MASS FLOW LIMITATION OF SUPERSONIC BLADE ROWS DUE TO LEADING EDGE BLOCKAGE

by

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ABSTRACT

The supersonic flow passing through the blade row of an axial flow compressor depends on the magnitude of the axial inlet Mach number. If the upstream stream tube is convergent, the axial Mach number remains subsonic and the unique incidence relation holds between inlet Mach number and inlet flow angle. This paper presents the influence of leading edge thickness and suction surface angle on the unique incidence and choking of supersonic blade rows. The theoretical results were experimentally verified in supersonic cascade wind tunnel tests.

NOMENCLATURE

c blade chord length
l_E length of limiting characteristic
L shock detachment distance
M flow Mach number
R_1 leading edge radius
t blade pitch
w flow velocity
a Mach angle
\beta flow angle with respect to cascade front
\beta_E surface angle at point E
\beta_N angle between nozzle axis and cascade front
\alpha stagger angle
\beta_{ss} suction surface angle at leading edge
n circumferential coordinate
\gamma_E \beta_E - \beta_{ss}
\gamma Prandtl Meyer angle

SUBSCRIPTS

1 uniform inlet condition
N uniform upstream nozzle flow condition
E condition along limiting characteristic
ax axial condition

INTRODUCTION

The mass flow passing through a supersonic blade row of a transonic or supersonic axial flow compressor depends on the inlet Mach number and on the cascade and blade geometry. This dependency was first realized in the 1940's, and the first calculation procedure for blades with curved suction surfaces was published in 1956 by Levine [1]. In the 60's and early 70's methods were developed for the flow calculation past blades with finite leading edges as published by different authors, e.g. Novak [2] and Starken [3]. Since that time the relation has been called the "unique incidence condition". Although the principle discussion on this subject as presented by Lichtfuß and Starken [4] included the axial-sonic and axial-supersonic inlet-flow conditions of pointed blades, a corresponding complete description and calculation of cascade blades having finite leading edges was still missing. This paper presents a solution to this problem with an approximate calculation of the unique incidence as well as the maximum mass-flow condition for blade rows with blunt leading edge blades.

THE UNIQUE INCIDENCE RELATION AT HIGH AXIAL MACH NUMBERS

In general, the hub and casing contours of axial flow compressors are designed such that the meridional flow is accelerated as
shown in Fig. 1. Upstream of the rotor blade leading edges a relative inlet flow Mach number \( M_1 \) is reached which depends on the axial and tangential Mach numbers as shown in Fig. 2. In the case of a relative supersonic inlet Mach number with an axial subsonic component, the relative inlet flow angle \( \beta_1 \) is directly related to the cascade and blade suction surface geometry as described in [4]. The wedge-type blades of Fig. 2, for instance, with sharp leading edges exhibit a constant relative inlet flow angle parallel to the suction surface up to axial-sonic inlet flow conditions as shown by the solid line in Fig. 3. Beyond this point the axial Mach number remains sonic and the mass flow must be constant, i.e. the mass flow is choked. Thus, a further increase of the relative upstream Mach number \( M_1 \) requires a simultaneous increase of the inlet flow angle \( \beta_1 \).

The sonic axial Mach number limitation is due to the fact that the meridional cross section is convergent, as shown in Fig. 1. The dependency of the inlet flow angle on inlet Mach number, as shown in Fig. 3, is called the "unique incidence relation", and is valid only for started supersonic blade sections with axial subsonic flow.

If the wedge-type blades have finite leading edges, as shown in Fig. 4, the inlet flow angle is increased and the choking of the through-flow occurs at a subsonic axial inlet Mach number. The resulting different unique incidence curve is shown as a dashed line in Fig. 3. In the past, only the left part of this curve was calculated by Starken [3], Novak [2], and York and Woodard [5]. In this relatively low supersonic Mach number range, the limiting characteristic of the supersonic entrance flow field emanates from the straight portion of the blade suction surface as sketched in Fig. 5. In other words, the limiting characteristic, which is also called the last captured Mach wave, connects the intersecting point between the detached shock wave and the stagnation streamline with point E on the suction surface of the adjacent blade. Therefore, the so-called neutral characteristic, which corresponds to the inlet flow conditions \( (M_1, \beta_1) \), starts from the blade leading edge. With increasing inlet Mach number, point E of the limiting characteristic moves upstream and finally reaches the leading edge, as shown in Fig. 6. Increasing the inlet Mach number beyond this point causes a more rapid change in the Mach number and flow angle of the limiting characteristic (line EC), this being due to the strong expansion around the rounded leading edge. There is also, simultaneously, a small shift in the location of the stagnation streamline at the detached shock wave of the adjacent blade (point C). The iterative calculation procedure, applied in the past, failed at this point and did not converge. However, by prescribing point E of the limiting characteristic on the leading edge instead of prescribing the Mach number \( M_E \), a converging solution procedure for the high Mach number range was also obtained.
UNIQUE INCIDENCE CALCULATION

The calculation procedure is based on the Levine method [1] extended with the Moeckel method [6] to include the effects of detached shock waves. In the Levine Method, the simplifying assumption of a simple wave flow in the entrance region of the cascade leads to straight left running characteristics, along which the flow properties are constant. The neutral and the limiting characteristics are therefore also straight lines. This facilitates the application of the continuity equation which was used by Levine together with the simple wave equation, far upstream of the cascade and along the limiting characteristic (Fig. 7).

\begin{align*}
(1) \quad \rho_1 \cdot w_1 \cdot \sin \beta_1 &= \rho_E \cdot w_E \cdot \sin \alpha_E \\
(2) \quad s_1 + v_1 &= \beta_E + v_E
\end{align*}

The length $l_E$ of the limiting characteristic (line BC) is determined with the aid of the Moeckel calculation which provides the shock detachment distance $L$. In the real flow, total pressure losses do occur due to the detached bow shock and all the upstream branches of the oblique shock waves induced by the adjacent blades. The contribution of the bow shock in front of the leading edge is dominant and therefore the left running characteristics near the blade surface are not really straight lines. In the present calculation the oblique shock losses are neglected and the left running characteristics are calculated as straight lines as in the Levine method. However, the blockage effect of the total pressure losses due to the detached shock wave between the points C and S is taken into account in the density calculation of equation (1) in the same approximate way as in the Moeckel calculation. This leads to an additional increase in inlet flow angle as shown in Fig. 8, where the calculated unique incidence curves of a simple flat plate cascade are presented for different leading edge radii $R_1/t$. 

Fig. 5 Supersonic inlet flow field of blunt cascade blades with limiting characteristic emanating from the suction surface

Fig. 6 Supersonic inlet flow field of blunt cascade blades with limiting characteristic emanating from the leading edge

Fig. 7 Control volume upstream and in the entrance region of a supersonic cascade

Fig. 8 Influence of upstream total pressure losses on the unique incidence
Fig. 9 Axial Mach number along unique incidence curves

Influence of Leading Edge Radius and Suction Surface Angle

The blockage effect due to the blade leading edges and the associated losses, both of which are responsible for the inlet flow angle increase, can be explained best in the related axial Mach number diagram of Fig. 9. Depending on the leading edge size, the axial Mach number reaches a maximum value at a certain inlet Mach number. This point corresponds to the flow pattern where point E of the limiting characteristic just reaches the leading edge.

Any further increase of the inlet Mach number does not increase the mass flow through the cascade. The calculation even indicates a small mass flow reduction, probably due to the increased shock losses at higher inlet Mach numbers. In Fig. 10 the maximum attainable axial Mach number is plotted as a function of the leading edge thickness parameter $R_1/t$ and suction surface angle $\beta_{ss}$ of the cascade. The diagram shows a strong influence of leading edge size on the maximum axial Mach number, with the most rapid change occurring at small leading edge radii. The suction surface angle, however, can be seen to have only a minor influence.

Fig. 10 Maximum axial Mach number of supersonic cascade blades as function of leading edge radius and suction surface angle

Fig. 11 Unique incidence curves of blunt flat plate cascade ($R_1/t = 0.03$) at different suction surface angles

The $90^\circ$ suction surface angle curve is included for comparison purposes only. It corresponds to subsonic relative inlet Mach numbers and may be considered as a leading edge blockage solution (choking) from a throughflow calculation using only the axial velocity component. Fig. 10 shows that this simplified calculation leads to much higher limiting axial Mach numbers than the two-dimensional blade to blade calculation.

The suction surface angle does have, of course, a considerable influence on the unique incidence curve in the low Mach number range where point E of the limiting characteristic is located on the blade surface. Fig. 11 shows this very clearly for a cascade with relatively thick leading edges. Around sonic inlet velocity, the unique incidence curve is dominated by the suction surface angle, whereas at higher inlet Mach numbers all the curves join into nearly one curve because here the whole entrance flow field is fixed by the leading edge as described in Fig. 6. The irrotational solution actually results in one single curve, whereas the consideration of the total pressure losses separates the curves slightly.

In cases of high axial Mach numbers the point E of the limiting characteristic (last captured Mach wave) is on the leading edge radius. That is, that the shape of the suction surface has no more influence on the upstream flow angle and Mach number relation. In cases of lower axial Mach numbers the unique incidence results depend on the near leading-edge suction surface curvature of the supersonic blading. In general, however, the suction surface curvature of supersonic cascade blades is small. Therefore, the presented curves in Fig. 10 - 11 are approximately valid for all types of supersonic blading and not restricted to the flat plate cascades used in this investigation.

Parametric flat plate unique incidence calculations have been performed for different leading edge suction surface angles $\beta_{ss}$ and leading edge thickness ratios $R_1/t$, and
Fig. 12 Unique incidence curves of flat plate cascades at different leading edge radii and suction surface angles.

Fig. 13 Blunt flat plate cascade.

Fig. 14 Schlieren picture of blunt flat plate cascade ($\beta_S = 135^\circ$) at nozzle conditions of $N_R = 1.91$, $\beta_N = 148^\circ$ and inlet conditions of $M_1 = 1.57$, $\beta_1 = 137.8^\circ$. 

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The unique incidence calculation for supersonic blades in cascade with blunt leading edges, based on the Levine and the Moeckel method, has been extended to high axial Mach numbers and large leading edge radii. Calculations show that the blockage effect of a finite leading edge leads to a limiting maximum axial inlet Mach number which depends on the leading edge size and the suction surface angle. In axial-flow turbomachinery this limit corresponds to mass-flow choking.

A parametric presentation of the unique incidence curves of flat plate cascades renders possible a rapid determination of the inlet conditions of supersonic cascades having low suction surface curvature.
Laser-Two-Focus measurements and an evaluation of Schlieren pictures confirmed the theoretical predictions.

REFERENCES


