Effects of the Tip Clearance Flow Field on the Secondary Losses

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ABSTRACT
In this paper an extension of our secondary flow calculation method is presented in order to estimate the influence of the tip clearance on the secondary flow field.

The impact of the tip clearance vortex is embodied in the method so that the secondary vorticity field, based on the complete form of the meridional vorticity transport equation, is properly modified. In this way the changes of the secondary flow field quantities are predicted along with the resulting additional losses imposed by the existence of the tip clearance.

The estimated tip clearance losses, based on the flow field structure, are compared to results obtained from various semi-empirical loss correlations found in the literature.

Several cases encountered in axial flow compressor configurations are investigated and the calculated results are favourably compared to experimental data and results of previous calculations.

Additionally, in order to clarify the influence of the tip clearance structure upon the secondary flow field, cases with and without tip clearance are also examined.

NOMENCLATURE
b blockage factor
B blade chord
Cp dissipation factor
Cl lift coefficient
Dl deficit force vector
et internal energy
ft total viscous force per unit mass
ghr gravity acceleration
ht blade passage height
ht total enthalpy
ks curvature of m-lines
Km curvature of m-lines
L length of blade's camber line
Mr relative Mach number
Pt pressure
Q specific heat flux
Q2 turbulent kinetic energy
Re Reynolds number based on the blade chord
RF reaction factor
s time; also blade thickness at the tip
f friction velocity
(u,v,w) components of velocity fluctuation with time
w absolute velocity vector
\( \Omega \) relative velocity vector
Z dist vertical distance from reference level
\( \beta \) angle between meridional -m and longitudinal direction -s
\( \beta ' \) blade angle
\( \gamma \) ratio of specific heats
(\( \theta , \phi , \rho \)) coordinates of an orthogonal curvilinear axisymmetric coordinate system
\( \kappa \) peripheral striction parameter
\( \mu \) meridional striction parameter
\( \nu \) total viscosity coefficient
\( \nu \) kinematic viscosity coefficient
\( \Pi \) vorticity vector
\( \sigma \) density
\( \sigma \) blade solidity
\( \tau \) clearance height
(\( \phi \)) components of viscous stresses
\( \phi \) flow coefficient
(\( \psi \)) stage loading
(\( \psi \)) angular velocity vector
\( \Delta \) tip loss coefficient
\( n_\Omega \) angular velocity vector

\[ \Delta \sigma_n = \int B' \left( \overline{\rho} \overline{w_m} - \overline{p} \overline{n_m} \right) \cdot dn \]

\[ \Theta_{\Omega n} = \int B' \overline{p} \overline{w_m} - \overline{w_m} \cdot dn \]

\[ \Theta_{\Omega n} = \int B' \overline{\rho} \overline{w_m} - \overline{n_m} \cdot dn \]

Subscripts
f external flow
e reference quantity
o,oo pressure, suction wall
(\( X, Y, N \)) components in intrinsic coordinate system
(\( \tau, \psi, \theta \)) (longitudinal, traverse, normal)
(\( X, Y, Z \)) components in curvilinear axisymmetric orthogonal coordinate system
(\( \psi, \theta, \phi \)) value at the wall
(\( \psi, \theta \)) value at the blade tip and hub

Superscripts
\( - \) mean peripheral value
\( \{ \} \) initial external flow field
\( = \) time average value
1. INTRODUCTION

The meridional flow model based on circumferentially averaged equations has been proven to be an appropriate approach of treating complex flow fields in single or multi-stage compressors when strong shock waves from axisymmetry appear or in the presence of strong shock waves.

During the last years, a well elaborated and reliable secondary flow model has been developed starting from basic principles. The method has been successfully tested for cases of highly loaded compressor cascades (Kaldellis et al., 1986a; 1986b) and high speed axial (Kaldellis, 1986b) and radial compressors (Dovikas et al., 1989).

Besides, the method has been used to analyze the flowfield through supersonic axial compressors when shock-Induced secondary flow interaction occurs (Kaldellis et al., 1989). Recently, three axial turbine cases, covering a wide range of loading (rt to rr) from flow acceleration to impingement, were investigated with excellent results (Kaldellis and R etenidis, 1990b). The realistic estimation of the lost and added work, through various single-stage compressors (Kaldellis et al. 1990a) was an additional requirement before extending the method in multi-stage environments.

Up to now the existence of the gap between the tip (or hub) of the blades and the corresponding endwall was either ignored (assumed negligible) or taken into account in a semi-empirical correlation. The present study is an attempt to carry out this research step through a tip clearance calculation method. It is evident, that for the majority of the test cases analyzed the tip clearance gap is small. Thus, as mentioned also by others (Bah. 1986; Steur and Wu, 1986) the tip leakage flow mainly disturbs locally the flow field, especially for axial compressor cases. However, even these limited flow disturbances in the tip leakage flow is an intrinsic and unfavourable way the efficiency (accounting for the loss of at least 2 points in efficiency) and the operational stability of an axial compressor.

In order to improve the quality of our computational results near the tip (or hub) regions, and to cancel incompatibilities related to the insufficient simulation of the tip leakage flow, we proceed to incorporate in our secondary flow code the effect of the tip clearance. A closed formulation based on recent information from various experimental (e.g. Booth et al., 1982, Chen et al., 1990, Moyle, 1988, Peacock, 1989, Pouaid and Delaney, 1986, Senoo and Ishida, 1986, Wadia, 1985) and numerical studies (e.g. Booth et al., 1982, Chen et al., 1990, Moyle, 1988, Peacock, 1989, Pouaid and Delaney, 1986, Senoo and Ishida, 1986, Wadia, 1985) devised. The resulting model, though approximate, keeps the order of magnitude which is vital for any secondary flow method, while special attention is paid to be consistent with the physics of the real flow field, i.e. the tips. This includes, mainly, the tip leakage vortex and its radial location from the tip vortex along with its radial location from the tip endwall.

In the early work of Conte et al. (1983) it is assumed that the existence of the tip clearance does not modify at all the velocity profile in the longitudinal direction. However, this approach leads to poor representative of the experimental data, especially near the wall. Using the information given by various experiments investigated we decided to describe the longitudinal velocity profiles by adopting a two-structure approach. This approach assumes that near the tip clearance region the flow field is dominated by the shear stresses imposed by the solid surfaces. Thus, a parabolic distribution is suitable enough to take into account the longitudinal mass flow. Therefore, the conservation of the mass flow contained in the tip clearance ring (rt to rr) from the leading to the trailing edge of the blade row, in the meridional direction.

Additionally, it is the secondary flow field which dominates the endwall shear layer behaviour for the region outside the tip vortex core. In the intermediate region the assumption of constant longitudinal velocity is not only simplifies the necessary boundary conditions, but also is in accordance with the assumption of constant velocity in the tip vortex region and verifies the available experimental data. Therefore we may write that:

\[
\vec{W}_{2} = a_{1} x^{2} + a_{2} x + a_{3} \quad \text{(1)}
\]

\[
\vec{W}_{3} = a_{4} x_{l}^{2} + a_{5} x_{l} + a_{6} \quad \text{(2)}
\]

\[
\vec{W}_{4} = a_{7} \delta x_{l} \quad \text{(3)}
\]

\[
\vec{W}_{5} = a_{8} x_{l} \quad \text{(4)}
\]

The necessary boundary conditions are provided by the non-slip condition at the endwall, the conservation of the mass flow throughout the tip clearance ring and the corresponding matching conditions at the joint points.

On the other hand, if the meridional vorticity distribution is considered, the vorticity related to the tip vortex and the one concentrated inside the gap have to be taken into account. The last component is related to the distribution of the peripheral velocity, induced by
the static pressure difference between the pressure and the suction side of the blades, and the relative motion of the endwall and the blade row. Ohayon (1979) has assumed that the peripheral velocity is constant inside the tip clearance, equal to its corresponding value at the tip section of the blade. However, the results obtained by using this approach overestimate by more than 20% the leakage flow rate, making necessary: the use of empirical coefficients to adjust the predicted value to the experimental one (Storer and Cumpsty, 1990). Additionally, such an approach does not take into account the relative motion of the endwall and also imposes zero vorticity value inside the tip clearance gap, which is unrealistic. On the contrary, the experimental data and the calculated results show that the total meridional vorticity distribution is semi-bounded and fully-bounded ones. Partially bounded regions (see Fig. [2]) are blade passages (S3 surfaces) which are the relative Mach numbers on the suction and on the pressure side of the blade. This fact can be accounted for by the work of Lakshminarayana and Horlock (1965) and by Lakshminarayana (1970), the tip clearance velocity in the peripheral direction occurs mainly due to the pressure difference between the two sides of the blade near the tip. Making use of figure [1] we can predict the value by considering the following equation:

\[ W_u = W_{suc} \frac{1}{B} \int_0^a \xi_s \mathrm{d}n \]  

where \( \xi_s \) is the algebraic sum of the secondary vorticity and the tip clearance one.

As described in Appendix One (see also Kaldellis et al. (1988a)), the secondary vorticity transport equation, \( \xi = \xi_0 \), concerning the boundary conditions of the secondary equations, \( \xi = \xi_0 \), takes into account the relative motion of the blade endwall. The total meridional vorticity value at the endwall and the leakage velocity at the blade tip endwall motion. The total meridional vorticity at the endwall is related, as described in details by Kaldellis et al. (1988a), to the peripheral blockage value, which is also linear. The peripheral velocity component is partial flow cases in addition to unbounded, semi-bounded and fully-bounded ones. Partially bounded regions (see Fig. [2]) are blade passages (S3 surfaces) where tip clearance exists. For the calculation of the "K" value we extend the "zero transverse" mass flow rate equation of the fully bounded cases to take into account the tip leakage mass injection, as described in details by Kaldellis (1988b).

In order to compute the complete meridional vorticity field, we assume also (see Lakshminarayana, 1970, Comte et al., 1983) that the tip induced vorticity is concentrated in a constant vorticity core. The tip vortex (with radius \( r_v \)) has its centre located at a distance \( n_f \) from the endwall and by from the blade suction surface. Only the radial location of the tip vortex is needed for the calculation of the circumferentially averaged flow field. With respect to the above mentioned remarks we derive that:

\[ \xi_0 = k^* \xi_v \]  

where \( \xi_v \) is a correction factor related to the maximum distance of the centre of the vortex from the endwall, while for the suction side the pressure at the tip section is considered. The conclusion is that the parameter \( k^* \) is strongly related to the tip airfoil section discharge coefficient and taken into account the fraction of blade lift, retained due to the tip airfoil section unloading, induced by the tip leakage flow. The correction parameter \( k^* \) expresses the experimental indication that the circulation of the tip vortex is less than the circulation of the tip section of the blade. The value of \( k^* \) depends on various factors (e.g. relative tip gap, relative wall speed, machine running conditions, blade loading etc.). A value of \( k^* \) equal to 0.8 has been suggested by many researchers (Lakshminarayana and Murthy, 1987, Storer and Cumpsty, 1990, Yaras and Sjolander, 1990).

Additionally, the location of the centre of the tip vortex is given by:

\[ r_v = r_i + \xi_v n_f \]  

where \( r_i \) is a correction factor related to the maximum distance of the centre of the vortex from the endwall, and \( r_i \) is located at a distance from the endwall. According to the analysis of Chen et al. (1990) the \( r_i \) value can be assumed constant. Therefore the main changes of the \( n_f \) value are due to the change of the \( n_f \) value, and due to the work of Lakshminarayana and Horlock (1965) we can assume that the mass flow traversing the tip clearance is included in the tip vortex core. This fact can be expressed as:

\[ n_f = \frac{m}{W_{suc}} \frac{dF}{d\theta} \]  

with

\[ m = \int_0^{\theta} \frac{dF}{d\theta} d\theta \]  

and

\[ \frac{dF}{d\theta} = \int_0^{\pi/2} W_{suc} \xi_s d\theta \]  

For simplicity the \( (\xi_0 W_{suc}) \) value is assumed constant inside the vortex core. It is also interesting to mention that since the complete vorticity is estimated, the spurious defect force distribution can be predicted using equation (1.10) with no additional assumptions. As described in details by Kaldellis et al. (1990a) the defect force distribution takes into account the modification of the blade loading due to the secondary and tip clearance effects, and is partially responsible for the redistribution of the work exchanged inside the rotor, called also as "added work".
Finally, various semi-empirical correlations were tested to estimate the total pressure loss related to the tip clearance existence. Only a brief presentation of the conclusions drawn from this analysis is given here.

The tip clearance loss depends on a large number of parameters, thus the total pressure drop, related to the tip clearance induced loss (see also Lakshminarayana and Holoric, 1983).

\[ \omega \cdot f(R/D, s, C, \gamma, \lambda, \rho, \Omega) \]  

(17)

Several researchers simplify the above functional dependence into various correlations formulae indicating the most important factors influencing the tip clearance loss. Some commonly used among them, related to the tip leakage flow, are the Mach number and the turning of the flow, and some flow coefficients, can be expressed in the following form:

\[ \omega \cdot f(t/h, \beta, \alpha, s, C_{h}/C_{l}) \cdot g(\Phi) \]  

(18)

\[ \omega \cdot f(R_{t}/D, \gamma, \beta, s, C_{h}/C_{l}) \cdot g(\Phi) \]  

(19)

\[ \omega \cdot f(R_{t}/D, \gamma, \beta, s, C_{h}/C_{l}) \cdot g(\Phi) \]  

(20)

\[ \omega \cdot f(R_{t}/D, \gamma, \beta, s, C_{h}/C_{l}) \cdot g(\Phi) \]  

(21)

\[ \omega \cdot f(R_{t}/D, \gamma, \beta, s, C_{h}/C_{l}) \cdot g(\Phi) \]  

(22)

\[ \omega \cdot f(t/h, \beta, \alpha, s, C_{h}/C_{l}) \cdot g(\Phi) \]  

(23)

\[ \omega \cdot f(t/h, \beta, \alpha, s, C_{h}/C_{l}) \cdot g(\Phi) \]  

(24)

\[ \omega \cdot f(t/h, \beta, \alpha, s, C_{h}/C_{l}) \cdot g(\Phi) \]  

(25)

Yaras & Sjolander (1990) (* taken from Schmidt et al., 1987)

Booth (1985) and Yaras and Sjolander (1990) calculate the complete total pressure loss as the sum of the gap, tip clearance and secondary flow loss, related either to the blade unloading due to the clearance or to the net modification of the secondary flow field.

Since the comparative study of the behaviour of the various tip loss correlations is beyond the scope of the present study, (see for example Booth, 1985, Schmidt et al., 1987, Moore, 1987, Peacock, 1989 and Yaras and Sjolander, 1990) we would just like to add that the results presented here are predicted using the RANS and VIVAX basic principles, properly adapted for axial compressor cases, according to the remarks of Yaras and Sjolander (1990) for axial turbine cases.

Finally, the spanwise averaged total pressure loss is equally distributed over the tip-influenced part of the blade span.

4. CALCULATION RESULTS AND DISCUSSION

The complete secondary flow calculation method, where the tip clearance effects are incorporated, is subsequently used to analyze several cases encountered in axial flow compressor configurations. Our main interest is focused on the effects of the structure of the tip clearance flow on the real flow field.

Highly Loaded Compressor Cascade (Plot Case C)

The first test case investigated concerns the experimental results given by Plot (1975) for a highly loaded NACA 65 (12A10)10 compressor cascade, with a turning of the flow direction from 55.2° to 24.6° (see fig. [3]) and with inlet Mach number equal to 0.14. The tip clearance gap existing at the one end of each blade is 1.05% of the blade chord. The blade chord is equal to 0.15m while the blade midspan height is equal to 0.157m.

The circumferentially averaged longitudinal and peripheral velocity profiles at various locations through the cascade passage are presented in comparison with the experimental data. The calculation results given by Comte et al. (1983), in order to verify the ability of the proposed tip clearance model to handle similar compressor cascade cases. In figures [4a], [4b] and [4c] the longitudinal velocity profiles at -22%, 22% and 66% of the blade chord of the cascade are successfully compared to the experimental data. The two-structure flow pattern is evident at 66% of the blade chord, since the tip vortex is already strong enough to disturb the secondary flow field. This phenomenon is more clear when the longitudinal profile near the outlet is examined (fig.[5a]).

The strongly related to the vorticity field peripheral component of the defect force (equ.(1.10)) is presented in figure [6]. The calculation results describe satisfactorily the experimental ones. Note that no additional assumptions have been made. The separation of the tip clearance, we compare the experimental and the calculated results of case C with the corresponding results of a similar test case (Plot Case C) with no tip clearance existing. The secondary flow field of this last case has been already analyzed by the authors (Kaldellis et al., 1988a). The aerodynamic loading of these two cases being almost the same, the occurring differences for the longitudinal velocity profile at 88% and 150% of the blade chord underline the net influence of the tip clearance upon the secondary flow field. According to the experimental data. Combining figures [5a], [5b] we can see also the extent of the tip clearance influenced region through the cascade passage.

Subsequently, the peripheral velocity components at -22%, 22% and 66% of the blade chord are shown in figures [6a], [6b] and [6c] respectively. As mentioned above, the existence of the tip vortex is clear after the midchord of the blade. The remarkable change of the peripheral velocity distribution is well described when the results at 22% and 66% of the blade chord are compared. Although the calculated results given by Comte et al. (1983) describe the trend of the experimental measurements, an overestimation of the experimental values of the peripheral velocity component at 66% by more than 50% is encountered. As it is well known for compressor cases, the tip vortex rotaties in the counter direction of the passage vortex. This results in an equilibrium between the secondary and the tip leakage flow. To make things clearer, we compare again the experimental and computational results of the peripheral velocity component for cases C and D at 88% and 150% of the blade chord in figures [7a] and [7b] respectively. As mentioned above, the existence of the tip vortex is clear after the midchord of the blade.

The circulation downstream of the cascade core, leads to an overestimation of the intensity of the tip vortex, which accordingly results in greater peripheral velocity values. This excessive mass flow rate is mainly due to the assumption of constant leakage velocity distribution (Comte et al., 1983) throughout the tip gap. Finally, the flow traversing the blade and subsequently included in the tip vortex core, leads to an overestimation of the intensity of the tip vortex, which accordingly results in greater peripheral velocity values. This excessive mass flow rate is mainly due to the assumption of constant leakage velocity distribution (Comte et al., 1983) throughout the tip gap. The flow mixing downstream of the cascade core, leads to an overestimation of the intensity of the tip vortex, which accordingly results in greater peripheral velocity values. This excessive mass flow rate is mainly due to the assumption of constant leakage velocity distribution (Comte et al., 1983) throughout the tip gap. Finally, the flow traversing the blade and subsequently included in the tip vortex core, leads to an overestimation of the intensity of the tip vortex, which accordingly results in greater peripheral velocity values. This excessive mass flow rate is mainly due to the assumption of constant leakage velocity distribution (Comte et al., 1983) throughout the tip gap. Finally, the flow traversing the blade and subsequently included in the tip vortex core, leads to an overestimation of the intensity of the tip vortex, which accordingly results in greater peripheral velocity values. This excessive mass flow rate is mainly due to the assumption of constant leakage velocity distribution (Comte et al., 1983) throughout the tip gap. Finally, the flow traversing the blade and subsequently included in the tip vortex core, leads to an overestimation of the intensity of the tip vortex, which accordingly results in greater peripheral velocity values. This excessive mass flow rate is mainly due to the assumption of constant leakage velocity distribution (Comte et al., 1983) throughout the tip gap.
clearance flow of a compressor cascade cannot be directly related to similar tip clearances existing in rotating machines, we proceed to investigate the flow field through an axial transonic rotor. In this way the effect of compressibility, relative motion between the casing and the blade tips are also examined. The present compressor case is analyzed for two tip gap values, a small one (\(r/c=0.411\%\)) and a large one (\(r/c=4.66\%\)). Only limited information about this machine is available. The rotor of the compressor was examined (Kaldellis, 1989) for the characteristics of the secondary flow field, tip clearance influence included. Note that both relative wall motion and compressibility effects were neglected in this early work.

The longitudinal and the peripheral velocity components in the relative to the blade coordinate system will be presented at the inlet and the outlet of the rotor for the two clearance values described above. The schematic representation of the machine is given in figure [9]. The compressor has a hub to tip ratio equal to 0.6 while the blade height is 0.31m and the mean blade chord is 0.14m. The rotation speed of the machine is approximately 3500rpm for the cases examined here, while the relative Mach number at the inlet of the rotor varies from 0.76 at the hub to 1.02 at the tip. Some more information concerning the machine under consideration and the experimental results of ECL1 is given by Kaldellis et al. (1988).

For both cases investigated, with small and large clearance, the inlet shear layer is quite thick ([10a], [11a]). As shown in figures [10a] and [11a] the two-structure flow pattern evident at the inlet is a result of the relative longitudinal velocity component at the rotor outlet is concerned. Especially for the large tip clearance case, the use of a modified longitudinal velocity profile for the whole tip shear layer leads to a more realistic representation of the flow field (Ohayon, 1979). Additional improvements in the method of flow compressibility is taken into account (Kaldellis, 1986b).

By neglecting the relative wall motion and assuming constant leakage velocity through the tip gap, unrealistic values of the peripheral velocity profile at the exit of the rotor are obtained and also with the corresponding transverse velocity profiles given by Ohayon (1979). On the contrary, if the proposed model is correctly adapted along the span of the rotor blade, taking into account the existence of the clearance and the effect of the relative motion of the casing and the blade tip, neglected in our previous calculation, the results obtained by approximating the relative motion of the tip with a uniform sheared flow overestimates the leakage flow and the tip vortex strength.

Another two cases will be examined here in order to clarify the improvement in the method of flow compressibility is taken into account (Kaldellis, 1986b). The corresponding results of the above mentioned calculation methods are also used to investigate the tip clearance flow field for the rotor of a transonic compressor. As in the previous transonic case examined, the results obtained by simulating the clearance effect will be compared for the IGV and the rotor blade rows of a modern axial transonic compressor (see also figure [12]). This compressor has been analyzed in previous studies as far as the secondary flow field quantities (Kaldellis et al., 1989) and the energy exchange process (Kaldellis et al., 1990) are concerned, but without taking into account the existence of either hub or tip clearances. The nominal rotation speed of the machine is \(n=1550\)rpm and the corresponding stage pressure ratio \(c=1.38\) (see also Leboeuf and Navier, 1983). The results given here are for the three rotor hub clearance cases investigated, with mass flow rate \(m=15.5\)kg/sec, rotation speed \(n=1550\)rpm and stage pressure ratio \(c=1.38\).

IGV of the Transonic Compressor ECL1

Figure 11a shows examples of averaged spanwise distributions of the longitudinal velocity, absolute flow angle and total pressure at the inlet ([15a], [15b] and [15c]) and the outlet ([16a], [16b] and [16c]) of the IGV of the IG1. We note that the hub clearance induced vortex is sufficiently strong to affect all the characteristics of the IGV flow field and the corresponding secondary flow field. The calculation results of the proposed improved model and the early results of the authors (Kaldellis et al., 1989) with the same hub clearance, are compared along with the experimental data. Note that the plots show only the hub region (30% of the blade span) in order to give a better picture of the hub clearance induced effects.

At the inlet of the IGV, the calculation results underestimate the longitudinal velocity profile near the hub, since the flow acceleration due to the curvature of the inlet shroud is not properly simulated (Kaldellis, 1989). A small reduction of the flow incidence angle near the hub is also predicted, while the inlet total pressure remains the same for both calculations. Although the calculation results at the inlet of the IGV are almost the same, this is not the case for the corresponding outlet results. Thus, a clear two-structure flow pattern is evident (figures [14a] and [14b]). The longitudinal velocity is almost constant for a distance up to 3% of the blade height from the hub, while the flow vortexing over the passage vortex is significantly reduced due to the influence of the counter rotating leakage vortex.

On the other hand the total pressure is only slightly modified and, in fact it increases. This means that the simple total pressure drop correlations, used to approximate the hub clearance total pressure drop, overestimate the corresponding losses. Summarizing, we would like to mention that even the limited improvement of the accuracy of the results obtained by the proposed model is encouraging, since the results already given by the authors (1989) are in very good agreement with the experimental data for the main part of the flow field. However, the results for the hub clearance case described in this work, will show the flow field for distances smaller than 6% of the blade passage from the hub.

Rotor of the Axial Transonic Compressor ECL1

The corresponding results of the above mentioned calculation methods are also used to investigate the tip clearance flow field for the rotor of a transonic compressor. As in the previous transonic case examined, the results obtained by simulating the clearance effect will be compared for the IGV and the rotor blade rows of a modern axial transonic compressor (see also Kaldellis et al., 1986a and Kaldellis, 1986b).

In figures [16a], [16b] and [16c] the corresponding calculated spanwise distributions of the longitudinal and peripheral velocity and total pressure along with the corresponding kinetic energy assumed lost. Before comparing our results to the experimental data for the tip region (30% of the blade height), the results obtained for the early version of our secondary flow method (Kaldellis et al., 1987) are also given for comparison purposes.

The relative longitudinal velocity profile, when the hub clearance effects are included, verifies the assumptions of the proposed tip clearance model. The velocity distribution near the tip is greatly improved, in comparison to the one obtained when the tip clearance is not taken into account. The relative motion of the casing and the tip vortex changes completely the calculated flow pattern near the casing of the machine (6% from the tip). Again the proposed model is usually more successful than the experimental data and are somewhat greater than the ones predicted using simplified semi-empirical correlations (Kaldellis et al., 1987, 1989). The cases not only ignore the compressibility effects but also accept constant peripheral velocity distribution inside the gap, as well as the corresponding kinetic energy assumed lost. Before completing the discussion of our results, it is interesting to compare the profiles of the relative velocity components for both cases investigated (figures [10a], [11a], [16a], [16b], [16c] and [16b]). More precisely, we can see that the results obtained for the ECL1 rotor are more similar to the ones...
obtained by the TS22 rotor for the large clearance case. However, the relative tip gap differs by one order of magnitude at least. On the other hand the ECL compressor has a rotation speed three times the one of the TS22 compressor while its blades are more heavily loaded. Consequently, we can say that for rotating transonic, the blade rows the relative tip clearance height, the rotation speed and the blade loading are three of the factors affecting the flow pattern near the tip region. Thus, oversimplified loss correlations based only on the relative clearance height, cannot realistically describe the complex tip clearance phenomena and therefore only fair representation of the losses can be expected.

5. CONCLUSIONS

An extension of our secondary flow calculation method is presented, including a suitable simulation of the tip (or hub) clearance effect. For this purpose an improved and extended form of the classical model proposed by Lakshminarayana and Horlock (1965) and Lakshminarayana (1970) is developed to take into account important flow aspects (e.g. relative endwall motion, compressibility effects etc.). Special attention is paid to the creation of a closed calculation procedure having the same order of approximations with the integral formulation of the secondary flow method. Accordingly, several tip leakage loss models are tested in order to select the appropriate one for the calculation of the total pressure flow field. Subsequently, the complete method is used to analyze a large number of compressors with a large tip clearance is successfully analyzed and the results of the proposed method are compared with results of previous calculations. A comparative study of the same configuration is made when zero clearance exists, and the influence of the tip clearance upon the complete flow field is emphasized.

Next, a transonic axial rotor is analyzed for two tip clearance heights, which differ by one order of magnitude. The predicted results are again successfully compared with the experimental measurements, while the results given in previous studies seem to represent poorly the tip clearance effects.

Finally, the flow field due to the hub clearance of the IGV and the tip clearance of the rotor of a modern transonic axial compressor stage are successfully examined. The calculated results with and without clearance are compared to the experimental data. Although the main flow field is not considerably modified by the leakage flow, the flow quantities change dramatically near the tip (or hub) region due to the tip (or hub) clearance effects. The aspect that the tip vortex and the relative wall motion have cumulative results for compressor cases is verified, since the results obtained for the transonic axial compressors, where the first one has a large clearance but a conventional rotation speed and the second one has a small clearance but a high rotation speed, are almost identical.

All the above cases verify with remarkable accuracy the basic assumptions of the proposed model. The assumptions are compatible with the ones of the main secondary flow method. This fact encourages us to apply the complete secondary flow method in multi-stage machines, since the method describes with accuracy not only the strong secondary vorticity field, the shock-secondary flow interaction and the highly accelerated turbine endwall shear layers, but also the effects of the tip clearance for stationary or rotating and for low subsonic to transonic cases.

Appendix One: Equations

### Mass Conservation Equation

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \tag{1.1}
\]

### Momentum Conservation Equation

\[
\frac{\partial (\rho \mathbf{V})}{\partial t} + \rho \mathbf{V} \cdot \nabla \mathbf{V} + \nabla (\rho \mathbf{V} - \rho \mathbf{F}) + \nabla (\mathbf{F} \cdot \mathbf{V}) = \frac{-\mathbf{V} \cdot \mathbf{F} - \nabla \rho}{2} \tag{1.2}
\]

### Energy Conservation Equation

\[
\frac{\partial (\rho e)}{\partial t} = -\nabla (\rho (\mathbf{V} \cdot \mathbf{V})) - \nabla \mathbf{F} \cdot \mathbf{V} - \nabla (\rho \mathbf{V} \cdot \mathbf{V}) + \nabla (\mathbf{F} \cdot \mathbf{F}) \tag{1.3}
\]

### Vorticity Transport Equation

\[
\rho (\mathbf{V} \cdot \nabla) \mathbf{V} + 2 \mathbf{V} \times \nabla \times \mathbf{V} = \nabla \cdot (\mathbf{F} \cdot \mathbf{F}) \tag{1.4}
\]

Using an orthogonal curvilinear axisymmetric coordinate system \((x_m, x_n, x_w)\) and after a peripheral integration (Kaldellis and Ktenidis, 1990b) from the suction to the pressure surface of the blade passage, we get the corresponding set of differential equations written in deficiet form (difference between the equations for the "external" and the real flow field), in order to describe the flow field of a steady, viscous and compressible fluid.

### PART ONE: DIFFERENTIAL EQUATIONS

#### Mass Conservation Equation

According to the analysis of Kaldellis et al. (1988a) and Kaldellis (1988b), equation (1.1) may be written as:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \tag{1.5}
\]

we derive the following equation for the peripheral component of the deficit force, compatible with the secondary vorticity field:

\[
D_{zz} = \frac{\partial (B \rho W_\eta) \mathbf{n}}{\partial t} + B \frac{\partial (\rho W_\eta) \mathbf{n}}{\partial t} + \frac{\partial (\rho \mathbf{F} \cdot \mathbf{n})}{\partial t} = \frac{\partial B (\rho W_\eta) \mathbf{n}}{\partial t} + \frac{\partial B (\rho \mathbf{F} \cdot \mathbf{n})}{\partial t} = \frac{\partial B (\rho W_\eta) \mathbf{n}}{\partial t} + \frac{\partial B (\rho \mathbf{F} \cdot \mathbf{n})}{\partial t} \tag{1.6}
\]

From the above equation we are able to calculate the correction of the blade force (deficit force) due to secondary flow field existence, which is directly related with the secondary vorticity field predicted by our method, and thus concentrated inside the area occupied by the endwall shear layers.

### Momentum Equation in the Normal Direction \((\mathbf{n})\)

Integrating the momentum equation in the normal direction, eq.(1.2), along a normal \((\mathbf{n})\) of our computational grid, from a point \(n\) to the shear layer edge, \(n_{ew}=n+b_{ew}\), we deduce:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho \mathbf{V} \cdot \mathbf{n})}{\partial t} + \mathbf{n} \cdot \mathbf{F} \cdot \mathbf{n} = 0 \tag{1.11}
\]

In the above equation the normal velocity component terms, quite small for axial machines, have been neglected.
Energy Conservation Equation

Accordingly equation (1.3) written in the deficit form gives:

\[ \frac{\partial \tilde{\Sigma}_1}{\partial m} + 2 \omega \frac{\partial \tilde{\Sigma}_1}{\partial n} = \Sigma \]

with

\[ \tilde{Z} = \frac{W_h}{W_m} \]

and

\[ \Sigma = \Sigma_1 + \Sigma_2 + \Sigma_3 \]

where

\[ \Sigma_1 = \frac{1}{2} \frac{\partial (p \tilde{W}_m^2)}{\partial n} - \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m \]

\[ \Sigma_2 = - \frac{1}{2} \frac{\partial (p \tilde{W}_m^2)}{\partial n} + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m \]

\[ \Sigma_3 = \frac{1}{2} \frac{\partial (p \tilde{W}_m^2)}{\partial n} - \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m \]

Transport of Vorticity Equation in the Meridional (m) direction

Finally using the analysis of Douvikas et al. (1987), Kaldellis et al. (1988a) and Kaldellis (1988b) equation (1.4) reads:

\[ \frac{\partial \tilde{\xi}_m}{\partial m} + \frac{\partial \tilde{\xi}_m}{\partial n} = \frac{\partial \tilde{\xi}_m}{\partial n} \tilde{W}_m + \frac{\partial \tilde{\xi}_m}{\partial n} \tilde{W}_m \]

where

\[ \tilde{\xi}_m = \frac{1}{W_m} \left( \frac{\partial (p \tilde{W}_m^2)}{\partial n} - \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m \right) \]

\[ \lambda = \frac{1}{W_m} \left( \frac{\partial (p \tilde{W}_m^2)}{\partial n} - \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m \right) \]

\[ \sigma = \frac{1}{W_m} \left( \frac{\partial (p \tilde{W}_m^2)}{\partial n} - \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m \right) \]

\[ \Pi = \Pi_1 + \Pi_2 + \Pi_3 \]

with

\[ \Pi_1 = \tilde{\xi}_m \left( \frac{1}{W_m} \frac{\partial (p \tilde{W}_m^2)}{\partial n} - \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m \right) \]

\[ \Pi_2 = - \frac{1}{2} \frac{\partial (p \tilde{W}_m^2)}{\partial n} + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m \]

\[ \Pi_3 = \frac{1}{2} \frac{\partial (p \tilde{W}_m^2)}{\partial n} - \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m \]

Note that equation (1.18), which is parabolic in nature, is a complete form of the non-linear equation of Burger (see Anderson et al., 1984). Applying now a second order of accuracy in both m and n directions, Mac Cormak predictor-corrector numerical scheme we compute the meridional vorticity distribution \( \xi_m(n) \), excluding the vorticity value at the wall.

PART TWO: INTEGRAL EQUATIONS

Integrating the corresponding circuuminferentially averaged differential equations along a normal (n) of our computational grid, from the wall to the edge of the endwall sheared layer, we get:

Integral Momentum Equation - Meridional Direction (-m)

\[ \frac{\partial \tilde{W}_m}{\partial m} + \frac{\partial \tilde{W}_m}{\partial n} = \tilde{\Sigma}_m \]

\[ + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m \]

\[ + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m \]

\[ + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m \]

Total Kinetic Energy Integral Equation

\[ \frac{\partial \tilde{W}_m}{\partial m} + \frac{\partial \tilde{W}_m}{\partial n} = \tilde{\Sigma}_m \]

\[ + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m \]

\[ + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m \]

\[ + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m + \frac{\partial \tilde{W}_m}{\partial n} \tilde{W}_m \]
Global Mass Flow Rate Equation

\[ \frac{\partial (\rho u)\Delta n_m + \mu \frac{\partial (\Delta n_m)}{\partial n_m} + \frac{\partial (\Delta n_m)}{\partial n_m} \Delta n_m + \frac{\partial (\Delta n_m)}{\partial n_m} \Delta n_m = \frac{\partial (\Delta n_m)}{\partial n_m} \Delta n_m}{\partial n_m} \]  

(1.29)

Note that equations (1.27) and (1.28) are assumed valid for both hub and tip shear layers. The semi-empirical frame utilized for the closure of the system of equations is given by Kaldellis (1988a) and Kaldellis et al. (1989).

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Figure 1: Schematic representation of the flow velocity vectors inside the tip clearance gap.

Figure 2: Schematic representation of the tip clearance flow model at a plane normal to the meridional direction and at a distance (m) from the trailing edge of the blades.

Figure 3: Schematic representation of the experimental cascade, indicating the computational stations where the comparison of the theoretical and the experimental results is performed.

Figure 4: Plot case C - Longitudinal velocity profiles at (a) -22% and (b) 22% of the axial chord. Comparison between theory and experiment.

Figure 4c: Plot case C - Longitudinal velocity profiles at 66% of the axial chord. Comparison between theory and experiment.

Figure 5: Plot case C and B - Longitudinal velocity profiles at (a) 88% & (b) 150% of the axial chord. Comparison between theory and experiments.
Figure 6: Plot case C - Peripheral velocity profiles at (a) -22%, (b) 22% and (c) 66% of the axial chord. Comparison between theory and experiment.

Figure 7: Plot case C and B - Peripheral velocity profiles at (a) 88% & (b) 150% of the axial chord. Comparison between theory and experiments.

Figure 9: Schematic representation of the TS22 transonic axial compressor (meridional view).

Figure 10a: Relative longitudinal velocity profiles at the inlet and the outlet of the rotor. Comparison between theory and experiment. Small clearance.
Figure 10b: Relative peripheral velocity profiles at the inlet and the outlet of the rotor. Comparison between theory and experiment. Small clearance.

Figure 11: Relative longitudinal (a) and peripheral (b) velocity profiles at the inlet and the outlet of the rotor. Comparison between theory and experiment. Large clearance.

Figure 12: Schematic representation of the ECLI transonic axial compressor (meridional view).

Figure 13a: Longitudinal velocity at the inlet of the IGV.

Figure 13b: Flow angle distribution at the inlet of the IGV.

Figure 13c: Total pressure distribution at the inlet of the IGV.

Figure 14a: Longitudinal velocity at the outlet of the IGV.

Figure 14b: Flow angle distribution at the outlet of the IGV.

Figure 14c: Total pressure distribution at the outlet of the IGV.
Figure 15a: Longitudinal velocity at the inlet of the rotor.

Figure 15b: Peripheral velocity at the inlet of the rotor.

Figure 15c: Total pressure at the inlet of the rotor.

Figure 16a: Longitudinal velocity at the outlet of the rotor.

Figure 16b: Peripheral velocity at the outlet of the rotor.

Figure 16c: Total pressure at the outlet of the rotor.