FLOW DEVELOPMENT IN RADIAL TURBINE ROTORS

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ABSTRACT

This paper surveys the development of the primary and secondary flows in the rotors of radial-inflow turbines. Information previously scattered throughout the literature has been brought together, and it has been possible to create a coherent picture and a good understanding of the complex flow processes which occur. The secondary flow is generated by cross-passage forces due to the turning of the blades, and Coriolis forces. Near the leading edge these give rise to a strong vortex adjacent to the pressure surface, moving low momentum fluid from hub to tip. This feature helps to explain why best efficiency occurs typically at 20°–30° negative incidence. Attempts to correlate the optimum incidence angle using traditional slip factor expressions can give quite misleading results, but a new approach based on the blade loading shows considerable promise. Nearer the exit there is motion of fluid from hub to tip near the suction surface, and a vortex in the suction surface-shroud corner, and this is linked to the highly non-uniform flow at exit. The latter effect makes the prediction and correlation of rotor deviation information very difficult, despite the development of a rational exit averaging procedure. The present deviation data are sparse and not easy to correlate.

NOMENCLATURE

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1. INTRODUCTION

The development of analysis models and design methods for radial turbines has lagged behind that of axial turbines, and axial and centrifugal compressors. Meanline, or one-dimensional, models which are used for performance prediction, design and analysis, usually have to resort to simple estimates of important modeling parameters such as loss, blockage and deviation. In almost all cases these parameters are specified as constants or are correlated as simple mathematical curve fits to limited sets of test data, without any real attempt to reproduce the actual flow physics. Occasionally such parameters are correlated against geometric and flow variables, but attempts to do this usually run up against a lack of understanding of the true flow features and inadequate data against which to check and validate the models.

The simple approach suffers from some major deficiencies. One is the range of applicability. With suitable stage performance data, a model may be "tuned" to reproduce accurately the performance of that stage by adjustment of the various constants or coefficients, but because it has no basis in the underlying physical processes, it is of quite uncertain accuracy when faced with another stage of different geometry, or even a different operating condition. It is therefore of very limited use in design. A second problem is more academic but also important: it is that the evolution and use of appropriate flow models, by challenging their developers' knowledge and understanding, actually encourages the development of a physical understanding of the actual flow processes which occur in machines. Simple models are, in this sense, an intellectual dead-end.

Over the past decade, knowledge of the flow field development in the radial turbine rotor blade passages has gradually begun to accumulate. The data are still sparse and come from a variety of unrelated test situations, but it is now possible to use them to formulate a realistic, qualitative picture of the flow processes and particularly the secondary flow development. Data are available from both low speed and high speed tests. From low speed tests, generally done on large scale models, blade surface flow visualizations and detailed traverses using multi-hole pressure probes and hot-wire anemometers are available. From high speed tests, some limited laser velocimeter measurements taken in a blade passage have been made. Further high speed test data is available in the form of exit traverses using multi-hole pressure probes, although in such cases only circumferentially-averaged data are available, due to limitations on the probe time response.

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Additionally, these experimental data are now complemented by flow field predictions using modern three-dimensional Navier-Stokes solvers. The interpretation of such data must always be undertaken with caution, since questions of grid geometry, boundary conditions, turbulence modeling, and the like will always arise. Nevertheless in many cases these predictions may be adequately validated by experimental data, and can give a more complete picture of the overall flow field than is available from experimental measurements which may be confined to relatively limited regions of the blade passage.

The principal focus of this paper is a discussion of the development of the flow processes which occur in the rotor of a radial turbine. It is helpful to divide the rotor conceptually into two regions: an inlet region where the meridional plane flow is primarily in the radial direction, and an exit region where it is primarily in the axial direction. This is an important simplification, and it is also essential to appreciate that it is an artificial division which ignores, for example, the turning process which the fluid must undergo between these two zones. As the basis for beginning to understand the flow development in qualitative terms (as opposed to modeling it quantitatively), it does prove helpful.

2. THE INLET REGION

In terms of influence on turbine design, the dominant effect in this region is the turning of the flow tangentially into the inlet, and the negative incidence at the point of best efficiency that this implies. This has been recognized for many years, and has frequently been deduced from measurements of the turbine performance. However, unless a full set of data from the region between the stator and rotor is available, the incidence angle can only be deduced after assumptions about the loss, blockage and deviation of various components have been made, and hence the results are dependent on the quality of these assumptions. In spite of this, the consensus has been clear that optimum incidence is in the region of $-20^\circ$ to $-40^\circ$. The incidence angle has also been occasionally measured directly by laser velocimeter. Yeo and Baines (1990) found it to be very nearly constant across the rotor span at about $-30^\circ$ for the particular turbine tested at its maximum efficiency point (Fig. 1).

The flow visualizations of Woolley and Hatton (1973) are helpful in understanding the inlet flow field. Here streaklines of the working fluid were visualized by means of entrained particles in a low-speed water flow experiment, for a purely radial-inflow rotor, and the results for three different flow angles are shown in Fig. 2. The most uniform flow distribution was found to be case (b) at an inlet angle of $-40^\circ$. The radial inflow condition (a) shows a strong recirculation on the blade suction surface, while the more negative flow angle (c) results in a recirculation on the pressure surface. It appears from this that the optimum flow condition is probably somewhere between those of cases (a) and (b).

The cause of this behavior is principally the pressure difference between the pressure and suction surfaces of the rotor blades which causes fluid to be drawn in, and as a result a secondary flow is set up in the blade passage, in the form of a circulation in the opposite direction to the passage rotation. If the circulation is sufficiently large, the flow will separate and stagnate on the pressure surfaces of the blades. This will happen at large negative incidence. Zero or positive incidence will reduce the strength of the circulation and reduce this tendency, but will also have the effect of reducing the cross-passage pressure gradient and make the flow more likely to separate on the suction surface.

This same effect can be seen in a high-speed turbine operating with a high speed flow, in which laser velocimeter measurements show a substantial movement of fluid from the suction to the pressure side of the passage (Fig. 3). This motion is particularly strong near the hub surface, but it is also apparent near mid-passage. Based on this data, Kitson (1992) shows computer flow predictions which illustrate clearly the development of a strong vortex near the pressure surface just downstream of the leading edge, with a migration of fluid up the pressure surface from hub to shroud (Fig. 4). This vortex is clearly the dominant secondary flow in the inlet region of the turbine. Its effect is particularly strong in rotors with sharp meridional plane curvature (low inlet-exit tip radius ratio). Baines and Yeo (1991) present evidence for its effect being felt upstream of the rotor itself, and influencing the flow in the volute of a nozzleless turbine.

The importance of the correct choice of optimum incidence angle is made clear in Fig. 5. Here a radial turbine constant speed line is plotted for several different optimum incidence angles. The basis of the meanline analysis model used for this purpose is described by Japikse and Baines (1994). For the purposes of this illustration, the turbine geometry used was that of one tested by Ricardo & Co. and reported by Hictt and Johnston (1963). Although of considerable age, this remains one of the most comprehensive sources of radial turbine data.
test data in the open literature. The experimental data are also plotted in Fig. 5 to check the basic accuracy of the model, but the crucial point about this figure is the comparison between the sets of results at different values of assumed optimum incidence.

The effect is most clearly seen on the efficiency. The choice of optimum incidence affects both the point of maximum efficiency (in terms of velocity ratio $U/C_p$), and also the efficiency values. Differences in efficiency become particularly marked at overspeed conditions. Figure 5 also shows $\beta_3$, the predicted gas flow angle relative to the rotor inlet, and this emphasizes that it is the choice of optimum incidence angle which influences the model results. It is clear that in stage design, the correct choice of optimum incidence angle is important for a realistic prediction of the stage performance.

It has often been suggested that the optimum incidence angle of a radial turbine be predicted using slip factor correlations originally derived for centrifugal compressors. For example Stanitz (1952) predicted the slip factor $\mu = C_2^2/U_2$ as a function of the blade number $Z$:

$$\mu = 1 - 0.63\pi/Z \quad (1)$$

This correlation comes from an inviscid solution of flow in the radial outflow of a compressor rotor. Such a solution is easily and rigorously reversed for an ideal inviscid turbine flow. Its accuracy in radial turbine cases was investigated by examining the data from a number of turbine tests. In each case it was assumed that the maximum efficiency point also corresponded to the point of optimum incidence, and the incidence angle was obtained from the test data. When plotted against blade number, a very poor correlation was obtained (Fig. 6). In many cases, the Stanitz correlation appears to predict a larger number of blades (or a very different slip factor) than was actually used, and even the modified form of the correlation introduced by Chen and Baines (1994), which was intended to reproduce the radial turbine flow field more realistically, did little better in correlating the data.

A criterion for the minimum number of blades necessary to avoid separation of fluid from the blade surfaces was obtained by Jamieson (1955) and based on assumptions that the flow velocity varied linearly across the blade passage from suction to pressure surface, and that separation would occur when the blade spacing was large enough to reduce the pressure surface velocity to zero at the blade tip. On this basis the minimum blade number was a simple function of approach flow angle:

$$Z_{\min} = 2\pi \tan \alpha_2 \quad (2)$$

For a typical stator exit angle of 70 degrees, this equation implies that at least 17 blades are required to avoid separation. This is more blades than is often used, which indicates that designers may have
Figure 5. The effect on performance of assumed optimum incidence angle (a) efficiency (b) rotor inlet flow angle

found from experience that Jamieson's method is pessimistic, either because the analysis is oversimplified or because radial turbine rotors are tolerant of some separation in the inlet region. In many cases, however, designers are limited by other considerations such as passing sufficient flow or maintaining enough hub-line separation for manufacturing. Glassman (1976) considered that the Jamieson relation gave too many blades, and preferred a purely empirical relation (for $\alpha_2$ in degrees):

$$Z_{\text{min}} = \frac{\pi}{30}(110 - \alpha_2)\tan\alpha_2$$  \hspace{1cm} (3)

In general, however, the loading coefficient provides a much more rational basis for estimating the optimum inlet angle and avoids the need for any empirical input other than the choice of loading and flow coefficients. The blade loading coefficient is here based on the inlet blade speed $U_2$, and can be expressed using the Euler turbomachinery equation as:

$$\psi = \frac{\Delta A_0}{U_2^2} = \frac{C_{\theta_2}}{U_2} - \frac{C_{\theta_1}}{U_2}$$  \hspace{1cm} (4)

where $\epsilon = r_2/r_1$ is the rotor radius ratio. The exit swirl is normally fairly small, so that the second term on the right hand side is small with respect to the first, and the loading coefficient can usually be approximated as:

$$\psi \approx \frac{C_{\theta_2}}{U_2}$$  \hspace{1cm} (5)

The flow coefficient is defined in terms of the exit meridional velocity, also non-dimensionalised by the inlet blade speed:

$$\phi = \frac{C_{\theta_3}}{U_2}$$  \hspace{1cm} (6)

These are related to the rotor inlet flow angle $\beta_2$ by:

$$\psi \approx \frac{C_{\theta_2}}{U_2} = 1 + v\tan\beta_2$$  \hspace{1cm} (7)

Figure 6. Optimum rotor inlet angle, compared with correlations. Data points show measured total-to-static isentropic efficiency (Chen and Baines 1994)

where $v = C_{m_2}/C_{m_3}$ is the meridional velocity ratio for the rotor. For chosen design point values of the loading and flow coefficients, Eq. (7) provides the design rotor inlet angle.

Figure 7 shows a correlation of radial turbine efficiency with these coefficients, using test data taken from some 40 different stages and a variety of sources. The reported total-to-static efficiency is shown in the figure against each performance point, and the correlation between them proves to be sufficiently good to allow lines of constant efficiency to be drawn over a wide area of the map. The data used here comes from a wide variety of stage designs, but this only serves to emphasize the universality of this chart.

Maximum efficiency occurs for flow coefficients in the range 0.2-0.3, and at loading coefficients between about 0.9 and 1.0. In many cases, of course, designers have used values significantly different from these optima. In some cases, particularly with the older data, this probably stems from a lack of awareness of these factors, but in other cases the application constrains the design. For example, a higher than optimum flow coefficient increases the meridional velocity at exit and reduces the exit area for a given mass flow rate, and this results in a more compact and lighter stage. For a loading coefficient of 0.9 and a flow coefficient of 0.25, and a meridional velocity ratio of unity, the optimum flow angle is $-21.8^\circ$. Similarly, a loading coefficient of 0.95 and the same flow coefficient give an optimum angle of $-11.3^\circ$. These values are quite consistent with experience.
3. THE EXDUCER REGION

In the exducer region of the rotor the flow is predominantly in the axial and tangential directions, and the high turning of the flow towards the trailing edge means that the latter is the largest principal component of velocity. This gives rise to a Coriolis acceleration now in the radial direction which tends to move fluid from hub to shroud. There is additionally the cross-passage acceleration acting between the blade surfaces as a result of the turning of the flow in the tangential direction. This is exactly the same as the mechanism at work in axial turbine blade passages, which gives rise to the classic picture of passage vortex development. The net result of these forces is a complex secondary flow development in this region, which typically results in non-uniform distributions of blade loading and of flow velocity at the trailing edges of the blades.

The blade surface flow visualization pictures in the low-speed radial turbine rig of Huntsman (1993) show clearly the motion of low-velocity fluid on the blade suction surface towards the shroud, but to a much lesser degree on the pressure surface where the streamwise velocity is much larger and the cross-stream velocities are proportionally lower (Fig. 8). The suction surface also shows evidence of some inward flow near the blade tips, creating a 'herring bone' pattern characteristic of a localized lift-off line. This inward flow is almost certainly caused by fluid leaking across the blade tip and forming a vortex in the shroud-suction surface corner of the passage.

Kitson’s (1992) laser velocimeter measurements in Fig. 9 taken from a high-speed turbine also show a very significant movement of fluid towards the suction surface under the effects of blade loading at several spanwise locations just downstream of the trailing edge. Flow field predictions in Fig. 10 also show many of these exducer effects. The extent of the spanwise movement on the suction surface is smaller than that shown in Fig. 8, but the trends are very similar. The
schematic diagram of Kitson (1992) shown in Fig. 10 is an attempt to interpret the cross-passage flows in this region, and shows the interaction of Coriolis, curvature and tip leakage flows.

A number of exit traverse measurements are consistent with this pattern of flow development in the rotor passage. The stagnation pressure loss measured at a plane immediately adjacent to the trailing edge shows clear evidence of a high-loss flow in the shroud suction surface corner and low loss elsewhere (Fig. 11, from Huntsman 1993). The graph on the right of this figure shows the mass-average total pressure loss at each radius, and emphasizes this point. Exit traverses of a similar low-speed turbine by Zanganeh et al. (1988) show a large positive exit swirl near the tip, indicating considerable under-turning in this region, but very small swirl values throughout the rest of the passage.

Data from high-speed tests are more difficult to interpret because only circumferentially averaged data from low response instruments are available, and any cross-passage effects are lost. Exit traverses by Baines (1992) and by Kofskey and Wasserbauer (1969) indicate that the flow is under-turned near the tip and over-turned towards the hub.

4. ROTOR DEVIATION

The foregoing discussion of the exducer flow field should have made it clear that considerable deviation angles can exist in the flow leaving the rotor, and also that the deviation angle varies in both the radial and the circumferential directions. It is an unfortunate fact that the performance of a turbine depends strongly on the exit angle, and that an accurate estimate of the deviation is important in predicting both the mass flow rate and the efficiency of the stage. Figure 12 is again based on a Ricardo turbine reported by Hiett and Johnston (1963), and shows the effect on the predicted performance of a changing the rotor deviation by ±5°.

At present good data of deviation angle are hard to come by, and harder still to correlate with rotor geometry and flow conditions. Mizumachi et al. (1975) demonstrate that the $\cos^{-1}$ (throat/spacing) rule often used for axial turbines is a poor correlator. Because of the variation in the flowfield at exit caused by secondary flow generation in the blade passage, the deviation angle varies with radius and between blades, and so a representative average must be determined from traverse data in order to be of use to a mean line analysis. A rational technique is to mass average the meridional component of velocity $C_m$ to ensure continuity, and momentum average the tangential component $C_0$ to conserve angular momentum. From these, first the absolute flow angle $\alpha$ and then the relative flow angle $\beta$ and the deviation $\delta$ can be obtained from the velocity triangle:

\[
C_m = \frac{1}{m} \int_0^R 2\pi r C_m dr
\]

\[
C_0 = \frac{1}{m} \int_0^R 2\pi r C_0 C_m dr
\]

\[
\alpha = \tan^{-1}\left(\frac{C_0}{C_m}\right)
\]

\[
\beta = \tan^{-1}\left(\frac{C_0 - \bar{C}}{C_m - \bar{C}}\right)
\]

\[
\delta = \beta - \beta_0
\]

Figure 9. Flow field measurements in plane C (see Fig. 3) (Kitson 1992)

Figure 10. Relative velocities near exit: prediction of vectors and schematic showing secondary flows (Kitson 1992)
in Table 1. In the case of the Ricardo turbines, the deviation angles are
deduced from the reported mean values of exit total pressure and total
temperature, but in all other cases the angle is obtained from full exit
traverse data, with measurement of exit total and static pressure, and
yaw angle. All exit flows were subsonic. It is apparent that a wide
range of values exist, both positive and negative. Some variation in
deviation angle is to be expected, given the different turbine
geometries and exit blade thicknesses, and in many cases there was
also a considerable variation in flow angle with radius which further
confuses the picture. There is clearly a major difficulty to be faced in
determining an appropriate value of rotor deviation for mean line predictions. The
deviation angle also changes with operating
condition, as shown in Fig. 13. At the design point of 100% speed,
\( U/C_s = 0.7 \), the deviation angle is about \(-7.5^\circ\), but away from design,
at lower speeds or lower pressure ratios it becomes less negative.

<table>
<thead>
<tr>
<th>Source</th>
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<td>Ricardo (1957) turbine B</td>
<td>-7.1</td>
</tr>
<tr>
<td>Ricardo (1963) turbine A</td>
<td>-7.8</td>
</tr>
<tr>
<td>Ricardo (1963) turbine C</td>
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<td>Miller and L'Ecury (1985)</td>
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<tr>
<td>Omichinski and L'Ecury (1988)</td>
<td>-3.2</td>
</tr>
<tr>
<td>Baines (1992)</td>
<td>-7.5</td>
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Table 1. Comparison of rotor mean deviation angle measurements

5. DISCUSSION

When one considers the performance prediction and modeling in
other types of turbomachines, it becomes apparent that developments
in meanline models have followed from the better understanding of
the flow through the machine. The realization by Dean and others
(see, for example, Dean and Senoo 1960) that the flow in a centrifugal
compressor impeller can rationally be divided into primary and
secondary zones formed the basis of the two-zone impeller model of
Japikse (1985), which has eliminated much of the uncertainty
associated with arbitrary modeling parameters, by largely dispensing
with them. It appears to be a very realistic expectation that a similarly
developing understanding of radial turbine rotor flow fields will form
the basis of new and better mean line models in just the same way.
The development of the secondary flow in the exducer has the effect of concentrating low-momentum fluid near the suction surface-shroud corner of the passage, and strongly suggests that a two-zone model can also be applied here. In this model, the two zones are considered separately: a primary zone, in which the flow expands without loss or with a very low level of loss, and a secondary zone, in which the flow is dominated by shear processes and the bulk or all of the rotor loss resides. The primary flow where the bulk of the work transfer occurs will typically underturn as the blade unloads at the trailing edge, while the secondary, low momentum fluid will tend to follow the blade and may even overturn to some extent (as happens in many centrifugal compressors). The two flows are mixed in a control volume of infinitesimal meridional extent to provide a final, mixed-out, state. In fact, this physical separation of the flows is not a necessary basis for the model, and it is entirely possible to construct a model on the assumption that the secondary flow is more widely dispersed through the primary flow. However, the development of the model is greatly helped by the knowledge that it well approximates the reality of the flow field.

6. CONCLUSIONS

Some of the major secondary flows which exist in the rotor of a radial turbine can now be identified. The principal feature which influences the rotor inlet region is a vortex near the pressure surface, which has its origin close to the hub and moves low-momentum fluid towards the shroud. This vortex is brought about by the Coriolis force acting in this region, and causes the incidence angle for maximum efficiency to be typically 20°–30° negative. No simple method to predict the optimum incidence for a turbine currently exists (those based on centrifugal compressor slip factors are not reliable), and it is suggested that a fundamental consideration of the rotor blade loading and flow coefficients offers the best approach to this problem.

In the exducer region the principal secondary flow is created by a combination of a cross-passage force due to the blade turning and a Coriolis force in the radial direction, and these produce a zone of low-momentum fluid near the suction surface-shroud corner of the passage. The effect of this is seen in a highly non-uniform flow field at the exit of the rotor. This secondary flow is a major contributor to the difficulty in correlating satisfactorily the rotor exit deviation angle, although it is most likely that other factors, such as rotor blade trailing edge thickness, are also important.

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