the modulus plot (the second figure from the top), two frequency ranges are clearly shown on the right part. The modulus of the lower frequency experiences gradual growth by the gradually intensifying black stripe (near the bottom of the modulus plot) from left to right, which can also be seen more clearly in the bottom plot of Fig.7 which is the modulus magnitude of the central line of the lower frequency stripe. This provides an earlier precursor of modal wave. This result comforms with previous studies. Moreover, the modulus pattern of fully developed rotating stall also shows some intermittant higher frequencies. it shows that the fully developed stall is actually a superposition of the lower frequency modal wave and the higher frequency

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species.

The third figure from the top is the phase plot. The emergence of the modal wave is difficult to be located because of its gradual growth. In the fully developed stall section, it is clear that the lower frequency phase lines regularly spread out into their double frequency phase lines, which is not the case of the pre-stall section. This forms a more regular phase pattern than the pre-stall section.

Fig.8 shows the results of Fourier analysis performing on the segments of the signals in the top plot of Fig.7. The growing process of the modal wave can be identified. These results are consistent with those of the magnitude plots of wavelet transform.

DISCUSSION

The inception signals of rotating stall are strongly non-stationary. Their frequency characteristics is often time-variant. The traditional Fourier analysis is theoretically circumscribed in periodical signals by its definition. It does not preserve any time-dependent information, and so it can not provide any imformation concerning with either a time evolution of spectral characteristics or a possible occurrence of time localized events.

In the field of signal processing, an alternative, short-time Fourier transform, has long been adopted in which some time window is introduced into the definition to concentrate the analysis in a short time range. By time translation, short-time Fourier transform can provide a time-fequency representation of signals. But it also has its drawback: because of its strict time-frequency window, it is not effective in detecting high fequency and low fequency signals (Chui, 1994).

The Wigner-Ville Distribution is also a choice in processing non-stationary signals. It provides a time-frequency representation without any restriction about time and frequency resolutions. Its advantage is that it does not introduce any assumptions on signals themselves. However, the bi-linearity of the definition has drastic consequences that it generates negative values in the time-frequency energy distribution of the signals. Moreover, its addition operation is a non-linear one. Therefore, the Wigner-Ville Distribution can not provide an easy representation of signals.

In analyzing the inception signals of rotating stall, one also can perform Fourier transform on each fraction of data (Chen et al, 1994). But the procedure requires priori knowledge of the signals and is time-consuming. As described in the previous section, wavelet transform is an effective candidate for processing non-stationary signals as inception of rotating stall. Through the dilation or compression by parameter a, the time-frequency windows are well-adapted to various frequency ranges with good resolution. Because of its advantageous property, further studies are expected to extend the analysis over phase analysis, autodetection and warning system.

CONCLUSIONS

Based on the applications of wavelet transform, there are several concluding remarks:

1. Wavelet transform can provide a visual representation of the process of rotating stall by time-frequency plot. The high and low frequency are both resolved distinctly.

2. The results show that wavelet transform has the ability to detect and locate the precursors of rotating stall in a very wide frequency range. It aslo has advantages in capturing relatively weak localized emergence of spike-like events.

3. The modulus of wavelet transform provides information about the magnitudes of various frequencies, and the emergence and growing process of the inception of rotating stall. The results of phases give a clear pattern of various frequencies and locate localized abrupt changes easily.

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