High-Speed Rotor Suspension Formed by Fully Floating Hydrodynamic Radial and Thrust Bearings

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The hydrodynamic suspension of a 44,000-rpm gas-turbine rotor is described with emphasis on interaction between the flexible rotor and its supports. Bearings with relatively large clearances are shown to allow continuous operation at the rotor first critical speed of 22,000 rpm. The apparent absence of hydrodynamic system instabilities is attributed to the use of simple floating-sleeve bearings. A parametric study of the influence of bearing clearances upon vibrational excitation relief is presented together with test data collected on actual system hardware.
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The reliability of high-speed, gas-turbine power plants is quite dependent upon satisfactory performance of the rotor/suspension system. A gas-turbine rotor must be capable of running for prolonged periods at any speed between the rated speed and a lower limit speed which is frequently less than one half of rated speed. Striving for weight reduction, the design of such rotors will have to anticipate occurrence of critical speeds within the operating range. The methodical system-design approach can be classified, with respect to suspension dynamics, into a few distinct phases. Firstly, the general arrangement of the rotor/suspension system must place the critical speeds, if unavoidable, where they hurt least. Then, the detailed suspension configuration has to provide rotor bearings capable of sustaining safely, and for an indefinite time, whatever force transmission level is established at any speed. Since bearing size limitation is always an important issue, the desirable suspension configuration is attained in a two-fold manner. Either damping is provided to limit force transmission for given excitation of the rotor or the rotor excitation is itself limited; preferably at all speeds, including the critical.

The purpose of the paper is to illustrate how these objectives were met for the suspension of a 44,000-rpm rotor by a fully floating, hydrodynamic bearing system. Interaction between rotor and suspension will be discussed for the case of the first critical speed at 22,000 rpm which is within the steady-state operating range of the rotor. Measured interaction data taken from bearing shells will clearly identify the resonance but will also evidence very weak response levels. Of the variety of analyses and tests done in behalf of this project, those will be presented which show either the engineering implications or the net results in simplest form.

GENERAL

The rotor under consideration takes physical shape as a component of the Model 520 engine shown in the cutaway view of Fig.1. The 520 "free-shaft" engine is suitable for several applications but is primarily intended as a lightweight airborne power plant in the 600-hp range with weight/power ratio less than 0.45 lb/hp. The power-output section of the engine (LH side of Fig.1) features an axial-flow turbine, reduction-gear box and some accessory drives. The gas-generator section includes an accessory drive section, two
Table 1 Test-Rotor Configuration Data

<table>
<thead>
<tr>
<th>Description</th>
<th>Accessory Gear Overhang (INCH)</th>
<th>Compressor Span (INCH)</th>
<th>Interspan (INCH)</th>
<th>Turbine Overhang (INCH)</th>
<th>Rotor Total (INCH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (INCH)</td>
<td>4.05</td>
<td>10.39</td>
<td>4.90</td>
<td>5.45</td>
<td>24.78</td>
</tr>
<tr>
<td>Weight (LB)</td>
<td>0.74</td>
<td>14.27</td>
<td>2.11</td>
<td>16.04</td>
<td>32.14</td>
</tr>
<tr>
<td>C. G. Location, FROM BEARING &quot;C&quot;</td>
<td>-16.96</td>
<td>-10.31</td>
<td>-2.56</td>
<td>-0.34</td>
<td>-2.32</td>
</tr>
<tr>
<td>Flexural Rigidity</td>
<td>-</td>
<td>1.31 x 10^7</td>
<td>8.00 x 10^7</td>
<td>4.00 x 10^7</td>
<td>-</td>
</tr>
<tr>
<td>About Design Axis</td>
<td>0.1084</td>
<td>0.0024</td>
<td>0.1084</td>
<td>0.3604</td>
<td>0.2713</td>
</tr>
<tr>
<td>Transversal Axis Through C. G.</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>4.29</td>
</tr>
</tbody>
</table>

**NOTE:**
1. Measured values
2. Turbine "neck" region
3. Compressor/inducer average, computed as in 2

**Fig. 2 Gas-generator rotor**

Cross-connected, reverse-flow combustion chambers and the rotor assembly. The rotor assembly consists of a double-entry, single-stage centrifugal compressor and a single-stage radial-inflow turbine. No mechanical coupling exists between rotors of the two sections. The radial-inflow turbine delivers about 870 hp at 44,000 rpm. A detailed view of the gas-generator rotor is shown in Fig. 2, as mounted for a shaker-table vibration test. The locations of the journal bearings A, B, and C are approximately indicated by the table mounting blocks. Numerical data for the test rotor configuration are given in Table 1, in a horizontal order which corresponds to the physical layout of Fig. 2.

**Rotor on Rigid Supports**

The results of a hand-computed vibrational analysis according to the Myklestad-Prohl ("remainder boundary value") method are shown in Fig. 3. The rotor properties, computed for a large number of cross sections, were lumped at eighteen stations. Measurements confirmed essentially these data (Table 1, ref) but the description of local effective rigidity in flexure remained somewhat indefinite in the blading region and was subsequently improved by comparison with hardware shaker-table tests. As is well known, the resonances, represented by "transparent planes" in Fig. 3, are found at frequency points with zero
"remainder" by interpolation between trial frequency points having non-zero remainder values. The mode shapes in Fig. 3 were derived by a transformation of computed data and yield more physical insight into rotor behavior throughout the frequency range. Such interpretation is apparently not utilized elsewhere and might be worth explaining. For given rotor boundary conditions and given frequency, the "remainder" quantity selected at Station 18 was the generally non-zero moment $M_{18}$ expressed in terms of the slope $\beta_0$ at Station 0. The rotor deflection $\psi$ at any station is also expressed in terms of $\beta_0$ and can be plotted with respect to the rotor axis. This is a common procedure and a mode-shape curve will result for any given frequency. However, it would not lend itself for easy quantitative comparison with mode shapes at other frequencies because the associated $M_{18}/\beta_0$ values would be considerably different. A "thought experiment" can be conducted by assuming that a definite physical moment

$$M_{18} = m\beta_0 \omega^2$$

might exist due to a purely "dynamic" unbalance $m\beta_0$ having a force couple acting about Station 18 which lies in the turbine disk of the discussed rotor. The dynamic deflection $\psi$ per "unit unbalance" $m\beta_0$ is then defined by

$$\frac{\psi}{m\beta_0} = \frac{(\psi/\beta_0)\omega^2}{M_{18}/\beta_0} = \frac{\psi}{M_{18}}$$  \hspace{1cm} (1)$$

The $\psi/\beta_0$ and $M_{18}/\beta_0$ values are available from the tabular method computation while the $m\beta_0$ factor is a constant defined by the assumed unbalance distribution in the rotor. The mode-shape curves gain mutually comparable proportions and the rotor is shown to "quiet down" between the resonances. This is now a forced undamped vibration; the associated sudden phase-angle changes are immaterial for deflection magnitude and can be "assigned" arbitrarily to either resonance. The ratio between turbine and compressor center deflection indicates that the first natural frequency is being controlled by the turbine while the compressor resonates predominantly at the second natural frequency.

Results shown in Fig. 3 pertain to a "point-mass" model of the rotor. Mass moment of inertia effects were also computed, using the tabular method, for the case of positive synchronous whirl ("gyroscopic effect") at the fundamental frequency only. The resulting increase in frequency was 12 percent, a magnitude which had been expected in view of the sizable mass moment of the overhung radial-turbine disk.

To provide comparison with the analysis, the
rotor shown in Fig. 2 was subjected to shaker-table tests on a Ling Mod. 182 electromechanical vibration system of 25,000 lb capacity. The inner faces of the rotor mounting clamps had an axial width of only 0.15 in. and the journals were clamped circumferentially at stations corresponding to bearing mid-lengths. Four accelerometers for rotor data as well as one table excitation monitor meter were installed, as shown in Fig. 2. The sine-wave excitation was kept at constant 1 g peak acceleration level throughout the frequency range. Data were recorded on magnetic tape, analyzed with a response analyzer as well as displayed on strip charts by means of a light-beam oscillograph. Fig. 4 shows the strip-chart record of a slow frequency sweep. The gains of all channels were adjusted to be approximately equal and their traces can be compared directly. The first resonance is associated with the turbine disk while at the second resonance the compressor is seen in control of the motion. It should be noted that the second resonance occurs at 960 cps only. The high amplitude at 902 cps is due to interaction between components of the system for the specific mounting configuration and disappeared at test runs with different mountings. Since the second natural frequency was outside the operating speed range, the fundamental frequency was of major interest. Fig. 5 shows an automatic recording of the response analyzer for the turbine end meter (misalignment of the chart shifted the frequency scale somewhat).

The analysis and static tests of the rotor on "rigid" supports provided data describing the overall trends of vibrational behavior; but they lacked satisfactory correlation with respect to determination of natural frequencies, even allowing for the transversal mass moments of inertia effects ("bending inertia") in the analysis. Finite stiffness of the shaker-table mounting setup had to be taken into account in order that the comparison between analysis and tests be more realistic. This was actually done by programming the rotor in 23 sections for a direct analog computer, utilizing the mass-capacitance analogy. The computer network included flexibility and damping of all supports as well as permitted simulation of longitudinal and transversal mass moment of inertia effects for each rotor section.

Fig. 4 Rotor vibration response (Shaker test)

Fig. 5 Turbine disk response (Shaker test)
Positive synchronous whirl condition data were obtained by iteration of intermediate results, inasmuch as inductors had been used to represent "negative capacitors." Parasitic damping of the network averaged 1.2 percent of critical for the "point mass" and "bending inertia" models, and 2 percent for the "gyroscopic effect" model. Simulating the shaker-table-rotor setup with mounting stiffness of \(5 \times 10^7\) lb/in, the analytical model reproduced test results with no change in turbine-end description and some corrections for compressor blading. Thus, a quite plausible "paper rotor" was arrived at. Resonant frequency data for the case of infinitely rigid supports are shown in Table 3.

**Table 3**

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Bending Inertia*</th>
<th>&quot;Point Mass&quot;</th>
<th>&quot;Gyroscopic Effect&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>(psa)</td>
<td>(rel)</td>
<td>(psa)</td>
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<tr>
<td></td>
<td>0.947</td>
<td>1.000</td>
<td>1.103</td>
</tr>
<tr>
<td>Second</td>
<td>(psa)</td>
<td>(rel)</td>
<td>(psa)</td>
</tr>
<tr>
<td></td>
<td>1.052</td>
<td>1.075</td>
<td>1.128</td>
</tr>
<tr>
<td></td>
<td>(rel)</td>
<td>(psa)</td>
<td>(psa)</td>
</tr>
<tr>
<td></td>
<td>0.978</td>
<td>1.000</td>
<td>1.029</td>
</tr>
</tbody>
</table>

Note: Computer repeatability ± 5 cps, approximately.

**Rotor on Flexible Supports**

The effect of support flexibility upon rotor natural frequency was investigated, assuming linear equivalent springs, over the stiffness range of \(10^3\) through \(10^5\) lb/in. Results for the "point-mass" rotor model with equal stiffness of all three supports are shown in Fig. 6. The natural frequencies are seen to remain constant for stiffness values higher than \(5 \times 10^7\) lb/in; i.e., the supports are practically rigid in that range. All frequencies drop off markedly at increased support flexibility. The first and second frequency would disappear eventually for vanishing support stiffness, while the third and fourth frequency are seen to blend into natural frequencies of the "free-free" rotor. The vibrational mode shapes of the deflected rotor axis are also shown in Fig. 6, superimposed upon circled reticles for different frequencies at representative stiffness values. A larger scale reticle (above the graphs) positions the undeflected rotor geometry with respect to the mode-shape curves. Mode shapes of the first two frequencies at high stiffness are identical to those of Fig. 3 (when drawn in inverted position). At low bearing stiffness, these two frequencies are associated with mode shapes recalling the coupled rocking and bobbing motion of a rigid beam on springs. For bearing stiffness of about \(5 \times 10^4\) lb/in., the rotor becomes very "stiff" with respect to the supports and the system natural frequencies can be computed with fair accuracy by neglecting the middle support and treating the rotor/support configuration as a two-degree-of-freedom system. The coupling coefficient is strong, owing to asymmetrical location of end bearings with respect to the rotor-mass center, and the vibrational modes are correspondingly blended. The mode shape of the third resonance curve at very low bearing stiffness has nodal points which almost coincide with mass centers of the turbine disk and compressor body, as would be expected from a free-free rotor.

The following observation was relevant for the suspension design. In order that the second critical be avoided at top rotor speed (45,000 rpm = 750 cps) the equivalent bearing stiffness would have to exceed \(6 \times 10^5\) lb/in., approximately. Gyroscopic effects were considered in this evaluation (large effect upon the first critical, minor effect upon the second; compare Table 3).

Since the first critical had to be sustained at steady-state operation, the assessment of bear-
ing force transmission (per unit rotor excitation) was quite important. Fig. 7 shows the bearing reactions as functions of equivalent stiffness and damping for conditions of the first resonance curve in Fig. 6. Inasmuch as the distribution of inertial excitation throughout the flexible rotor is not well defined, data in Fig. 7 are referred to an identical unit acceleration of the rotor at the support points. As the first critical speed was shown to be associated with turbine resonance, the turbine-end bearing C sustains the heaviest load. However, the load distribution between bearings B and C is considerably altered at low stiffness (even without damping) owing to the change in vibrational mode shape, described before. A significant attenuation of bearing loads becomes feasible by simultaneous existence of flexibility and damping. That combined effect is shown at bearing-stiffness values corresponding to the lowest design target, discussed in conjunction with Fig. 6. The conditions of high damping at that stiffness, physically unlikely in bearing systems, or any damping at high stiffness, where attenuation becomes ineffective, are also shown in Fig. 7 for completeness of functional description.

FULLY FLOATING BEARING SYSTEM

The basic concept of the Model 520 gas-generator rotor-bearing system was patterned after that used on several types of previously designed gas-turbine engines in the Model 502 series. Reasons for incorporating self-acting hydrodynamic bearings in lieu of rolling-contact bearings in these engines, as well as the original development of a pivotless tilting-pad thrust bearing, were described by Hill (1). The performance of floating-ring journal bearings under laminar flow, low speed conditions was analyzed by Shaw and Nussdorfer (2). Here, attention will be drawn upon the preference for floating bearings, rather than plain hydrodynamic bearings, from the viewpoint of high-speed-rotor dynamics.

The physical configuration of these bearings, as used on the test rotor, is shown in Table 2. The callout sketch atop of the tabulation is representative for bearings A and C. Bearing C lacks the outer circumferential groove and is shown in Fig. 8. Dimensions and callouts for bearing B in-

1 Underlined numbers in parentheses designate References at the end of the paper.
dicate a ring with almost square cross section, without grooves; such shape is determined by the thrust bearing, (Fig.13, ref). The lubricant is conducted to all three bearings through the hollow rotor shaft. In order to minimize friction losses and cooling requirements, the bearing system is designed for high-temperature operation. Under test conditions encountered, the synthetic lubricant MIL-L-7808 drops then to viscosity levels less than those of water. Two geometrical characteristics, shared by these journal bearings, are noteworthy. The total diametrical clearance, averages 0.0080 in. and is large for the shafting size. The effective bearing area is very "slender"; its L/D ratio averaging 0.20, is unusually small.

The considerations leading to such design are closely linked with the prime objective of the floating-bearing system and will be explained in a conceptual manner, concentrating on only two major design requirements. Both relate to interaction between the described rotor and its suspension. First, dynamic load conditions must be solved either by sheer bearing strength, or force-transmission attenuation, or force-excitation relief. Secondly, the half-frequency whirl has to be eliminated since the first critical speed had been predicted at about half rated speed and resonant whirl might develop at rated speed.

Consider bearing-load capacity conditions. Dynamic deflections of the quite flexible rotor (Fig.3, ref.) would create large unbalances across the critical speed range. This powerful excitation would result in high bearing forces as was shown parametrically in Fig.7. Steady-state operation in that range would be questionable with bearings of any size. On the other hand, the problem is put into better perspective by recalling that "unbalances" and associated "inertia" forces merely indicate that the rotor is being spun about an axis which is not one of its principal axes of inertia. If the principal axis is allowed to become the spin axis, no forces whatever, except those counteracting mass center acceleration, are needed to stabilize the rotor motion. Inasmuch as the design axis (in the general "unbalance" case of a rotor) is skewed with respect to the principal axis, a possible spin about the latter involves revolution of the former with a kinematic envelope which is then a single sheet hyperboloid of revolution. While rotation about the principal axis is practically unattainable, the less deviation from it, the easier would the supporting job seem theoretically. To maintain the desired motion, the kinematic envelope would have to develop freely within the bearing clearance space. One aspect of the task of any "clearance" bearing, in a very idealized sense, would be to facilitate that freedom of motion and large design clearances would appear a logical consequence. Of course, forces in the lubricant film have to be accounted for. Viewing forced vibrations of a flexible unbalanced rotor on flexible supports from the bearing vantage point, the stable synchronous whirl must be contemplated. In this motion the rotor whirls about the bearing-center axis under action of centrifugal forces and the journals orbit about the bearing centers encountering hydrodynamic pressures which exactly balance the disturbing force. The question arises whether bearings with large clearances do a better job in sustaining the rotary forces resulting from such motion. Reinterpreting an available analysis of the synchronous whirl (2), large bearing clearances do appear favorable in the relevant part of the range for which that analysis is valid. Compared with the performance of a bearing having a "reference" clearance, larger mass "eccentricity" of an unbalanced rotor could be tolerated at same journal eccentricity if the bearing clearance were made larger, all other parameters of the system remaining equal. The inference of constant journal eccentricity in the foregoing statement implies a constant Sommerfeld number, which in conjunction with the larger clearance indicates a smaller applied force. As this force is the centrifugal force of the rotor, a suitably designed hydrodynamic-clearance bearing is seen to possess the capability of rotor-excitation relief. Since the claim made here on the benefit of large clearances is not self-evident and since the subject is
quite important for turbomachinery design, the referenced analysis, including its limitations, will be appraised in the Appendix.

Consider a "large-clearance" bearing which, under flexible rotor unbalance loads, does a good job of relieving the disturbing forces and maintaining low journal eccentricity. Such a system is prone to be hydrodynamically unstable through occurrence of half-frequency whirl at low eccentricity and low force level. However, interposition of a free sleeve between the journal and the bearing shell will subdivide the clearance into an outer and an inner clearance. The "effective" clearance, in the context discussed before, will be less than the sum of the actual outer and inner clearance but more than either of them taken separately. Under operating conditions; this sleeve will be propelled in the direction of shaft rotation by friction in the inner film and dragged due to friction in the outer film. Its steady-state angular speed will be a fraction of the shaft speed. The half-frequency whirl in a single film bearing could be interpreted as lubricant film "whirl" conditioned by the curvilinear translation of the moving journal boundary with respect to the stationary shell boundary. The kinematic relationship between the boundaries will be altered by making the former stationary boundary free to move. Moreover, this movement is then dependent upon hydrodynamic conditions in two films. Since a film instability changes the hydrodynamic behavior of the film, it will also alter the sleeve motion. This, in turn, must have a feedback effect upon the instability. When the mechanism or trends of that feedback could be merely guessed at, high-speed-rotor tests did not indicate occurrence of instability effects and the stabilizing function of free-floating elements was taken for granted.

With respect to practical bearing design, two features of floating-sleeve journal bearings evolved partly from parametric performance considerations and partly from developmental test experience. The load-carrying surfaces of the sleeves were made slender. This allows larger operating eccentricities under nonrotating load which serve as film instability inhibitors. Slenderness is achieved for long sleeves by use of centered circumferential oil grooves. The second design feature is the outer versus inner clearance distribution. Analytical considerations indicate that the floating speed ratio \( \omega_s/\omega \) (where \( \omega_s \) is the sleeve speed and \( \omega \) the shaft speed) should become lower whenever \( C_o/C_1 \) increases, given \( D_o/D_1 \). Associated theoretically with larger \( \omega_s/\omega \) and \( C_o/C_1 \) is a better cooling characteristic of the bearing (cf. reference 2). Tests conducted with configurations having \( D_o/D_1 = 1.2 \) and \( 0.9 \leq C_o/C_1 \leq 5.0 \) indicated the highest achievable speed ratio, independent from shaft speed, of \( \omega_s/\omega = 0.32 \) at \( C_o/C_1 = 1.5 \). Also, for

\[
C_o/C_1 \leq 1.5, \quad d\omega_s/d\omega = \omega_s/\omega = \text{const.}
\]

However, the lowest \( \omega_s/\omega = 0.24 \) was obtained at \( C_o/C_1 = 5.0 \) for shaft speeds larger than 35,000 rpm. The large \( C_o/C_1 \) bearings tried to approach higher floating speed ratios by having, for \( C_o/C_1 = 5, \omega_s/\omega = 0.36 \) up to about 14,000 rpm shaft speed but decreased that gradient to \( d\omega_s/d\omega = 0.16 \) at 35,000 rpm.

Rotation of floating sleeves was very reliable, practically no intentional test abuse could stop them. The load-carrying function for constant simulated rotor weights was satisfactory in all cases, even at \( C_o/C_1 = 5 \) where the outer film would appear weak indeed if compared to the inner film \( (C_o/D_o = 0.0100, C_o/D_1 = 0.0024) \). Owing to earlier onset of turbulence in the outer film of configurations with large \( C_o/C_1 \), the slower speed of their sleeves could be tentatively explained. Since turbulence occurred always within the rated speed range, the friction and temperature-rise parameters were completely different from those predicted according to the isoviscous laminar-flow theory. At the same time, they were not very different for test bearings of given overall shape but having various \( C_o/C_1 \) ratios. The insensitivity of performance to bearing micro-detail configuration, for a given job, eased manufacturing tolerances and made material selection more flexible with respect to the required large temperature spread of \(-60 \text{ through } +300 \text{ F.} \)

**ENGINE TESTS**

The first critical speed of the described rotor, predicted to be lower than 31,000 rpm (cf. Table 3, "gyroscopic effect" model, infinitely rigid supports) was not observable by monitoring gas-generator-housing vibrations on several test engines. One engine gas generator, without the power-output section and reduction-gear box, was selected for precision instrumentation. Six piezoelectric accelerometers were screwed directly into the bearing shell retainers, two at each bearing, one in the vertical and the other in the horizontal direction. The accelerometers were "ENDEVCO" Mod 2229, chosen because of small size (3/8 in. hexagon base, 32/64 in. height) and small temperature sensitivity deviation (15 percent or less, from -65 to 350 F). Their sensitivity was 5 mV/G and resonant frequency 30,000 cps. The output of the six channels, after passing a cath-
ode follower and an additional preamplifier system to obtain optimum recording quality, was fed into an "AMPEX" Mod 100A FM record/reproduce magnetic tape machine. Also, this output was continuously displayed on voltmeters for each channel and run off on a strip chart by means of a light-beam oscillograph. A cathode-ray oscilloscope was selected to individual channels. The rotor speed was monitored by a variable-reluctance magnetic pickup facing one of the accessory gears. Its output was displayed on an SPMT counter as well as recorded on magnetic tape simultaneously with the six vibration traces. Each bearing was instrumented for temperature, and the lubrication system for temperature, flow, and pressure measurement.

The gas-generator structure was mounted to ground by means of flexible supports. The test rotor was assembly balanced in two planes at 1200 rpm on a Gishold Dynetric Mod S balancing machine. The residual unbalances were calibrated per Thearle's field balancing procedure, in several trial runs with good repeatability. The magnitudes and phases of residual unbalances at 1200 rpm were (cf. Fig.2) 0.037 ± 0.002 oz-in., 0 ± 3 deg, in the rear face plane of the turbine disk and 0.035 ± 0.001 oz-in., 9 ± 2 deg, in the inlet plane of the compressor next to bearing A.

Before the rotor test runs and vibrational data taken at the bearings are reviewed, it is necessary to appraise which rotor variable was implied in the actual measurements. To that end, consider two linear structural systems (connected in series) by means of which is a bearing shell linked to ground (taken as quasi-inertial reference). Let the first system be the overall engine structure of large mass $m_1$ tied to ground by such springs that the natural frequency of that system is small compared to the rotor running frequency; that is, the frequency ratio $r_1 = \omega_1 / \omega$ is the natural frequency and $\omega$ the shaft frequency. Let the second system be the bearing-shell retainer and the immediately adjacent structure, the extent of which is undefined but such that its combined effective mass $m_2$ is very small compared to $m_1$. If the structural "springs" connecting the second system to the first one are large enough, the natural frequency of the second system would be higher than the shaft frequency; i.e., $r_2 < 1$. With respect to the bearing radial plane, the motion of the connected masses $m_1$ and $m_2$ under action of the periodic bearing force $F_B$ is that corresponding to a two-degrees-of-freedom system and the ratio between their displacements is given by

$$\frac{X_2}{X_1} = \frac{m_1}{m_2} \left( 1 - \frac{1}{r_1^2} \right) r_2^2 + 1$$

In high-speed gas turbines and for thin-walled structures found in such machinery, $m_1/m_2$ can be sufficiently larger than $1/r_2^2$, so that

$$\left( \frac{m_1}{m_2} \right) r_2^2 \gg 1, X_2 / X_1 \gg 1$$

and the engine-base motion can be neglected when force transmission to the bearing-shell system is considered. Because of negligible damping in that system, as well as $r_2 < 1$, the bearing force $F_B$ is in phase with the system spring force $k_2 X_2$. Inasmuch as the accelerometers were measuring $X_2$, the following relationship would result (including mass acceleration effects):

$$F_B = -k_2 \omega^2 \left( 1 - r_2^2 \right)$$

in which $k_2$ is unknown but constant. For structural configurations where equation (3) is valid, occurrence of the shaft speed in the denominator of the equation indicates that if measured accel-
erations increase with shaft speed, the bearing forces need not do so.

The rotor-vibration test runs covered the range between 6000 and 44,000 rpm. Two different types of runs were utilized across the range; namely, the constant-speed run in 500-rpm increments and the slow-sweep run. In order to gather steady-state data in the range under the self-sustaining burner operation limit of 18,000 rpm, the gas generator was run from 6000 to 19,000 rpm on high-pressure cold air. Such runs were scheduled immediately after hot runs and speeds in that range were maintained until excessive icing at the turbine took place. Normal self-sustained burner-supported runs covered the range from 18,000 to 44,000 rpm. To improve the useful measurement output, filters were used with either preset bandwidth (500 to 300 cps) for sweep runs or hand-tuned narrow band (20 cps) for constant-speed runs.

Yet, no clear resonance was visually obtainable from the instruments throughout the speed range, but a gentle spread-out hump was faintly indicated between 20,000 and 24,000 rpm. Finally, the tape recordings were processed in a "Technical Products" Mod 626 spectrum analyzer system. Each data-sampling loop had a length corresponding to 10 sec of rotor running time. The spectrum scanning filter was 5 cps wide; the analysis results were normalized to 1 cps and presented in the form of "acceleration density" in \( G^2 \text{/cps} \) as function of component frequencies. Spectrum analysis graphs for typical data samples are shown in Figs.9 and 10. The actual acceleration components for each frequency, in G rms, amount to the square roots of the corresponding acceleration densities. The range of the analyzer from saturation was about 3 decades (10^3) as can be seen from these graphs. The output resolution tolerance was approximately ±10 percent over the top 2 decades.

The gradual buildup of the rotor first critical speed was now clearly discernible. Since this critical speed was shown to be associated solely with turbine-end resonance, the turbine bearing C provided the best measurements. That bearing, after 204.7 hr running time in a test engine, is shown in Fig.8. Fig.9 shows the bearing-shell acceleration component, due to the transmitted rotor force \( F_B \), as that spectrum component which is synchronous with the rotor speed of 296 cps. Its magnitude of 0.087 G rms is shown in the squared form (7.6 x 10^-2) and can be seen to amount to only 31 percent of total acceleration of 0.258 G rms measured at the bearing. Among the remaining "accelerations" picked up by the instruments, the 60-cps electrical noise and its 120-cps harmonic are readily distinguishable. While Fig.9 showed a subcritical shaft speed, the data sample in Fig.10 is taken almost exactly at the critical speed. The synchronous component rose to 7.2 x 10^-2 G^2/cps in terms of acceleration density or 0.268 G rms actual value.

When consecutive synchronous components of the acceleration were plotted versus the shaft speed, the critical speed interval becomes outlined in Fig.11. A 5th order least-square polynomial fitted to the test points describes the response shape and, based on this shape, an "equivalent" damping ratio of about 8.5 percent with respect to critical could be derived. However, in view of the complexity of conditions in the lubricating film, equivalent damping can hardly be assessed by comparison with the response of any linear system. This can be seen considering the response in the horizontal direction at the same bearing, as shown in Fig.12. The magnitude of the horizontal response is considerably smaller than that in the vertical direction. Moreover, the dashed outline of the response shape might not appear plausible at first glance. Yet, Lund and Sternlicht (4) show in a study of rotor-bearing dynamics force-transmission response shapes strikingly similar to those in Figs.11 and 12. Even the relative mag-

![Fig. 10 Bearing-vibration spectrum, first critical speed](https://proceedings.asmedigitalcollection.asme.org)
nitudes between the vertical and horizontal responses are comparable considering that the force response here is shown in squared form, and the equivalent eccentricity ratio of our turbine bearing C at the critical speed can be estimated to 0.5 < ε < 0.7.

Vibration data at higher shaft speeds showed nothing unexpected. For example, the data sample analysis for bearing C at 41,800 rpm yielded 0.87 G rms synchronous component out of 3.86 G rms total acceleration in the vertical direction, and 0.36 G rms synchronous out of 10.3 G rms total in the horizontal direction. Note that the vibrations attributable to the rotor (synchronous components of the acceleration) were low compared to the total measured acceleration at all speeds, even though the accelerometers were directly in the force-transmission path as close to the rotor as was physically possible. Vibrational data taken at bearings A and B were generally of low intensity and were less relevant inasmuch as the rotor components supported by these bearings did not reach resonance within the engine-speed range.

All test data shown heretofore were dealing with the radial equilibrium of the rotor/suspension system and the vibrational characteristics were of prime importance. However, the described rotor is subjected also to axial forces of about 600 lb which originate from the aerodynamic configuration; i.e., the double-inlet compressor is axially balanced while the turbine is not. These loads are taken up by a floating pivotless tilting-pad thrust bearing which was originated by Hill (1).

While reference (1) reviews the concept of the floating slipper bearing primarily from the friction-loss viewpoint, its specific load-carrying capacity and ruggedness as a component of a floating suspension will be demonstrated here by test data.

Fig.13 shows schematically the principle of operation, the configuration of the envelope and the range of test conditions. The bearing of size shown in Figs. 13, 14 and 15 was endurance tested with 1000 lb at 38,000 rpm in a high-speed bearing test machine. Its performance exceeded
considerably the engine design requirements and, as a consequence, the bearing size was reduced for actual applications. The compactness of envelope for given shafting size is worth notice. With 1.57-in-dia (40 mm) shafting at 45,000 rpm, the characteristic DN number amounts to 1.8 x 10^6 and this hydrodynamic bearing would well compare in size with rolling-contact bearings as shown in Fig.13. More relevant is its comparison with other concepts of hydrodynamic thrust bearings, especially with the Kingsbury-Michell pivoted tilting pad and the Rayleigh step bearing, both of which possess higher specific-load capacity. However, within such small envelopes, the mechanical design of pivots and load-equalizing elements proved a rather formidable task which was abandoned after several unsuccessful attempts. The simplicity of the Rayleigh step concept had been very appealing to the author who tried it out in various configurations, on the running thrust collar, floating rings, and so on, under the same test conditions and envelopes as were those for the floating-slipper bearing. Yet, whenever a continuous thrust plate was utilized, the thermal and load distortions of the hardware made the load-carrying capability of the bearing quite unreliable. On the other hand, the independent slipper segments were insensitive to hardware distortions as can be seen from Fig.14. The runner thrust collar was statically tested under area distributed load totaling 1000 lb and the runner tip deflected 0.016 in. Under operating conditions and the same load, the slipper segments adjusted themselves to the conical space formed by the deflected runner and the retainer thrust plate which was deflected in the opposite direction. At the same time, the slipper segments were pressed outward radially by their own inertia load and the bright wear marks at the edge of their radial faces indicate the pattern of envelope deflection. Nevertheless, the segments continued rotating at 23 percent of shaft speed, carried their axial load of 200 lb per segment as well as sustained 155 lb per segment in the radial direction, at a bearing temperature of 407 F (cf. Fig.13).

Both Fig.14 and 15 refer to the condition after that run. Notice the radial surface curvature of the segments (1.14 in. radius) as well as the bearing-retainer radius (1.235 in. radius) in Fig. 14. In terms of a radial-bearing geometry, this amounts to a "clearance" ratio of 0.080 and the corresponding Sommerfeld number value was exceedingly low, \( S = 0.00002 \). The specific load characteristics were almost as severe on the axial faces as shown in Fig.15 in terms of the dimensionless load number \( \mu P / U^2 \) where \( \mu \) is the viscosity, \( U \) the linear velocity at segment mid-radius (0.995 in.) and \( F \) the thrust load per unit radial width (0.393 in.) of the segment. It should also be noted that the wear pattern on any surface (except the spot wear mentioned in conjunction with envelope deflection) did not penetrate the 0.0005 in. tin layer with which the bronze segments were coated.
CONCLUSIONS

From the viewpoint of general high-speed turbomachine rotor/flexible suspension system design, the following observations deserve attention:

1. Properly designed hydrodynamic bearings are capable of relieving mass unbalance excitation of flexible rotors at any speed to such extent that safe steady-state running at the critical speed is practically always possible.

2. The simplest bearing-design parameter which can be employed to achieve "self-balancing" is the bearing clearance which should be made large.

3. Floating sleeves which subdivide the large clearance into an inner and an outer film are apparently well suited to avoid film instabilities associated with relief of exciting forces by means of large clearances.

4. It is advantageous to make the load-carrying surfaces of floating bearings slender, \( L/D < 0.5 \).

5. In conjunction with a floating-bearing system, the transmission of axial loads can be effected by fully floating thrust bearings which offer several design, manufacturing and installation advantages.

APPENDIX

The synchronous whirl of a vertical rotor was analyzed by Sternlicht, Poritsky and Arwas (3) in conjunction with a study of dynamic bearing behavior. The same analysis was published subsequently by Pinkus and Sternlicht (5) in a virtually unaltered form. The analyzed configuration consists of a single disk rotor supported on two symmetrically located identical full journal bearings. Lumped parameters of the system are defined by the disk mass \( m \), mass "eccentricity" (distance between mass center and geometrical center of the disk) \( \delta \) as well as a shaft of diameter \( D \), length \( L \) and flexural rigidity \( EI \). The bearings, of length \( L \), have radial clearance \( c \) and are lubricated by a lubricant of viscosity \( \mu \). Shaft journals are displaced radially from the bearing centers for \( e = \delta \phi \) (where \( \phi \) is the relative eccentricity) and the rotor system orbits about the bearing center axis with angular velocity \( \omega \) which is identical to the rotational speed of the shaft. The equation of motion for steady-state conditions is given in equation (68) of reference (3). It will be redefined and interpreted as follows:

\[
\delta^2 = \epsilon^2 c^2 \left( 2 \frac{\kappa \gamma}{\delta} \right) - \epsilon \cos \phi \frac{1}{c} + \left( \frac{\kappa \gamma}{\delta} \right)^2 \frac{1}{\epsilon^4} \tag{4}
\]

where

\[
\kappa = \frac{i}{4 \pi} \mu \frac{L D^3}{r^2}
\]

\[
\gamma = \frac{1}{m} \omega_{n} \left( 1 - r^2 \right) \frac{1}{r^2} \tag{5}
\]
\[
\omega_n = \left(\frac{48 \pi^2}{m L^2}\right)^{1/3}, \quad \omega_n \text{ is the critical speed of the undamped simply supported rotor}
\]

\[
r = \frac{\omega}{\omega_n}, \quad r \text{ is the speed (frequency) ratio}
\]

\[
S = \frac{1}{8\pi} \frac{d^2 \omega}{E}, \quad S \text{ is the Sommerfeld number}
\]

\[
P = \text{applied force per unit projected bearing area}
\]

\[
\phi = \text{is the journal "attitude" angle, defined in equation (4) as directed arc from the tip of the journal displacement unit vector to the tip of the applied force unit vector; positive when in direction of rotation}
\]

For given type of motion, S and \( \phi \) are functions of \( \varepsilon \) and \( L/D \) only. Moreover, for synchronous whirl, \( 0 < \phi < \pi/2 \). All quantities in equation (4), except \( \gamma' \), are inherently positive. The parameter \( \gamma' \) is a function of the speed ratio \( r \) and is positive when \( r < 1 \), negative when \( r > 1 \), equation (5).

When equation (4) is solved for

\[
\gamma' = \frac{\alpha^3 S}{L} \left[ \varepsilon \cos \phi \pm \left( \frac{S^2}{\alpha^2} - \varepsilon^2 \sin^2 \phi \right)^{1/2} \right] \quad (6)
\]

the relationship between journal locus (defined by \( \varepsilon \)) and the shaft speed (defined by \( \gamma' \)) of a given rotor/bearing system can be determined for discrete values of \( \varepsilon \), as suggested in references (3) and (5). However, in order to evaluate critically the possible motion as well as the practically important relationship between \( \alpha \) and \( \delta \), some physical requirements must be observed:

(a) The shaft speed is real, \( \gamma' \) cannot take complex values.

(b) The bearing "fails" if \( \varepsilon = 1 \).

(c) The journal locus must be a single-valued function of the shaft speed, for stable motion.

It follows, from (a) and equation (6), that the assumed motion can persist only at such \( \varepsilon \) that

\[
\frac{\delta}{c} \geq \varepsilon \sin \phi \quad (7)
\]

From (b) and equation (6), the rotor can pass throughout its speed range without bearing failure only if

\[
\frac{\delta}{c} < 1 \quad (8)
\]

since at \( r = 1 \), \( \gamma' = 0 \)

\[
\varepsilon = \frac{\delta}{c} \quad (r=1) \quad (9)
\]

The consequences of requirement (c) can be appraised by drawing a few facts from the lubrication theory.

The function \( \varepsilon \sin \phi \) has an analytical extreme (maximum) for any finite length full journal bearing. The highest value of \( (\varepsilon \sin \phi)_{\max} \) is less than 0.54 (for \( L/D \to \infty \)); the least value of \( (\varepsilon \sin \phi)_{\max} \) is more than 0.43 (for \( L/D \to 0 \)). If \( \varepsilon_m \) is the argument of \( (\varepsilon \sin \phi)_{\max} \), then \( 0.52 < \varepsilon_m < 0.82 \) and \( \cos \phi > \sin \phi \) whenever \( \varepsilon > \varepsilon_m \).

Thus, with respect to a bearing set of given \( L/D \), the following cases are relevant:

\[
\frac{\delta}{c} < (\varepsilon \sin \phi)_{\max} \quad (10)
\]

Practically all turbomachine hardware assemblies are released for service with \( \delta/c < 0.43 \); i.e., they are subject to equation (10). Equation (7) indicates that there will be such \( \varepsilon \) on both sides of \( \varepsilon_m \) where the assumed motion cannot persist.

That is, the operating \( \varepsilon \) must be either low [with maximum value \( \varepsilon_1 = (\delta/c)/\sin \phi_1 \) or very high with minimum value \( \varepsilon_2 = (\delta/c)/\sin \phi_2 \)]. This becomes clearly evident when \( \varepsilon \sin \phi \) is plotted versus \( \varepsilon \). However, high \( \varepsilon \) for systems subject to equation (10) are ruled out per requirement (c) because there would appear up to three values of \( \varepsilon \) for given shaft speed, \( \gamma' \to 0 \), \( r < 1 \), as can be seen from equation (6). Physically, the impossibility of high \( \varepsilon \) becomes obvious for the limit transition \( \delta/c \to 0 \), because then \( \varepsilon_2 \to 1 \). The second case occurs when

\[
(\varepsilon \sin \phi)_{\max} < \frac{\delta}{c} \quad (11)
\]

which might correspond in turbomachine practice to a blade loss. All values of \( \varepsilon \) are then analytically accessible for real \( \gamma' \). The operating eccentricity \( \varepsilon \) could always be brought arbitrarily close to 1 at \( \gamma' \to 0 \) [even for the smallest \( \delta/c \) corresponding to equation (11)] by considering the Sommerfeld number in equation (6), since \( \delta \to 0 \) when \( \varepsilon \to 1 \). However, this can be ruled out per (c) because there will appear [per equation (6)] up to three values of \( \varepsilon \) for given shaft speed when \( \gamma' \to 0 \).

An important analytical conclusion can be drawn from the discussed analysis about the role of bearing clearance upon unbalance-effect relief. A large clearance appears beneficial in a quite wide shaft speed range across \( r = 1 \). First, this can be spot-checked by considering equations (9), (10) and (11). In equation (9), a larger \( \delta \) allows a larger \( \varepsilon \) given \( \varepsilon \), or a smaller \( \varepsilon \) given \( \delta \). Comparing equation (11) with equation (10), it is seen that the former can always be converted (for given \( \delta \)) into the latter if \( c \) is made large enough; a definite ceiling upon the operating eccentricity \( \varepsilon \) would be thus imposed. More general-
ly, a grapho-numerical analysis of all parameters at $L/D = 1/5$ showed the same trends. On the other hand, for $r \ll 1$ and $r \gg 1$, large clearances might not be advantageous. Consider equation (4); its RH side (with $c$ as the only variable) is controlled by $c^2$ if $c$ is "large" but by $1/c^4$ if $c$ is "small." Therefore, if one starts out with too small $c$ values, enlarging $c$ would decrease the permissible unbalance radius $\delta$. Obviously, it is desirable that $\delta$ increase when $c$ increases; this will happen for all $c > c_m$ where $c_m$ is found by solving equation (4) for an analytical extreme:

$$c_m^3 = \frac{\phi Y}{2\epsilon} \left[ -\cos \phi \pm \left( 8 + \cos^2 \phi \right)^{1/2} \right]$$

(12)

Keeping in mind the overall consequences of the quoted analysis (e.g., that if the bearing does not fail at $T=0$ it will not fail at any other speed) equation (12) reconfirms at $T=0$ the advantage of large clearances for purely dynamic load conditions.

REFERENCES


NOMENCLATURE


$[L]$ - $c$ - radial clearance;
$[L]$ - $C$ - diam. clearance;
$[L]$ - $e$ - displacement;
$[L^2F]$ - $E$ - elasticity modulus;
$[F]$ - $F$ - force;
$[L^2T^2]$ - $G$ - gravity acceleration;
$[L]$ - $A$ - couple arm;
$[L]$ - $I$ - inertia moment;
$[L^2F]$ - $k$ - spring constant;
$[L]$ - $L$ - shaft length;
$[L]$ - $L$ - bearing length;
$[L^2F]$ - $m$ - mass;
$[L F]$ - $M$ - moment;
$[L^2P]$ - pressure;
$[L^2F]$ - $P'$ - force parameter;
$[L]$ - $r$ - radius;
$[L^2F]$ - $S$ - Sommerfeld Number.

$[L]$ - $U$ - velocity;
$[L]$ - $X$ - displacement;
$[L]$ - $y$ - deflection;
$[L^2F]$ - $\chi$ - parameter (Eq.5, ref);
$[L^2F]$ - $\theta$ - parameter (Eq.4, ref);
$[L^2F]$ - $\mu$ - viscosity;
$[L]$ - $\tau$ - 3.1416;
$[L]$ - $\phi$ - angle;
$[L]$ - $\omega$ - circular frequency.

Subscripts:

- $i$ - inner;
- $o$ - outer;
- $rms$ - root mean square.

all other subscripts explained in text.