A FULLY THREE-DIMENSIONAL INVERSE METHOD FOR TURBOMACHINERY BLADING WITH NAVIER-STOKES EQUATIONS

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ABSTRACT

A new numerical method for solving fully three-dimensional inverse shape design problem of turbomachinery blading has been developed. The general inverse problem refers to the problem in which the pressure distributions on suction and pressure surfaces of blade are given, but the corresponding blade profile is unknown. In this paper, the calculations are based on the 3D Navier-Stokes equations expressed in terms of nonorthogonal curvilinear coordinates and corresponding nonorthogonal velocity components, and the explicit time marching algorithm and Baldwin-Lomax turbulence model are adopted. A special treatment for boundary conditions on blade surfaces is employed to satisfy the given pressure distribution. In computational process, an initial blade profile is supposed at starting, and then the blade surfaces will move regularly with time steps in the time marching process until the convergence is reached. The movement velocities at every point of blade surfaces are obtained from the solution of the Navier-Stokes equations. After each revision of the blade profile, the grid is reconstructed, and the aerodynamic parameters need to be transferred between the old and new grid points by an accurate interpolation method. Thus the viscous inverse problem is solved in a new process. The computational results for two test cases indicate that the method presented in this paper is very effective.

NOMENCLATURE

Contravariant metric tensor of \( x^i \) coordinate system

\( \epsilon^i \) base vector of \( x^i \) coordinate system

\( \hat{\epsilon}^i \) reciprocal base vector of \( x^i \) coordinate system

\( \sqrt{g} = \hat{\epsilon}_i \cdot (\hat{\epsilon}_2 \times \hat{\epsilon}_3) \)

\( g^i \) covariant metric tensor of \( x^i \) coordinate system

\( l \) relative stagnation rothermal

\( p \) static pressure

\( R \) gas constant

\( r, \theta, z \) cylindrical coordinates

\( T \) absolute temperature

\( W \) relative velocity

\( w^i \) contravariant physical component of relative velocity

\( w_j \) covariant component of relative velocity

\( x^i \) nonorthogonal curvilinear coordinates

\( \Gamma^i_{jk} \) Christoffel symbol

\( \lambda \) coefficient of thermal conductivity

\( \mu \) coefficient of viscosity

\( \pi_{ij} \) elements of the viscous stress tensor

\( \rho \) density

\( \omega \) angular velocity of rotor

\( \omega^j \) contravariant component of angular velocity

SUBSCRIPTS

0 stagnation

i inlet boundary

INTRODUCTION

In order to develop high-performance turbomachinery, one of the problems to which the researchers pay great attention is how to reduce the various losses in flow field, e.g., blade profile loss and secondary flow loss etc. It can be easily understood that the inverse methods of aerodynamic design presented for solving this problem may play a very important role in the optimization of blade shapes.

In recent many years, a variety of schemes for solving two-
dimensional inverse and mixed-type shape design problems have been developed for improving the velocity or pressure distributions along blade surfaces (e.g., Meauze (1981), Dulikravich and Sobieczky (1982), Dunker et al. (1984), Hobbs and Wignold (1984), Wang (1985), Giles and Drela (1987), Cai (1987), Luu and Viney (1987), Sun and Xu (1990), Zedan and Sehra (1990), Korakianitis (1993)). In practice, better results have been obtained by combining direct problem (analysis) schemes with the inverse (design) problem schemes (e.g., Wang et al. (1989)). However, most of these methods are based on the viscous flow models. To adequately represent the physics of the flow, it is necessary to consider the fluid viscosity in the solution of the inverse problem as well as in the direct problem. The two usual methods for considering the influence of fluid viscosity are: (1) the method of iteration between the inviscid main region and boundary layer (e.g., Dulikravich and Sobieczky (1983), Hassan and Dulikravich (1987), Wang (1990)); (2) the method of directly solving Navier-Stokes equations (e.g., Malone and Swanson (1991), Wang and Dulikravich (1995)). The former has the advantage of being simpler and faster, but also has the disadvantage that some details of flow field cannot be exactly obtained from the calculation of this method. It is quite evident that the latter is a more effective method than the former for solving viscous inverse problem of cascade flow.

Because it is difficult to control accurately fully three-dimensional performance of blade row with two-dimension design and optimization, some fully three-dimensional inverse design methods for inviscid flows have been developed by BORGES (1990), ZANGENEH and HAWTHORNE (1990), CHEN et al. (1990), DANG (1993), BRAEMBUSCHE and DEMEULENAERE (1996), etc. in recent years. In this paper, the numerical method presented by Wang and Dulikravich (1995) for solving two-dimension viscous inverse problem is extended into fully three-dimensional viscous flow. Based on the Navier-Stokes equations in conservative form expressed with nonorthogonal curvilinear coordinates and corresponding nonorthogonal velocity components, the explicit time marching algorithm is adopted and the boundary conditions on blade surfaces are treated in a special manner. In the time marching process, the blade surfaces are movable like the elastic films the shapes of which can change when subjected to the pressure distributions prescribed in fully three-dimensional inverse problem.

**GOVERNING EQUATIONS**

Based on nonorthogonal curvilinear coordinates and corresponding nonorthogonal velocity components, the following governing equations derived in conservative form are employed (see Wu (1976), Chen (1994), Wang (1997)).

**Continuity equation:**

\[
\frac{\partial}{\partial t} \left( \frac{\sqrt{g}}{\sqrt{g_{ij}}} \right) + \frac{\partial}{\partial x^i} \left( \frac{\sqrt{g}}{\sqrt{g_{ij}}} \rho \nu^j \right) = 0
\]  

**Momentum equations:**

\[
\frac{\partial}{\partial t} \left( \frac{\sqrt{g}}{\sqrt{g_{ij}}} \rho \nu^i \right) + \frac{\partial}{\partial x^j} \left( \frac{\sqrt{g}}{\sqrt{g_{ij}}} \rho \nu^i \nu^j \right) = - \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^j} \left( \frac{\sqrt{g}}{\sqrt{g_{ij}}} \rho \nu^j \nabla \cdot \mathbf{W} \right)
\]

\[
\frac{\partial}{\partial t} \left( \frac{\sqrt{g}}{\sqrt{g_{ij}}} \rho \nu^i \nu^j \nu^k \right) + \frac{\partial}{\partial x^j} \left( \frac{\sqrt{g}}{\sqrt{g_{ij}}} \rho \nu^i \nu^j \nu^k \nu^l \right) = - \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^j} \left( \frac{\sqrt{g}}{\sqrt{g_{ij}}} \rho \nu^j \nabla \cdot \mathbf{W} \right)
\]

\[
\frac{\partial}{\partial t} \left( \frac{\sqrt{g}}{\sqrt{g_{ij}}} \rho \nu^i \nu^j \nu^k \nu^l \right) + \frac{\partial}{\partial x^j} \left( \frac{\sqrt{g}}{\sqrt{g_{ij}}} \rho \nu^i \nu^j \nu^k \nu^l \nu^m \right) = - \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^j} \left( \frac{\sqrt{g}}{\sqrt{g_{ij}}} \rho \nu^j \nabla \cdot \mathbf{W} \right)
\]

The eight equations, (1), (2), (3), (4), (5), (6), (7), (8), (9), (10), contain...
eight unknown scalar quantities, $W^1, W^2, W^3, W, p, p, T, I$, and may be solved. The turbulence model developed by Baldwin and Lomax (1978) is adopted in calculations of this paper.

**NUMERICAL METHOD**

The explicit predictor-corrector time marching scheme presented by MacCormack (1969, 1982) is used in the numerical solution of the Navier-Stokes equations. In the calculation of inverse problem, because the blade profile is unknown beforehand, an initial blade shape needs to be provided. In principle, the initial blade may be selected within a wider range of realistic configurations, but in engineering practice, the numerical solution of fully three-dimensional inverse problem is usually applied to the improvement of the blade generated in quasi-three-dimensional design, thus it is most convenient to use the original blade as the initial blade profile. The computational process for the inverse problem is similar to the more common direct problem, but the manner of treating boundary conditions on blade surfaces is different. In the direct problem, the no-slip condition is imposed at the blade surface, that is $W^1 = W^2 = W^3 = 0$. In the inverse problem, the pressure distributions specified along blade surfaces are enforced and let $W^3 = W^4 = 0, W^1 ≠ 0$. The $W^3$ at blade surfaces are obtained from the local solutions of the Navier-Stokes equations. Hence, the blade surfaces could be moved in the $x^1$ direction by the amount $W^1 \Delta t$ with each iteration time step until the convergence is reached, that is, until $W^1 = 0$ is satisfied on the final surface configuration. This idea requires only minor modification in any standard flow analysis code since it is essentially a surface-movement concept where the movement velocity could be chosen to equal $W^1$. In the time marching process, the computational grid needs to be changed with each update of the blade profile, and the aerodynamic parameters need also to be transferred between the old and new grid points by an interpolation method.

The boundary conditions at other boundaries of the computational domain except blade surfaces are the same as in the case of an analysis problem. The no-slip condition is still imposed on the annular walls. At the inlet boundary, the stagnation pressure, the stagnation temperature and the inlet flow angle are specified, while the density is extrapolated along $x^2$ direction from the interior of the computational domain. At the exit boundary, the static pressure is imposed, while the density and the contravariant physical components of velocity are extrapolated from the adjacent interior grid points. Along periodic boundaries, the standard periodic conditions are used.

**COMPUTED RESULTS**

Based on the method presented in this paper, a fully three-dimensional inverse design code using Navier-Stokes equations has been developed. In order to verify this method, two examples were tested.

For the convenience of generating the input data, both test cases were arranged in following manner. First, the flow field of a three-dimensional annular cascade was calculated by the code of solving 3D viscous direct problem, then, the pressure distributions on blade surfaces obtained by analysis code were adopted as the input conditions of 3D inverse problem, and the blade profiles generated after the thicknesses of original blade were adequately changed (in test case No.1, the thickness was increased; in test case No.2, the
Fig. 2 Comparison of initial blade profile, target blade profile and new blade profile generated by inverse method at different spans along radial direction (test case No.1)

Fig. 3 Comparison of non-dimensional pressure distributions on blade profile surfaces at different spans along radial direction (test case No.2)

Fig. 4 Comparison of initial blade profile, target blade profile and new blade profile generated by inverse method at different spans along radial direction (test case No.2)
thickness was decreased) were used as the initial blade profiles. At the inlet boundary of the annular cascade, the stagnation pressure is 0.1266 MPa, the stagnation temperature 307 K, the flow angle 30°, and they are uniform along radial and circumferential directions. At 50% blade span of the exit boundary, the static pressure is taken as 0.1059 MPa. The aim of the tests is very clear, that is, it is desired that when the different initial blade profiles and the same pressure distributions are used, the new blade profiles obtained in the solutions of 3D inverse problem can be very good in agreement with the original blade profile. The computational results for the both test cases indicate that the method presented in this paper is very effective.

Figures 1-(a), 1-(b), 1-(c) show the comparison of non-dimensional pressure distributions \( \frac{\Delta p}{p_{\infty}} \) on blade profile surfaces at three spans along radial direction for initial blade profiles and target blade profiles of test case No.1. Figures 2-(a), 2-(b), 2-(c) show respectively at three spans the initial profiles, the target profiles, and the new profiles obtained by solving 3D inverse problem for test case No.1. For test case No.2 figures 3-(a), 3-(b), 3-(c) and 4-(a), 4-(b), 4-(c) are the similar results to figures 1 and 2. It can be clearly seen from figures 2 and 4 that based on the different initial blade profiles and the same target pressure distributions on blade surfaces, the new blade profiles generated by 3D inverse design code are almost coincident with the target blade profiles. This is a powerful verification of the reliability of the method developed in this paper. Figures 5-8 show some computed results of fully three-dimensional flow fields respectively for the initial blade profile of test case No.1, for the initial blade profile of test case No.2, and for the target blade profile. The non-dimensional pressure contours on suction surfaces of the blades and their extended surfaces are shown in the figure 5. Figures 6-(a), 6-(b), 6-(c) show the pressure contour distributions of flow field at 50% blade span. Figures 7-(a), 7-(b), 7-(c) show the distributions of total pressure recovery coefficient contours on the sections perpendicular to the axis at 25% of axial chord length downstream of the blade trailing edge. Figures 8-(a), 8-(b), 8-(c) show the secondary flows on the sections perpendicular to the axis at 50% of axial chord length. The vectors of the secondary flows in figure 8 are based on the composition of \( \mathbf{W}_1 \) and \( \mathbf{W}_2 \). It can be discovered from the results in figure 7 that the total pressure loss for the initial blade of test case No.1 is somewhat higher than that for the initial blade of test case No.2 and the target blade.

In the calculations, the leading and trailing edges of real blades have been sharpened and the \( 23 \times 47 \times 23 \) H-type mesh is used. The number of total time marching steps was taken as \( n = 8 \times 10^4 \). Before \( n = 2 \times 10^4 \), it is the flow field calculation for initial blade by solving the 3D direct problem to create a better steady initial field of inverse problem. When \( n > 2 \times 10^4 \), the special method solving the inverse problem was activated. In the process of solving direct problem for initial blade, the local time steps were used to accelerate the convergence, while the process of solving inverse problem began, the identical time steps must be used to actually practice the unsteady process. In practice, the CPU time needed for completing a total process of fully 3D viscous inverse problem was about 100 hours by using SGI computer system.

CONCLUDING REMARKS

In this paper, a numerical method for solving fully three-dimensional viscous inverse problem of turbomachinery blading was developed that is based on a complete set of Navier-Stokes equations in conservative form expressed with nonorthogonal curvilinear coordinates and corresponding nonorthogonal velocity components for unsteady, compressible turbulent flows. The non-dimensional pressure distributions on blade surfaces were taken as given boundary conditions in the inverse problem. The computed results of two examples manifest the reliability of the method presented in this paper. Obviously, the viscous inverse method using Navier-Stokes equations may consider the effect of fluid viscosity more accurately and reasonably. It may be expected that in the future the present inverse
Fig. 6  Non-dimensional pressure contours of flow fields at 50% blade span

Fig. 7  Total pressure recovery coefficient contours on sections perpendicular to axis at 25% of axial chord length downstream of blade trailing edge

Fig. 8  Secondary flows on sections perpendicular to axis at 50% of axial chord length  
(Velocity scale: 50 m/s)
design method will be conveniently applied to the improvement and optimization of actual blades.

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REFERENCES


