THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
345 E. 47th St., New York, N.Y. 10017

98-GT-48

A COMPARISON OF THE STREAMLINE THROUGHFLOW AND STREAMLINE CURVATURE METHODS FOR AXIAL TURBOMACHINERY

Anthony J. Gannon
Theodor W. von Backström

Department of Mechanical Engineering
University of Stellenbosch
South Africa

ABSTRACT

The axial flow turbomachinery throughflow equation states that radial gradients of rothalpy, entropy and moment of momentum affect the conservation of tangential vorticity. The streamline throughflow method (STFM) transforms this equation, expressed in terms of stream function in a radial-axial co-ordinate system, to an equation for streamline radial position in a stream function-axial co-ordinate system. The paper assesses the accuracy and efficiency of the STFM relative to the streamline curvature method (SCM) by comparing streamline positions and velocity profiles to analytical results. Test cases include flow through a single actuator disc, flow through twin actuator discs using a coarse computational grid, compressible flows through an almost choked nozzle, through single and twin actuator discs, and swirling flow using sloped stations. Results from the STFM and SCM agreed about equally well with analytical solutions for the same number of streamlines. The STFM, however, was much more tolerant of distorted computational grids and used an order of magnitude less computer time to converge. The test cases show that the STFM is suitable for annuli with large variations in hub and tip radius, for highly swirling and compressible flow, and is more robust and converges faster than the SCM. To demonstrate the practical applicability of the STFM a multistage compressor was simulated and STFM results compared with experiment.

NOMENCLATURE

a	=	tangential velocity factor for actuator disc flow; sonic velocity
ADT	=	actuator disc theory
b	=	tangential velocity factor for actuator disc flow;
	=	unblocked fraction
C	=	absolute velocity
c_p	=	specific heat at constant pressure
F	=	force
I	=	rothalpy
h	=	enthalpy
LHS	=	left hand side
M	=	Mach number
MTFM	=	matrix throughflow method
P	=	pressure
r	=	radial co-ordinate, radial position of streamline
R	=	gas constant
RHS	=	right hand side
s	=	entropy
S	=	source term
SCM	=	streamline curvature
STFM	=	streamline throughflow
T	=	absolute temperature
W	=	relative velocity
x, y	=	orthogonal co-ordinate system components
z	=	axial co-ordinate
γ	=	specific heat ratio
ρ	=	density
ψ	=	stream function

Subscripts

b	=	body
in	=	inlet
out	=	outlet
o	=	stagnation property
r	=	radial component
z	=	axial component
θ	=	tangential component

Presented at the International Gas Turbine & Aeroengine Congress & Exhibition
Stockholm, Sweden — June 2–June 5, 1998

INTRODUCTION

During the last decade the main emphasis in turbomachinery flow analysis has been on computational fluid dynamic (CFD) development. This has led to the ability to simulate the three-dimensional flow in multistage machines. However, due to the possibility of including empirical information from extensive data bases, meridional methods, in which axial symmetry is assumed, and blade rows are represented by actuator discs, are still used as preliminary design tools. (Sanger, 1996, Damle et al., 1997). Development of the associated deviation and loss models also continues as presented by Camp and Horlock (1994) and König *et al* (1996a, 1996b). This leaves scope for further investigation into the improvement of meridional methods.

Requirements of a useful meridional method will now be mentioned. The method should be able to handle large changes in radial co-ordinate and flows with high swirl, high throughflow Mach numbers and high pressure ratios. It must allow sloped quasi-orthogonals to be placed on the leading and trailing edges of the blade rows. The method must be easy to programme and the code should be robust and converge quickly, as throughflow methods are often used to generate entire performance maps consisting of many data points. The method must be accurate, showing good agreement with analytical and experimental results.

This paper will attempt to show that the Streamline Throughflow Method (STFM) has the above properties and could thus be considered as a useful alternative to existing meridional methods. An outline of the STFM development and implementation will be given. The performance of the method in simulating compressible and incompressible flows is investigated by comparing STFM flow fields to flow fields obtained with the well known streamline curvature method (SCM). Both STFM and SCM results will be compared to analytical test cases as absolute reference.

STFM DEVELOPMENT.

The most popular meridional throughflow method, the SCM as described by inter alia Smith (1966) and Novak (1967), is used in many commercial and in-house codes. The main advantages of the method, as stated by Davis and Millar (1975) are the small computer memory requirements and that it is streamline based: in general properties are determined by the actual flow path or conserved along streamlines as in inviscid flow. The use of streamlines also leads to easy interpretation of computational results without interpolation. The SCM suffers from two inherent disadvantages: firstly, mass conservation must be enforced in a separate loop, since it solves a radial momentum equation, and secondly, the results are dependent on the curve fit used for the streamlines. As a curve is fitted to each streamline independently, there is no guarantee that the chosen curve fit represents the flow accurately between the quasi-orthogonals, and uneven spacing of quasi-orthogonals may even cause computational instability.

Another throughflow method is the Matrix Throughflow Method (MTFM), also investigated by Davis and Millar (1975). It solves the tangential vorticity equation expressed in terms of the stream function as an independent variable on a fixed irregular grid. The method yields values of stream function from which velocities and eventually other variables can be calculated at each grid point. Advantages of the method are the use of standard CFD discretization schemes and solution methods, and its inherent stability, as it is solved as an elliptic Poisson equation of the form:

$$\frac{\partial^2\psi}{\partial z^2}+\frac{\partial^2\psi}{\partial r^2}=\frac{1}{r}\frac{\partial\psi}{\partial r}+S \qquad (1)$$

Consideration of the SCM and the MTFM has led to the question of whether the use of streamlines could be combined with a CFD type approach. Boadway (1976) presented such a method for general irrotational (non turbo-machinery) flows. The transformed tangential vorticity equation, with S = 0, has co-ordinate r exchanged for ψ and dependent variable ψ for r, giving:

$$\left(\frac{\partial r}{\partial\psi}\right)^2\frac{\partial^2 r}{\partial z^2}+\left(1+\left(\frac{\partial r}{\partial z}\right)^2\right)\frac{\partial^2 r}{\partial\psi^2}=-\frac{1}{r}\left(\frac{\partial r}{\partial\psi}\right)^2+2\frac{\partial r}{\partial z}\frac{\partial r}{\partial\psi}\frac{\partial^2 r}{\partial z\partial\psi} \qquad (2)$$

This is called the expanded formulation because the first term on the right hand side (RHS) is not absorbed in the LHS, as in the compact formulation to be discussed later.

Turbomachinery Throughflow Equations

Hirsch and Warzee (1976) derived the general turbomachinery throughflow equation for inviscid compressible flow:

$$\frac{\partial}{\partial z}\left(\frac{1}{\rho r b}\frac{\partial\psi}{\partial z}\right)+\frac{\partial}{\partial r}\left(\frac{1}{\rho r b}\frac{\partial\psi}{\partial r}\right)=\frac{1}{W_z}\left[\frac{\partial I}{\partial r}-T\frac{\partial s}{\partial r}-\frac{F_{b,r}}{\rho}-\frac{W_\theta}{r}\frac{\partial}{\partial r}(rC_\theta)\right] \qquad (3)$$

$$\text{where } \frac{\partial\psi}{\partial r}=\rho r b W_z,\text{ and } \frac{\partial\psi}{\partial z}=-\rho r b W_r. \qquad (4)$$

The formulation of the equation given by Davis and Millar (1975) is similar except that on the LHS of the equation only the Laplacian of the streamline radial position is retained, while the terms containing density gradients are included in the RHS source terms.

Equation (3), being formulated in terms of relative velocities and rothalpy, is valid within blade rows and between blade rows. For the sake of simplicity and for the purposes of comparison to analytical solutions certain assumptions will now be made.

Assumptions

1. The radial force term $F_{b,r}$ is negligible.
2. The blade rows are completely represented by actuator discs.
3. Consequently all the flow occurs in blade free space, and the blade blockage factor b is equal to unity (no blockage).
4. As a consequence of point 2 above, the flow field can be solved in the absolute frame of reference. (Relative velocity, W replaced by absolute velocity, C and rothalpy, I by stagnation enthalpy, h_0).
5. Also, gradients with respect to radius in the turbomachinery source term can be replaced with gradients with respect to stream function by using equation (4).

Under these assumptions the turbomachinery equation reduces to the form given by Oates (1988):

$$\frac{\partial}{\partial z}\left(\frac{1}{\rho r}\frac{\partial\psi}{\partial z}\right)+\frac{\partial}{\partial r}\left(\frac{1}{\rho r}\frac{\partial\psi}{\partial r}\right)= \rho r\left[\frac{\partial h_o}{\partial\psi}-T\frac{\partial s}{\partial\psi}-\frac{C_\theta}{r}\frac{\partial}{\partial\psi}(rC_\theta)\right] \quad (5)$$

where

$$\frac{\partial\psi}{\partial r}=\rho rC_z,\ \frac{\partial\psi}{\partial z}=-\rho rC_r \quad (6)$$

Boadway's (1976) transformation is applied to the LHS, with the equation multiplied by $-\rho r(\partial r/\partial\psi)^3$. The result is the transformed compact formulation of the turbomachinery throughflow equation:

$$\left(1+\left(\frac{\partial r}{\partial z}\right)^2\right)\frac{1}{\rho r}\frac{\partial}{\partial\psi}\left(\rho r\frac{\partial r}{\partial\psi}\right) +\left(\frac{\partial r}{\partial\psi}\right)^2\rho r\frac{\partial}{\partial z}\left(\frac{1}{\rho r}\frac{\partial r}{\partial z}\right)=2\frac{\partial r}{\partial z}\frac{\partial r}{\partial\psi}\frac{\partial^2 r}{\partial z\partial\psi} -(\rho r)^2\left(\frac{\partial r}{\partial\psi}\right)^3\left[\frac{\partial h_o}{\partial\psi}-T\frac{\partial s}{\partial\psi}-\frac{C_\theta}{r}\frac{\partial(rC_\theta)}{\partial\psi}\right] \quad (7)$$

The STFM solves this equation for r in a field with z and ψ as co-ordinates. Oates *et al.* (1976) solved an equivalent but more complicated equation by means of a finite element method. The question of compatibility of the flow fields on opposite sides of an actuator disc was addressed by them and also by Von Backström and Roos (1993).

The method described is called the streamline throughflow method since it resembles the streamline curvature method in having streamline position as the dependent variable, but is based on the same turbomachinery throughflow equation as the matrix throughflow method.

Discretization and Solution.

The equation is discretized using a non-uniform grid spacing which allows a fine grid tn be used in regions of extreme velocity gradients and coarse grids in areas of small gradients. The discretized equation is solved using a Gauss-Seidel method using relaxation factors between 0.9 and 1.8, the finer the grid the smaller the relaxation factor required. The code was written in FORTRAN and run on a Pentium 75 personal computer. The convergence criteria was that the absolute streamlines shift should be less than 1e-5 between iterations (for 1 m maximum streamline radius).

For compressible flows the density of the fluid is influenced by the pressure and temperature. In the isentropic flow between the actuator discs, however, the density can be directly calculated from the velocity, by employing the stagnation speed of sound. The local speed of sound is not used because it is not constant between actuator discs.

$$\rho=\rho_o\left[1-\frac{\gamma-1}{2}\left(\frac{C_\theta^2+C_z^2+C_r^2}{a_o^2}\right)\right]^{\frac{1}{\gamma-1}} \qquad a_o^2=\gamma RT_o \quad (8)$$

The method has been extended to handle sloped stations through a scheme described by Harms *et al.*(1996). This allows a local rectangular grid to be used around each point with the neighbouring values being calculated using linear interpolation.

Description of comparative SCM code

The study required a SCM code for comparative purposes. The comparative SCM code was based on the paper of Novak (1967) as implemented in the masters degree thesis of Heyns (1982). It has options to use either cubic spline curve fits or to directly discretize the gradients and curvatures in the method. There was little difference in the convergence times achieved by the two options. The first, more typical SCM type approach was used.

TEST CASES

A series of test cases, starting with simple ones, systematically introduces the various requirements and phenomena to be modelled. To eliminate experimental error, test cases with analytical solutions are selected. The exclusion of blade cascade models, boundary layer blockage, mixing, bleed flows etc focuses attention on the essential problem of correctly predicting streamline and velocity profile shape.

There is however, no reason why these effects cannot be included in the STFM as shown later by the 10 stage compressor simulation.

Incompressible flow

Test case 1. was the simulation of non-swirling incompressible flow through a parallel walled annulus. The expanded formulation based on the method of Boadway (1976) as reported by Von Backström and Roos (1993) was inaccurate near the inner wall. The new, compact formulation completely eliminated this inaccuracy and placed the streamlines at exactly the same radii as the simple analytical solution.

Test case 2. was non-swirling incompressible flow over a sphere. It introduced radial flow and streamline curvature. This test case is useful as the flow is similar to that of the nose bullet on a turbo-machine hub. The classical inviscid solution was used for comparison of results. The test case used a sphere of radius 0.4 m, with the outer annulus shaped to follow the shape of the analytically calculated stream surface with a maximum radius of 1.0 m. A non-uniform grid spacing was used with a finer grid being used in the region of large radial flows as shown in Figure 1.

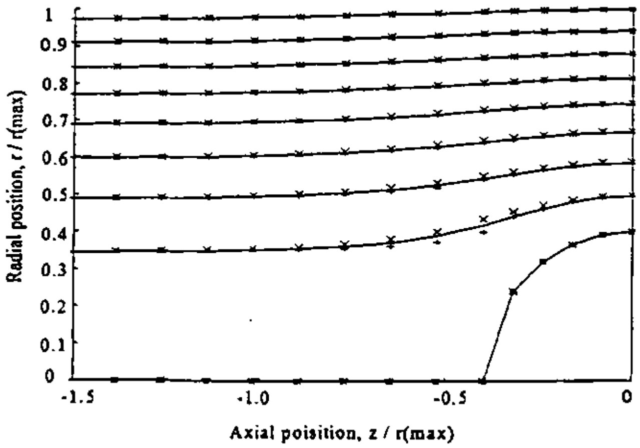

Fig. 1 Streamline positions for flow over a sphere.

For both methods the maximum error of 4 % was on the inner streamline near the front of the circle. For the STFM, convergence time was 2 s and for the SCM 40 s. Increasing the number of internal streamlines of the STFM to 31 increased the convergence time to 8 s and reduced the error on the same streamline to 1.7 %.

Test case 3. introduced tangential velocities in the simulation of incompressible flow through a single actuator disc in a parallel annulus. The outer diameter was 1 m and the inner 0.4 m with the disc is placed at z = 0. The up and down stream vortex distributions were of the form below, with $a = 6.0\ s^{-1}$ and $b = 6.0\ m^2/s$. In all the test cases the tangential velocity at the disc is taken as the average of the upstream and downstream value.

$$C_\theta = ar \pm b / r \qquad (9)$$

The axial velocity at the hub inlet was 10 m/s. Results were compared to the simple actuator disc theory (ADT) as described by Dixon (1978). Figure 2 shows the radial positions of the 7 internal streamlines at each axial station as computed by the STFM, as well as the median internal streamline from the simple ADT solution.

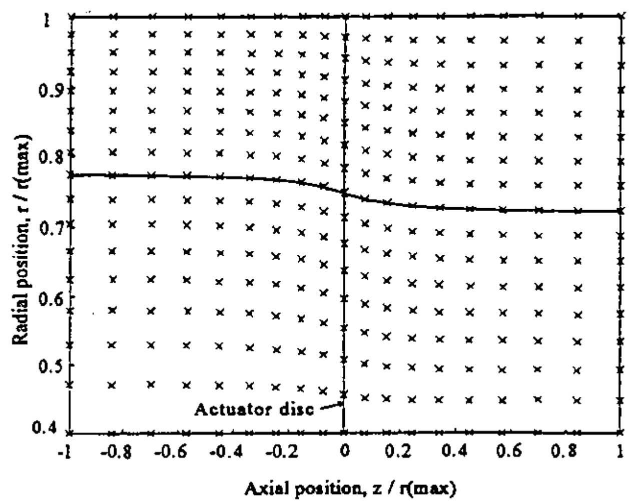

Fig. 2 Incompressible flow through a single actuator disc.

The maximum difference of 0.7 % in median streamline position occurs immediately after the actuator disc. Since simple ADT disregards radial velocities, it is not accurate in the immediate vicinity of the disc, but the general agreement in streamline shape is good. Figure 3 gives an indication of the accuracy of the STFM in predicting velocity profiles in incompressible flows. The test case is similar to the previous but there are 15 internal streamlines. The objective in this test case is to predict the far upstream and downstream velocity profiles where ADT is accurate. The maximum error in inlet velocity far upstream is 0.3 % at the hub and in exit velocity far downstream it is 0.6 % at the casing.

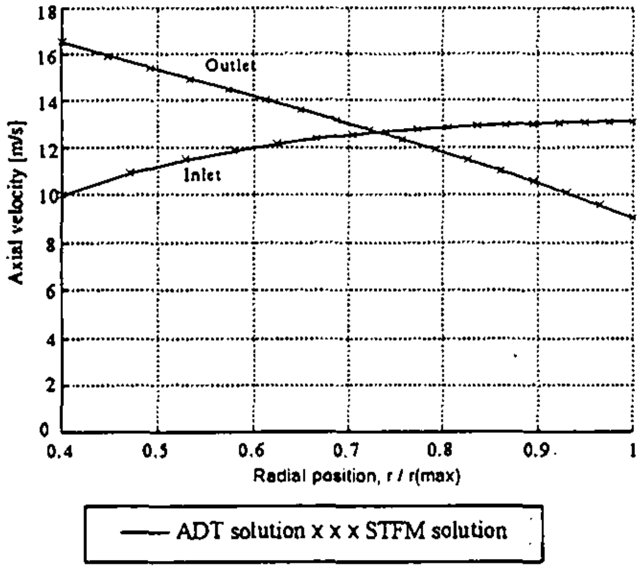

Fig. 3 Inlet and outlet axial velocity profiles of incompressible flow through a single actuator disc.

Test case 4. is the same as test case 3 except that a second actuator disc removed the swirl introduced by the first, and the axial velocity at the hub inlet was 5.5 m/s (Dixon, 1978). This test case represents throughflow modelling with one axial station on the blade leading edge and one on the trailing edge. The tangential velocity at the discs is taken as an average of the up and downstream values. There were 7 streamlines and 7 stations in the flow field.

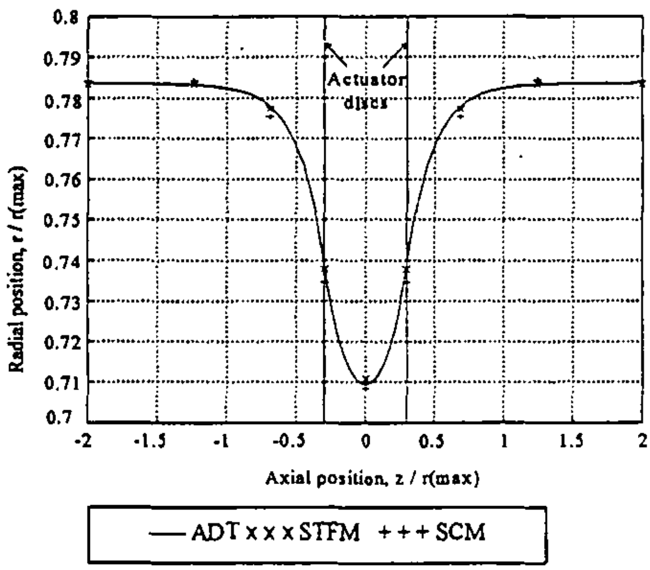

Fig. 4 Median Streamline position for twin actuator disc case using coarse axial grid.

Figure 4 shows only the median streamline as calculated by simple ADT, STFM and SCM. The general impression is that STFM agrees better with the analytical solution. The maximum differences compared to the ADT solution were 0.5 % for STFM between the discs, and 0.6 % for SCM at the discs. STFM took 1 s to converge and SCM took 5 s.

Compressible Flow

Test case 5 is compressible non-swirling flow through a nozzle. The nozzle dimensions and fluid properties are given in the following table.

Table 1. Gas constants and nozzle radius at inlet and outlet for compressible flow nozzle.

T_o	300 K	R	287.14 J/kg.K
P_o	600 kPa	c_p	1005 J/kg.K
ρ_o	6.965 kg/m^3	Mass flow	274.5 kg/s
r_{in}	1 m	r_{out}	0.25 m

The nozzle area decreases by a factor of 1/16 starting at z = 3 m. The radius varies according to a cosine function over a length equal to 2.5 upstream diameters. A comparison of selected variables of the STFM and one dimensional solution far downstream is shown in Table 2.

Table 2. Comparison of STFM and 1 dimensional analysis of compressible flow nozzle.

Outlet	STFM	1-D analysis	error
$C_{z\ out}$	306.7202 m/s	306.7152 m/s	0.002 %
ρ_{out}	4.55780 kg/m^3	4.55787 kg/m3	0.002 %
M_{out}	0.96140	0.96138	0.002 %

The STFM results for this test case agree extremely well with the one dimensional solution. Figure 5 shows that the highest velocity in the nozzle is just below sonic The results prove the method to be reliable for non-swirling flow up to just below Mach 1.0, but the present discretisation cannot handle supersonic throughflow velocities. Computation time was 16 s.

Test case 6. is a compressible flow through an inlet guide vane represented by an actuator disc. The flow upstream of the actuator disc has no tangential velocity, while downstream the following distribution was assumed with the tangential Mach number at mid radius being sonic,

$$C_\theta = b / r \qquad (10)$$

Although the throughflow velocity in this test case is always less than Mach 1 there are regions of sub, trans and supersonic flow. It is interesting to note that had the flow been incompressible in this test

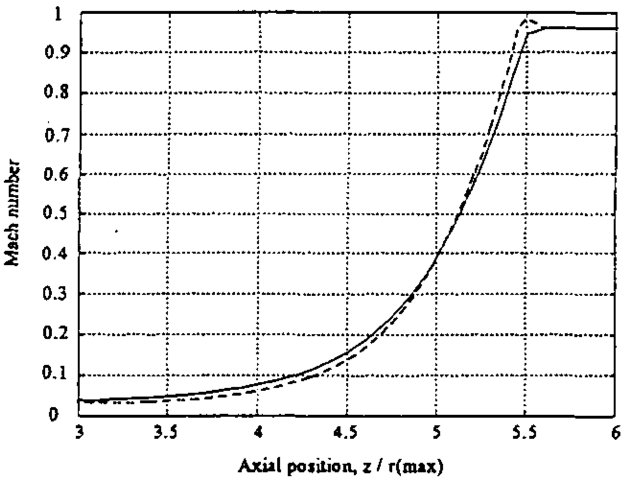

Fig. 5 Compressible flow through a nozzle.

case, no streamline shift would have occurred through the disc, due to the free vortex distribution of the swirl . The shift is entirely due to changes in density, making this a good test case for investigating the effect of compressibility in the method. The problem simulated was the same as that used by Horlock (1978) with the inlet properties and geometry listed in Table 3. There were 15 internal streamlines and 51 axial stations in the STFM model.

Table 3. Constants used for compressible flow through an actuator disc.

T_o	273 K	R	287 J/kg.K
P_o	98 490 Pa	c_p	1004 J/kg.K
$C_{z\ in}$	60 m/s	b	281.5 m^2/s
hub-tip ratio	0.7	Disc co-ordinate	0
Outer radius	1 m	Mean radius tangential Mach number	1.0

Figure 6 shows the variation in axial velocity along the hub and casing (tip) and along the median streamline compared to the analytical solution of Horlock (1978). The STFM predicts a 3.2 % higher velocity at the hub just upstream of the disc when compared to the Horlock (1978) solution that assumes radial velocities to be zero. Using the same grid, the SCM solution was almost identical, predicting a 2.9 % higher velocity. The convergence time using the STFM, however, was 16 seconds, using a relaxation factor of 1.6, while the SCM relaxation factor had to be reduced to 0.002 to obtain convergence to 1×10^{-5}, and consequently took much longer to converge.

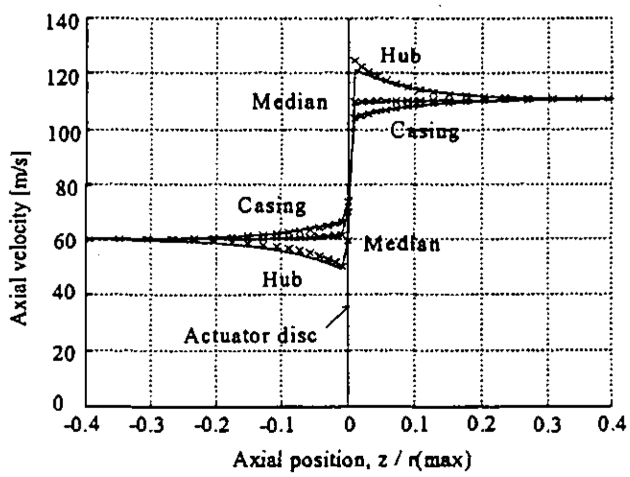

Fig. 6 Axial velocities for compressible flow through a single actuator disc.

Test case 7. simulates compressible flow through a nozzle guide vane and turbine blade row using two actuator discs. This is an extension of test case 6, with the second actuator disc at one third of the blade span downstream. Across the second (turbine) disc the flow is turned back to the axial direction with a stagnation temperature drop of 49 K. Figure 7. shows STFM and SCM solutions, for 15 internal streamlines and 96 axial stations, superimposed on a compressible simple ADT solution given by Hawthorne and Ringrose (1963-4).

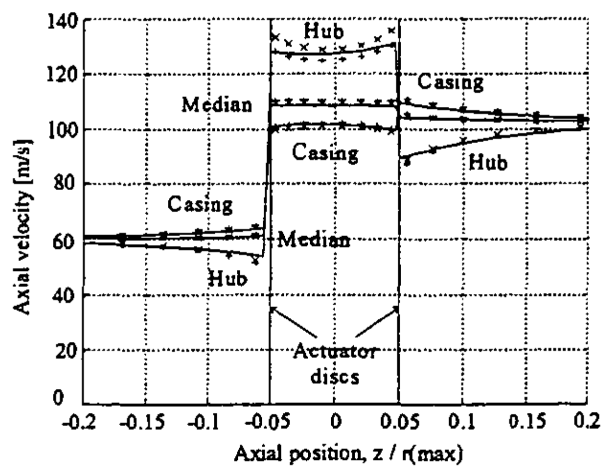

Fig. 7 Axial velocities for compressible flow through twin actuator discs.

Table 4 shows velocity differences compared to ADT theory for STFM and SCM. It is worth noting that the differences between STFM and SCM are typically 2 % but when both methods are compared to ADT the differences are about 4 %. STFM converged to 1.0e-5 in 26 s with a relaxation factor of 1.8, while SCM converged to 7.0e-5 in 10 000 iterations in 3 hours using a relaxation factor of 0.00005.

Table 4. Percentage difference in axial velocity compared to ADT

Position ⇓ Method ⇒	STFM	SCM
Upstream of disc 1	-4.5	-6.5
Downstream of disc 2	4.6	3.0
Upstream of disc 2	4.5	3.5
Downstream of disc 2	-1.5	-3.9

Test case 8 investigates the effects of sloping the axial stations on the performance of the STFM for swirling flow through an expanding annulus as shown in Figure 8. The grid geometry of the reference case consisted of 19 evenly spaced internal axial stations. The geometry and streamline positions of the simulation with sloped stations are superimposed in the figure. There is no discernible difference in streamline position. The sloped station solution required 16 iterations for convergence compared to 14 for the vertical stations.

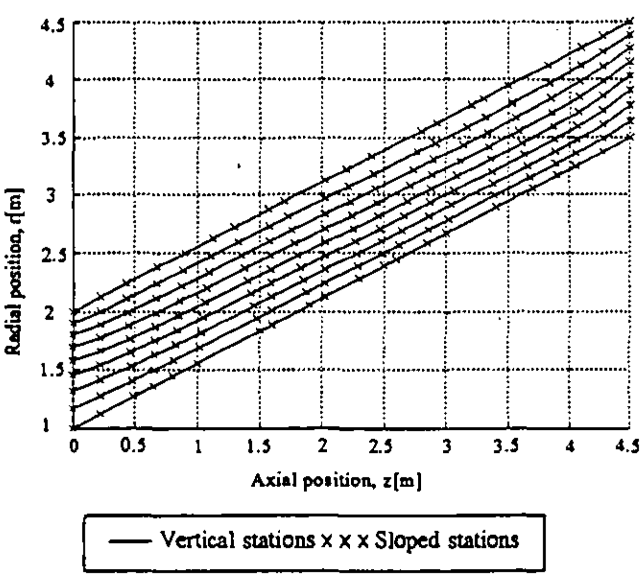

Fig. 8 Swirling flow comparing results from vertical and sloped stations

COMPRESSOR SIMULATION

For a numerical method to be useful it must be able to be applied in practice. For this Howell's (1942, 1945) correlation for loss and deviation were used with the STFM to simulate a repeating 10 stage (21 blade row) process gas axial compressor. Experimental results for air pumped at scaled conditions were available and these were compared to the STFM results. The design pressure ratio for these conditions was 1.3 with a density ratio of 1.21. The inlet casing and hub diameters were 0.246 m and 0.16 m respectively with an operating speed of 4350 rpm. The inlet casing and hub solidities were 1.1636 and 0.7630 respectively.

The grid used for the simulation consisted of three axial stations ahead of the inlet guide vanes and three after the outlet. One axial station was placed at the leading and trailing edge of each blade row while 11 internal streamlines were used. This reasonably coarse grid is similar to that used in many SCM applications. A typical STFM simulation took 45 seconds. The performance map shown in Figure 9 has three curves corresponding to different stagger blade angles as indicated. The experimental results are superimposed over the numerical simulation. There is good agreement between the experimental and numerical results on flow resistance lines through and below the design point but the predicted curves tend to be steeper than the experimental results at higher pressure ratios. The likely cause of these deviations is the simple boundary layer blockage model employed. It is one suggested by Cumpsty (1989, p80) who gives as rule of thumb for design point calculations a blockage increase of 0.5 % per blade row up to a maximum of 4 %.

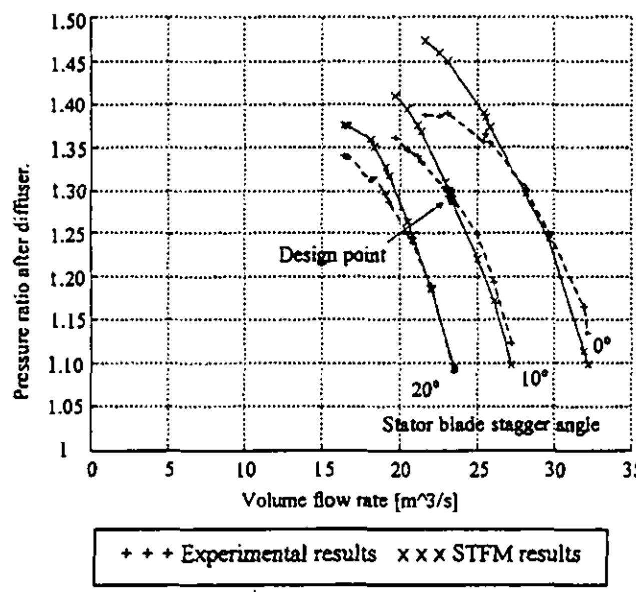

Fig 9. Comparison of numerical and experimental results for multistage compressor.

CONCLUSION

Compared to analytical solutions the accuracy of the STFM for the same number of streamlines was similar to the SCM. The STFM, however, used an order of magnitude less computer time to converge to the same criterion because its stream function formulation automatically satisfied mass conservation and did not need a separate mass conservation loop. STFM was also highly tolerant of high aspect ratio grid cells, as it did not require a separate curve fit procedure to calculate streamline gradient and curvature. The more robust STFM would converge with typical over-relaxation factors of up to 1.5 compared to between 0.00005 and 0.2 for the SCM. for the more complex problems the STFM convergence times were orders of magnitude less than for SCM. Similar results were found by Davis and Millar (1975) in their comparison of the matrix throughflow method and the SCM.

The test cases show that the STFM is suitable for annuli with large variations in hub and tip radius, for highly swirling and compressible flow. The ability of the STFM to be practically implemented has also been demonstrated. The discretisatioo method used at present cannot handle supersonic throughflow.

REFERENCES

Boadway, J.D., 1976, "Transformation of elliptic partial differential equations for solving two-dimensional boundary value problems in fluid flow," *International Journal for Numerical Methods in Engineering*, Vol. 10, pp. 527-533.

Camp, J.R., and Horlock, J.H., 1994, "An analytical Model of Axial Compressor Off-design Performance," Vol. 116, pp 425-434.

Cumpsty, N.A., 1989, "Compressor Aerodynamics," Longman Scientific and Technical, Harlow.

Damle, S.V., Dang, T.Q. and Reddy, D.R., 1997, "Throughflow Method for Turbomachines Applicable for All Flow Regimes," *Transactions of the ASME Journal of Turbomachinery*, pp. 256-262.

Davis, W.R. and Millar, D.A.J., 1975, "A comparison of the Matrix and Streamline Curvature Methods of Axial Flow Turbomachinery Analysis, From a User's Point of View," *Transactions of the ASME Journal Engineering for Power*, pp. 549-560.

Dixon, S.L., 1978, "*Fluid Mechanics, Thermodynamics of Turbamachinery*," Third Edition, Pergamon Press, pp 174-180.

Harms, T.M., von Backström, T.W. and du Plessis, J.P., 1996, "Simplified Control-Volume Finite-Element Method," *Numerical Heat Transfer*, Part B, 30: pp 179-194.

Hawthorne, W.R. and Ringrose, J., 1963-4, "Actuator Disc Theory of the Compressible Flow in Free-Vortex Turbo-Machinery," *Proc. Inst. Mech. Eng.*, Vol 178, Pt 3I(ü), pp 1-13.

Heyns, P.S., 1982, "*An investigotion of a Turbine for a Low-Cost Turbojet Engine*," M.Eng Thesis, University of Pretoria, Pretoria, South Africa.

Hirsch, C. and Warzee, G., Sept. 1976, "A finite element method for through flow calculations in turbomachines," ASME Journal of Fluids Engineering, pp403-421.

Horlock, J.H., 1978, "*Actuator Disk Theory Discontinuities in Thermo-Fluid Dynamics*," McGraw-Hill, U.K., pp 143-148.

Howell, A.R., 1942, 'The Present Basis of Axial Flow Compressor Design: Part I - Cascade Theory and Performance', *Aeronautical Research Council Reports and Memoranda No. 2095.*

Howell, A.R., 1945, "Design of Axial Compressors", *Proceedings of the Institution of Mechanical Engineers*, London, Vol 153.

Howell, A.R., 1945, 'Fluid Dynamics of Axial Compressors', *Proceedings of the Institution of Mechanical Engineers*, London, Vol 153.

König, W.M., Hennecke, D.K., and Fottner, L., 1996, "Improved Blade Profile Loss and Deviation Angle Model for Advanced Transonic Compressor Bladings: Part I - A Model for Subsonic Flow. Part II - A Model for Supersonic Flow," ASME Journal of Turbomachinery, Vol 118, pp 73-87.

Novak, R.A., October 1967, "Streamline Curvature Computing Procedures for Fluid-Flow Problems," *Transactions of the ASME, Journal of Engineering for Power*, Vol 89 No 4, pp. 478-490.

Oates, G.C., Knight, C.J. and Carey, G.F., January 1976, "A Variational Formulatioo of the Compressible Throughflow Problem," *Transoctions of the ASME, Journal af Engineering for Power*, pp. 1-8

Oates, G.C., 1988, "*Aerothermodynamics of Gas Turbine and Rocket Propulsion*," Revised and Enlarged, 2nd Edition, AAIA Educatioo Series, New York.

Sanger, N.L., 1996, "Design of a Low Aspect Transonic Compressor Stage Using CFD Techniques," *ASME Journal of Turbomachinery*, Vol. 118, pp. 479-491.

Smith, L.H. JR., 1966, "The Radial-Equilibrium Equation of Turbomachinery," *Transactions of the ASME Journal Engineering for Power*, pp. 1-12.

Von Backström, T. W. and Roos, T.H., September 1993, "The Streamline Throughflow Method for Axial Turbomachinery Flow Analysis," *Presented at ISABE XI*, Tokyo, Japan.