Summary

Improving the performance of high speed axial compressors through low speed model compressor testing has proved to be economical and effective (Wisler, 1984). The key to this technique is to design low speed blade profiles which are aerodynamically similar to their high speed counterparts. The conventional aerodynamic similarity transformation involves the small disturbance potential flow assumption therefore its application is severely limited and generally not used in practical design. In this paper, a set of higher order transformation rules are presented which can accommodate large disturbances at transonic speed and are therefore applicable to similar transformations between the high speed HP compressor and its low speed model. Local linearization is used in the non-linear equations and the transformation is obtained in an iterative process. The transformation gives the global blading parameters such as camber, incidence and solidity as well as the blade profile. Both numerical and experimental validations of the transformation show that the non-linear similarity transformations do retain satisfactory accuracy for highly loaded blades up to low transonic speeds. Further improvement can be made by only slightly modifying profiles numerically without altering the global similarity parameters.

1. Introduction

The development of the modern aero-engine compressor in the last two decades shows a trend of using increasingly highly loaded and low aspect ratio blading to achieve high stage performance. This leads the designer to pay more attention to the blade end regions where flow three dimensionality is predominant—particularly for the rear stages of a high pressure (HP) compressor. Many efforts have been made in order to understand the nature of the complex flow and to reduce the high loss concentrated in the region. The fact that the flow is viscous and fully three dimensional greatly complicates numerical simulations and detailed measurement of the flow structures is almost impossible for rear stages because it is extremely difficult to instrument such tiny geometries. To get out of this dilemma, Wisler presented a successful approach which uses a large scale low speed research compressor to simulate the rear stages of the high speed compressor to be studied and improved. This technique enables a remarkable reduction of loss in the end wall region and flow separation at blade corners, and a substantial increase in stall margin. Compared with the high speed test, the low speed simulation has the advantages of greater accuracy, low cost, and low risk (Wisler, 1984). In order to achieve a correct simulation, the low speed model compressor should be aerodynamically similar to the high speed compressor stages to be studied. When the problem regions in the low speed compressor are detected in testing and removed by custom-tailoring the blade shape, these modifications to the low speed blade can be transformed back to its high speed counterpart. Wisler specified solidity, aspect ratio, velocity triangles, reaction, Reynolds number, airflow surface pressure (coefficient) distribution, clearance—blade height, and axial spacing—to-chord ratio as the similarity parameters and made them the same for the high and low speed compressors.

Aerodynamic similarity has long been used for two geometrically similar bodies at the same aerodynamic conditions such as Reynolds number, Mach number, incidence, etc. to achieve similar aerodynamic forces as, eg., a scaled wind tunnel testing. However, if aerodynamic
force similarity between two bodies is to be achieved, the geometrical similarity has to be relaxed in some manner analogous to the way isolated aerofoil profiles are according to Prandtl-Glauert rules. Woolard (1950) developed a set of similarity transformation rules for aerofoils in cascades at different speeds. However, because it was based upon linear small disturbance potential equations, its use was strictly limited. For the modern high pressure compressor with high stage loading, it is obvious that the linear assumption is no longer valid. Hence, Wisler had to use a numerical approach for the low speed compressor blade design in stead of a similarity transformation which would have reduced the design work significantly.

In this paper, a set of similarity transformation rules of different orders of approximation are ranging from an improved (over Woolard's) transformation based on the linear small disturbance potential equation to that for the non-linear full disturbance potential equation. The three dimensional effect can also be accounted for by a similar transformation of the stream tube contraction in the meridional plane. Because the higher-order non-linear terms are taken into account in the transformations, they can be used for blades of high speed and loading and are therefore applicable to the transformation of high speed HP compressor and low speed model blading. Both numerical and experimental validations for the transformations have been made and it is shown that a satisfactory accuracy can be obtained even for highly loaded blades.

The advantage of using this similarity transformation in the low speed simulation technique is not only that it is fast and simple, but also that it may cope with the very complex flows in the end-wall regions. Conventional two/ quasi three dimensional flow calculations may fail in these regions and give erroneous results. With the assumption that viscous behaviour is controlled by similar pressure distributions if other aerodynamic parameters are the same, two aerodynamically similar blades should exhibit similar boundary layer behaviour. Therefore, the viscous effects are implicitly taken care of by the inviscid similarity transformation. Further more, because the governing equations are valid at every point in the flow field, the transformation is a point-to-point one (in a sense). Therefore, it can still be used (with degraded accuracy) in cases where three dimensionality is strong and two/quasi three dimensional calculations are no longer trustworthy.

2. Similar transformation of different levels

It has been shown that only for linearized two dimensional flows does there exist exact similarity and analytical transformations between flows become possible (Zhu, 1990). Therefore, efforts have concentrated on the study of linearized /locally linearized potential flows. It is obvious that the accuracy of the simulation is tied to the order of approximation of the governing equations of the flow. Therefore, different levels of complexity of the governing equations are considered and presented in order of complexity. The derivation of the similarity rules is based on two dimensional flows and variations of the height and thickness of the stream tube can be accounted for by a separate two dimensional similarity consideration.

2.1 Subsonic small disturbance case

A linearized small disturbance potential equation (SDP) can be used for thin and slightly cambered blades. A coordinate system is chosen as shown in Fig.1 where the x-axis coincides with the vector averaged flow direction which is defined as the main flow direction. In this sense, the far field parameters such as $M_a$ and $V_a$ are defined as the vector averaged inlet and exit values and all variations in the flow are regarded as disturbances superimposed on this main flow. The SDP equation has the form of $(1-M_a^2)\phi_{xx} - \phi_{yy} = 0$ (e.g. Kuethe and Chow, 1986) where $\phi$ is the disturbance potential and $M_a$ is the vector averaged Mach number. Following to Woolard (1950), the transformation can be effected according to Prandtl-Glauert rule:

$$\begin{align*}
x' &= x \\
y' &= \Omega y \\
u' &= ku \\
v' &= (k / \Omega) v
\end{align*}$$

where $k$ is a constant to be determined and $\Omega = \sqrt{1-M_a^2}$.

Then we have the transformation to an incompressible flow field where the corresponding disturbance velocity components are:

$$\begin{align*}
u' &= ku \\
v' &= (k / \Omega) v
\end{align*}$$

If the pressure coefficient $C_p$ is chosen as the similarity criterion, to the first order it can be expressed as

$$C_p = -2u / V_a$$

For the $C_{p}'$s at corresponding points in two flow fields to be identical, the velocity ratio should be equal, ie.

$$u' / V' = u / V_a$$

It follows that $k$ is equal to the ratio of the main flow velocity magnitudes of the two flows:

$$k = V_a / V_a$$

If the coordinates of two blades are expressed as

$$\begin{align*}
Y_m &= f[x - mh] \tan \beta + a] + mh' \\
Y_n &= g[x - mh] \tan \beta + a] + mh
\end{align*}$$

Where $m = 0, \pm 1, \pm 2, \pm 3, \ldots$ etc, is the ordinal number of the blades in cascade, then to a first order approximation, the tangents of the blade surfaces are

$$\begin{align*}
dY_m / dx &= dg / dx \approx (Y_m / V_a)_{-ma} \\
dY_n / dx &= df / dx \approx (Y_n / V_n)_{-ma}
\end{align*}$$

From equation (2) we have the blade shape relation

$$df / dx = (1 / \Omega) dg / dx$$

and the similarity relation between the incidences (inlet flow angle) im-
mediately follows:
\[ t^2 \sigma_s = (1/\Omega) t^2 \sigma_s \]  
(9)

Equation (8) also gives the transformation relation for blade thickness and camber angle of two sets of blades.

According to eq.(1)
\[ m^t = \Omega m_h \]  
(10)

and the stagger angles are related by
\[ \beta' = t^2 \left[ 1 - \frac{1}{\Omega} t^2 \right] (\beta + \alpha_s') \]  
(11)

and the pitch should be transformed accordingly
\[ \Omega d \cos (\beta + \alpha_s) = d' \cos (\beta' + \alpha_s') \]  
(12)

The solidity of the transformed cascade turns out to be
\[ t' = \frac{\cos (\beta + \alpha_s)}{\cos (\beta' + \alpha_s')} \]  
\[ \left( \frac{\Omega}{\Omega_1} \right)^2 + \frac{1}{\Omega_1} \]  
(13)

With these relations, we have
\[ C_{p}' (\beta', \alpha_s' , \tau') = C_{p} (\beta, \alpha_s, \tau, M_o) \]  
(14)

This means that for a compressible cascade in a flow field at a mean Mach number \( M_o \) and at mean incidence \( \alpha_s \), with cascade stagger angle \( \beta \) and solidity \( t \), and for a cascade in an incompressible flow field of which the parameters \( \beta', \tau' \) and \( \alpha_s' \) etc. are determined by equations (8), (9), (11) and (13) respectively, the pressure coefficient (defined as eq.(3)) distributions of the two are the same, and all other aerodynamic force parameters of two cascades are the same as well. Thus, the above equations give the transformation rules between cascades in subsonic compressible and incompressible flows under small disturbances.

If the two flows in question are both compressible but with different vector mean Mach number \( M_{a1} \) and \( M_{a2} \), then for the two to be aerodynamically similar, the following should be satisfied:
\[ \frac{df_i}{dx} \Xi_1 \frac{df_i}{dx} \Xi_2 \]  
(15.1)

\[ \frac{tg \sigma_s'}{tg \sigma_s} = \frac{\Xi_1}{\Xi_2} \]  
(15.2)

\[ \beta'' - t^2 \left[ \frac{1}{\Xi_1} \right] t^2 (\beta' + \alpha_s') = \alpha''_s \]  
(15.3)

\[ \tau'' = \frac{\cos (\beta' + \alpha_s)}{\cos (\beta' + \alpha_s')} \]  
\[ \left( \frac{\Xi_1}{\Xi_2} \right)^2 + \frac{1}{\Xi_2} \]  
(15.4)

where \( \Xi_1 = \sqrt{1 - M_{a1}^2} \), \( \Xi_2 = \sqrt{1 - M_{a2}^2} \).

It should be noted that the transformation rules (15) not only guarantee the similarities of the equation and the blade surface boundary conditions, but also those of periodicity and inlet and exit boundary conditions. Further more, the definition of \( C_p \) here is based upon the vector averaged mean velocity but not inlet velocity, which is the common practice in compressor design. However, it has been shown (Zhu, 1999) that, under the small disturbance assumption, if \( C_p \) distributions based upon the vector mean velocity are the same, those based upon inlet velocity are also the same.

2.2 Non-linear disturbance cases
More generally, the transonic small disturbance potential (TSD) equation has the form
\[ \Omega^2 \phi_{\tau \tau} + \phi_{\gamma \gamma} = R H S \]  
(16)

and the second order full disturbance potential (FDP) equation can be expressed as:
\[ \Omega^2 \phi_{\tau \tau} + \phi_{\gamma \gamma} = R H S \]  
(17)

where
\[ R H S = 2M^2 \left( \frac{v}{V_o} \right) \left( 1 + \frac{u}{V_o} \right) \phi_{\tau \tau} \]

and
\[ \Omega^2 = 1 - (\gamma - 1) M^2 \left( \frac{u}{V_o} \right) \]

for eq.(16)
\[ \Omega^2 = 1 - (\gamma - 1) M^2 \left( \frac{u}{V_o} \right) \]

for eq.(17) (18)

(e.g. Kuether and Chow, 1986). Note that here \( \Omega \) is no longer a constant over the whole flow field but rather a point function in the flow field, \( \Omega = \Omega(x,y) \). In order to obtain an analytical solution, it is necessary to maintain the problem at the one-dimensional level. Therefore, pitchwise averaging is applied to the field variables \( u / V_o \) and \( v / V_o \)

\[ \left( \frac{u}{V_o} \right) = \frac{1}{Pitch} \int_{0}^{Pitch} \left( \frac{u}{V_o} \right) dy \]

and when these pitchwise averaged values are substituted equation (18), \( \Omega \) becomes a function of \( x \) only, i.e., \( \Omega = \Omega(x) \). Thus it is possible to write out the transformations
\[ \begin{pmatrix} x' \cr y' \cr \gamma' \cr \phi' \cr \theta' \end{pmatrix} = \begin{pmatrix} X \cr Y \cr \gamma \cr \phi \cr \theta \end{pmatrix} \]

(20)

and to substitute these into equations (16) and (17) respectively. Notice \( \Omega \) has different expressions for these two equations (eq.(18)). The transformed equations can be written as
\[ \phi_{\tau \tau} + \phi_{\gamma \gamma} = -\frac{1}{R H S'} \]

(21)

for eq.(16)
\[ \phi_{\tau \tau} + \phi_{\gamma \gamma} = -\frac{1}{R H S'} \]

for eq.(17)

In order to obtain the form of the Laplace equation which is essential for similarity transformation, we have to drop the RHS' term in the second equation of eq.(21). Doing this inevitably introduces error into the transformation and the exact second order accuracy is lost. However, since in the linearized equation the cross term \( \phi_{\tau \gamma} = dv/dx \) is locally frozen and is of the same order of the disturbance velocity \( v \) in general, the RHS' term is actually of second order therefore its neglect is partially justified.

With the original equations (16) and (17) transformed into the form of the Laplace equation (equation (21)), the same steps outlined in section 2.1 can be taken and the transformed blade geometry can be obtained according to equation (8). However because \( \phi \) is locally linearized and varies with \( x \), the similarity of the two flow fields is only true in a correspondingly small region \( \delta x \), i.e., the flows are similar in piceemal. In order to obtain the overall blade parameters such as stagger angle, solidity, etc, as well as inlet flow angle \( \alpha \) using equations (9) through (13), a global rather than piecemeal \( \Omega \) is needed. To achieve this, an average along the axial chord for \( \Omega(x) \) is performed. For \( (v / V_o) \) and \( (u / V_o) \) obtained in equation (19), a further average is made:
The results are substituted into equation (18) to obtain an averaged \( \Omega \).

The final obstacle to the non-linear transformation is that in general the disturbance velocities \( u \) and \( v \) are unknown beforehand. For the results presented in this paper, the following approach has been adopted:

In the transformation from an incompressible flow to a high speed one, the linear transformation of section 2.1 is used first to get an approximate cascade, an inviscid flow field calculation is then performed for this cascade to get \( u \) and \( v \), these are substituted in equations (19) and (22) and the similarity rules to obtain a new cascade for which the effect of the disturbance velocities has been taken into account. Then a new flow field is calculated and a new transformation is made and the iteration is continued until \( \Omega \) converges. In practice the numerical results show that the convergence is very rapid and only two iterations are sufficient.

For the case of the high speed–to–incompressible transformation, the components of the disturbance velocity can be obtained from direct calculation of the high speed cascade; therefore no iteration is needed.

It should be noted that the transformations of SDP and TSD are exact in the sense that both the equation and the \( \text{Cp} \) are approximated to first order. However, that of FDP is actually not exact because, while the final form of the equation (with non–homogeneous RHS neglected) still retains most of the second order terms, the similarity parameter \( \text{Cp} \) is only of first order approximation. Therefore, the transformation cannot attain the accuracy indicated by the equation. Nevertheless, some higher order influences can be included in the equation to improve the quality of the transformation as shown later in the section on numerical validation.

The present similarity transformations are for plane cascades. The extension to annular cascades is straightforward and the transformation rules retain exactly the same form as those for the plane cascade. This is because the term containing quasi–three dimensional effects variations of thickness and height of streamtube all go to the non–homogeneous RHS of equation 17 and are dropped in the transformation. These quasi–three dimensional effects are grossly ignored in the transformation based on SDP, but for those based on TSD and FDP those effects can be included in iterative calculation of \( \Omega(x,y) \).

### 2.3 Quasi–three dimensional case

In a real compressor, because of the existence of the streamtube thickness and radius variations, the effect of axial velocity density ratio (AVDR) on the transformation should be considered. Based on the fact that in a high pressure compressor the streamtube variation is not very large, two basic assumptions are made: that in the meridional surface the linearized small disturbance equation can be used and that the disturbance on this surface has no correlation with that on the blade-to-blade surface.

The streamtube thickness ratio, the following relation is obtained:

\[
\frac{dt}{d_1} = 1 - \Omega^2 + (d'_{f}/d^2)\Omega^2
\]

where \( g(z) \) and \( g' \) represent the stream surface coordinates for incompressible and compressible flow fields respectively. In term of the streamtube thickness ratio, the following relation is obtained:

\[
\frac{dt}{d_1} = 1 - \Omega^2 + (d'_{f}/d^2)\Omega^2
\]

Because it is assumed that the disturbances in two orthogonal surfaces are independent, equation (25) is also applicable for the flow described by the TSD or FDP equations.

It is interesting to notice that equation (25) actually says that for a low speed compressor, in order to achieve a similar pressure distribution at the edges of a streamtube (casing and hub as the extremes), the latter should converge more than its high speed counterpart. This is against the common practice as the low speed compressor usually has the constant casing and hub. Therefore, in this kind of simulation, the pressure distributions at the end walls cannot be simulated according to the similarity rules; instead, this is relaxed to achieve the similarities of other more important parameters as pointed out by Wisler (1984). The significance of the above transformation on the meridional plane is that it presents a relation for AVDRs at different speeds. Therefore, the effect of the non-similar end-wall transformation on the pressure distributions of the blade-to-blade flow surface can be accounted for.

### 3. Numerical validation of the transformation rules

The transformation rules obtained in the previous sections have been examined numerically for their validity and applicability. In doing this, Denton's inviscid two- or quasi–three dimensional time dependent method (Denton, 1982) was used. There are good reasons for choosing an inviscid code for the validating purpose. Firstly, because the transformation is of the potential type, an Euler code, which in shockless flows involves virtually the same approximations to the flow, is better for validations than a viscous one. Secondly, although the development of the boundary layers on the blade surfaces is an important issue to consider in the design, it is expected that under similarity transformations, the behaviour of the boundary layers should also be similar at different speeds. Indeed it has been verified that at the same Reynolds number and free stream turbulence, the Cp distribution on the blade surfaces is the predominant factor in boundary layer development (Zhu and Xu 1990).

![Fig.2 SDP linear transformation from low speed to high speeds](image-url)
Fig. 2–4 show the sample transformations for different levels of approximations for two NACA 16 series aerofoil cascades. Fig. 2 illustrates the SDP transformation from low speed to high speeds (M = 0.4 to M = 0.6 and to M = 0.75) for a cascade of thin aerofoil (8% thickness/chord) with a small stagger angle of 20° and a moderate inlet flow angle of 35°. Although in general the Cp distributions agree well on the suction surface, the transformation accuracy deteriorates at higher Mach number, as the flow non-linearity increases and the linear assumption involved in the transformation becomes unrealistic. This is especially true at the leading edge where the disturbance is expected to be the largest and is also seen on the pressure surface. To increase the camber and stagger angle will make it even worse. Thus a non-linear transformation is necessary in the cases of high speed or large disturbance.

Fig. 3 gives an example for a cascade with a thicker aerofoil (12% thickness/chord), larger stagger angle (30°) and inlet flow angle (55°) from incompressible flow to M = 0.6. It can be seen that the non-linear transformation (FDP) improves the solution remarkably compared with that of SDP. Fig. 4 shows the effect of the quasi-three dimensional consideration on the transformation. The original cascade has an AVDR of 0.85 at M = 0.4; when transformed to M = 0.6, an AVDR of 0.91 is obtained via the transformation on the meridional flow surface. The parameters for the high speed aerofoil are obtained from the SDP transformation on the blade-to-blade surface. The Cp transformations for the two blades can be seen to agree very well. The calculation without the corresponding adjustment for streamtube contraction produces a much poorer result.

In summary, the numerical validation for the transformations of different levels shows that the accuracy of the transformation is associated with the approximation of the governing equation to the actual flow to be transformed. In general, the transformation based on the full disturbance potential (FDP) equation is found to be the most accurate and suitable for the most situations but it involves more calculations. The transformations based upon the analytical similarity rules prove to be an effective and fast means to obtain the similar aerofoil coordinates and cascade parameters although it is seen throughout the numerical validation that there still exists some discrepancies in the Cp distributions. These discrepancies may come from the difference between the Euler approximation used in the numerical scheme and the linear or locally linear approximation used in the transformations, but mostly result from the one dimensional approximation involved in the transformation which is inherent in the analytical approach. To achieve higher accuracy, some kind of numerical correction (inverse method for blade design) can be used to generate more accurate blade shapes (Zhu, 1990). However, this inevitably takes more computer time yet is often unnecessary for most cases encountered in engineering design practice.

4. Experimental validation of the transformations

A set of cascade tests were designed to verify the transformation rules in real flow situations, including the viscous effect which is assumed to be modelled implicitly in the analytical similarity transformation by keeping Reynolds number, free stream turbulence and pressure coefficient distributions on the blade surfaces the same. The high speed original cascades were taken from the literature: a NACA65 series cascade tested by Williams (1952) at an inlet Mach number of 0.71 and a
I cascade ranges continuously from zero to 34 m/s. The tunnel has a blower driven by a 40 kW DC motor, and the air speed at the inlet of the Propulsion Department, BUAA. The air supply of the tunnel is from a 5 of the high speed tests (Re = 2.4 \times 10^5 and 6 for the transformed DCA cascade. This sacrifices the periodicity, the closest possible Reynolds number to those of the high speed tests on-

working cross section of only 124mm \times 250mm. So in order to achieve and tested in a low speed blowdown linear cascade wind tunnel at the Jet

cascades yet the Reynolds number achieved is still only half that of the high speed original and a high speed cascade with an AVDR which when transformed to low speed will equal that of the low speed experiment.

For example, the AVDR in the low speed experiment of the transformed NACA65 profile is 1.23. When transformed to its high speed counterpart, the latter should have an equivalent AVDR of 1.13. Since the original high speed test of Williams was two dimensional, \( \delta \text{CP}_{AVDR} = \text{CP}_{AVDR_{\text{low}}} - \text{CP}_{AVDR_{\text{high}}} \) accounts for the discrepancy caused by AVDR differences in the two tests.

Table 1. The Parameters of pre- and post- transformed cascades

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<th>Cascade</th>
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<th>DCA</th>
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The latter effect, i.e., the numerical and transformation error, is more complex. Fig. 9 shows the calculated and measured Cp distributions of the DCA cascade of Hoheisel and Seyb at an inlet Mach number of 0.6. Although in general the agreement is good, the discrepancies at leading edge and on the pressure surface are of the same order as those that appeared in the low-to-high speed transformations. In incompressible flow calculations such discrepancies are expected to be larger. If the Cp distributions of the corresponding low and high speed

If the errors in Cp caused by the various factors mentioned above (and others left unmentioned) can be separated out, the following relation should be valid:

\[ \Delta C_p = \delta C_{p_{\text{av}}} - \sum \delta C_p \]

where \( \delta C_{p_{\text{av}}} \) is the measured Cp difference between the original and transformed blades at a particular point on the blade surfaces and \( \sum \delta C_p \) is the summation of discrepancies from all sources. For the ideal situation, \( \Delta C_p \) would equal zero. It has been estimated that viscous effects are not significant on the surface pressure distributions in the present cases whilst there are two effects related to the transformation and the numerical scheme which can be accounted for, i.e., AVDR and the implicit discrepancy between the transformation and the numerical calculation. The former is estimated by calculating the \( \delta C_p \)s between the high speed original and a high speed cascade with an AVDR which when transformed to low speed will equal that of the low speed experiment.

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5. Discussions and Concluding Remarks

With the introduction of local linearization in obtaining the similar equation, a set of higher order similarity transformation rules have been derived and the severe restrictions imposed upon the conventional similarity transformations have been removed. These rules provide a very efficient means of designing similar blades with an acceptable accuracy. In the FDP case the expression for the similarity parameter \( \Delta P \) contains only first order term whilst the governing equation retains most of the second order terms (only the non-homogeneous term in equation (17) has been discarded). Thus although it is not a second order transformation in an exact sense, it does get rid of the restrictions imposed by the SDP and TSD equations and can therefore be used universally up to low transonic speeds.

The similarity rules of Equations (10) through (13) give the global cascade transformation parameters such as camber angle, incidence, solidity, etc. In design practice, solidity is an important parameter in loss correlations and should be dictated by the similarity of loss. However, neither criterion is exercised precisely because in practice the meanline diameter of the low speed model compressor and the number of the blades determines the solidity although the number of the blades can be adjusted to approach the closest value possible for the target solidity. Fortunately, the variation of solidity given by equation (13) is usually very small. Typical values can be seen in the examples presented in the cascades are calculated and a \( \delta C_p_{num} \) is defined as \( \delta C_p_{num} = C_p_{num} - C_p_{num} \) at corresponding points on the blade surfaces, this \( \delta C_p_{num} \) accounts for the relative errors in numerical calculation and analytical transformation.

Thus the comparison is made based upon the relative scale, ie,
\[
\Delta C_p = \delta C_p_{num} - \Delta C_p_{num} - \delta C_p_{num} - \delta C_p_{num}
\] (26)
rather than the \( C_p \) distribution itself. It represents the error in two aerodynamically similar cascades with the same AVDR, but for which viscous effect is not included. Figs 10 and 11 show the calculated \( C_p \) distributions at high speed for different AVDRs and the chordwise \( \Delta C_p \) distributions defined in equation (26) respectively for both cascades tested. From Fig.11 it is seen that after the AVDR and numerical effects have been deducted, the discrepancy between high and low speed tests is reduced remarkably; \( \Delta C_p \) falls within \( \pm 0.05 \) which is of the same order as of the analytical transformation.

From this indirect comparison it is seen that the similarity transformation can be used despite of the existence of viscous effects. Just as in the present experiment, the viscous effect only contributes part of the residual \( \Delta C_p \) and therefore should be very small. Thus the applicability of the similarity transformation is verified in a broader sense.
if the radial pressure distributions are similar for two compressors, even gaps between blade rows (typically near the end regions) can gradient in the gap between blades. Recent studies of the three distributions, which is very important when there exists a radial pressure calculation prior to the blade profile design. This not only avoids numerical inverse design is that it gives the global cascade parameters as value for solidity which is not necessarily modelled precisely. Therefore it is concluded that the transformation gives only a reference will have very little effect on blade surface pressure distributions. numerical experiment has shown that such a small difference in solidity modulus (Wisler, 1984) is seen to be of the same order and the preceding section. The variation presented in Wisler’s low speed modelling (Wisler, 1984) is seen to be of the same order and the numerical experiment has shown that such a small difference in solidity will have very little effect on blade surface pressure distributions. Therefore it is concluded that the transformation gives only a reference value for solidity which is not necessarily modelled precisely.

One of the advantages of the similarity transformation over a purely numerical inverse design is that it gives the global cascade parameters as well as the blade profile. The velocity triangles at different radii come out of the transformation rather than from an iterative meridional flow calculation prior to the blade profile design. This not only avoids arbitrary artificial adjustment of the radial flow angle distribution, the outcome of which may not conform with the similarity requirements, but also automatically ensures similarity of the radial static pressure distributions, which is very important when there exists a radial pressure gradient in the gap between blades. Recent studies of the three dimensional effects on optimal blade design (eg Wadia and Beacher, 1989) show that the rapid adjustment of radial pressure gradient in the gaps between blade rows (typically near the end regions) can significantly alter the flow angle distributions specified by the two-dimensional calculation. From this point of view it is expected that, if the radial pressure distributions are similar for two compressors, even though the flow angles may turn out to be different from design intent, the real flow deviating from design should be similar, i.e. the flows maintain similarity (although of an unexpected value)! However if the similarity of the radial pressure distribution is overlooked in a numerical redesign, the similarity of the flow may be easily overwhelmed by the three dimensional effects.

In the preceding two sections it was illustrated that the higher order transformation in general can provide satisfactory transformation accuracy. However, there is still room for improvement especially at the blade leading edge region. If further improvement must be made, this transformed profile can be used as the starting profile for an inverse design. Because only very small amounts of correction are to be made in order to achieve an exact match of the Cp distribution from this starting profile, the inverse design calculation can be numerically very efficient if a simple iteration inverse-direct method based on residual correction, such as that of Miton et al (1984), is used. By using such kinds of profile modification a cascade with both correct global similarity parameters and exact surface Cp distributions can be obtained.

References