ABSTRACT

This paper describes a combined computational and experimental study of the turbulent flow between two contra-rotating discs for -1 \leq \Gamma \leq 0 and \Re = 1.2 \times 10^6, where \Gamma is the ratio of the speed of the slower disc to that of the faster one and \Re is the rotational Reynolds number. The computations were conducted using an axisymmetric elliptic multigrid solver and a low-Reynolds-number k-\epsilon turbulence model. Velocity measurements were made using LDA at nondimensional radius ratios of 0.6 \leq x \leq 0.85. For \Gamma = 0, the rotor-stator case, Batchelor-type flow occurs: there is radial outflow and inflow in boundary layers on the rotor and stator, respectively, between which is an inviscid rotating core of fluid where the radial component of velocity is zero and there is an axial flow from stator to rotor. For \Gamma = -1, anti-symmetric contra-rotating discs, Stewartson-type flow occurs with radial outflow in boundary layers on both discs and inflow in the viscous nonrotating core. At intermediate values of \Gamma, two cells separated by a streamline that stagnates on the slower disc are formed: Batchelor-type flow and Stewartson-type flow occur radially outward and inward, respectively, of the stagnation streamline. Agreement between the computed and measured velocities is mainly very good, and no evidence was found of nonaxisymmetric or unsteady flow.

NOMENCLATURE

- \a: inner radius of disc
- \b: outer radius of disc
- \Cw: nondimensional mass flow rate (= m/\rho \b)
- \G: gap ratio (= \a/\b)
- \Gs: shroud-clearance ratio (= s/b)
- \r: radial coordinate
- \Re: rotational Reynolds number (= \rho \Omega \b^2/\mu)
- \s: axial gap between discs
- \sg: shroud clearance
- \Vn, \Vs, \Vz: velocity components referred to stationary cylindrical coordinates
- \x: nondimensional radius (= r/\b)
- \xg: nondimensional radius of stagnation ring
- \z: axial coordinate
- \Gamma: ratio of speed of slower disc to that of faster one
- \mu: dynamic viscosity
- \rho: density
- \Omega: angular speed of disc

1. INTRODUCTION

In gas turbines, the turbine disc usually rotates close to either a stationary casing or another rotating disc. The essential features of the flow and heat transfer can be modelled using rotating-disc systems in which plane discs represent the more complex turbine discs in an engine. The rotor-stator system, which is used to model the flow between a turbine disc and an adjacent casing, has been and still is studied extensively (see Owen and Rogers 1989), and the rotating cavity, which is used to model the flow between corotating discs, has also received considerable attention (see Gan et al. 1993a for a recent review). Contra-rotating discs are not so widely used as the other configurations and, as a consequence, have received less attention. They may be used in future generations of ultra-high-bypass-ratio engines, in which contra-rotating...
fans are employed, and a consequential advantage is that contra-rotating turbines require one fewer row of stator nozzles thereby reducing the weight and size of the engine.

The turbulent flow structure for contra-rotating discs is quite different from that for a rotor-stator system, as shown schematically in Fig. 1. (Here and elsewhere in the paper, the rotation ratio \( \Gamma \), where \( \Gamma = \Omega_2/\Omega_1 \), will be used to define the system: \( \Gamma = 0 \) refers to the rotor-stator system, and \( \Gamma = -1 \) to discs rotating at equal and opposite speeds.) Referring to Fig. 1a, fluid moves radially outward in a thin boundary layer on the rotating disc (disc 1) and radially inward in a boundary layer on the stationary one (disc 2). Fluid flows from disc 1 to 2 in a thin boundary layer on the shrouds, and returns from disc 2 to 1 as a weak axial flow in the inviscid core between the boundary layers. This core rotates in quasi-solid-body rotation (with \( V_r/\bar{r}_2 = 0.4 \)); as a consequence of the Taylor-Proudman theorem (see, for example, Batchelor 1967), \( V_r = 0 \) and both \( V_\phi \) and \( V_z \) are invariant with \( z \) in the core.

At this point, it is convenient to comment on the so-called Batchelor-Stewartson paradox. Batchelor (1951), from consideration of laminar flow between infinite rotating discs, predicted that for the rotor-stator system (\( \Gamma = 0 \)) there would be an inviscid rotating core between the two boundary layers; Stewartson (1953) concluded that, for \( \Gamma = 0 \), there would be no such core and rotation would fall to zero outside the boundary layer on the rotor. For \( \Gamma = -1 \), Batchelor forecast that there would be radial outflow in boundary layers on both discs and radial inflow would be confined to a thin shear layer in the mid-plane, on either side of which was a rotating core of fluid; again Stewartson concluded that there would be no such cores, and radial inflow would occur in the (nonrotating) region between the two boundary layers. Gan et al (1993b) showed, from computations for laminar flow in a finite contra-rotating-disc system, that Batchelor’s predictions were correct: for \( \Gamma = -1 \), there was indeed a thin shear layer with contra-rotating cores of fluid. However, computations for turbulent flow, and experimental measurements, showed that Batchelor-type flows are intrinsically unstable and are unlikely to occur in practice. For turbulent flow, therefore, Stewartson-type flow occurs for \( \Gamma = -1 \), as illustrated schematically in Fig. 1b.

In an engine, it is possible for the ratio of the speeds of contra-rotating turbine discs to be in the range \(-1 < \Gamma < 0 \). The question then arises: how does the flow structure change from Batchelor-type flow at \( \Gamma = 0 \) to Stewartson-type flow at \( \Gamma = -1 \)? In other rotating-disc systems (\( \Gamma = 0 \), \( \Gamma = +1 \)), laminar computations provide good insight into the flow structure that occurs in both the laminar and the turbulent cases; for contra-rotating discs, however, this is not necessarily the case, and studies of laminar flow must be treated with caution.

The code used was a modified version of a finite-
volume, multigrid, elliptic solver developed at Sussex University. Simplex pressure-correction smoothing was carried out with a V-cycle, full-approximation-storage multigrid scheme as employed by Lonsdale (1988) and extended by Vaughan et al. (1989). The axisymmetric, Reynolds-averaged, steady-state Navier-Stokes equations were solved in conjunction with a low-Reynolds-number (LR) k-ε turbulence model.

The code was used by Gan et al. (1993b) for the case of contra-rotating discs with $\Gamma = -1$. In that paper, a variant of the LR k-ε model of Morse (1991a,b) was used; in this paper, the (original) LR k-ε model of Launder and Sharma (1974) is used. Kilic (1993) has used both turbulence models for rotating-disc systems and, although neither was ideal for all flows, both were satisfactory for most of the cases considered. Both models were found to handle transition from laminar to turbulent without the need to specify transitional criteria, or to "trip the flow" artificially, but the Launder-Sharma and Morse models predicted transition to occur later and earlier, respectively, than that shown by the experimental data. The main advantage of the Launder-Sharma model is that it is more "robust"; that is, it is easier to achieve converged solutions of the multigrid elliptic solver at high rotational Reynolds numbers.

For $\Gamma = -1$ and $C_w = 0$, Gan et al. (1993b) took advantage of the symmetry about the mid-plane ($z/s = 1/2$) to halve the size of the computational domain to $0 \leq r \leq b$, $0 \leq z \leq 1/2s$; the computed velocity profiles provide "mirror images" for $1/2s \leq z \leq s$. For $-1 < \Gamma \leq 0$, no such symmetry exists. For the solid boundaries ($r = a$, $r = b$, $z = 0$, $z = s$), no-slip conditions were used. For the small clearance, $s_0$, between the two contra-rotating shrouds, it was assumed that $V_r$ and $V_z$ were zero and $V_s$ varied linearly with $z$.

The dimensions of the computational domain were the same as those of the experimental rig: $G = 0.12$, $G_0 = 0.016$, $a/b = 0.128$. Grid-dependency tests were conducted for nonuniform grids with up to $91 \times 115$ (axial $\times$ radial) nodes, and it was found that a grid with $67 \times 67$ nodes (contrasted to the boundaries) gave sensibly grid-independent results. Morse (1991a) recommended that the grid point closest to the wall should be located to ensure that $y^+ < 0.5$, where $y^+$ is the wall-distance Reynolds number, and this condition was achieved for the computations presented here. The computations were conducted on one of the 16 i860 nodes of a Meiko parallel computing facility. The time for a converged solution depended on the rotational Reynolds numbers and on $\Gamma$, and typical times ranged from 1 to 2 hours for $91 \times 115$ nodes.

Further details of the computational methods are given by Kilic (1993).

Fig. 2. Computed streamlines for $Re_s = 1.25 \times 10^6$.
As $\Gamma$ changes from -0.4 to -0.6 to -0.8, the stagnation ring on disc 2 moves radially outward from $x = 0.45$ to 0.75 to 0.94, and Stewartson-type flow occupies an increasingly large part of the domain. For $\Gamma = -1$, the streamlines are symmetrical about the mid-plane, and Stewartson-type flow occurs throughout the domain with radial outflow in boundary layers on both discs and inflow between them.

An alternative way of viewing this transition from Batchelor-type flow to Stewartson-type flow is shown in Fig. 3 where the computed radial and tangential components of velocity, corresponding to the conditions in Fig. 2, are shown. Referring to $x = 0.8$, the following observations can be made.

(i) For $-0.4 \leq \Gamma \leq 0$, there are separate boundary layers on both discs with radial outflow on disc 1 and inflow on disc 2. In the core between the boundary layers, $V_r = 0$ and $V_\theta$ is virtually invariant with $z$. This corresponds to Batchelor-type flow.

(ii) For $-1 \leq \Gamma \leq -0.8$, there are separate boundary layers with radial outflow on both discs. Between the boundary layers, $V_r < 0$ and $V_\theta$ is sheared continuously from a positive value near disc 1 to a negative value near disc 2. This corresponds to Stewartson-type flow.

(iii) For $\Gamma = -0.6$, $V_r$ is sheared continuously from a positive value near disc 1 to a negative value near disc 2 whereas $V_\theta$ shows the existence of separate boundary layers and core rotation. This indicates transition between Batchelor-type flow and Stewartson-type flow.

For $x = 0.6$, transition occurs between $\Gamma = -0.4$ and $\Gamma = -0.6$; for $x = 0.4$, transition occurs at $\Gamma = -0.4$. These velocity profiles are consistent with the flow structure shown by the streamlines in Fig. 2: transition from complete Batchelor-type flow at $\Gamma = 0$ to complete Stewartson-type flow at $\Gamma = -1$ takes place by the development of a two-cell structure.

Fig. 3 Computed velocity profiles for $Re_\gamma = 1.25 \times 10^6$.

Fig. 4 Computed and measured velocity profiles for $Re_\gamma = 1.25 \times 10^6$, $\Gamma = 0$
- Experimental data --- Computed profile
Fig. 5 Computed and measured velocity profiles for $Re_\theta = 1.25 \times 10^6$, $\Gamma = -0.4$
- Experimental data --- Computed profile

in which Batchelor-type flow occurs in the outer cell and Stewartson-type flow in the inner one.

The above solutions were obtained for axisymmetric, steady flow using the Launder-Sharma turbulence model. There is, of course, no guarantee that the real flow will be either steady or axisymmetric, and, even if it were, the real flow is not constrained to follow the solutions obtained with the turbulence model. Having found a possible solution for the transition process, it is now convenient to test it on the experimental data.

3. COMPARISON BETWEEN COMPUTATIONS AND MEASUREMENTS

3.1 Experimental apparatus

The apparatus is identical to that described by Gan et al. (1993b), and the salient details are given below for convenience.

The rotating-disc rig comprised two discs of 762 mm diameter which were independently rotated at up to 1500 rev/min in either direction. A peripheral shroud was attached to each disc, and the axial clearance between the two shrouds was less than 4 mm; the axial gap ratio for the discs was $G = 0.12$. A radial outflow of air could be supplied through rotating gauze tubes, of 100 mm diameter, attached to the centre of the discs; for the experiments described below, no superposed air was supplied and the tubes acted as rotating solid surfaces with $a/b = 0.13$.

One disc was made from transparent polycarbonate, and a single-component TSI laser-Doppler anemometer, with an IFA 750 burst correlator, was used to measure the radial and tangential components of velocity. Micron-sized oil particles were used to "seed" the air in the immediate...
vicinity of the discs, and enough particles were ingested into the "wheel-space" between the two discs to produce acceptable Doppler signals. With the optical "probe-volume" of the LDA system inside the polycarbonate disc, the tangential component of velocity could be compared with the independently-measured angular speed of the disc: the difference was typically less than 0.5% of the speed.

3.2 Computed and measured velocity profiles

Fig. 4 shows the comparison between the velocity profiles, computed using the method described in Section 2 and measured using the apparatus outlined above, for $\Gamma = 0$, $Re_\theta = 1.25 \times 10^6$ and $0.6 \leq x \leq 0.85$: the rotor-stator case. Apart from some differences near the stationary disc, the agreement between the computed and measured velocities is mainly very good. The expected Batchelor-type flow structure is clearly visible: $V_\theta = 0$ and $V_\phi$ is invariant with $x$.

Fig. 7 shows the equivalent results for $\Gamma = -0.4$, and again the agreement between computations and experiments is mainly very good. The computations shown in Figs. 2 and 3 indicate that, for $\Gamma = -0.4$, Batchelor-type flow occurs for $x \geq 0.4$, and the experimental results for $x \geq 0.6$ support this. No measurements were made for $x < 0.6$, and so the existence of Stewartson-type flow at the smaller radii could not be confirmed.

The co-existence of Batchelor-type flow and Stewartson-type flow can be seen in Fig. 6 for $\Gamma = -0.6$: Batchelor-type flow occurs at $x = 0.85$, Stewartson-type flow at $x = 0.6$ and 0.7, and transitional flow at $x = 0.8$. Agreement between computation and experiment is again mainly very good. It is interesting to see that radial inflow between the boundary layers is associated with $V_\phi/\Omega < 0.1$ in the core: for $V_\phi/\Omega < 0.1$, the Taylor-Proudman theorem appears to hold such that there is no radial flow in the (inviscid) core. For $x \leq 0.7$, where Stewartson-type flow...
occurs, both the radial and tangential components of velocity are sheared across the (viscid) core between the boundary layers.

Fig. 7, for \( \Gamma = -0.8 \), shows that Stewartson-type flow exists for \( x \leq 0.85 \); Fig.8, for \( \Gamma = -1 \), shows the final stage of the transition.

In conclusion, it can be seen that the experimental data have confirmed the two-cell transition from Batchelor-type flow to Stewartson-type flow predicted in Section 3. Although, no LDA measurements were made for \( x > 0.85 \) or \( x < 0.6 \), measurements were made at other values of \( \Gamma \) and \( \text{Re}_\text{u} \) in addition to those shown here. Although the agreement between computations and experiment was not as good at lower values of \( \text{Re}_\text{u} \) where transition from laminar to turbulent flow complicates the picture, the basic mechanism remains the same. Also, apart from regions of laminar-to-turbulent transition, both the Launder-Sharma and the Morse turbulence models yield similar results for the flow structure. Whilst it would be unsafe to conclude that the turbulent axisymmetric flow never becomes three-dimensional or unsteady, the authors have seen no evidence to suggest that it does: transition from Batchelor-type flow to Stewartson-type flow appears to be a continuous steady process.

4. CONCLUSIONS

A combined computational and experimental study of the turbulent flow between contra-rotating discs has been conducted for \( C = 0.12 \), \( \text{Re}_\text{u} = 1.2 \times 10^6 \) and \(-1 \leq \Gamma \leq 0\).

The axisymmetric computations show that Batchelor-type flow occurs for \( \Gamma = 0 \), the rotor-stator case, with radial outflow and inflow in boundary layers on the rotor and stator, respectively, and axial flow from the stator to the rotor in the inviscid rotating core. For \( \Gamma = -1 \), anti-symmetrical contra-rotating discs, Stewartson-type flow occurs with radial outflow in boundary layers on both discs and inflow between the boundary layers in the viscid nonrotating core. For intermediate values of \( \Gamma \), the computations show a two-cell structure separated by a streamline that stagnates on the slower disc: Batchelor-type flow occurs radially outward of the stagnation streamline, and Stewartson-type flow occurs radially inward.

Velocity measurements, using LDA for \( 0.6 \leq x \leq 0.85 \), provide evidence to support the computations, and agreement between computations and measurements is mainly very good. No evidence of unsteady or nonaxisymmetric flow was found for the range of variables considered, and measurements and computations carried out at other values of \( \text{Re}_\text{u} \) show similar results.

The above study was conducted without a superposed flow. The effect of a radial outflow of air on both the flow structure and heat transfer will be reported elsewhere.

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