A NEW ENDWALL MODEL FOR AXIAL COMPRESSOR THROUGHFLOW CALCULATIONS

John Dunham
Propulsion Division
Defence Research Agency
Pyestock, Farnborough, Hampshire
United Kingdom

ABSTRACT

It is well recognised that the endwall regions of a compressor - in which the annulus wall flow interacts with the mainstream flow - have a major influence on its efficiency and surge margin. Despite many attempts over the years to predict the very complex flow patterns in the endwall regions, current compressor design methods still rely largely on empirical estimates of the aerodynamic losses and flow angle deviations in these regions. This paper describes a new phenomenological model of the key endwall flow phenomena treated in a circumferentially-averaged way. It starts from Hirsch and de Ruyck's annulus wall boundary layer approach, but makes some important changes. The secondary vorticities arising from passage secondary flows and from tip clearance flows are calculated. Then the radial interchanges of momentum, energy and entropy arising from both diffusion and convection are estimated. The model is incorporated into a streamline curvature program. The empirical blade force defect terms in the boundary layers are selected from cascade data. The effectiveness of the method is illustrated by comparing the predictions with experimental results on both low speed and high speed multistage compressors. It is found that the radial variation of flow parameters is quite well predicted, and so is the overall performance, except when significant endwall stall occurs.

NOMENCLATURE

- $s'$: staggered pitch, $s \cos \alpha$
- $S$: distance along streamline
- $v$: velocity
- $\Delta v_r$: radial velocity difference across the trailing edge of a blade
- $W$: axial velocity
- $x$: axial distance
- $\alpha$: flow angle
- $\delta$: boundary layer thickness
- $\delta^*$: displacement thickness
- $\theta_{xx}$: axial momentum thickness
- $\theta_{en}$: tangential momentum thickness
- $\mu_t$: eddy viscosity
- $\xi$: vorticity
- $\rho$: density
- $r_f$: distance measured across staggered pitch
- $\Omega$: angular velocity

Subscripts

e: at edge of boundary layer
mid: at mid-pitch
r: radial
s: streamwise
sec: due to secondary flow
wall: relative to the wall
x: axial
\theta: tangential
\phi: normal to the streamline

1. INTRODUCTION

The flow through an axial compressor is three-dimensional, viscous, and unsteady. Research is in progress on algorithms for computing such flows, but meanwhile the design and
analysis of turbomachines will continue to be done in two interlinked phases: computation of the "throughflow" - the time-averaged and circumferentially-averaged flow through the whole machine, known as the "S2" solution - and computation of the "blade-to-blade" flow through individual blade rows in more detail, known as the "S1" solution.

Considerable advances in the blade-to-blade (S1) computations are still being made. Quasi-three-dimensional steady viscous-inviscid calculations or viscous (Reynolds-averaged Navier-Stokes) calculations are in regular use and so are fully three-dimensional steady viscous calculations. Unsteady codes are emerging. Less effort is being devoted to parallel improvements in the throughflow (S2) codes, which are needed to supply the boundary conditions for the blade-to-blade codes, and which provide the framework of all multistage designs. Codes are now available, however, which solve the viscous throughflow equations (Adkins and Smith, 1982, Gallimore and Cumpsty, 1986, de Ruyck and Hirsch, 1988, Howard and Gallimore, 1992, Ucer and Shreeve, 1992, Dunham, 1992), and it is time to improve the boundary conditions employed in doing so.

The boundary conditions required by any throughflow code are:

(1) the pitchwise-average stagnation pressure at every calculation station, normally expressed as a loss coefficient to be applied to some upstream value;
(2) the pitchwise-average flow angle;
(3) an effective blockage, to account for boundary layers.

In principle, these boundary conditions should be supplied (except of course at inlet) from the blade-to-blade solution. In practice, quasi-three-dimensional blade-to-blade calculations (that is, calculations along a meridional stream surface produced by the throughflow calculation) are used to generate two-dimensional (2D) loss and angle values. Then, recognising that the annulus wall boundary layers, secondary flows, and tip clearance flows have important effects in the endwall regions, three-dimensional (3D) corrections are applied and also a blockage is introduced. It should be possible - and in due course it will become possible - to predict the full 2D+3D loss, angle, and blockage from fully-3D computations. At present, empirical 3D corrections are used.

The 3D corrections now in use have either been obtained by correlating measurements made in cascades (Roberts et al., 1988) or by indirect deductions from multistage tests - that is, by seeing what endwall assumptions seem to give the observed throughflow (Calvert et al., 1989; Dunham, 1992). These ways of arriving at 3D corrections have serious defects. It is obvious that the incoming endwall boundary layer must affect the result, as must the tip clearance, but few systematic measurements are available to correlate. Furthermore, it is believed that tailoring the aerofoil shape in the endwall region ("endbending") and tailoring the endwall shape itself can improve the performance and surge margin, presumably by changing the "3D corrections", and the correlation approach is fundamentally incapable of predicting such changes.

Leboeuf (1984) presented a complex analytical approach to predicting endwall phenomena which appears to be heading towards fully-3D computations. De Ruyck and Hirsch (1988) proposed a simpler analytical scheme which was successful for a single stage and forms one of the starting points of the present method.

The purpose of the research reported here is to produce an analytically-based endwall model, using both annulus wall boundary layer (AWBL) theory and secondary flow theory, which replaces wholly empirical 3D corrections by explicit calculations of loss, angle, and blockage in the endwall regions. By this means, the plan is to make the S1/S2 design and analysis system (Calvert and Ginder, 1985) predict the correct overall performance and radial distributions without using empirical losses and gas angles.

Section 2 of this paper describes the new endwall model. Sections 3 and 4 describe its application to a linear cascade and to a rotating cascade respectively, utilizing extensive experimental data. Then in Section 5 some complete multistage compressors are analysed.

2. THE ENDWALL MODEL

2.1 The Physical Phenomena Modelled

A complex three-dimensional boundary layer develops along the annulus walls. For the purposes of a throughflow calculation, the circumferential average boundary layer is modelled. The displacement thickness, in particular, determines the "blockage".

Leylek and Wisler (1991) have shown that the flow within a compressor is significantly affected by the radial transfer of momentum, energy, and entropy across the meridional streamlines by two physical mechanisms: turbulent diffusion and turbulent convection. Both are modelled.

Secondary flow (Fig 1) is a well-known phenomenon which generates streamwise vortices which alter the pitchwise-average flow angles. At the exit from a blade passage the flow near the wall turns more than it would in mid-stream ("overturning") and the flow some way away from the wall (just outside the boundary layer) turns less than it would in mid-stream ("untwisting"). Although secondary flow is an inviscid phenomenon it only occurs to a significant extent because the incoming AWBL constitutes the normal vorticity needed to create it. The AWBL model itself, however, only predicts the crossflow within the boundary layer; a separate secondary flow model is provided to predict the overturning outside the boundary layer.

Finally, tip clearance effects are modelled. When there is a clearance between the end of the blade and the wall, air flows from the pressure side of the aerofoil to the suction side through the clearance, creating a tip clearance vortex in the opposite sense to the passage secondary vortex (Fig 1). Measurements show that both a secondary vortex and a clearance vortex develop, but the latter is stronger. So the net vorticity generates overturning outside the AWBL (which the secondary flow model predicts) and overturning near the wall (which the AWBL model predicts).
2.2 The Annulus Wall Boundary Layer Model
The AWBL model chosen is that of Hirsch and de Ruyck (1981), with modifications. This is an integral boundary layer method, which avoids the need to use a fine grid in the AWBL regions. The key assumption in this type of method is the blade defect forces - that is, the extent to which the blade lift and drag change through the boundary layer. Hirsch and de Ruyck proposed algebraic expressions for the defect forces, in a form justified by the physical phenomena involved but with four empirical constants chosen to fit available experimental data. These expressions have been retained but with different empirical constants. The details are given in Dunham (1993), including an account of how the constants were chosen.

2.3 The Secondary Flow Model
There are two steps in the calculation of secondary flow: the calculation of the streamwise vorticity, and the calculation of the pitchwise-average tangential velocity induced by it. The equation given by Marsh (1974) is used to calculate the passage secondary vorticity, and the finite difference scheme of de Ruyck and Hirsch (1988) is used to calculate the pitchwise-average changes in flow angle. The details are given in Appendix 1.

2.4 The Tip Clearance Model
A new simple model is proposed (Fig 2). The bound vorticity on each blade is assumed to be shed at its geometric tip. This is represented in the streamline curvature model by a vorticity at the nearest grid point adjusted to give the same induced circulation as the actual tip vortex (Dunham, 1993). This vorticity is added to the passage secondary vorticity before solving as explained in Section 2.3 for the secondary deviation. Since the tip clearance vorticity is usually larger than the passage secondary vorticity and of opposite sign, it reverses the direction of the net circulation.

2.5 The Spanwise Mixing Model
The viscous form of the streamline curvature equations, as derived by Gallimore and Cumpsty (1986), is solved, to enable spanwise mixing effects due to turbulent diffusion to be included. Following Dunham (1992), the freestream eddy viscosity is calculated from the equation

$\nu = \text{density} \times \text{axial velocity} \times \text{span} \times \text{GAL}$,

where GAL is an empirical constant assumed to be the same throughout the cascade or compressor.

Within turbomachinery blading, however, the strong passage vortices located near the edge of the annulus wall boundary layers, shown in Fig 1, stir up the boundary layers, convecting mainstream fluid into them and convecting low energy fluid into the mainstream instead. Using the radial velocities calculated in Section 2.3, an estimate is made of the mass flow exchanged between the boundary layers and the mainstream and hence the radial interchanges of momentum and entropy due to turbulent convection. The details are given in Appendix 2.

3. APPLICATION TO A LINEAR CASCADE
Salvage (1974) tested a large number of low speed cascades over a range of conditions, and reported measurements of the spanwise variation of the pitchwise-mean local deflection and pressure loss, concentrating on the endwall region. (The local deflection is the change in gas angle from the upstream to the downstream traverse plane at that distance from the wall, and the local loss coefficient is similarly defined with respect to the upstream flow at that distance from the wall.) These measurements were used to choose values of the two empirical constants in the AWBL method which do not involve tip clearance, and also to select the value GAL=0.0004. Several examples are given in Dunham (1993); one is shown in Fig 3, a 45° camber cascade tested with 8=13% span (run 35) and 25% span (run V12). The predictions are remarkably good. Notice the change in mid-span deflection caused by the
secondary flow at the low aspect ratio of unity.

4. APPLICATION TO AN ISOLATED ROTOR

Inoue et al (1987) conducted a systematic set of tests on a low speed isolated rotor, measuring the inlet and outlet flows, and reporting the spanwise variation of pitchwise-averaged values. They varied the tip clearance from 0.5 to 5 mm (blade height 89.5 mm). One illustration of their results has been selected. Runs 1A and 1C were with 0.5 mm and 2 mm tip clearance respectively, at a lift coefficient of 0.5 and an inlet casing boundary layer thickness $\delta = 3.5\%$ span. (The hub inlet boundary layer was not measured in detail.) The constants chosen from the Salvage runs were retained, including GAL, and the tip clearance-related constants were chosen to match the Inoue results as far as possible.

Fig 4 compares predictions and measurements for runs 1A and 1C. The small tip clearance results are well predicted, and the changes due to the larger tip clearance are quite well followed. Notice the extra effective blockage increases the axial velocity. The tip loss is much bigger.

5. APPLICATION TO COMPLETE COMPRESSORS

Robinson (1991) tested a low speed four stage compressor with stage loadings representative of an aero-engine core compressor. He reported pitchwise-average pressure ratios and flow angles after stage 3 rotor and stator.

Fig 5 compares the measurements on his free vortex datum build near peak efficiency with predictions using the present method. (The pressure rise coefficient is defined as the stagnation pressure rise divided by (density $\times$ mean blade speed$^2$).) In making the predictions the two-dimensional loss coefficients were chosen to match the observed mid-span results, but the four constants and GAL chosen in Sections 3 and 4 were retained. In general the endwall effects are well-predicted except for the deflection at the casing end of the stators. The predicted polytropic efficiency is 86.4% which compares well with the measured 86.8%.

Calvert et al (1989) tested a four-stage high speed compressor with cantilevered stators, designated C147, representative of the rear stages of an aero-engine core compressor, and undertook inter-stage traverses. Their results were analysed by Dunham (1992) using fitted empirical endwall corrections. Fig 6 shows the predictions obtained using the present method employing 2D losses and gas angles calculated using the method of Calvert (1982), and again the same empirical constants. This is therefore a direct test of the S1/S2 system with the present endwall model. 39 axial planes and 21 equally-spaced radial stations were used in the S2
It will be seen that the overall temperature and pressure rise at the design mass flow are under-predicted. The predicted overall polytropic efficiency is 90.1% as against the measured 89.1%. (To reach the measured pressure ratio the flow has to be reduced by about 3%.) Also, the boundary layers predicted in stages 3 and 4 are much too thick, especially at the casing. These inaccuracies are directly linked. As can be seen in Figs 6c and 6d, the excessively thick predicted casing boundary layer gives rise to exaggerated secondary deviation in the mainstream and hence too low an enthalpy rise. Without the spanwise convection model this problem was much worse. Fig 6e shows the efficiency distribution. (The "measured" values were evaluated along the predicted streamlines.) It will be seen that the predictions are good at stage 2 outlet, but the losses in the outer half of the annulus in the later stages are underestimated.

The AWBL is not predicted to separate anywhere, but very high shape factors are predicted at both ends of stage 1 rotor: 1.82 at the hub, where a corner separation was seen in the test results (Dunham,1992) and 1.83 at the casing. Large increases in the boundary layer thickness occur on both walls. Since the annulus height is reducing, the calculated boundary layers at the OGVs fill two-thirds of the span. They are clearly too thick.

The problem is believed to lie in an underprediction of the
The radial variations of the stator outlet angles are predicted satisfactorily, though the "hump" in the gas angle at OGV exit is exaggerated and misplaced radially. (There were no traverses after the rotors.)

The low speed results on cascades and compressors are not very sensitive to the value of the turbulent diffusion constant $G_{AL}$. This is not surprising, as its main effect is to prevent high temperatures accumulating near the walls of a high speed compressor (Dunham, 1992). The level $G_{AL}=0.0004$ chosen from the Salvage results appears satisfactory for the C147 unit.

Doubling it reduces the pressure ratio by 0.025 and the efficiency by 0.5%. This value of $G_{AL}$ is far smaller than the value 0.005 needed in Dunham (1992) because in the present method the AWBL treatment prevents the high temperatures at the walls.

6. DISCUSSION

Section 5 has shown that the turbulent mixing constant and the four "universal" constants in the AWBL blade force defect
terms, chosen from cascade data, appear acceptable for the low speed compressor cases tested, but predict too thick a boundary layer in the high speed compressor. A wider range of test cases at high speed is needed to resolve this question.

Salvage's measurements were made on lightly-loaded cascades. There is no suggestion of endwall or corner stall in any of the measurements or calculations. But other tests on cascades and compressors in adverse aerodynamic conditions have shown clear indications of corner stall. Papailiou et al (1976) tested a cascade similar to one of Salvage's but at higher incidence and found corner stall. Inoue's cascades were also lightly-loaded, but Fig 7 shows how the outlet velocity profile changed as the tip clearance was increased. At 5 mm tip clearance the velocity profile suggests AWBL separation at the tip. Dring (1992) pointed out that corner stall greatly increases the spanwise convection. The present method cannot predict the onset or the consequences of corner stall.

Apart from that, the comparisons (of which a selection have been shown) suggest that the method is capable of predicting the three-dimensional effects in a compressor realistically. There is a need for a more extensive assessment of its accuracy - and perhaps some refinement of the empirical assumptions - especially in the context of high speed compressors.

7. CONCLUSIONS
The application of annulus wall boundary layer theory together with secondary flow calculations and the present spanwise diffusion and convection models appears to offer a promising foundation for a more satisfactory and more accurate way of predicting the endwall effects than any published purely empirical scheme.

The application of the method to a variety of cases draws attention once again to the significance of endwall corner separations, which the method is unable to predict. Much more research is still needed on endwall flows.

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APPENDIX 1
CALCULATION OF THE SECONDARY FLOW

![Fig 8 NOTATION](image)

The secondary flow is calculated in the plane normal to the mainstream flow leaving the blade row (the Trefftz plane, Fig 8). The coordinates are S, r\(\phi\), and r (radial), with velocity components \(v_r\), \(v_\phi\), and \(v_s\) respectively, and \(s' = s \cos \alpha\) is the staggered pitch.

The continuity equation is

\[
\frac{1}{r} \frac{\partial}{\partial r} (rv_r) + \frac{1}{r} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_s}{\partial s} = 0 \quad \ldots (1)
\]

The streamwise vorticity is defined as

\[
\zeta_s = \frac{1}{r^2} \frac{\partial}{\partial r} (rv_\phi) - \frac{1}{r} \frac{\partial v_r}{\partial \phi} \quad \ldots (2)
\]

Making assumptions similar to those of de Ruyck and Hirsch,

\[
\frac{\partial v_r}{\partial s} = 0 \quad \ldots (3)
\]

\[
v_r = \frac{1}{s} \int_0^s (\phi - \phi_{mid}) \quad \ldots (4)
\]

where \(\Delta v_r\) is the difference between the radial velocities on the pressure and suction surfaces of the aerofoil at radius r and \(\phi_{mid}\) is the value of \(\phi\) at mid-pitch; and

\[
v_\phi = \frac{3}{2} \bar{v}_s \int_0^s \left[ 1 - \frac{4r^2}{s^2} (\phi - \phi_{mid})^2 \right] \quad \ldots (5)
\]

where \(\bar{v}_s\) is the pitchwise mean value of \(v_r\).

Since \(s' = 2\pi r/N \cos \alpha\) for a row of N blades, inserting equations (3), (4), and (5) into (1) gives

\[
\frac{d}{dr} (r Av_r) = \frac{6N}{\pi r \cos \alpha} \bar{v}_s \quad \ldots (6)
\]

Inserting (3), (4), and (5) into (2) and averaging pitchwise gives

\[
\bar{v}_s = \frac{1}{r} \frac{d}{dr} (r v_{s,\infty}) = \frac{N}{2\pi r \cos \alpha} \Delta v_r \quad \ldots (7)
\]

Substituting for \(\bar{v}_{s,\infty}\) from (6) into (7),

\[
\frac{1}{r} \frac{d}{dr} \left( \frac{d}{dr} \frac{d}{dr} (r Av_r) \right) = \frac{N}{2\pi r \cos \alpha} \Delta v_r = \bar{v}_s
\]

To simplify the solution by avoiding a \(d/dr\) term, put \(G = r^2 \Delta v_{r,-}\)

\[
\frac{d^2 G}{dr^2} + \left( \frac{1}{\alpha^2} \frac{3N^2}{\pi^2 \cos^2 \alpha} \right) G = \frac{6N r \bar{v}_s}{\pi \cos \alpha^2}
\]

In order to calculate the secondary flow velocity field from the known streamwise vorticity distribution \(\zeta_s\), this equation is solved for \(G\) using Thomas' algorithm (Na, 1979), as recommended by de Ruyck and Hirsch (1988). Then \(\bar{v}_{s,\infty}\) is calculated from equation (6).

Thomas' algorithm is a numerically efficient method of integrating an ordinary differential equation between two points at which the boundary conditions are known.

APPENDIX 2
SPANWISE CONVECTION BY SECONDARY FLOW

The radial velocity due to the secondary flow is assumed to vary linearly across the passage from \(\frac{1}{2} \Delta v_r\) on one side of the passage to \(-\frac{1}{2} \Delta v_r\) on the other side (eqn 4), so the mass flow transferred across the meridional streamlines in a streamwise distance \(dS\) is

\[
\xi_r = \frac{1}{r} \frac{\partial}{\partial r} (r v_r) - \frac{1}{r} \frac{\partial v_r}{\partial \phi} \quad \ldots (2)
\]
Fig. 9 MOMENTUM EXCHANGE MODEL

\[ dm = \rho r^2 \left( \Delta v_\alpha \right) \frac{s'}{2} \|	ext{d}S. \]

where \( s' = \text{staggered pitch}. \)
But \( dS = dx \sec \alpha \) and \( \text{N}_S' = 2\pi r \cos \alpha, \) so

\[ dm = \frac{1}{4} \rho r^2 \| \Delta v_\alpha \\| dx. \]

\( \Delta v_\alpha \) is calculated only at the trailing edge of the blade row. If it is assumed that the streamwise vorticity (which gives rise to it) develops linearly through the passage, the total \( \Delta m \) across the row is

\[ \Delta m = \frac{1}{8} \rho r^2 \| \Delta v_\alpha \\| \Delta x. \].....(8)

As pointed out in Section 2.1, if there is an endwall clearance both a secondary vortex and a tip clearance vortex develop. The model assumes that each vortex convects mass separately; the total mass flow convected \( \Delta m \) is taken to be the sum of the amounts convected by each vortex.

The exchange of momentum and enthalpy across the edge of the boundary layer is illustrated in Fig 9.
The exchange of momentum and enthalpy across the edge of the boundary layer is

\[ \int_0^h 2\pi r \rho v^* dn = 2\pi r \rho v^*[v_\alpha(\delta - \delta') - v_\alpha(\theta_\alpha)] \]

and it is increased by an amount

\[ \Delta \dot{m} v_\alpha \frac{\theta_\alpha}{\delta - \delta'} \]

so there is a reduction in \( \theta_\alpha \) by an amount

\[ \frac{1}{16} \Delta v_\alpha \| \Delta x \frac{\theta_\alpha}{\delta - \delta'}. \].....(9)

The enthalpy in the boundary layer is

\[ \int_0^h 2\pi r \rho v^* H dn \]

which can be integrated knowing that \( H_{\text{wall}} \), the enthalpy relative to the wall, is constant across the boundary layer (Schlichting, 1968), to give

\[ 2\pi r H_{\text{wall}} \left( H_\alpha(\delta - \delta') - H_{\text{wall}} \right) \left( \theta_\alpha \right) \]

Hence the change in boundary layer enthalpy \( \Delta H_{\alpha} \) due to the mass flow exchange is given by

\[ \rho r W_\alpha (\delta - \delta') \Delta H_{\alpha} = \rho r W_\alpha W_{\text{wall}} \left( \theta_\alpha \right) + \frac{1}{16} \Delta v_\alpha \| \Delta x \frac{\theta_\alpha}{\delta - \delta'}. \]

But from equation (9) the right hand side of this equation is zero. So there is no change in boundary layer enthalpy.
Turning now to the momentum and enthalpy transfers from the boundary layer into the mainstream, the mass flow \( \Delta m \) exchanged with a streamtube at radius \( r \) is

\[ \frac{1}{8} \pi r \rho \frac{d}{dr}(r \Delta v) dr \Delta x \]

so using equation (6)

\[ \Delta m = 0.75 \rho N \sec \alpha \frac{\dot{v}}{\text{sec}} \Delta x. \]

Since \( \dot{v}_{\text{sec}} \) is known from eqn (6), this allows the momentum exchanges with all the streamtubes outside the boundary layer to be evaluated in much the same way. Enthalpy interchanges occur only if \( H \neq H_{\text{wall}} \), so are assumed to be catered for by the turbulent diffusion terms in the streamline curvature equations.

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