Secondary Flow Mixing Losses in a Centrifugal Impeller

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Detailed flow measurements made in a 1-m dia shrouded centrifugal impeller running at 500 rpm are presented. All 3 mutually perpendicular components of relative velocity and rotary stagnation pressures were measured on 5 cross-sectional planes between the inlet and the outlet, using probes which were traversed within the rotating impeller passage. The reduced static pressures were also calculated from these flow measurements. The measurements were made for an impeller flow rate corresponding to approximately zero incidence at the blade leading edges. Shroud boundary layer separation and secondary flow were observed to lead to the formation of a wake in the suction-side/shroud corner region. It is concluded that the turbulent mixing associated with the shroud boundary layer separation and the strength of the secondary flow strongly influence the size and location of the wake respectively.

NOMENCLATURE

\( p \) 
static pressure

\( P_r \)
reduced static pressure, \( p = p - \frac{1}{2} \rho \omega^2 r^2 \)

\( p^* \)
rotary stagnation pressure, \( p^* = p + \frac{1}{2} \rho \omega^2 r^2 \)

\( P_r \)
dimensionless reduced static pressure, \( P_r = \frac{p}{P_r} \)

\( P_r \)
dimensionless rotary stagnation pressure, \( P_r = \frac{p^*}{P_r} \)

\( r, \theta \)
cylindrical polar co-ordinates

\( R_n \)
radius of curvature of streamline

\( s, n \)
streamwise, normal and binormal directions - streamline co-ordinates

\( W \)
relative velocity

\( x/x_o \)
meridional inlet to outlet co-ordinate, measured along the shroud

\( y/y_o \)
pressure-side to suction-side co-ordinate

\( z/z_o \)
hub to shroud co-ordinate

\( \omega \)
angular velocity

\( \Omega_s \)
component of absolute vorticity along streamline

\( \rho \)
fluid density

SUBSCRIPTS

\( \min \) 
minimum value

\( \max \) 
maximum value

INTRODUCTION

Many research workers, for example (1,2,3), have studied the outlet flows from centrifugal impellers and most have observed a 'jet-wake' pattern. Eckardt (2) was able to show that potential flow theory was capable of describing the jet flow, which he measured in some detail. However, although impeller geometry, flow rate and operating speed are known to influence the size and position of the wake, wake flows cannot be predicted reliably by current design methods. An improvement in our understanding of the wake flow could therefore help in optimising the performance of centrifugal compressors.

In order to predict the development of the wake it is necessary to investigate in detail the secondary flows which contribute to the thickening of the boundary layers on the suction surface and shroud walls.

Objective

The principal objective of this work was to determine the flow phenomena which influence the development of the wake and its size and location in the impeller discharge flow. To this end, the authors measured all three mutually perpendicular velocity components and the rotary stagnation pressure \( p^* = p + \frac{1}{2} \rho \omega^2 r^2 \) on five cross sectional planes between the inlet and outlet of the impeller.

Secondary Flow

Let us now consider how secondary flow develops in centrifugal impellers.

The generation of streamwise vorticity along a streamline in inviscid, incompressible flow in a rotor is governed by an equation derived by Smith (5), but more recently presented by Hawthorne (6) in the following simple form:

\[
\frac{\partial \Omega_s}{\partial t} = \frac{2}{\rho W^2} \left[ \frac{1}{R_n} \frac{\partial p^*}{\partial \theta} + \omega \frac{\partial p^*}{\partial z} \right]
\]  

where \( p^* = p + \frac{1}{2} \rho (W^2 - \omega^2 r^2) \) is the rotary stagnation pressure, \( W \) is the fluid velocity relative to the rotor.

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Rotary Stagnation Pressure $p^*$

The rotary stagnation pressure $p^*$ is conserved along a streamline in steady, inviscid, incompressible flow in a rotor. Viscosity reduces $p^*$, but for small distances along streamlines one may identify a fluid particle by its value of $p^*$. This is particularly useful as it offers a method of following the development of the flow by tracking fluid particles. The migration and accumulation of low $p^*$ fluid which will be found in the boundary layers and the wake, can thus be observed directly. The large gradients of $p^*$ which will occur in the boundary layers will give rise to secondary flows as indicated by equation (1).

Stable Locations for Low $p^*$ Fluid

Low $p^*$ fluid will migrate under the influence of secondary flows towards regions of low reduced static pressure, which Johnson (7) has termed 'stable locations'. However, the low $p^*$ fluid will not necessarily accumulate at these stable locations because, as discussed by Johnson, the inertia of the secondary flow may carry this fluid beyond the stable location. Nonetheless, the stable location for each cross section of the impeller passage, is the approximate position where the wake flow should develop.

Secondary Flow in Centrifugal Impellers

The Ghost impeller and Eckardt's impeller have similar geometries (8) and both have two bends, the inducer bend and the axial-to-radial bend. In the inducer bend, secondary flow develops which convects fluid with low $p^*$ from the shroud and hub surfaces towards the suction surface. Secondary flow in the axial-to-radial bend moves low $p^*$ fluid towards the shroud from the suction and pressure surface boundary layers.

Rotation also induces a streamwise component of vorticity whenever a gradient of $p^*$ exists in the axial direction. Such gradients occur on the blade surfaces in the inducer of the centrifugal impeller and also on the shroud and hub surfaces once the flow has a radial component of velocity in the lateral part of the passage. Anand and Lakshminarayana (9) have investigated inducer flow and observed the convection of low momentum boundary layer fluid towards the shroud on both the suction and pressure surfaces of their axial inducer blades. The migration of low $p^*$ fluid towards the suction surface in the radial part of an impeller passage was studied by Moore (10), who used a rotating radial flow channel.

It is interesting to note that in the axial-to-radial bend, the rotation and the curvature do not produce similar 3-dimensional flows, but compete for the dominating influence on the stable location of low $p^*$ fluid. This competition must contribute to the observed differences in wake location in the discharge flow from different centrifugal impellers. From equation (1) it can be seen that the Rossby Number \( \frac{n}{\omega_{n}} \), is a measure of the relative influence of curvature and rotation. For impellers operating with a low Rossby Number for the axial-to-radial bend, rotation effects will dominate and the wake fluid will most likely be found on the suction surface. However, if the Rossby Number is high, the wake fluid may be expected on the shroud wall at the impeller exit as the bend's curvature will dominate the flow.

The axial-to-radial bend of both the Ghost and Eckardt's impeller have Rossby Numbers close to unity and so in these impellers, curvature and rotation are likely to be of approximately equal importance and so the wake might be expected close to the suction-side/shroud corner region, as was indeed observed by Eckardt.

Turbulent Mixing

Just as rotation and curvature can influence large scale secondary flow in centrifugal impellers through processes described by equation (1), so small scale fluid motions can be modified also. Turbulence modifications by curvature and rotation has been studied in detail by Johnston (11), but here we simply note a rule for determining whether turbulent mixing is suppressed or enhanced. If, in a boundary layer or shear layer, the cross-stream gradient of rotary stagnation pressure $p^*$ is in the same direction as the gradient of reduced static pressure $p_c$, then turbulent mixing will be suppressed. However, if the gradients are in opposite directions, turbulent mixing will be enhanced. In the centrifugal compressor impeller this will mean that, in general, mixing will be suppressed in the boundary layer close to the stable location for low $p^*$ fluid.

THE TEST RIG

The centrifugal impeller rig, as described in Johnson and Moore (12) and Johnson (13), is shown in a schematic diagram (see Fig. 1). Measurements were made on five cross-sectional planes as shown in Fig. 2, using probes which rotated with the impeller.

![Fig. 1 Schematic of test rig](https://example.com/fig1.png)

(1) Impeller rotating at 500 r.p.m.
(2) Pulley driven by motor
(3) Screens (30 mesh, 27 gauge)
(4) Honeycomb
(5) Inlet duct
(6) Pressure transducers and amplifiers
(7) Slip rings
(8) Seal
(9) Shroud attached to impeller
(10) Boundary layer trips

The three mutually perpendicular components of relative velocity were measured using a 5-hole pressure probe. The probe was calibrated for flow direction and velocity and the estimated accuracies...
were ± 1 m/s and ± 5 degrees for a typical velocity reading of 10 m/s.

\[ P_r = \frac{p_r - p^*_{\text{min}}}{p^*_\text{max} - p^*_{\text{min}}} \]  \hspace{1cm} (2)

and

\[ p^* = \frac{p^* \text{min} - p^*_{\text{min}}}{p^*_\text{max} - p^*_{\text{min}}} \]  \hspace{1cm} (3)

A value of one for \( P^* \) occurs in the inviscid potential flow and \( P_r = 1 \) is associated with stagnation of the potential flow, at the blade leading edges for example.

**Presentation of the Relative Velocity Results**

The passage direction is defined as the direction in which the ratio of the distances from each pair of walls remains constant. The relative velocity is then resolved into two parts; the component in the passage direction and the remaining component perpendicular to this, that is, in the cross-passage direction. The velocity in the passage direction is represented by contours in the results, whereas the cross-passage component is represented by an arrow at each measurement point. These arrows therefore indicate the combined effects of diffusion and secondary flow.

**DISCUSSION**

**Station 1**

The flow angles at Station 1 (approximately 6 mm downstream of the blade leading edge) show that the incidence is relatively small (see Fig. 3). The blade leading edges present a blockage for the inlet flow and this varies from 46% at the hub to 12% at the tip. Thus there is an elliptical flow, in the blade-to-
blade plane, around the blade leading edge and the incidence angles, relative to the blade camber line, are therefore positive on the suction side and predominantly negative on the pressure side.

The velocity contours in Fig. 5 represent contours of relative velocity at Station 1. These show that the fluid near the shroud has developed a higher axial velocity than the fluid near the hub. This is probably an upstream effect of the axial-to-radial bend. The velocity vectors show fluid in the lower half of the passage moving in the hub-to-shroud direction to make up the increased flow.

The reduced static pressure (Fig. 6) shows a gradient of $p_r$ in the hub-to-shroud direction due to the centrifugal acceleration. The pressure is also slightly higher on the pressure side than on the suction side, so the blades are only lightly loaded.

The minimum $p_r$ occurs in the shroud/suction-side corner region ($p_r < 0.05$ at $y/y_0 = 1.0$, $z/z_0 = 1.0$). Within this region at the wall, fluid with the lowest $p^*$ ($p^* < 0.05$) at Station 1 will occur. This does not appear on the $p^*$ diagram (Fig. 4) however, as the Kiel probe could not be brought close enough to the wall to measure $p^*$ within the thin boundary layer. Figure 4 also shows little or no losses have occurred and the flow is therefore essentially a potential flow.

**Fig. 5** Station 1. Relative velocities - contours in m/s

**Fig. 6** Station 1. Dimensionless reduced static pressure $p_r$.

Close to the shroud there is a large adverse pressure gradient of reduced static pressure, particularly between Stations 2 and 3, as shown by the minimum values of $p_r$ (0.05 and 0.3, respectively) in the two reduced static pressure diagrams.
(Figs. 9 and 12). This pressure rise occurs as the fluid reduces its tangential velocity and hence its relative velocity as it turns through the inducer bend.

Due to the adverse pressure gradient, low velocity fluid in the boundary layers close to the suction-side/shroud corner region at Station 2 will possess insufficient rotary stagnation pressure ($P^* < 0.3$) to reach Station 3. Consequently, this fluid must mix with higher $P^*$ fluid in order that it may increase its momentum and proceed to Station 3. However, turbulent mixing will be suppressed by the pressure gradients induced by the curvatures in the inducer and axial-to-radial bends. The measured pressures in this region at Stations 2 and 3 were observed to be very unsteady and it therefore seems probable that the mixing, which must occur in this region, was associated with some unsteady flow separation. This may also be associated with periodic reversed flow.

On the hub surface, the boundary layer experiences little or no pressure rise in the streamwise direction. In addition, secondary flows cause boundary layer fluid to move from the hub wall onto the suction and pressure surfaces as shown in Figs. 8 and 11. The region of low $P^*$ fluid observed on the suction side wall in Fig. 7 is due to the thickening of the boundary layer caused by these secondary flows. Figure 8 shows that this low $P^*$ fluid is moving up the suction side wall to a point at approximately $z/z_o = 0.65$.

The stable location for this low $P^*$ fluid is close to the suction-side/shroud corner, where the reduced static pressure is lowest. However, as $P_r$ is increasing in the streamwise direction in this corner region, the flow here is decelerating and diverging. Hence cross velocities are induced, away from the corner, which oppose the secondary flow up the suction surface and so prevent the low $P^*$ fluid from reaching the corner region.

![Fig. 7 Station 2. Dimensionless reduced static pressure $P^*$](image)

![Fig. 8 Station 2. Relative velocities - contours in m/s](image)

![Fig. 9 Station 2. Dimensionless reduced static pressure $P_r$](image)

It is interesting that the large change in the distribution of $P^*$ between Stations 1 and 2 is not
reflected in a similar change in the velocity distributions as shown in Fig. 8. This is because there has been no significant rise in $P_r$ in the suction-side/shroud corner region between Stations 1 and 2 (see Figs. 6 and 9) as the blades have loaded.

The blade-to-blade pressure gradient for the inducer bend can be approximately modelled by the streamline curvature equation

$$\frac{\Delta p}{\Delta r} = \frac{\alpha W^2}{R_n}.$$  \hfill (4)

Taking $R_n$ for the inducer bend at Station 2 as 200 mm and $W = 15$ m/s, the calculated value for the blade-to-blade pressure gradient at about mid-height is $\Delta P_r = 0.25$, which is of the same magnitude as the measured value. This pressure difference is larger near the shroud, where the relative velocities are higher ($W = 17$ m/s) and the blade-to-blade distance is larger. The calculated value is $\Delta P_r = 0.48$, which again compares well with the measurements.

Station 3

Station 3 is approximately halfway along the passage and the flow has turned through $58^\circ$ of the axial-to-radial bend.

The reduced static pressure gradient required to turn the flow around the axial-to-radial bend can again be calculated from the streamline curvature equation (4). For the axial-to-radial bend, $R_n = 200$ mm and $W = 12$ m/s, which gives a calculated hub-to-shroud pressure difference of $\Delta P_r = 0.34$, close to the measured value in most of the potential flow region (see Fig. 12).

The velocity diagram (Fig. 11) shows that the axial-to-radial bend is dominating the secondary flow pattern. The passage is occupied by two opposing vortices; an anticlockwise vortex on the suction side and a second, weaker, clockwise vortex on the pressure side of the passage near the hub. This is the pattern which is characteristic of a stationary axial-to-radial bend, but here there is evidence of a secondary flow along the shroud towards the suction side corner, which is generated by Coriolis forces due to the passage rotation. This flow along the shroud, together with the flow up the suction side, has convected the low $\rho^*$ fluid on these two walls at Station 2 into the shroud/suction side corner region, as shown in Fig. 10.

The flow now has a jet-wake flow pattern. In the jet, where $\rho^*$ is approximately unity (see Fig. 10), the potential flow has changed very little between Station 2 and 3, as can be seen from the velocity and pressure diagrams (Figs. 8, 9, 11 and 12). But a wake has formed as the low $\rho^*$ fluid has accumulated and slowed down. This gives the first indication in the velocity diagram of a large-scale departure from a potential flow and as such, it corresponds remarkably closely with Eckardt's first observation of the wake, in his impeller, at his operating condition and his Station 3. In this study, however, the thickening of the suction side and shroud wall boundary layers has already been observed in the $\rho^*$ diagram at Station 2 (Fig. 7) and the subsequent migration of this low $\rho^*$ fluid between Stations 2 and 3.

Station 4

At Station 4, the passage is almost radial. The reduced static pressure distribution is therefore dominated by the tangential, blade-to-blade gradient of pressure due to the Coriolis acceleration. This is seen in Fig. 15, where the static pressure falls almost linearly from the pressure side to the suction side near the hub. Near the shroud, however, the pattern is broken by a uniform pressure region in the wake.

This pressure gradient can be modelled fairly simply as due to a radial flow with a mean velocity $W = 12$ m/s. The tangential pressure gradient is then given by the simple expression:

$$\frac{1}{r} \frac{\Delta p}{\Delta \theta} = 2\rho^* W.$$  \hfill (5)

In the impeller at Station 4, the tangential distance between the suction and pressure wall is about 0.11 m. Hence we obtain a simple estimate for the pressure difference between the blade surfaces of 0.51. This agrees well with the measured static pressure difference of about 0.50.
The stable location for low $p^*$ fluid in the radial section of the passage is the suction side, which is also the position of the lowest reduced static pressure (see Fig. 13), as would be predicted for a straight rotating channel. However, although the axial-to-radial bend has a small influence on the $p_r$ distribution, its influence remains in the inertia of the twin secondary flow vortices observed at Station 3 (see Fig. 14). The flow up the suction and pressure surfaces to the shroud has weakened considerably, but low $p^*$ fluid is still being convected up the suction surface to the shroud. As illustrated by the velocities (Fig. 14) and the $p^*$ contours (Fig. 13), the wake has moved further along the shroud, its core now being at $y/y_0 = 0.7$, even though the stable location for low $p^*$ fluid has moved the other way! This movement must be attributed to the inertia of the anticlockwise vortex on the suction side of the passage. The second vortex, the clockwise one on the pressure side of the passage, has been greatly augmented along the shroud by the action of Coriolis forces. The low $p^*$ fluid from the secondary flows up the suction side and along the shroud collide on the shroud at $y/y_0 = 0.7$ and this causes low energy fluid to move away from the wall.

In the strong shear layer between the wake and the potential flow region at Station 4 turbulent mixing is probably suppressed by the action of curvature and Coriolis forces.

Station 5

Station 5 is close to the impeller exist and the passage is still radial as shown in Fig. 2.

The $p^*$ contours (Fig. 16) show that the wake area had increased considerably between Stations 4 and 5. The minimum value of $p^*$ at Station 5 is higher than that obtained at Station 4 ($p^* = 0.51$ compared with $p^* = 0.46$). The only way the rotary stagnation pressure of a fluid particle can increase along a streamline is by mixing with neighbouring fluid with a higher value of $p^*$. Therefore, the lowest $p^*$ fluid at Station 4 must have mixed with higher $p^*$ fluid.

Mixing was suppressed at Station 4 by the strength of the Coriolis forces but, as can be seen from the $p_r$ diagram (Fig. 18), these forces have weakened as the static pressure gradient has reduced, due to the blades unloading close to the impeller discharge. The reduced static pressure on the suction side has risen while that on the pressure side has dropped only slightly. This has been achieved in the
wake region by mixing, as no further diffusion was possible with the very low momentum of the wake fluid. However, fluid in the suction side hub quarter of the passage has diffused considerably, decelerating from 18 to 13 m/s, while the fluid in the pressure-side/shroud corner region has accelerated to take the excess flow from the suction side.

The mass averaged $P^*$ (Fig. 19) shows that the main loss occurs between Stations 1 and 2 and this must be attributed to the boundary layer separation and associated mixing which takes place in this region. Figure 19 also shows that the diffusion of the flow through the impeller achieves a fairly steady rise in the mass-averaged $P^*_r$.

The secondary velocities (Fig. 17) show the divergence of the highest velocity fluid and this fluid is the most likely source of the high $p^*$ fluid required to mix out with the wake fluid. The cross velocities also show that the flow up the suction side is carrying this high $p^*$ fluid into the wake region and a flow from the centre of the high velocity zone at $y/y_0 = 0.8$, $z/z_0 = 0.25$ is carrying similar fluid to the other side of the wake ($y/y_0 = 0.4$, $z/z_0 = 0.9$). The general trend in the cross velocities from the suction to the pressure side of the passage is due to the slip of the flow near the outlet. This is most marked on the pressure side of the flow near the outlet. There is some evidence of the secondary flow along the shroud, due to the Coriolis forces, which has not been entirely effaced by the slip.

The wake has moved closer to the suction-side/shroud corner, so the secondary flow along the shroud must have been stronger than the flow up the suction surface between Stations 4 and 5. However, the wake is still some distance from its stable location on the suction side of the passage.

The dimensionless reduced static pressure $P_r$ corresponds to the pressure coefficient of potential flow which has diffused from the maximum relative velocity in the inducer. This pressure coefficient is used by many designers as a simple criterion for two-dimensional boundary layer growth in a diffusing impeller flow. Clearly, the boundary layer flow in this study is not two-dimensional but nevertheless, it is interesting that the value of 0.5 for the wake is representative of the pressure coefficient necessary to cause separation of a two-dimensional turbulent boundary layer.

Mass Averaged Pressures

The mass averaged $P^*$ (Fig. 19) shows that the main loss occurs between Stations 1 and 2 and this must be attributed to the boundary layer separation and associated mixing which takes place in this region. Figure 19 also shows that the diffusion of the flow through the impeller achieves a fairly steady rise in the mass-averaged $P^*_r$.

CONCLUSIONS

1. The wake flow observed in the present study is an accumulation of fluid with a low rotary stagnation pressure $p^*$, which forms in the suction-side/shroud corner region of the impeller passage. The major flow phenomena which contribute to the formation and development of the wake are:
(a) The adverse gradient of reduced static pressure between Stations 2 and 3 near the shroud/suction-side corner, which results in a substantial increase in the quantity of low $p^*$ fluid in this region at Station 2, and the convection of low $p^*$ fluid by the secondary flows, which are generated in the boundary layers due to the curvature and rotation of the impeller passage.

(b) The adverse gradient of reduced static pressure between Stations 2 and 3 causes the separation of the shroud boundary layer and also substantial turbulent mixing, which accounts for approximately two-thirds of the total loss in the impeller. The magnitude of this separation will therefore, to a large extent, determine the size of the wake in the discharge flow. The authors consider the separation to be an unsteady phenomenon which may exhibit periodic reversed flow.

The results show the build up of low $p^*$ fluid upstream of the point where the first large-scale departures from potential flow are observed in the contours of relative velocity. Low $p^*$ fluid from the boundary layer is observed to migrate to the shroud/suction-side corner region, where the wake is formed. It is therefore concluded that secondary flow contributes to the formation of the wake.

The axial-to-radial bend dominates the secondary flow pattern through most of the impeller. Two opposing passage vortices are induced, one on the suction side and one on the pressure side of the passage. On the suction side, low $p^*$ fluid is carried by the vortex along the wall to the wake region. The passage vortex on the pressure side creates a secondary flow along the shroud towards the suction surface. This flow is enhanced by the action of Coriolis forces due to the passage rotation. The low $p^*$ fluid in these two flows collides in the wake region and moves away from the shroud wall.

The inertia of the suction side passage vortex, induced in the axial to radial bend, prevents the wake from moving from the shroud to its stable location on the suction surface at the outlet. The authors therefore conclude that the secondary flows strongly influence the position of the wake in the outlet plane of a centrifugal impeller.

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