A NUMERICALLY SUPPORTED INVESTIGATION OF THE 3D FLOW IN CENTRIFUGAL IMPELLERS

PART II: SECONDARY FLOW STRUCTURE

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ABSTRACT

The three-dimensional flow in centrifugal impellers is investigated on the basis of a detailed analysis of the results of numerical simulations. An in-depth validation has been performed, based on the computations of Krain's centrifugal compressor and a radial pump impeller, both with vaneless diffusers and detailed comparisons with available experimental data, discussed in Part I, provide high confidence in the numerical tools and results. The low energy, high loss 'wake' region results from a balance between various contributions to the secondary flows influenced by tip leakage flows and is not necessarily connected to 3D boundary layer separation. A quantitative evaluation of the different contributions to the streamwise vorticity is performed, identifying the main features influencing their intensity. The main contributions are: the passage vortices along the end walls due to the flow turning; a passage vortex generated by the Coriolis forces proportional to the local loading and mainly active in the radial parts of the impeller; blade surface vortices due to the meridional curvature. The analysis provides an explanation for the differences in wake position under different geometries and flow conditions. A secondary flow representation is derived from the calculated 3D flow field for the two geometries validated in Part I, and the identified flow features largely confirm the theoretical analysis.

NOMENCLATURE

\( u \) entrainment velocity \( u = \omega r \)
\( \nabla \) absolute velocity vector
\( W \) relative velocity
\( x, y, z \) Cartesian coordinates
\( \beta \) relative flow angle
\( \delta \) boundary layer thickness
\( \rho \) density
\( \phi \) meridional flow angle
\( \omega \) angular velocity
\( \Omega_3 \) streamwise vorticity

Subscripts

b binormal direction
m meridional distance
n normal distance
\( p_{\text{r}} \) pressure, pitch
\( p^* \) rotary stagnation pressure \( p^* = p + pW^2/2 + pu^2/2 \)
r local radius
r,\( \theta, z \) cylindrical coordinates
\( R_b \) blade-to-blade radius of curvature
\( R_m \) meridional radius of curvature
s streamwise distance

INTRODUCTION

As stated in the introduction to Part I of this paper, many open questions are still left concerning the 3D flow structure in centrifugal compressors. This is largely due to the complex geometry of centrifugal impellers, formed by long twisted 3D passages with strongly varying radii of curvature and low aspect ratios. Therefore, it is to be expected that the essential flow structure will be dominated by viscous effects. The jet/wake pattern introduced by Bob Dean and his co-workers, Dean and Senoo (1960), Dean(1971), recognizing the formation, in the downstream parts of the impeller, of a region of low energy fluid in the shroud/suction side area and high velocity fluid in the hub/pressure side area, is now generally accepted and confirmed by numerous experimental...
details unreachable through experiments. As shown in Part 1, the calculations performed on Krain's impeller and on the SHF pump impeller, one of the essential questions seems to be: how much 3D flow separation, if any, is present due to reverse flows, are largely present along the solid surfaces.

From the presently available literature, see also recent interesting textbooks, Japikse and Baines (1994), Brennen (1994), a large uncertainty still remains as to the precise mechanisms generating the wake flow and more particularly, the interaction with possible 3D separated regions. The wake concept has indeed been 'marked', from its origin on, as strongly related to boundary layer separation on the suction surfaces as well as on the shroud. One of the essential questions seems therefore to be: how much 3D flow separation, if any, is involved with the wake and what is the relative strength of the different vortices contributing to its formation. In addition, we might hope, from an answer to these questions, to define guidelines for the designer in order to predict and control the position and strength of the high loss regions.

Due to the impossibility of obtaining sufficiently detailed measurements of boundary layer and secondary flows in the complex flow channels of centrifugal machines, the only way to answer these questions is to rely on detailed numerical simulations, which are able to provide many details unreachable through experiments. As shown in Part 1, the calculations performed on Krain's impeller and on the SHF pump with fine meshes, provide good confidence in the computed results as to be used as a reliable database for the understanding of the basic mechanisms behind the formation of the wake region.

Part II of this paper attempts therefore to analyse the secondary flow structure, including qualitative evaluations of the various secondary flow components. Although these components are well known and discussed, in particular by the above mentioned authors, it does not seem that the detailed mechanisms of their interaction, as occurring in real life centrifugal compressor and pump geometries, have been analysed in detail.

The first section contains a reminder of the basic forces acting on the flow, while the next section discusses and evaluates the secondary flow contributions. In the following sections, the secondary flow structure, as derived from the computed results is analysed for the test cases of Part I.

THE BASIC FORCES ON IMPELLER FLOWS

The inviscid balance of forces equals the pressure gradient to the sum of the inertia and the centrifugal forces resulting from the flow turning plus the centrifugal force due to the rotation and the Coriolis force.

In a blade-to-blade surface generated by the rotation of the meridional streamline m with angle \( \sigma \) to the axial direction, Fig. 1, the balance of forces can be expressed in the streamline coordinates (s,n,b), where s is along the streamline, n is normal to the streamline in the blade-to-blade surface and b is the binormal perpendicular to s and n.

\[
\frac{1}{\rho} \frac{\partial p}{\partial s} = -W \frac{\partial W}{\partial s} + \frac{W^2}{R_b} n + \omega^2 b - 2 \omega W \sin \sigma n
\]

The first term on the right hand side is the streamwise inertia force, while the second term is the centrifugal force due to the curvature. Note that the sign of this term corresponds to a convex streamline s, as shown on Fig. 1, while for a concave streamline, the sign would be reversed. The third and fourth terms correspond to the centrifugal and Coriolis forces.

In the blade-to-blade surface with back swept blades, the radius of curvature \( R_b \) is generally very large and the Coriolis term dominates the centrifugal term, (for radial blades \( \sin \beta \) is close to zero and when looking at the pressure gradients in the \( \theta \) direction the centrifugal term does not contribute). Hence a transverse pressure gradient is generated with the high pressure on the pressure side due essentially to the Coriolis term. In the inducer part, where the meridional angle \( \sigma \) is small, the curvature term is directed towards the pressure surface and is responsible for the transverse pressure gradient towards the pressure side.

In the meridional section, where the curvature \( 1/R_m \) is large in the inducer and inlet regions of impellers, while the absolute tangential velocity \( v_\theta \) is small, a pressure gradient from shroud to hub balances the centrifugal force due to the meridional curvature, Fig. 1b. The normal balance of forces along \( n_m \) leads to

\[
\frac{1}{\rho} \frac{\partial p}{\partial n_m} = \frac{v_\theta^2}{R} \cos \sigma - \frac{v_{\text{m}}^2}{R_m}
\]
Hence, the inviscid balance of forces leads to a velocity increase from hub to shroud and from pressure to suction side in a nearly linear variation. The iso-velocity lines are therefore oriented from suction side/hub to the shroud/pressure side corner. The isolines are of course closed in the boundary layers, as seen in Fig.4a, planes I and II, in Part I of this paper. The impeller flow will be controlled by the inviscid forces in the inlet region, where viscous effects are not yet developed. This is confirmed by all experimental observations and Navier-Stokes computations of centrifugal machines. In the radial parts of the impeller both terms on the rhs of Eq. (2) will become very small and tend to zero.

SECONDARY FLOW COMPONENTS

It is known from the basic studies of Acosta and Bowerman (1957), Moore (1973), Johnson and Moore (1983), as also confirmed by the analysis of Part I, that the viscous effects progressively dominate the flow behaviour when moving downstream in the impeller. An appropriate way of understanding the various flow contributions due to viscous effects, is to follow the rate of increase of the streamwise vorticity, describing the secondary flows generated by the rotary stagnation pressure gradients in the boundary layers, see e.g. Van den Braembussche (1985). This relation was derived initially by Hawthorne (1974), in an unpublished report and can also be found in Johnson (1978). This basic relation is expressed as follows:

\[
\frac{\partial}{\partial s} \left[ \frac{W}{\omega} \right] = \frac{2}{\rho W^2} \left[ \frac{1}{R_n} \frac{\partial p^*}{\partial s} + \frac{\omega}{W} \frac{\partial p^*}{\partial z} \right]
\]

where \( R_n \) is the radius of curvature in the direction \( n \), normal to the streamline. The first term is responsible for the passage vortices due to flow turning, while the second term has its origin in the Coriolis forces. Other vortices, having a local influence on the flow, such as the horseshoe, corner and tip clearance vortices, are not described by Eq. (3).

Due to the geometry of centrifugal impellers, the blade-to-blade curvature will generate secondary flows due to the hub and shroud boundary layers and the streamline curvature (the passage vortices, PV), while the meridional curvature will induce secondary flows due to the boundary layers developing along the blade surface (the blade surface vortices, BV). The second term, originating from the Coriolis forces, will be effective if a boundary layer gradient exists in the axial direction. This will be the case for the end-wall boundary layers in the radial parts of the impeller, where they will be contributing to the passage vortices (Coriolis passage vortices, CV). If the inducer blade geometry is at an angle to the axial direction, the blade boundary layers will contribute an axial component of...
the blade stagnation pressure loss gradients. This contribution is expected to be small and will not be considered here. Figure 2 shows a sketch of the different components to the secondary flow.

The passage vortices drive low energy fluid from the pressure towards the suction surface along hub and shroud walls, while the blade surface vortices generate flow components along the blade surfaces from hub to shroud. The resulting motion leads to an accumulation of low velocity, high loss fluid towards the suction surface region, called the wake region, and a high velocity region around the pressure surface. The resulting position of this wake region will depend on the balance between these different vortices. In addition, for unshrouded impellers, the tip clearance flow from suction to pressure surface, is acting in a direction opposite to the shroud passage vortex. As seen in Part I, for Krain's impeller, this can have a major effect on the wake position.

**Passage vortex from blade-to-blade curvature:** Considering the first term with the blade-to-blade curvature, the total pressure gradient generating the passage vortex is on the hub and shroud endwalls. The intensity of this vortex is estimated as follows from Eq. (3), considering that the reduced pressure \( (p-p_{u}^2/2) \) does not vary significantly in the boundary layers

\[
\frac{\partial}{\partial s} \left[ \frac{\Omega^2}{W} \right]_{PV/h,s} = 2 \frac{1}{R_m} \left( \frac{\partial W}{\partial b} \right)_{h,s} \tag{4}
\]

With \( d\beta = ds/R_m \), the variation of the streamwise vorticity is proportional to the local turning

\[
\Delta \beta = \frac{2}{W} \left( \frac{\partial W}{\partial b} \right)_{h,s} \tag{5}
\]

Hence, this contribution will vanish locally for radial ending blades, in absence of backsweep. This equation can be integrated to obtain an estimate of this passage vortex at impeller exit, with \( 2/W \) replaced by the average \( (1/W_1+1/W_2) \) and where \( \Delta B \) is the overall turning from inlet to exit.

\[
\left[ \frac{\Omega^2}{W} \right]_{PV/h,s} = (1 + \frac{W_2}{W_1}) \left( \frac{\partial W}{\partial b} \right)_{h,s} \Delta \beta \tag{6}
\]

Assuming a circular streamline, \( W_1=W_2 \), one recovers the formula of Squire and Winter for the passage vortex intensity. Estimating the velocity gradients by the ratio \( W/\delta \), where \( \delta \) is the boundary layer thickness, the passage vortex can be expected to be stronger in the regions where the velocity is higher, that is at the shroud in the inlet part of the impeller. However, this can be reversed towards the downstream part of the impeller, when the velocity gradients between hub and shroud reverse, with higher velocities and thinner boundary layers at the hub.

**Blade surface vortices:** The blade surface vortices can be estimated in a similar way, leading to

\[
\frac{\partial}{\partial s} \left[ \frac{\Omega^2}{W} \right]_{BV/ps,ss} = \frac{2}{W} \frac{1}{R_m} \left( \frac{\partial W}{\partial b} \right)_{ps,ss} \tag{7}
\]

This contribution will not grow in the radial parts of impellers, where \( 1/R_m \) tends to zero. In integrated form, with \( \Delta \sigma \) the flow turning in the meridional surface, Eq. (7) takes the form

\[
\left[ \frac{\Omega^2}{W} \right]_{BV/ps,ss} = (1 + \frac{W_2}{W_1}) \left( \frac{\partial W}{\partial b} \right)_{ps,ss} \Delta \sigma \tag{8}
\]

With \( \Delta \sigma=\pi/2 \) and the velocity gradients estimated as \( W/\delta \), the blade passage vortices will be proportional to the blade surface velocities and therefore increase with the mass flow rate.

**Passage vortex from Coriolis forces:** The second term of Eq. (3) is significant where the hub and shroud normals have axial components, that is in the radial portions of the impeller. With \( \partial \Theta = \sin \sigma \partial \delta \), referring to Fig. 1, one can write this contribution as

\[
\frac{\partial}{\partial s} \left[ \frac{\Omega^2}{W} \right]_{CV/h,s} = \frac{2}{W} \frac{\omega}{W_m} \sin \sigma \cos \beta \left( \frac{\partial W}{\partial \delta} \right)_{h,s} \tag{9}
\]

The factor \( 2\cos \sigma \sin \sigma \cos \beta \) is the main contribution to the blade loading, see for instance Cumpsty (1989, p. 239), expressed as the velocity difference \( (W_s-W_p)/p= \Delta W_{ps}/p \) between suction and pressure surface divided by the pitch \( p \). Replacing the velocity gradient by the ratio \( W/\delta \), where \( \delta \) is the boundary layer thickness on hub or shroud, leads to

\[
\frac{\partial}{\partial s} \left[ \frac{\Omega^2}{W} \right]_{CV/h,s} = \frac{2}{W_m} \frac{\Delta W_{ps}}{p \delta_{h,s}} \tag{10}
\]

Hence, this contribution to the passage vortex variation is proportional to the loading and inversely proportional to the meridional velocity, that is to mass flow rate. This component of the secondary flow will tend to become dominant in radial parts of impellers. An estimation of the vortex intensity is obtained by integrating over the streamline of length \( L \) as above, leading to

\[
\left[ \frac{\Omega^2}{W} \right]_{CV/h,s} = W_2 \frac{L}{p} \frac{\Delta W_{ps}}{W_{ps}} \tag{11}
\]

the overbar indicating an average value over the blade length. The ratio between the Coriolis passage vortex and the blade surface vortex can be written as

\[
\frac{\Omega_{s,CV}}{\Omega_{s,BV}} = \frac{1}{W_m} \frac{\Delta W_{ps}}{p \delta_{h,s}} \tag{12}
\]
Fig. 3 Secondary flow structure in Krain's impeller with tip clearance, mass flow = 4.08 kg/s

Fig. 4 a) isolines of entropy loss on shroud, and b) velocity vectors near the trailing edge of midpitch hub-shroud plane, of Krain's impeller with tip clearance, mass flow = 4.08 kg/s
Fig. 5 Secondary flow structure in SHF radial impeller at nominal flow rate, a) plane I, b) plane II, c) plane III, d) plane IV, e) plane V, f) plane VI, g) plane VII, h) plane VIII and i) cutting plane locations.
The first factor is the ratio of meridional curvature to radius, the second factor is a ratio of boundary layer thicknesses along blade surfaces and endwalls and the third factor is the relative loading referred to the meridional velocity. A high value of this ratio indicates that the passage vortex due to the Coriolis forces is dominating and that the main secondary flow will be along the end wall towards the suction surface. If this ratio is low, the blade vortex dominates and the main vorticity component drives the flow along the blade surfaces towards the shroud.

As can be seen from Eq.(12), many factors can influence this ratio. Highly curved endwalls (small $R_m$) will strengthen the radial movement along the blades towards the shroud, while higher blade loadings will reinforce the movement along the endwalls towards the suction side. On the other hand, increasing the mass flow will favour the blade surface vortex and the low energy fluid transport towards the shroud. The thin boundary layers on the pressure surface will reinforce the preponderance of the radial motion along this surface towards the shroud, while the thin hub boundary layers, compared to the suction surface boundary layer thickness will reinforce the transport towards the suction side along the hub.

**CALCULATED SECONDARY FLOWS**

One way of looking at the secondary flow structure is to display the velocity vectors induced by the streamwise vorticity in cross sectional planes. However, there is no unambiguous way to extract these velocity components out of the 3D flow field. Therefore, an approximate approach is taken whereby the local direction of the streamwise oriented mesh lines, which are roughly aligned with the blade surfaces, are taken as indicative of the primary flow direction which would exist in absence of secondary flows. The secondary velocity vectors are then defined by their components perpendicular to this 'primary' direction. This is displayed in Fig. 3 for the unshrouded Krain impeller at design conditions, and Figs. 5 and 9 for the shrouded SHF pump at nominal and partial flow rates, respectively.

**Krain's impeller**

This impeller has a continuous meridional curvature of both end walls and it is to be expected that the blade surface vortices, Eq. (8), will play an important role in the secondary fluid transport, as already observed in part I. Close to the inlet of the curved inducer, section II, the counter clockwise blade surface vortex on the suction side...
Fig. 7 Comparison of predicted static pressure coefficient along blade surfaces at midspan of SHF radial impeller, with experimental data, at partial flow rate.

Fig. 8 Limiting streamlines in SHF radial impeller, a) suction surface (SS) and shroud, and b) pressure surface (PS) and hub, at partial flow rate.

Fig. 9 Secondary flow structure in SHF radial impeller at partial flow rate, a) plane V, b) plane VI, and c) plane VII. Their locations are indicated in Fig. 5i.
is dominating, reinforced by the hub passage vortex, while on the pressure side two smaller clockwise vortices are noticed, generated by the meridional curvature, a smaller vortex at the hub and a larger vortex at the shroud, strengthened by the shroud passage vortex. Passing to section III and IV, it can be seen that both blade vortices are growing in strength, with the pressure side vortex, associated to the growing passage vortex at the shroud, mainly fed by the Coriolis passage vortex, Eq.(10), progressively gaining in importance. The traces of the counter-rotating vortices on the hub are identified by the reattachment and separation lines seen on Fig. 5 of Part I.

Between sections IV and V the shroud passage vortex, mainly contributed by the blade loading and supported by the pressure side vortex, covers the major part of the section, while the suction surface vortex, formed by high energy fluid, is pushed towards the hub and reduced in size. The accumulated low energy fluid forms the core of a clockwise vortex with a second clockwise vortex formed by higher energy fluid coming from the pressure side region. The formation of a small hub/shroud side corner vortex can be observed in the last sections. It is seen from Fig.3 how the leakage flow counterbalances the influence of the shroud passage vortex, pushing progressively the low energy region towards the pressure side, while entraining high velocity fluid up to the shroud along the suction surface.

Figure 4a shows the isolines of entropy loss on the shroud wall for the unshrouded case. It is noted from that a small, localized high entropy region is produced at the inducer tip due to the leakage vortex. Behind the mid chord position, a strong increase in entropy is observed, created by a reverse flow near the shroud resulting from an unloading of the tip sections, as seen on Fig.4b, where the velocity vectors on the mid pitch hub-shroud plane are displayed. It is interesting to observe that an important entrainment of fluid from suction to pressure side, takes place along the shroud in the downstream part of the blade section. The maximum entropy values are located near the trailing edge of the suction side.

### SHF pump

The shrouded SHF pump has blades with 62.5° back sweep with hub and shroud walls formed by a short curved part, followed by an essentially radial main and exit part. Hence the passage vortex due to the Coriolis forces, Eq.(11), proportional to the loading, will be dominating the viscous effects, with rather weak radial fluid transport along the blade surfaces and the development of a shroud/suction surface corner vortex. This is clearly to be seen in the Fig. 12 of Part I. Figure 5 shows the secondary flows in the cutting planes I to VII, indicated in Fig. 5i, confirming these features. In cutting plane I, Fig. 5a, situated in the curved part of the impeller, one observes the growing blade surface vortices on the pressure and suction sides, due to the meridional curvature, with weak shroud and hub passage vortices from the blade-to-blade curvature. Since the velocity in this plane is larger on the shroud side, the shroud passage vortex is stronger than the hub vortex. From cutting planes II to IV, one can distinguish the growing blade surface vortices, while the effects of the Coriolis passage vortex is gradually reinforcing the shroud and hub secondary flows. At plane V, the growing shroud passage vortex and the weakening blade surface vortices results in the observed flow feature in the suction side/shroud corner. In plane VI, the upward flow along the suction surface has nearly vanished due to this process and an anti-clockwise vortex is formed in the suction side/hub corner. Planes VII and VIII show the corner vortices in the suction side/endwalls corners and the growing hub and shroud passage vortices. The contours of losses, shown in Fig. 6a for several mesh surfaces, summarize the formation and extension of the low energy wake region in the suction side/shroud corner.

It is instructive to look at the flow features for the same pump, but at reduced mass flow, namely at 60% of the nominal flow rate. From Fig. 6b, displaying the total pressure loss distribution in several mesh surfaces, section k=33 shows that a high loss region is formed in the shroud/suction side corner, originating from an inlet separation due to the high incidence, creating a separated region on the shroud, as seen from the plot of the limiting streamlines on Fig. 6b. However, this loss region appears as a local phenomena, not contributing really to the wake formation, as seen from the distribution of losses in section k=38. As observed in the previous section, the blade surface vortices decrease with decreasing mass flow, while the Coriolis passage vortex increases in importance when reducing the mass flow, for constant loading. From Fig. 7, it is seen that the loading is nearly constant over the blade length. Hence, this explains the evolution of the high loss region, where the dominating shroud passage vortex pushes the high velocity region towards the shroud and the low velocity region towards the hub, since the radial motion along the suction surface is not strong enough to counterbalance this tendency. In addition, Fig. 8a and 8b show that local areas of reverse flow from one passage to the next appear, close to the suction surface at the hub and moving towards the pressure side at the shroud. The back flow with the trailing edge separation forms the hub/suction surface corner separation.

This is confirmed by the secondary flow plots in cutting planes V to VII shown on Fig.9. In plane V one sees the nearly vanished blade surface vortex at the suction side, with the growing endwall vortices from the Coriolis forces. In planes VI and VII, the increasing intensity of the passage vortex, particularly at the hub, can be observed while the vertical upward suction surface flow results from the separated region in the suction side trailing edge region as seen on Fig. 8a.

### CONCLUSIONS

A detailed analysis of two centrifugal compressors, based on Navier-Stokes solutions on meshes fine enough to resolve the wall boundary layers and validated in Part I, supported by a quantitative estimate of the contributions of the various secondary vortices, has been performed. This analysis gives an insight into the basic mechanisms leading to the formation of the jet/wake flow structure in centrifugal machines. The development of a low energy, low momentum region close to the shroud wall, associated to a high velocity region near the pressure surface, results from the predominant viscous dominated flow in the narrow channels of centrifugal impellers. The main mechanism for
the accumulation of low energy fluid in the shroud area is
the radial transport of boundary layer material along the
passage surfaces and its location results from a balance
between shroud passage vortex, blade surface vortices and
tip leakage flows and their relative strengths are estimated
quantitatively in terms of mass flow, loading, radii of
curvature and boundary layer thicknesses. In addition, a
comparison of the flow structure for the SHF pump at
design and off-design mass flows (60% of design flow)
sheds an interesting light on the mutual interactions of the
various components of the secondary flow.
Complex patterns of 3D separation and reattachment lines
are largely present along the solid surfaces and in particular
corner vortices and corner separation regions are observed,
much along the same pattern as in axial compressors. In
addition, inducer tip separation close to the shroud can
appear in the inlet region, as a consequence of incidence
effects. However, these regions do not appear to be playing
an important role in the generation of the wake flow.
The analysis of full Navier-Stokes solutions provides a
powerful tool for a detailed understanding of the complex
centrifugal compressor flows, provided the boundary layers
are finely resolved. The present results may not be totally
mesh independent, nor has the influence of different
turbulence models and of rotation and curvature on the
turbulent flow been taken into account. However, the
excellent level of validation obtained seems to indicate that
these effects are not predominant in the establishment of
the main flow properties.

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