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FLOW FIELD DIAGNOSIS IN MULTISTAGE AXIAL COMPRESSORS

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ABSTRACT

A mathematical model is developed for the flow field diagnosis problem in multistage axial compressors. In view of the ill-posedness of the diagnostic problem, an effective measure is adopted to transfer the diagnostic problem into a variational problem which is solved by a regularization method. Two numerical results demonstrate the rationality of the flow field diagnosis problem for the compressors running near the design point and the effectiveness of the computational method.

NOMENCLATURE

C_p	specific heat at constant pressure
G	flow rate
h^*	rothalpy
i	enthalpy
i^*	stagnation enthalpy
k	ratio of specific heats
K_{BK}	blockage factor
K_e	effective thermal conductivity
L_S	axial stage length
m	meridional direction
M	Mach number
P	static pressure
P^*	total pressure
r, θ, Z	cylindrical coordinates
S	entropy
S_L	entropy, related to ω
T	static temperature
T^*	total temperature
V	flow velocity
W	relative velocity
W_m	meridional velocity
y	quasi-orthogonal direction

β	flow angle
σ	angle between streamline and Z direction
ρ	density
λ_l	angle between y direction and z direction
δ	deviation angle
ω	rotary speed
ω	total pressure loss coefficient
ϵ	spanwise mixing coefficient

SUBSCRIPTS

$2d$	two-dimensional
$3d$	three-dimensional
h	hub
m	meridional
t	tip

INTRODUCTION

With the rapid development of aviation technology, compressor design requirements are becoming more and more stringent. During the design of a compressor, a large number of tests and modifications are necessary to meet the required performance. Owing to the complexity of the internal structure of a compressor and the limitations of testing instruments and testing methods, test information describing the flow field is inadequate. Therefore the design of a compressor is always difficult and time consuming. On the other hand, turbomachinery design is being revolutionized by the rapid development of Computational Fluid Dynamics (CFD). However, at present it is still very difficult and more expensive to compute the 3D flow field in a multistage axial compressor using the Navier-Stokes equations. In engineering practice, aerodynamic analyses of a multistage axial compressor still rely on simple models with semi-empirical correlations.

Therefore, methodology to use simple physical models with suitable correlations to rapidly obtain correct and detailed information

about the flow field in multistage axial compressor is very important and practical. This is the purpose of flow field diagnosis (Ye and Yuan, 1989).

The use of a flow field diagnosis system can provide detailed information about the flow field and how to adjust the compressor design. Therefore, the compressor design period can be shortened with significant reduction in the required manpower and material resources. This is of important practical significance.

MATHEMATICAL MODEL FOR FLOW FIELD DIAGNOSIS

At present, quasi-three dimensional calculations using various correlations are still the most frequently used method for aerodynamic analysis of multistage axial compressors. In multistage axial compressors, each blade row works cooperatively and it is very difficult even if possible to measure between stages during testing. However, the pressures at the inlet and outlet of the compressor, as well as the static pressures at the endwall are easy and convenient parameters to measure. Therefore, flow field diagnosis uses this easily measured data with some simple numerical models to get as much flow information as possible.

The following equations, which are widely used in engineering practice, are used as the basic equations for flow field diagnosis in the meridional plane using the streamline curvature method. (The flow is assumed to be steady-state.)

Control Equations

Integral continuity equation for use with the streamline curvature method:

$$G = 2\pi \cdot K_{BK} \cdot \int_{y_h}^{y_l} y \cdot \sin \lambda_1 \cdot \rho \cdot W_m \cdot \sin(\lambda_1 - \sigma) \cdot dy \quad (1)$$

Momentum equation used in the streamline curvature method:

$$\partial W_m / \partial y = \left\{ -W_m / (1 - M_m^2) \cdot (A_1 + A_2 M_m^2) + W_m A_3 - W_m A_4 - A_5 \sin \lambda_1 + A_6 / W_m - T / W_m \cdot \partial S / \partial y \right\} / A_7 \quad (2)$$

where

$$A_1 = \cos(\sigma - \lambda_1) \left[\frac{\sin \sigma}{r} + \frac{1}{\sin(\lambda_1 - \sigma)} \frac{\partial \sigma}{\partial y} - \cot(\lambda_1 - \sigma) \frac{\partial \sigma}{\partial m} \right]$$

$$A_2 = \cos(\sigma - \lambda_1) \sin \sigma / r$$

$$A_3 = \sin(\sigma - \lambda_1) \partial \sigma / \partial m$$

$$A_4 = \tan \sigma / r \cdot \partial(r \cdot \tan \beta) / \partial y$$

$$A_5 = 2\omega \tan \beta$$

$$A_6 = \partial h^* / \partial y$$

$$A_7 = (1 + \tan^2 \beta)$$

$$h^* = i + W^2 / 2 - r^2 \omega^2 / 2$$

$$i = Cp \cdot T$$

The form of the momentum equation is the same as the inviscid gas flow equation but viscous effects can still be included by variations of the rothalpy, h^* , and the entropy, S , along each streamline.

Energy equation:

$$\partial h^* / \partial m = 1 / r \rho W_m \cdot \partial(r \rho \epsilon \cdot \partial h^* / \partial r) / \partial r \quad (3)$$

Spanwise mixing can be taken into account in this equation using the spanwise mixing coefficient, ϵ , which includes the effects of turbulence and three-dimensional flows including secondary flows. ϵ is calculated in the same way as that of Gallimore (1986).

Entropy equation:

$$T \cdot \partial S / \partial m = 1 / r \rho W_m \cdot \partial(r K_e \cdot \partial T / \partial r) / \partial r + T \cdot \partial S_L / \partial m \quad (4)$$

where K_e is the effective thermal conductivity coefficient and S_L is the entropy related to the empirical total pressure loss coefficients. The entropy equation is not a complete viscous equation as the momentum equation. It is assumed that the dissipation function is included in the entropy S_L .

State equation (Ideal Gas):

$$P = \rho \cdot R \cdot T \quad (5)$$

Other equations or relations:

$$S_2 - S_1 = Cp \ln(T_2^* / T_1^*) - R \ln(P_2^* / P_1^*)$$

$$Cp \cdot T^* = Cp \cdot T + V^2 / 2$$

$$h^* = i^* - \omega \cdot r \cdot V_\theta \quad (6)$$

$$P^* / P = (T^* / T)^{k/(k-1)}$$

Boundary Conditions

- Geometric data of meridional flow path
 - Total temperature and total pressure distribution at the inlet
 - Flow angle at the inlet
 - Flow rate G and rotary speed ω
- Boundary conditions are used for the direct problem calculation. (7)

Additional Conditions

- Measured data of static pressures on the endwall
 - Measurement data of total temperatures and total pressures at the outlet
- (8)

Additional conditions are used to determine the following diagnostic parameters, which are different from boundary conditions in use.

Diagnostic Parameters

- Blockage factor distribution, K_{BK}

- Deviation angle distribution, δ , at every blade exit
 - Total pressure loss distribution, ω , at every blade exit (9)
- K_{BK} , δ and ω are three important parameters of flow field.

Variations of these parameters gives viscous corrections to the above 2-D inviscid model. These parameters are usually calculated by empirical models or given directly by experience. Once they are given, the solution of the flow field can be obtained using Eqs. (1)-(6) and the boundary conditions, Eq. (7). If the parameters are given incorrectly or the model is not accurate, the solution of the direct problem may be incorrect. In flow field diagnosis problem, these parameters will be determined using the additional measured data so as to obtain a more accurate solution to the problem.

It will be discussed below that the solution of diagnosis problem is a optimum seeking process. The three groups of diagnostic parameters are modified during each direct problem calculation, and the additional conditions are treated as a target with the corresponding values through direct problem calculation close to it to the greatest extent.

ANALYSIS OF CONDITIONING

Equations (1)-(9) describe a typical inverse coefficient problem with differential equations. Inverse problems with differential equations are very important but difficult because most are ill-posed (ill-posedness means that no unique solution exists or the solution does not depend continuously on the measured data).

In the flow field diagnosis problem, if there is insufficient additional measurement data available, the solution is not unique. In order to overcome this difficulty, either more additional measurement data must be obtained or the content of parameters to be determined must be reduced. Due to the limitations of the test instruments and testing methods, it is very difficult to get more measurement data; therefore the content of parameters must be decreased.

One way to reduce the content of parameters is to make full use of some physical knowledge or experience. In the development of compressors, many principles or empirical formulas about loss and deviation angle have been obtained. These formulas are useful but have restricted applications and, so, must be modified for different applications. Flow field diagnosis uses these empirical formulas as the basic formulas. By varying coefficients in these formulas the modified formulas can be used to approximate the needed parameters. It is these coefficients which will be recovered in the diagnosis problem by the additional measurement data. Then the problem to identify the parameters K_{BK} , δ and ω is replaced by the need to identify these coefficients. In this way, the content of unknown parameters is greatly reduced. This is especially useful for inverse problems where the additional measured data are few.

In this paper, the loss model (Roberts et al., 1988) and deviation angle model (Roberts et al., 1985) are used as the basic models. In these models, the loss and deviation angle are divided into two parts:

$$\delta = \delta_{2d} + \delta_{3d} \quad \text{and} \quad \omega = \omega_{2d} + \omega_{3d}$$

In multistage compressors, the flow in the middle stages will differ greatly from the flow in stages near the entrance or near exit. Since most empirical formulas are derived for the middle stages, additional modifying coefficients are needed in the formulas when they are used in stages near the entrance and near exit. In addition, blockage factor is modeled by a formula according to its value given in engineering practice. So:

$$\delta = (\delta_{2d} + \delta_{3d}) \cdot K_{\delta}(J)$$

$$\omega = (\omega_{2d} + \omega_{3d}) \cdot K_{\omega}(J)$$

$$K_{BK} = 2.0 - (1.0 + K_R \cdot I)^{0.02} \quad (\text{Rotor})$$

$$K_{BK} = 2.0 - (1.0 + K_S \cdot I)^{0.02} \quad (\text{Stator}) \quad (10)$$

$K_{\delta}(J)$, $K_{\omega}(J)$, K_R and K_S are the modifying coefficients, J is the stage number, I is the number of calculating station.

These modifying coefficients must be determined in the flow field diagnosis of multistage axial compressors. They are just the unknown coefficients in diagnostic problem.

For simplicity, the unknown coefficients are represented by $K = (K_1, K_2, \dots, K_m)^T$; where m is the number of unknown coefficients which is often smaller than the number of additional conditions. Therefore, the diagnostic problem could have a unique solution.

The flow field diagnosis problem in multistage axial compressors can be written in the following simple form:

$$\begin{cases} \text{Control equation:} & L(K) \cdot u = f & (11) \\ \text{Boundary condition:} & B \cdot u = g & (12) \\ \text{Additional condition:} & A \cdot u = h & (13) \\ \text{Diagnostic factor:} & K & (14) \end{cases}$$

where $L(K)$ is the differential operator in the control equations; B is a boundary operator; A is a measurement operator; f and g are given functions, h is a vector of measurement data and u represents the flow parameters. For this inverse problem, when the parameter K is given, a unique solution can be obtained from Eq. (11) and the boundary condition, Eq. (12). The diagnostic problem, Eqs. (11)-(14) can also be written in the following operator form:

$$Q(K) = h \quad (15)$$

where the nonlinear operator Q is defined by:
 $\forall K \in \Omega, Q(K) = A \cdot u(K, Z_0)$, Ω is the admissible domain of parameter K , and Z_0 is the measured points.

DIAGNOSTIC PROBLEM SOLUTION

It is usually impractical to solve Eq. (15) for several reasons. First, due to modeling and measurement errors, a 'true' parameter K does not necessarily exist. Hence, determination of the unknown coefficients is usually treated as an optimization problem of the form

$$\min_{K \in \Omega} \|Q(K) - h\|^2 \quad (16)$$

where $\|\cdot\|$ is the Euclidean norm. Secondly, as indicated above, the problem is ill-posed in the sense that solutions K (provided they exist) may not depend continuously on the data, h . Hence, the solution of Eq. (15) does not depend continuously on the measured data, h . Therefore,

discretized versions of the problem are likely to be highly ill-conditioned. Consequently, some sort of regularization (i.e. stabilization) is required to obtain an accurate approximation for K and Eq. (16) is replaced by the minimization problem

$$\min_{K \in \Omega} T_{\alpha}(K) \quad (17)$$

Where

$$T_{\alpha}(K) = \left\{ \|\varrho(K) - h\|^2 + \alpha \Gamma(K) \right\} / 2$$

$\alpha > 0$ is a regularization factor which controls the trade-off between goodness of fit to the data and stability. The penalty functional, $\Gamma(K)$, provides stability and allows the inclusion of a priori information about the true parameter K' or about the solution, u (Tautenhahn, 1994). A state range regularization method is used here.

In the diagnostic problem, the target function and its gradient can not be explicitly obtained. Therefore, the Flexible Tolerance Method (FTM), in which the target function's gradient is not needed, is used. This method is simple and stable.

In solving Eq. (17), the solution of the direct problem, Eqs. (11)-(14), is used repeatedly and the solution convergence and accuracy of Eq. (17) are greatly affected by the solution of the direct problem. Therefore, the numerical solution of the direct problem is very important. The direct solution of the meridional flow is calculated using the streamline curvature method which is widely used in engineering practice. Because the direct problem is solved independently, the solution method can be easily updated with a better numerical method in the future.

NUMERICAL RESULTS

Two numerical examples are presented to verify the effectiveness of flow field diagnosis solution method.

Example 1: A two-stage fan (Cunnam, 1978) with a total pressure ratio of 2.4 and a total temperature ratio, flow rate and rotary speed of 1.335, 33.248 kg/s, 16042.8 rpm, respectively. The minimization value is 1.3428×10^{-2} and the optimization time is 27. Figures 1-4 show the computational results and measured results. Figure 1 and 2 show the total pressure and total temperature at the exit of rotor 1. Figure 3 and 4 show the same parameters for rotor 2. The results show good agreement between the diagnostic results and the measurements.

Example 2: A seven-stage axial compressor with test pressure ratio of 4.06, and an efficiency, flow rate and rotary speed of 0.876, 24.8 kg/s, 10479 rpm, respectively. The final values of the modified coefficients are:

Table 1 Values of Modified Coefficients

$K_{\delta}(1)$	$K_{\omega}(1)$	$K_{\delta}(2)$	$K_{\omega}(2)$	$K_{\delta}(3)$	$K_{\omega}(3)$
1.304	1.250	1.174	0.988	1.121	1.109
$K_{\delta}(4)$	$K_{\omega}(4)$	$K_{\delta}(5)$	$K_{\omega}(5)$	$K_{\delta}(6)$	$K_{\omega}(6)$
0.910	0.821	0.986	0.885	1.023	1.049
$K_{\delta}(7)$	$K_{\omega}(7)$	K_R	K_S		
1.067	1.607	0.858	0.756		

Figures 5-7 show the diagnostic computational results, the direct computational results and the measurements of the static pressure at the endwall and the total pressure and total temperature at the outlet, respectively. Since the measured endwall static pressure lies in the space between blade rows, the values from diagnosis and direct calculation are only given at the same positions. It is shown that if the empirical models are not modified, the direct computational results do not compare well with the measurements. The agreements between the diagnostic results and the measurements are better, but their discrepancies are not satisfactory. As mentioned above, the optimum seeking process of diagnostic solution may go on to get a smaller target function value.

Figure 8 and 9 show the diagnostic computational predictions of the deviation angles and losses at the exits of some blade rows. Some strange behavior occurs in the last stage, for example, rotor 7 has a high loss and low deviation. This shows that the 7th stage does not work at its design point, so the Roberts models result in great errors here. It is one of the aims of flow field diagnosis to find the worse working condition of the compressor. The compressor is a developing one, some modifications can be taken according to the diagnostic results.

CONCLUSIONS

The diagnostic problem for the flow field in a multistage axial compressor is discussed fully. The governing equations for meridional flow using the streamline curvature method are supplemented by models to determine the unknown coefficients from experimented data. The inverse problem is then solved using minimization method to determine the best values of the unknown parameters for the measured data. The method is well posed and stable. The Roberts models apply at design point conditions, and the diagnostic method above is suitable near design point of the compressors. During the adjusting process of the compressor, the technique of flow field diagnosis can provide directions to improve the performance. Further works will include using widely usable correlations in the method to get more realistic results and adding a blade-to-blade flow field computation program to get detailed information of the 3-D flow field in the multistage axial compressor. A flow field diagnostic system will then be developed.

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REFERENCES

- Cunnam, W. S., Stevans, W., Urasek, D. C., 1978, "Design and Performance of a 427 Meter Per Second Tip Speed Two Stage Fan Having a 2.40 Pressure Ratio," NASA TP-1314.
- Gallimore, S. J., 1986, "Spanwise Mixing in Multistage Axial Flow Compressors: Part II - Through flow Calculations Including Mixing," ASME Paper 86-GT-21.
- Roberts, W. B., Serovy, G. K., Sandercock, D. M., 1985, "Modeling the 3-D Flow Effects on Deviation Angle for Axial Compressor Middle Stages," ASME paper 85-GT-189.
- Roberts, W. B., Serovy, G. K., Sandercock, D. M., 1988, "Design Point of 3-D Loss and Deviation for Axial Compressor Middle Stages," ASME paper 88-GT-57.

Tautenhahn, U., 1994, "Error Estimates for Regularized solutions of Non-linear Ill-posed Problems," *Inverse Problems*, 10, pp. 485-500.

Ye Dajun, Yuan Xin, 1989, "Development of Flow Field Diagnosis," *Collections of aerothermodynamics development conference reports on special topics*, pp. 126-130.

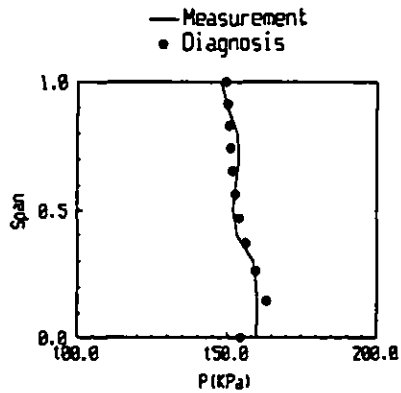


Fig. 1 Total Pressure at Exit of Rotor 1

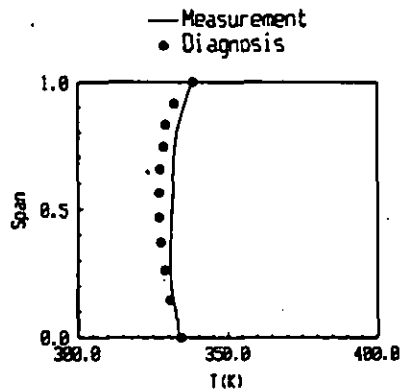


Fig. 2 Total Temperature at Exit of Rotor 1

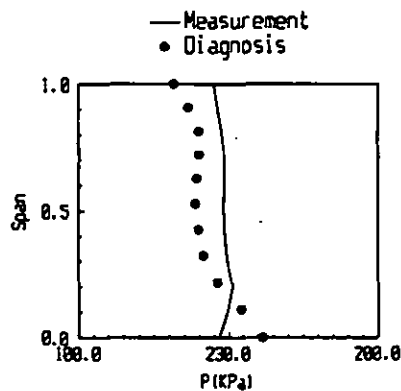


Fig. 3 Total Pressure at Exit of Rotor 2

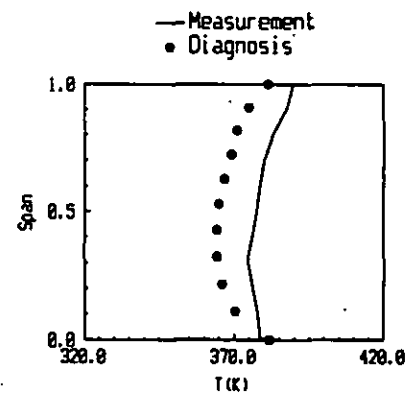


Fig. 4 Total Temperature at Exit of Rotor 2

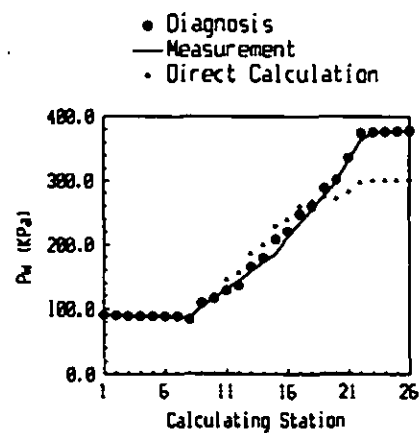


Fig. 5 Endwall Static Pressure

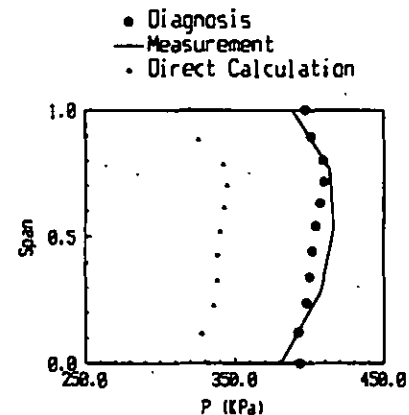


Fig. 6 Total Pressure at Exit of compressor

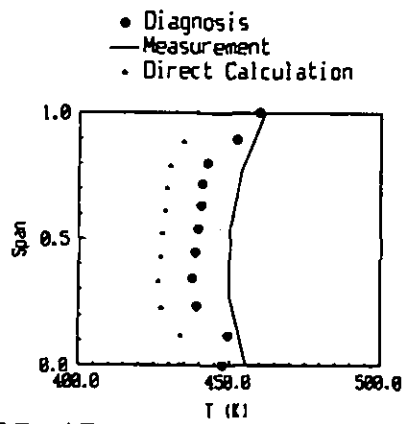


Fig. 7 Total Temperature at Exit of compressor

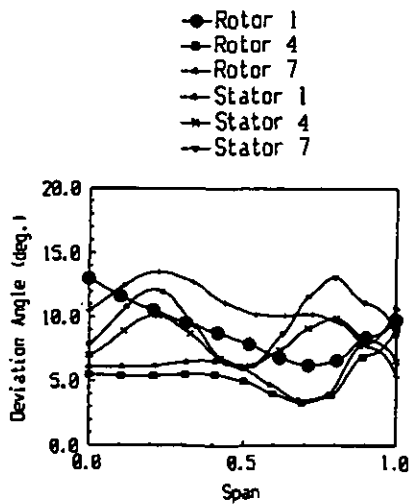


Fig. 8 Deviation Angle Distribution

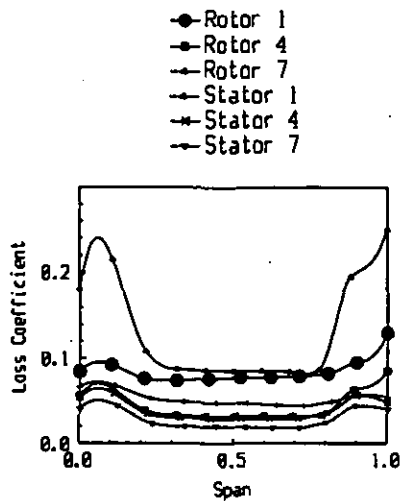


Fig. 9 Loss Coefficient Distribution